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We consider contractual alliances between firms that seek to take advantage of complementarities of the firms on a specific project, such as alliances between biotech and pharmaceutical firms to develop promising molecules. We argue that an alliance has two distinct stages. In the first stage, each firm invests in the project at some cost. In the second, the activities needed to complete the project are executed. We denote these stages the investment and execution stages.

The firms entering an alliance have divergent interests and alternative projects and these can lead to under-investment in the investment stage and to shirking in the execution stage. We show how these forces interact to generate an equilibrium contract.

Our theory quantifies the ability of each firm to extract rents from a contract in the execution stage; we refer to this as bargaining power. We show that excessive bargaining power can prevent an alliance from forming because of the excess incentive to free ride. But investment by firms can moderate this bargaining power enough to overcome the defection incentive and thereby enable alliance formation. We quantify the resulting contract in terms of investment, the cost of that investment, payoffs, and net profits, all of which can be asymmetrically distributed across the firms.

Keywords: Theory of the Firm, Cooperative Alliances Organizational Incentives, Under-investment

JEL classification: L23, L24

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# 1 Introduction

Two pirates, Captain Flint and Long John Silver, want to row to an island that lies across a strait in order to dig up a treasure chest. There are rival pirates who are trying to get there first. Flint and Silver have a boat, but it lacks oars. They must decide whether or not to make oars cooperatively, each knowing that he cannot reach the island before the rivals if he rows alone. Flint's and Silver's respective investments will impact the quality of the oars they make and thereby the speed with which they cross the strait, and whether they cross it at all.

If they decide to build the oars, each must subsequently also decide how hard to row across the strait. Because they are pirates, they understand that the other will have incentives to shirk when rowing. Each can pretend to row hard, but the other cannot verify effort instantaneously. However, both can observe that the other has deviated from a pattern of rowing and rest.

They must therefore solve two incentive problems. They may have the incentive to underinvest in building the oars, and they may also have the incentive to shirk as they row. These incentive problems seem unconnected, but we will show that they are linked.

Flint's and Silver's problem is analogous to that of two firms considering whether to undertake investments in relationship-specific complementary assets in a world of incomplete contracting. How firms solve such incentive problems is central to the theory of the firm.[1]

[1]Ultimately, the world's first biotechnology drug, Humulin, was the outcome of this alliance. Humulin is still a major drug with 2015 sales of over $1 billion—the treasure!.

## 1.1 Formation

To show how to solve the contracting problem we create an analytical framework that incorporates the following elements:

- The incentive to invest and the cost of investing during the oar-construction phase.
- The incentive to cooperate during the rowing stage.

Unlike previous formulations that focus on one or the other of these incentive problems, our framework examines the link that arises because the pirates anticipate how their investments will affect their incentives when they are rowing. A "contract" is then the equilibrium of a non-cooperative game that is understood by both parties (i.e., in the rational expectations sense).

We denote the successful resolution of the contracting problem as *formation.* This resolution entails two stages: the stage in which the oars are made, which we denote the investment stage, and the rowing stage, which we denote the execution stage. In the investment stage the pirates decide not only whether to make the oars but also how big to make them. These decisions will be influenced both by the costs of making oars and by incentives in the rowing stage. Unless the incentive problems at both stages are resolved, there could be sub-optimal investment. In the extreme, formation might not even occur: the oars are not made, leaving Flint and Silver stranded.

In our framework, the outcome—whether the pirates will cooperate, their investments and their payoffs—will be influenced by Flint's and Silver's relative initial bargaining powers, their relative leverage over each other's bargaining power, their relative costs of investment and the technological uncertainty associated with their cooperation. Their profits are comprised of the treasure itself, which they must decide how to split in advance, adjusted by the effort each expends in getting to the island.

## 1.2 Reducing bargaining power to achieve alliance formation

Suppose Silver is the stronger rower—or has the bigger and more effective oar. If he rows diffidently—he shirks—while Flint rows hard, they might never get to the island. Silver's relative bargaining power is higher than Flint's, because he has a greater influence on their speed, and on whether they get to the island at all. This will translate into his getting a greater share of the payoffs.

In effect, each pirate's bargaining power reflects his relative gain from free riding in the rowing stage. Our framework allows us to quantify this bargaining power and to show how it is influenced by investment. First, we observe that if the combined bargaining power of the two pirates is initially too high—in our model the relevant combined bargaining power is the *product* of the individual bargaining powers—formation cannot occur without investment. Specifically, we will show how investment by one pirate reduces the relative bargaining power of the other. This in turn reduces the *combined* bargaining power sufficiently to achieve formation. Precisely because of its influence on the other's bargaining power to enable formation, the investment by the first pirate is *strategic*.

The magnitude of investment required to achieve formation is crucially influenced by technological uncertainty (such as unobservable currents in the strait that speed up or slow down the progress of the boat randomly). We translate this technological uncertainty as impatience, and show that greater impatience will create greater demands on the alliance, necessitating higher investment for formation to occur.

### 1.3 Who invests, payoffs and profits: The influence of leverage and costs

The weakening of one pirate's bargaining power via the partner firm's investment can enable formation to occur. But the exact combination of investments by the two pirates that yields formation also depends on the relative impact of their investments on the gains to the other from cooperation in the rowing stage, as well as their costs of investment. Our theory allows us to quantify these investments in relation to initial bargaining power and the other parameters of the model.

Suppose Flint's investment in oar-building has a stronger impact on Silver's gains in the rowing stage. We translate this as the higher leverage of Flint's investment on Silver's bargaining power. Whether the investments in oars occur in sufficient measure to achieve formation, and the relative sizes of the oars, will be dictated by the pirates' oar-making skills, and their skills might be very different. We translate these differences in skills as differences in their costs. If their leverage or costs are very different, the oars they make will be different sizes, and consequently their bargaining power as they row will be different, as will be their final payoffs and profits.

Viewing the pirates as representing firms contemplating alliances, our theory thus not only rationalizes the formation of alliances, but also articulates the potential impediments to formation. It predicts the potentially asymmetric investments that the participating firms undertake, and it also specifies exactly how profits are split, which also can be highly asymmetric. Moreover, the requirement that the contract induce cooperation causes inefficiency in general, and we quantify this inefficiency. We therefore have a positive theory of alliance contracts.

### 1.4 An example

To demonstrate a business setting wherein strategic investments by alliance partners may resolve the contracting problem, consider the Lilly-Genentech R&D alliance formed in August 1978 to develop recombinant human insulin at an industrial scale. This venture was initiated after Genentech scientists had demonstrated their capability to create human insulin in the laboratory, so that technological uncertainty had been reduced to a level where investments in an alliance could be contemplated. While Genentech was expected to pursue the science of producing economically viable human insulin, Lilly would contribute its capabilities in process engineering for large-scale manufacturing, quality control, and clearing regulatory hurdles.[2]

In the context of our framework, Genentech and Lilly would each need to decide whether and how much to invest in the alliance, as well as the level of effort they would devote to the collaboration. Both firms were aware that the other could divert its efforts to other projects ("shirking"), signifying high initial bargaining power that impeded alliance formation. Lilly had higher potential gains from diverting effort to other projects and thereby higher

[2]See Hall (1987).

bargaining power, but also had higher leverage over Genentech's bargaining power, i.e., its potential investment in the alliance had a stronger effect on Genentech's gains from collaboration in the alliance. Our framework suggests that because of its greater leverage, Lilly would make the bulk of the investment in the alliance, thereby reducing Genentech's bargaining power sufficiently to achieve alliance formation. Genentech would commit itself to reduced shirking to forestall Lilly's temptation to defect. While Lilly's investment to achieve formation would increase its bargaining power and payoffs relative to Genentech, its profits would be reduced by the amount of its investment. Nonetheless, both parties would voluntarily hew to the arrangement for its incentive properties.

# 2 Relationship to the literature

Since the seminal work of Ronald Coase (1937), the theory of the firm has represented a central issue of interest for research in numerous fields including economics, strategy, law, finance and accounting. In its essence, such a theory must consider the question of how to create incentives for entities that possess complementary assets, but can enjoy private gains at the expense of the other, to cooperate in activities that yield joint benefits. Property rights theory is the prevailing approach to analyzing this problem (Grossman and Hart, 1986; Hart and Moore, 1988).

## 2.1 The Property Rights Theory approach

The Property Rights Theory (PRT) formulation of the oar-building problem is as follows. Flint owns a boat but only one oar: he needs another oar. Will

Silver contract with Flint to make it for him or will Flint make it himself? Silver is concerned that even if he did enter into a contractual agreement with Flint to make an oar, Flint might ex post find an alternative supplier of oars and then try to renegotiate or walk away from this agreement: this is the classic holdup problem.

More generally, as Hart and Moore (1988] show in their canonical study of a buyer-seller relationship involving specific investments: if ex post renegotiation cannot be prevented by the parties, the holdup problem characterized by underinvestment results. PRT models predict that ex ante decisions about asset ownership will focus on resolving this holdup problem. In the PRT formulation, all possible contingencies cannot be perfectly anticipated, so that all possible uses of an asset cannot be pre-specified in a manner that can be contractually enforced. Under incomplete contracting, the residual control rights conferred by asset ownership is of central importance to resolve the holdup problem. This often implies ownership of all assets by one party—the party whose marginal investment is more productive—even though this is not the first-best solution. Allocation of control rights creates appropriate incentives for investment, thus producing a theory of vertical integration.

PRT thus considers firms as defined by the group of assets they own. Ownership over assets confers control rights, so that a firm can specify exactly how the asset will be used. The value of these control rights is a function of the outside opportunities of the assets, i.e., the value in alternative uses. The strength of the outside opportunities determines the threat points—the payoff they can guarantee themselves by not participating in the collaborative effort (Noldeke and Schmidt, 1998) or prices at which the parties trade

(Hart and Moore, 1988). So, the value of control rights over their respective assets creates bargaining levers for firms and hence influences how the payoffs from investment in relationship-specific complementary assets will be divided.

In effect, the bargaining power of the firms is associated only with their ownership of assets. Any differences in bargaining power that are not related to asset ownership are explicitly assumed away by utilizing a perfect Nash bargaining solution as a way of dividing the surplus earned above the threat points of the individual parties (Hart and Moore, 1988).

## 2.2 Some limitations of PRT

While PRT models have developed valuable insights on the resolution of the holdup problem via the control rights associated with asset ownership, various features limit their applicability. For example, in motivating their model of firm scope, Hart and Holmstrom (2010) note that the assumption that ex post conflicts are resolved through bargaining with unrestricted side payments does not appear to correspond to correspond to casual empirical observation. Perhaps their most serious limitation is an inability to take into account the second kind of incentive problem described in our formulation of the problem of cooperation: that of inducing appropriate effort, and the bargaining power considerations that influence this problem. As Holmstrom and Roberts (1998:92) note, "...power derives from other sources than asset ownership and other incentive instruments than ownership are available to deal with the joint problems of motivation and coordination." They conclude that the PRT literature is unable to explain a wide range of governance forms where aspects like principal-agent problems, reputation, monitoring and measurement problems and knowledge transfer play a crucial role.

Gibbons (2005a, 2005b) recommends a system approach that involves joint optimization over asset ownership and incentive contract parameters. He suggests that asset ownership can be an instrument in a multi-task incentive problem that includes both the direct effects of incentives from asset value, and indirect effects that arise from changes in the optimal incentive contract under different ownership arrangements.

## 2.3 Our approach and recent extensions of PRT

In our approach, residual control rights associated with asset ownership are the foundation of investments, similar to PRT. In contrast to PRT, however, our model demonstrates how investments may overcome obstacles to alliance formation, thus obviating the need for integration. Second, traditional PRT models focus only on how cooperation may be achieved in the investment stage of a project, but downplay the problem of cooperation in the execution stage. In contrast, our analytical framework incorporates the incentive to invest during the investment stage as well as the incentive to cooperate during the execution stage. Thus, our framework lies within the class of models described as "ex ante incentive alignment" by Gibbons (2005a) but also addresses considerations of both property rights and incentive systems. Third, in contrast to PRT which assumes bargaining power is static and exogenous (tied to the ownership of assets), in our model investments can impact and modify bargaining power.

Our paper is similar in spirit to recent work that builds upon and extends the traditional PRT approach. In one such extension, Baker, Gibbons and Murphy (2002) incorporate spillovers from relational contracts—informal agreements and unwritten codes of conduct that influence behavior—in their repeated game model of an ongoing supply relationship between an upstream and downstream party. The downstream party desires the upstream party to take actions to improve the value of the supplied good; similar to PRT, these actions are either unobservable or non-contractible so that they cannot be verified by a third party. However, as opposed to holdups motivated by specific investments, the focus is on incentive problems in relational contracts ex post. To induce efficient actions, relational contracts must be self-enforcing: the value of the future relationship must be sufficiently large that neither party wishes to renege. The main proposition of the analysis is that the temptation to renege on the contract (i.e., the extent to which the payoff from defection exceeds the payoff from cooperation) depends who owns the asset. So, integration versus non-integration, as well who owns the asset, crucially depends on which arrangement facilitates the superior relational contract.

We emphasize that our use of the term "bargaining power" is somewhat removed from the notion of bargaining power in the axiomatic bargaining literature. Our construction is of a noncooperative game, and our notion of bargaining power is attached to this context. However we point out that the model on which our model is founded, Taub and Kibris (2004) (henceforth TK), establishes an equivalence between the noncooperative and axiomatic bargaining solutions (Kalai-Smorodinsky, not Nash) approaches.

Thomas and Worrall (2011) set out a dynamic model of relational contracts that is similar in spirit to ours: two agents invest, and share post-investment output. Unlike our model, the investment is "physical" in the sense that an agent can exit the contract with some investment. By contrast, in our model, investment is one-shot, but the one-shot investment permanently alters the payoffs. After that, payoffs depend only on actions. Also the benefits of defection are short term, namely the one-shot payoff; that payoff may, however, have been increased by prior investment. Thus, we emphasize the two-stage nature of firm formation and operation.

# 3 The model

We use the prisoner's dilemma to capture the complementarity between Flint and Silver in the rowing stage (i.e., between firms in the execution stage).[3] Our modeling approach contains two innovations to enable us to address the question of alliance formation while incorporating issues of investment and effort. First, in our approach to the prisoner's dilemma, players can move the game frontier. Second, the specialization of prisoner's dilemma to a parallelogram structure allows us to get closed form solutions and do comparative statics

It is well known that in a static prisoner's dilemma, cooperation cannot be achieved. By playing the game repeatedly and over an infinite horizon,

[3] We emphasize that we assume no technological complementarity between the firms' investments, nor do we assume any cost complementarity. That is to say, if firm 1 invests, firm 2's payoffs are unaffected by that investment. This is distinct from the technological complementarities arising from their *actions*: if firm 1 acts cooperatively, firm 2's payoffs are enhanced, due to the prisoner's dilemma structure of the game, so there is endogenous complementarity driven by the incentives in the rowing stage. The PRT literature does assume technological complementarities. However, incorporating technological complementarities here would obscure the complementarities arising from incentives.

though, cooperation is possible. However, in the standard repeated prisoner's dilemma game, a continuum of cooperative equilibria arise. By adding a realistic modification of the repeated prisoner's dilemma, we collapse this continuum to a single point that represents a unique equilibrium. In the rowboat setting, suppose the strait that Flint and Silver want to cross has unpredictable currents that affect the progress of the boat; also recall that the rival pirates are pressing to get to the treasure first, but their progress is unknown. If either Flint or Silver deviates from the equilibrium pattern of rowing-and-rest, then in response the other too ceases to cooperate as punishment. The boat is then swept off course and they fail to arrive before the rival pirates. This is analogous to the situation wherein, if firms deviate from the pattern of cooperation that they initially contracted to, they precipitate the legal termination of the contract.

We capture this situation with the following device (detailed in TK): deviations from a fixed mixed strategy that is chosen at the initiation of the contract trigger termination of the game. This game maps directly into a repeated prisoner's dilemma game, and the admission of mixed strategies means that the equilibria of the game comprise a continuum that can be expressed geometrically as convex combinations of payoff pairs in the plane (see Figure 1).

With sufficient patience, the folk theorem applies, and at the threshold at which the folk theorem applies, the equilibrium of the game is a unique Pareto-optimal equilibrium point.[4] This equilibrium point is determined by

[4]One must add the assumption that the Pareto optimal point is always chosen from the set of equilibria. In the setting that we employ, if there is a single equilibrium on the

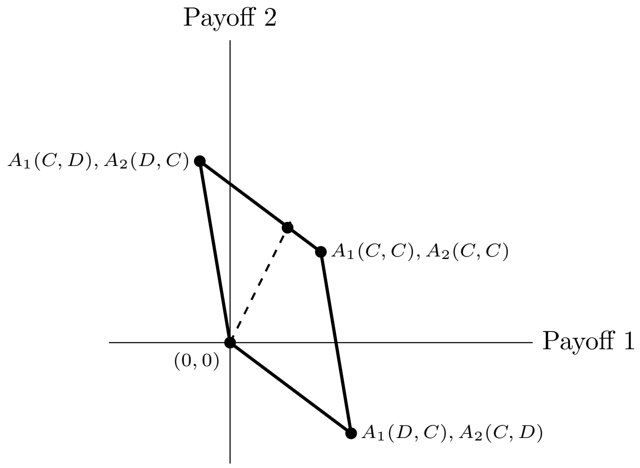


Figure 1: Parallelogram payoff structure. All payoff combinations are attainable via mixed strategies.

the relative bargaining power of the two players, which is in turn determined by the structure of the payoffs in the underlying game. It is also true, and crucial to our argument here, that if there is insufficient patience (or equivalently an excessively high probability of random termination) the only equilibrium is the static non-cooperative outcome.

In the literature on the repeated prisoner's dilemma and the folk theorem, it is standard to quantify patience via the discount factor, and to characterize the equilibria of the game as a function of the discount factor. In our model, we depart from TK and from the standard approach, in which payoffs

Pareto frontier of the game, then that point is the unique Pareto dominant equilibrium. As detailed in TK, this solution is identical to an axiomatic solution that is related to the Kalai-Smorodinski bargaining solution. In more general settings, equilibria on the Pareto frontier of the game might not Pareto dominate all equilibria. See Conley, Chakravarti and Taub (1996) for details.

are fixed. Instead we hold the discount factor fixed and vary the payoffs of the game. We show how modifying those payoffs alters the bargaining powers of the players, making it possible to achieve the unique Pareto-optimal equilibrium.

The changes in payoffs are achieved via investments, which in our model are endogenous. In our model, contracting on investment has a specific meaning. Such a contract has a legal implementation, but the legal implementation does not alone solve the incentive problem. The contracts are actually self-enforcing, in the sense that in principle a firm could deviate by deviating from a fixed mixed strategy but chooses not to, not because of legal strictures per se, but out of the fear of the lost surplus this would precipitate.

Each firm's investments increases its own payoffs. The change in payoffs stemming from investment alters bargaining power. Specifically, one firm's investment increases its payoffs, and at the same time reduces the bargaining power of the partner firm. To forestall the investing firm's temptation to defect, the rival firm must compensate the investing firm with reduced shirking, which is an expression of its reduced relative bargaining power.

The intuition is straightforward: if in the investment stage Silver builds a large oar, he can go faster and his expected payoff increases. But in addition, due to the parallelogram structure of the game, Silver's investment causes Flint's marginal gain from free-riding in the rowing stage to be reduced. To forestall Silver's temptation to defect, Flint must compensate him with reduced shirking. This is the expression of a reduction of Flint's bargaining power. If done in sufficient measure, this increases the propensity to act cooperatively, thereby effecting formation.[5]

### 3.1 Some technical details of the model

Our model allows us to quantify bargaining power. The formal definition of bargaining power is as follows (see TK, p. 456): The bargaining power $\beta_i$ of firm $i$ is $i$'s marginal gain from switching from cooperation to defection, relative to the loss this change induces in the rival firm's payoff, holding the action of $i$'s rival fixed.

This in turn enables us to determine the ratio of payoffs between the two firms. As demonstrated in TK, if $\beta_1$ is the bargaining power of Firm 1 and $\beta_2$ the bargaining power for Firm 2, the equilibrium ratio of their payoffs is

$$\frac{x_2}{x_1} = \sqrt{\frac{\beta_2}{\beta_1}} \tag{1}$$

The simplicity of this formula stems from our central assumption: that the payoffs are structured so that the game frontier is a parallelogram (*ibid*); see Figure 1. A parallelogram structure is equivalent to requiring that the marginal gain from defection is independent of the actions of the other player.

As developed in TK, bargaining power is determined by the slopes of the facets of the game frontier: the steeper the slope of a facet of the game frontier, the less is the bargaining power for the player associated with that facet. For example, in Figure 1, Player 1's bargaining power is linked to the slope of the right facet; his bargaining power is low relative to that of player

[5]The outside opportunities of each firm are incorporated into the payoff function at the $(0, 0)$ point. So, the payoffs we consider are net of the outside opportunities and stem only from the interactions of the firms.

2. The reason is that his marginal gain from defection is low relative to that of player 2.

If the pirates make their oars and they begin rowing to the island, at each instant some outcome can occur. They can get to the island, which happens at a random time because of random currents and winds, go to the site of the treasure, and dig up the treasure. They can also get to the site of the treasure only to find it gone, the other pirates having arrived there first. Indeed, Flint and Silver might see the other pirates pull up to the shore of the island before they even arrive. Finally, they might simply continue to row, with none of these other outcomes.

In the game representation of these possibilities, we quantify the potential for the game to end by $\delta$, the probability that the game will end in the current round, and correspondingly the probability that it continues until the next round with $1-\delta$. If the game does end, the actions chosen by the pirates in the current round—cooperate or defect—will be implemented, and the payoffs associated with those actions are realized. As is evident from the pirate story, the outcome for Flint and Silver might be good (they get the treasure) or bad (the other pirates get there first).

When the game is modeled with this structure, the probability that the game will continue to the next round, $1-\delta$, corresponds to a standard discount factor, which in intuitive terms is the patience of the players, and $\delta$ can thus be viewed as their impatience.

As in the standard repeated prisoner's dilemma, there is an upper bound on the impatience firms can have in order for cooperation to occur in this game. We denote this maximum impatience $\delta^*$. Figure 3 includes the ray (dashed line) that constitutes the equilibrium set if the actual value of $\delta$ is equal to $\delta^*$, and the shaded area is a measure of the degree to which impatience exceeds that needed to attain cooperation if the actual $\delta$ exceeds $\delta^*$.

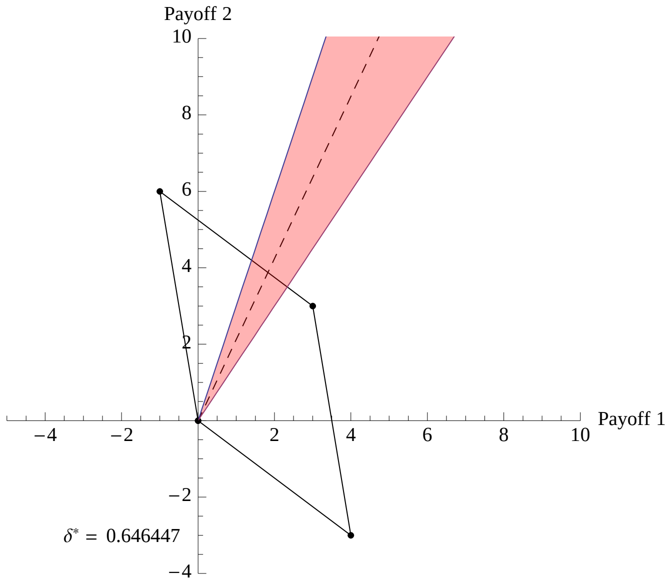


Figure 2: The equilibrium set with $\delta^*$

The initial parallelogram has an implicit $\delta^*$, determined by the initial bargaining powers as follows (*ibid*, p.456):

$$\delta^* = 1 - \sqrt{\beta_1 \beta_2} \tag{2}$$

However, the $\delta$ that determines the equilibrium of the game is given exogenously and might exceed $\delta^*$. In a more standard approach to modeling repeated games, we would then carry out thought experiments in which we altered $\delta$. Instead, we interpret the actual value of $\delta$ to be determined by technological factors (such as unobservable currents in the strait that speed up or slow down the progress of the boat randomly in our pirate example) that cannot be changed. We assume that this exogenously given $\delta$ (which we will subsequently denote $\delta^F$ to distinguish it from $\delta^*$) exceeds the $\delta^*$ in the pre-investment situation and therefore, without further modification, no cooperation is possible.

Investment achieves this modification. In particular, investment by firm 1 reduces the bargaining power $\beta_2$ of firm 2, while leaving the bargaining power of firm 1 unaffected. (This asymmetry in the impact of investment is a direct consequence of the parallelogram structure of the game.) This in turn increases the implied $\delta^*$. Similarly, firm 2 can invest, lowering $\beta_1$ and thereby increasing $\delta^*$.

With sufficient investment by one or both firms, it is possible that $\delta^*$ attains the level of $\delta^F$, then enabling cooperation. This is *formation.*

#### 3.1.1 The link between investment and bargaining power

We equate investment by firm 1 to an outward movement of the right facet, corresponding to an increase in firm 1's payoffs when firm 2 is cooperating. So, if firm 1 invests $I_1$, then its payoffs increase by $I_1$ when firm 2 is cooperating. We assume that payoffs when firm 2 is not cooperating are unaffected by that investment, reflecting the necessity of the interaction of the firms to obtain the payoffs.

This investment leaves the slope of the right facet of the game frontier unaffected—it is a parallel shift, thereby leaving firm 1's bargaining power unaffected. Moreover, the resulting game frontier is still a parallelogram. However, the slope of the upper and lower facets of the game frontier is reduced, so that firm 2's bargaining power is reduced.

The influence of the incentives in the investment stage can result in asymmetry: one firm will invest more than the other, and at higher cost, in order to achieve formation. Also, this firm will achieve payoffs proportionally greater than the payoffs of the other firm. Nevertheless, firms voluntarily hew to this arrangement, because it enables them to cope with the incentive problems and achieve cooperation.

Thus, we have a rudimentary model in which firms which initially might not find it in their interests to cooperate end up achieving cooperation by carrying out investment. Our subsequent development explores this in detail. Investment is costly and the cost might be asymmetric across firms. When added to the asymmetry in the payoffs for the firms we find a rich set of predictions for cooperation, including the circumstances in which the cost structure blocks formation.

## 3.2 Mechanics of the model

We next set out an example detailing how investment results in formation. We begin with a set of payoffs such that the firms are too impatient, that is $\delta^F > \delta^*$ so that cooperation is initially impossible. We then construct the equilibrium in two main steps. In the first step, firm 1 invests until formation is achieved, that is, $\delta^*$ increases to $\delta^F$. This is not the final outcome however. By investing, firm 1 has lowered firm 2's bargaining power. Because the relative payoffs of the firms is determined by the ratio of bargaining powers, firm 2's relative payoff will shrink.[6] Firm 2 might also want to invest so as to increase its relative payoff. In order to exactly maintain formation, that is $\delta^* = \delta^F$, firm 1 must partially disinvest. When neither firm's marginal gain exceeds its cost in lost marginal payoffs from marginal costs incurred, equilibrium is attained.[7]

We emphasize that these two stages—the initial formation stage and the subsequent adjustment stage—are atemporal, with the equilibrium actually being attained in one step.

In our initial experiment, we alter the initial state of the game. Specifically, we move the right facet of the parallelogram in a particular way, namely, rightward in a parallel way and we thus maintain the parallelogram properties. This corresponds to investment by firm 1. We illustrate this in Figure 3.

Figure 3 shows that because firm 2's bargaining power has been diluted by the movement of the frontier, the equilibrium ray that could be achieved if $\delta^F$ were small enough has tilted in firm 1's favor.

A mathematical consequence of this structured movement is that the possible payoff combinations that just achieve formation are restricted to a rectangle, and this rectangle is not altered by the change in the initial parallelogram brought on by firm 1's investment. We denote this rectangle

[6] However, it should be noted that its absolute payoff might actually increase, because the game frontier has been moved out by firm 1's investment.

[7] The game payoffs can be considered as revenues accruing to the firms. Investment is costly, so that each firm invests until its marginal cost of investment equals its marginal increase in the payoff, maximizing profit. In our initial experiments we will consider the marginal cost of investment to be unitary, so that the total cost of investment is equal to the quantity of investment. In later experiments we explore the impact of higher marginal costs.

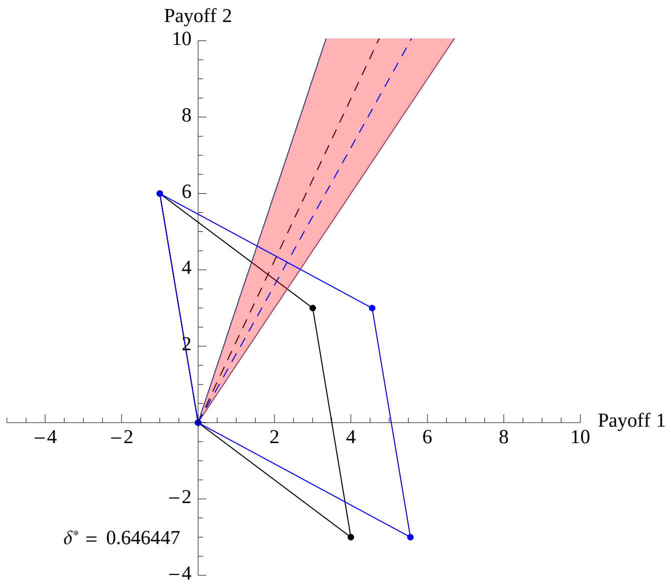


Figure 3: Parallelogram before and after investment by firm 1

the *formation rectangle.*

## 3.3 The formation rectangle

To establish the rectangle property we recall some algebraic relationships from TK. Examining Figure 1 of TK, we see that the formation ray intersects the payoff frontier at point $\pi(\delta^*)$. We want to characterize this point algebraically

Consider the non-formation case. If firm 1 invests sufficiently, which is represented as a parallel rightward shift of the right facet of the game frontier, then for sufficient investment formation will be achieved and the equilibrium set will be a ray. Further investment by firm 2 and concomitant disinvestment by firm 1 will maintain a balance of bargaining powers such that alliance is just on the cusp of formation and the equilibrium set remains a ray, but with a different slope.

The equation for the ray is given by equation (5) in T-K:

$$\frac{x_2}{x_1} = \left(\frac{A_2(D,C)A_2(C,D)}{A_1(D,C)A_1(C,D)}\right)^{1/2} \tag{TK-5}$$

which we can rewrite as

$$(A_1(D,C)A_1(C,D))^{1/2}x_2 = (A_2(D,C)A_2(C,D)))^{1/2}x_1 \tag{3}$$

There is a second equation, equation (4), which relates the probability of termination $1-\delta^*$ to the bargaining powers, resulting in

$$\delta^* = 1 - \left(\frac{A_2(C,D)A_1(C,D)}{A_2(D,C)A_1(D,C)}\right)^{1/2} \tag{TK-4}$$

where the asterisk denotes the $\delta$ such that the equilibrium set is a ray rather than a point at the origin (non-formation) or a cone.

Because $\delta$ is given in equation (TK 4), investment is needed to make the equation hold with equality. This is achieved by solving for the investment $I_1$ needed to make the equation hold:

$$\delta^* = 1 - \left(\frac{A_2(C,D)A_1(C,D)}{A_2(D,C)(A_1(D,C)+I_1)}\right)^{1/2} \tag{TK-4}$$

It is important to note from the parallelogram property that this investment is in fact a *parallel* shift because $A_1(C,C)$ increases by the same amount due to the parallelogram identity $A_1(C,C) = A_1(C,D) + A_1(D,C)$.

If we then look at the intersection of the ray with the facet once sufficient investment has taken place and then adjust $I_1$ and $I_2$ to maintain formation, the intersection moves either vertically or horizontally.

**Proposition 3.1** *If $A_i(x, y)$ and $\delta^*$ are such that formation exactly holds and $I_1$ and $I_2$ are adjusted so that $\delta^*$ is constant then (i) A parallel shift of the payoff frontier causes $\pi(\delta^*)$ to shift either horizontally or vertically. (ii) The apex of the horizontal and vertical movements of $\pi(\delta^*)$ is also the apex of the appropriately shifted game frontier.*

The proof is provided in the appendix. The rectangle appears in Figure 4 and subsequent figures.

**Lemmas about the rectangle**

With the main rectangle result in hand we can state the following corollary:

**Corollary 3.2** *Suppose (a) that there is a unitary marginal cost of investment*[8] *and (b) that the formation rectangle has been attained via investment. Then*

- *There is a unique Pareto-optimal point on the rectangle and this point coincides with the cusp of the corresponding game frontier;*
- *If the unique Pareto-optimal point has not been attained then it is Pareto improving for one firm to invest, and this moves the equilibrium point closer to the Pareto-optimal point.*

The proof is in the appendix.

[8] That is, holding all else equal, an increase in investment yields a one-for-one increase in payoffs, yielding a net increase in payoffs of zero. Of course the investment alters bargaining power, and therefore all else is not held equal in general.

Given that the derivative is zero in one direction, that is,

$$\left.\frac{dx_1}{dI_1}\right|_{dI_2=-\frac{(A_2(D,C))}{(A_1(D,C))}dI_1} = 0,$$

we can also immediately conclude that

$$\left.\frac{dx_2}{dI_1}\right|_{dI_2=-\frac{(A_2(D,C))}{(A_1(D,C))}dI_1} = 0,$$

or equivalently

$$\left.\frac{dx_2}{dI_2}\right|_{dI_2=-\frac{(A_2(D,C))}{(A_1(D,C))}dI_1} = 0,$$

characterizes the impact of investment on the payoff of investment by firm 2. Thus, when costs are added to the mix, the marginal tradeoff of the marginal cost of investment versus the marginal improvement in the payoff can be calculated (whilst keeping in mind that the rival firm's gain from reducing investment in this calculation is not included, that is, there is a positive externality.)

It is possible to construct the formation rectangle directly from the information available from the shape of the game frontier. In the absence of costs, recall from Corollary 3.2 that the equilibrium will ultimately be located at the apex of the formation rectangle. The locus of the apex is determined by a simple formula:

**Proposition 3.3** *The locus of the formation rectangle apex is*

$$-\Big(\frac{1}{1-\delta}-1\Big)(A_1(C,D),A_2(C,D))$$

**Proof:** See the appendix. □

Thus, when the game frontier lacks symmetry in the dimensions we will discuss below, it is still possible to identify the equilibrium point.

## The second stage

The second stage is deciding which point on the rectangle is chosen. If the equilibrium payoff combination lies on the horizontal segment of the rectangle, it indicates that the game payoff for firm 2 is fixed. Correspondingly, if the equilibrium payoff combination lies on the vertical segment of the rectangle, it indicates that the game payoff for firm 1 is fixed.

Our example is constructed so that the initial formation ray intersects the rectangle on its vertical segment. By firm 2 increasing its investment, with firm 1 decreasing its investment so as to remain on the rectangle, then firm 2's own game payoff can be increased without affecting firm 1's game payoff.[9] However, firm 2 increases its costs by doing this.

Recall that formation is characterized by the equilibrium set being a ray rather than a cone. The point where this ray intersects the formation rectangle is the combination of equilibrium game payoffs. Investment by firm 2 increases the slope of this ray, and therefore increases its own payoff. If firm 2 were acting selfishly, it would continue this process until the marginal increase in its game payoff was just equal to the marginal increase in its costs.

[9] There is a technical issue. If firm 1 continues its investment beyond the quantity needed to achieve formation, then the equilibrium set will expand from a ray to a cone. The $\delta^*$ ray associated with this payoff set will tilt even more to the right, so that the *minimum* payoff for firm 2 falls below the payoff on the formation ray. Firm 2 will therefore have an incentive to invest so that the formation rectangle is again achieved.

Additionally, it is fairly obvious that either firm would want to invest to achieve formation.

# 4 The basic properties of the model

We next conduct a series of numerical experiments to demonstrate the impact of various parameters of our model and to illustrate the effects of bargaining power, leverage, and investment cost, each of which can potentially be asymmetric across the firms.

## 4.1 The basic symmetric game and unitary investment costs

In our initial and benchmark example we suppose that the firms are symmetric, that their bargaining powers are relatively high, and that the cost of investment is unitary. This results in a game frontier that is diamond shaped (see Figure 4). As a result of the high bargaining power the firms initially do not achieve formation.

Figure 4 shows the final outcome of investment: the green game frontier is bigger in size than the initial black game frontier, and its shape has moved closer to a rectangular shape. The bargaining powers have decreased to the point of allowing formation, and the equilibrium point is on the cusp of the game frontier and is thus efficient. In addition the cusp coincides with the cusp of the formation rectangle in accord with Corollary 3.2.

The result of the investment is that the apex of the game frontier is attained: the marginal increase in the payoffs matches the investments made. The key result of the investment is that the apex of the game frontier is now attainable due to formation, which means that the net payoffs of the original game can now be attained, whereas they were not attainable before. Subtracting those investments from the total payoffs yields the firms' net

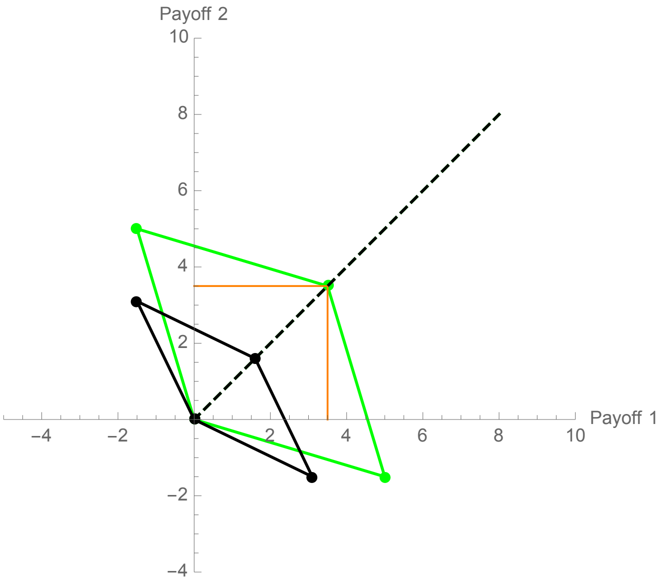


Figure 4: High bargaining power and low $\delta$.

| $\delta$ | $a_1$ | $a_2$ |
|---|---|---|
| .7 | .001 | .001 |

| | Initial BP | Final BP | Investment | Final payoff | Net profit |
|---|---|---|---|---|---|
| Firm 1 | .484 | .3 | 1.9 | 3.5 | 1.6 |
| Firm 2 | .484 | .3 | 1.9 | 3.5 | 1.6 |

profits, relative to their initial payoffs of zero in the non-formation initial state.

## 4.2 Low bargaining power

We next analyze a second example in which the game frontier is symmetric, the value of $\delta$ is unchanged, but in which the initial bargaining power of the firms is reduced. The initial game frontier is now closer to a square (see Figure 5).

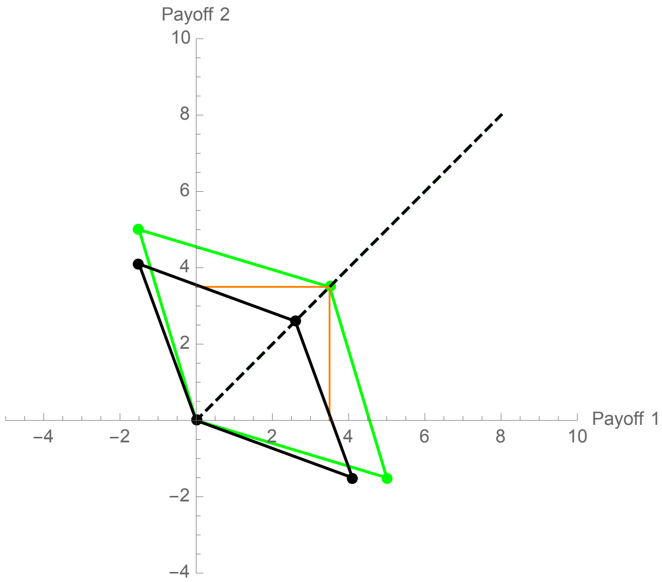


Figure 5: Low bargaining power and low $\delta$.

| $\delta$ | $a_1$ | $a_2$ |
|---|---|---|
| .7 | .001 | .001 |

| | Initial BP | Final BP | Investment | Final payoff | Net profit |
|---|---|---|---|---|---|
| Firm 1 | .366 | .3 | .9 | 3.5 | 2.6 |
| Firm 2 | .366 | .3 | .9 | 3.5 | 2.6 |

The result is that the firms again invest, expanding the game frontier in the process, and attain the green game frontier. However, because initial bargaining power is low, less investment is needed to attain the formation apex (.9 by each firm as opposed to 1.9 in the initial example).

Notice that the final game frontiers (the green game frontiers) in both Figure 4 and Figure 5 are identical: they have identical bargaining powers. This is because $\delta$ is identical in the two situations, so to exactly achieve formation the bargaining powers must attain the same levels.

As in the previous example, net profit is simply the initial payoffs: investment by each firm is rewarded one-for-one by increased payoffs, but the investment enables formation to take place so that the payoffs are attainable in equilibrium.

Thus, our initial examples establish that investment can overcome the resistance to formation, and high initial bargaining power requires more investment.

## 4.3 The effect of impatience

In our next examples we analyze the impact of patience, via variation in $\delta$. Increasing $\delta$ and thus decreasing patience puts greater demands on the firms: they are initially less willing to cooperate, and their bargaining power must concomitantly be reduced more than in the initial examples in order to achieve formation. Replicating the setting of Figure 4 except for the higher $\delta$ yields Figure 6.

This example highlights that in the presence of greater impatience, higher investment is needed to achieve formation—4.4 instead of .9. The result is that the final (green) game frontier is enlarged and much closer to a square shape than the final game frontier in the initial example, that is, bargaining power has been weakened to a much greater degree: final bargaining power

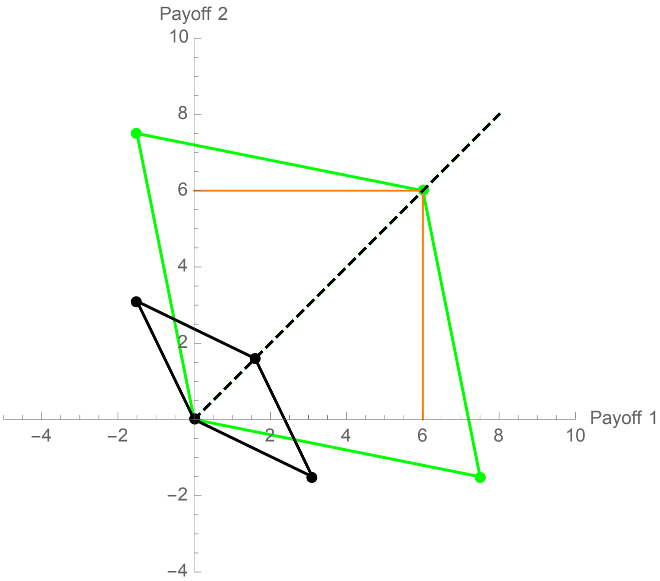


Figure 6: High symmetric bargaining power and high $\delta$.

| $\delta$ | $a_1$ | $a_2$ |
|---|---|---|
| .8 | .001 | .001 |

| | Initial BP | Final BP | Investment | Final payoff | Net profit |
|---|---|---|---|---|---|
| Firm 1 | .484 | .2 | 4.4 | 6.00 | 1.6 |
| Firm 2 | .484 | .2 | 4.4 | 6.00 | 1.6 |

is .2 in contrast to its value of .3 in the initial example.

This example also highlights the influence of technological uncertainty on the timing of alliance formation. In our framework, higher technological uncertainty implies high impatience as captured by $\delta$, necessitating greater strategic investment for alliance formation. In the face of very high technological uncertainty, firms will desist from making these investments, but may do so when technological uncertainty declines. Recall that the Lilly Genentech R&D alliance was only formed after Genentech demonstrated that it could create human insulin in the laboratory.

### 4.4 The effect of leverage

Our next experiment introduces asymmetry in the "aspect" of the game frontier. We can stretch the game to the right and shrink it downward, so that it approaches a line with negative 45 degree slope, all without changing the slopes of the game faces, and so also not changing bargaining power. This asymmetry differentially affects the incentives of the firms to invest.

Recall that when firm 1 invests, the right facet of the game frontier moves to the right. This reduces firm 2's bargaining power by reducing the slope of the upper facet of the game frontier. However, unlike the symmetric examples, the foreshortening of the upper facet makes its slope more sensitive to firm 1's investment: firm 1's investment now has a strong effect on the slopes of the upper and lower facets, thus strongly affecting bargaining power with a small investment. Conversely, firm 2's investment will have a lower marginal impact on the right and left slopes. Thus, firm 1 has more "leverage" on firm 2's bargaining power than it did in the symmetric examples, and firm 2's leverage is lower.

The consequence of this asymmetric leverage is that firm 1 ends up investing more than firm 2. Recalling from equation (2) that $\delta^*$, the endogenous value of $\delta$ needed for formation, is a function of the *product* of the bargaining powers, asymmetric investments by the two firms can still result in formation.

Figure 7 presents an example, which has approximately the same initial bargaining powers as in Figure 4. Firm 2's payoffs are bigger to start with.

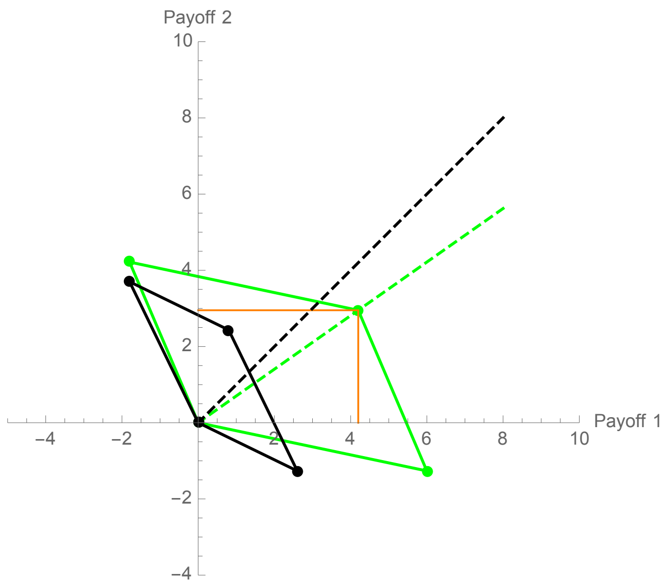


Figure 7: High bargaining power and low $\delta$: asymmetric firm payoffs.

| $\delta$ | $a_1$ | $a_2$ |
|---|---|---|
| .7 | .001 | .001 |

| | Initial BP | Initial leverage | Final BP | Investment | Final payoff | Net profit |
|---|---|---|---|---|---|---|
| Firm 1 | .486 | .194 | .427 | 3.4 | 4.20 | .8 |
| Firm 2 | .486 | .139 | .211 | .517 | 2.95 | 2.435 |

We see from this figure that firm 1 does most of the investing, because it has more "leverage:" a unit of investment by firm 1 twists the game frontier so that the upper facet becomes flatter, thus reducing firm 2's bargaining power, but because the upper facet is shorter than the right facet, the impact of a unit of investment on the slope is bigger. The end of the investment process also yields much-reduced bargaining power for firm 2 relative to firm 1. Thus, the firm with the greater leverage, firm 1, carries the main burden of investment to achieve formation.[10]

We can quantify leverage, that is, the sensitivity of a rival firm's bargaining power, $\beta_{-i}$, to a firm's investment, $I_i$. Denoting firm $i$'s leverage by $\lambda_i$, we show in Appendix B that leverage is

$$\lambda_i = \frac{\beta_{-i}}{A_i(D,C)} \tag{4}$$

that is, the rival firm's bargaining power is moderated by a firm's defection value.

The impact of leverage on outcomes can be seen by looking at the ratio of the firms' leverages, $\frac{\lambda_i}{\lambda_{-i}}$. Using the formula for $\beta_i$, it is straightforward to establish that the ratio is

$$\frac{\lambda_i}{\lambda_{-i}} = \frac{A_{-i}(C,D)}{A_i(C,D)}$$

which is identical to the slope of the apex of the formation rectangle. Leverage determines the ultimate outcome of the alliance in this sense.

It is evident that the asymmetry in leverage induces asymmetry in investment and in net profit as well. Despite the fact that firm 1 carries the greater burden of investment, and also ends with greater relative bargaining

[10] We can also interpret the example in light of Proposition 3.3. The asymmetry in leverage is equivalent to reducing the absolute value of $A_2(C,D)$, which by Proposition 3.3 shifts the apex of the formation rectangle downward.

power as a result, and with gross payoffs that are tilted in its direction, its ultimate net profit is lower than firm 2's profit; this is because firm 2 has not incurred the cost of investment.

## 4.5 The impact of costs

So far, our experiments have assumed unitary investment costs for both firms, that is, an investment by firm 1 increases its payoff by exactly its investment, holding other things equal. Our next experiment introduces asymmetry in costs, so that firm 2 has a positive cost of investment, that is, an additional per-unit cost of investment is subtracted from its payoff. This will impede its investment.

Our notion of cost is analogous to the cost of an input into a production function. The expenditure on a unit of capital in a standard model would then translate into a marginal change in output; due to diminishing returns, the marginal change in output is dependent on the quantity of capital already present. Similarly here, the marginal impact of an investment is not necessarily one-for-one with the investment: investment shifts a facet of the game frontier, but the change in the payoff for the firm making the investment is then affected by the response of the other firm.

Figure 8 illustrates cost asymmetry. The initial game frontier is symmetric, with equal bargaining powers and leverage. Any asymmetry in the investment and payoffs that ensue is therefore driven entirely by the asymmetry in costs.

Firm 1 has the main burden of investment, because its marginal cost of investment is lower than firm 2's cost. Because it invests more, it reduces firm 2's relative bargaining power. Firm 2, on the other hand, invests up until its marginal increase in the payoff is equal to the marginal cost of investing. The final ray no longer passes through the apex of the modified game or the apex of the formation rectangle. Rather, the payoffs are now tilted toward firm 1. (Notice that $\delta$ has been increased to .75 in order to make the effect visible.)

The example illustrates that the efficient outcome, the apex of the formation rectangle, might not be attained. Indeed, if costs are high enough, formation will not occur as it entails negative profit for the investing firm.

### 4.6 Asymmetric initial bargaining power

Finally, we consider the more general case when initial bargaining power is asymmetric across the two firms. Figure 8, viewing the final game frontier as the initial game frontier rather than the end of the formation process, would be an example of this.

We begin by noting that if the costs are unitary, then the equilibrium of the game ends at the cusp of the formation rectangle, but the locus of the cusp as stated in Proposition 3.3 does not depend directly on bargaining power. The formation rectangle itself is driven by the investments of the firms. Therefore we would like to characterize how that investment is driven by bargaining power.

Harking back to Figure 7, we see that the initial payoff structure of that game had symmetric bargaining power but asymmetric leverage; we then saw that the higher-leverage firm undertook the greater investment. By manipulating initial investment in a game that initially has asymmetric

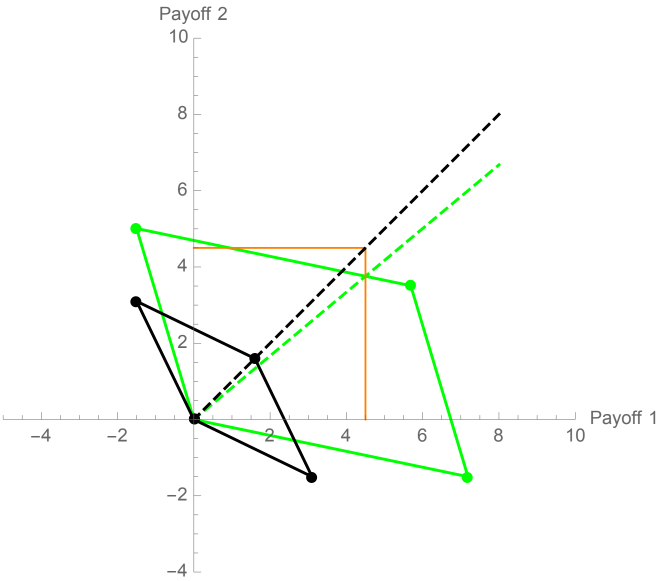


Figure 8: High bargaining power asymmetric cost.

| $\delta$ | $a_1$ | $a_2$ |
|---|---|---|
| .75 | .001 | .197 |

| | Initial BP | Final BP | Investment | Final payoff | Net profit |
|---|---|---|---|---|---|
| Firm 1 | .484 | .3 | 4.088 | 4.500 | .412 |
| Firm 2 | .484 | .209 | 1.908 | 3.756 | $1.848 - .197 * 1.908$ |
| Firm 2 | .484 | .209 | 1.908 | 3.756 | $= 1.472$ |

bargaining power such as the larger parallelogram of Figure 8, we can recreate this initial state by translating the initial payoff structure of the model into an equivalent one in which the asymmetry in bargaining power is removed. All of the effect of bargaining power is translated into leverage, which is explicitly a function of bargaining power in formula (4). Once that is done, we can predict that the high-leverage firm will undertake the higher investment, as we saw in Figure 7.

To see how this works, consider the larger game frontier as the *initial* game frontier in Figure 8. For the given value of $\delta$, the initial game frontier has an equilibrium ray penetrating one facet of the frontier; formation has already been achieved for that value of $\delta$. By running investment *backwards*, that is, with negative investment, we arrive at a game frontier in which bargaining power is symmetric. That game frontier might, however, have asymmetric leverage. Therefore the high-leverage firm will invest more.

Once the game frontier has been run backwards to attain an asymmetric leverage game frontier, it can again be run forward, but potentially with a different value of $\delta$. In particular, the initial asymmetric-bargaining-power game frontier might not already be at formation. In that case, the backwards run can be carried out for a value of $\delta$ that is low enough so that formation is just attained initially. Then, after running the game backwards to the symmetric bargaining power, asymmetric leverage state, the game can then be run forward with positive investment at the higher value of $\delta$ until formation and the equilibrium on the formation rectangle is attained. Recall that the formation rectangle can be established directly using Proposition 3.3.

# 5 Conclusions

We have a theory of why firms sometimes cooperate with each other, an explanation for why they might fail to agree to cooperate at all, and the inefficiency that can accompany cooperation when it does occur. The theory rests on incentives, and resides fully in a dynamic framework that is necessary to account for those incentives.

In our theory firm cooperation is delineated by parameters of bargaining power, leverage, cost, technological uncertainty, and payoff structure. When costs are low enough, firms will undertake strategic investments that dampen the other's incentives to shirk. These strategic investments then enable them to form alliances in which they each act selfishly to maximize their own profits. Our evidence is that if it is costly for firms to invest, the alliances will in general be inefficient in the sense that they do not maximize joint output, even though there is no physical impediment to this maximization. For costs that are high enough firms will fail to form alliances. This is exacerbated in the presence of high technological uncertainty, with requires higher investment to achieve formation.

In the real world firm alliance contracts have significant asymmetries in investment and in the division of profits. Our theory rationalizes these asymmetries and the diversity of contract structure. It has tight predictions about contract structure: it prescribes the investments required of each firm to achieve formation, and dictates a clear—and not necessarily symmetric—division of payoffs. . A counter-intuitive implication of our theory is that the firm with the greater leverage, while it can receive the greater payoff, also can receive a lower profit from the alliance (because it undertakes the strategic investment that enables formation). Nonetheless, it does so since in the absence of investment, it would receive no profits at all.

It seems obvious that our idea of formation can be greatly generalized. Firms in general can be viewed as ongoing alliances that stem from some sort of initial investment that is surrounded by the efforts of firm participants in production. Our theory not only rationalizes such arrangements, it also explains why there is not one grand firm for the entire economy, and at the other extreme, why the economy does not just consist of individual proprietorships.

The empirical correlates of our model can in principle be measured. It is straightforward to measure costs, profits and investment. Any empirical test must also have measures of bargaining power, which is at the heart of the model. Bargaining power is determined by the structure of the parallelogram, which in turn reflects the payoffs stemming from the actions of the firms. These payoffs can also be measured: they are the gains and losses from cooperative and noncooperative actions. We are therefore confident that our theory can be put to the test.

# A Proofs of formation rectangle propositions

**Proof:** (of Proposition 3.1) We can begin by algebraically characterizing the movements of $I_1$ and $I_2$ such that $\delta^*$ does not change:

$$d\delta^* = 0 = \frac{1}{2}\frac{(A_2(C,D)A_1(C,D))^{1/2}}{(A_2(D,C)+I_2)(A_1(D,C)+I_1)^{3/2}} \times ((A_2(D,C)+I_2)dI_1 + (A_1(D,C)+I_1)dI_2) \quad (5)$$

which easily translates to

$$\frac{dI_2}{dI_1} = -\frac{(A_2(D,C)+I_2)}{(A_1(D,C)+I_1)} \quad (6)$$

If we start from formation, then this simplifies to

$$\frac{dI_2}{dI_1} = -\frac{(A_2(D,C))}{(A_1(D,C))} \quad (7)$$

or in differential terms,

$$dI_2 = -\frac{(A_2(D,C))}{(A_1(D,C))}dI_1 \quad (8)$$

Thus, all we need to demonstrate is that the intersection point with the game facet moves either horizontally or vertically as we change $I_1$ and $I_2$ with this constraint.

The proof of part (i) consists of taking the derivative of $\pi(\delta^*)$ with respect to a parametric parallel shift of the facet, but subject to the constraint (6), and demonstrating that that derivative is either zero or infinite.

There are two equations needed to characterize $\pi(\delta^*)$: the first is the equation for the ray, equation (5) of TK. Recalling that $x_i$ is the expected payoff of player $i$:

$$\frac{x_2}{x_1} = \left(\frac{A_2(D,C)A_2(C,D)}{A_1(D,C)A_1(C,D)}\right)^{1/2} \quad \text{(TK-5)}$$

which we can rewrite as[11]

$$(A_1(D,C)A_1(C,D))^{1/2}x_2 = (A_2(D,C)A_2(C,D)))^{1/2}x_1 \tag{9}$$

The second equation needed is the equation for the facet of the payoff frontier. There are two facets. We begin with the upper facet:

$$x_2 = A_2(C,C) + \frac{A_2(D,C) - A_2(C,C)}{A_1(C,D) - A_1(C,C)}(x_1 - A_1(C,C)) \tag{10}$$

(Notice that the slope is negative.) The next step is to use the parallelogram property. From equation (1) in TK,

$$\begin{aligned} A_1(C,D) - A_1(C,C) &= A_1(D,D) - A_1(D,C) = -A_1(D,C) \\ A_2(D,C) - A_2(C,C) &= A_2(D,D) - A_2(C,D) = -A_2(C,D) \end{aligned} \tag{11}$$

or

$$\begin{aligned} A_1(C,C) &= A_1(D,C) + A_1(C,D) \\ A_2(C,C) &= A_2(D,C) + A_2(C,D) \end{aligned} \tag{12}$$

The facet equation, equation (10) then becomes

$$x_2 = A_2(C,C) + \frac{-A_2(C,D)}{-A_1(D,C)}(x_1 - A_1(C,C)) \tag{13}$$

These two equations can be solved for $\pi(\delta^*) = (x_1, x_2)$.

We can write the equations in matrix form:

$$\begin{pmatrix} -(A_2(D,C)A_2(C,D))^{1/2} & (A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & A_1(D,C) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ A_1(D,C)A_2(C,C) - A_2(C,D)A_1(C,C) \end{pmatrix} \tag{14}$$

[11]Because $A_1(C,D)$ and $A_2(C,D)$ are negative, it is technically more proper to write

$$(-A_1(D,C)A_1(C,D))^{1/2}x_2 = (-A_2(D,C)A_2(C,D)))^{1/2}x_1$$

We will suppress the altered signs in the subsequent derivations.

and using the parallelogram property in (12) we can write

$$\begin{pmatrix} -(A_2(D,C)A_2(C,D))^{1/2} & (A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & A_1(D,C) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ A_1(D,C)(A_2(D,C)+A_2(C,D)) - A_2(C,D)(A_1(D,C)+A_1(C,D)) \end{pmatrix} \tag{15}$$

A rightward parallel shift *that preserves formation* can then be represented by taking the derivative with respect to $A_1(D,C)$ which implicitly captures the equal shift of $A_1(C,C)$ via the parallelogram property we already used, along with subtracting a corresponding constant from $A_2(D,C)$ as expressed in the differential constraint (8):

$$\begin{pmatrix} \frac{1}{2}\frac{(A_2(D,C))}{(A_1(D,C))}A_2(D,C)^{-1/2}A_2(C,D)^{1/2}dI_1 & \frac{1}{2}A_1(D,C)^{-1/2}A_1(C,D)^{1/2}dI_1 \\ 0 & dI_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ (A_2(D,C)+A_2(C,D))dI_1 - A_1(D,C)\frac{(A_2(D,C))}{(A_1(D,C))}dI_1 - A_2(C,D)dI_1 \end{pmatrix} \tag{16}$$

Now take the total differential of the system in equation (15):

$$\begin{pmatrix} -(A_2(D,C)A_2(C,D))^{1/2} & (A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & A_1(D,C) \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix}$$
$$+ \begin{pmatrix} \frac{1}{2}\frac{(A_2(D,C))}{(A_1(D,C))}A_2(D,C)^{-1/2}A_2(C,D)^{1/2}dI_1 & \frac{1}{2}A_1(D,C)^{-1/2}A_1(C,D)^{1/2}dI_1 \\ 0 & dI_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ (A_2(D,C)+A_2(C,D))dI_1 - A_1(D,C)\frac{(A_2(D,C))}{(A_1(D,C))}dI_1 - A_2(C,D)dI_1 \end{pmatrix} \tag{17}$$

Now substitute the solution of $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ from (15):

$$\begin{pmatrix} -(A_2(D,C)A_2(C,D))^{1/2} & (A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & A_1(D,C) \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix}$$
$$+ \begin{pmatrix} \frac{1}{2}\frac{(A_2(D,C))}{(A_1(D,C))}A_2(D,C)^{-1/2}A_2(C,D)^{1/2}dI_1 & \frac{1}{2}A_1(D,C)^{-1/2}A_1(C,D)^{1/2}dI_1 \\ 0 & dI_1 \end{pmatrix}$$
$$\times \begin{pmatrix} -(A_2(D,C)A_2(C,D))^{1/2} & (A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & A_1(D,C) \end{pmatrix}^{-1}$$
$$\times \begin{pmatrix} 0 \\ (A_1(D,C))(A_2(D,C)+A_2(C,D)) - A_2(C,D)(A_1(D,C)+A_1(C,D)) \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ (A_2(D,C)+A_2(C,D))dI_1 - A_1(D,C)\frac{(A_2(D,C))}{(A_1(D,C))}dI_1 - A_2(C,D)dI_1 \end{pmatrix}$$

or

$$\begin{pmatrix} -(A_2(D,C)A_2(C,D))^{1/2} & (A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & A_1(D,C) \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix}$$
$$= -\begin{pmatrix} \frac{1}{2}\frac{(A_2(D,C))}{(A_1(D,C))}A_2(D,C)^{-1/2}A_2(C,D)^{1/2} & \frac{1}{2}A_1(D,C)^{-1/2}A_1(C,D)^{1/2} \\ 0 & 1 \end{pmatrix}$$
$$\times \begin{pmatrix} -(A_2(D,C)A_2(C,D))^{1/2} & (A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & A_1(D,C) \end{pmatrix}^{-1}$$
$$\times \begin{pmatrix} 0 \\ (A_1(D,C))(A_2(D,C)+A_2(C,D)) - A_2(C,D)(A_1(D,C)+A_1(C,D)) \end{pmatrix} dI_1$$
$$+ \begin{pmatrix} 0 \\ (A_2(D,C)+A_2(C,D)) - A_1(D,C)\frac{(A_2(D,C))}{(A_1(D,C))} - A_2(C,D) \end{pmatrix} dI_1$$

and finally, the solution for $\begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix}$ is

$$\begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix}$$
$$= -\begin{pmatrix} -(A_2(D,C)A_2(C,D))^{1/2} & (A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & A_1(D,C) \end{pmatrix}^{-1}$$
$$\times \begin{pmatrix} \frac{1}{2}\frac{(A_2(D,C))}{(A_1(D,C))}A_2(D,C)^{-1/2}A_2(C,D)^{1/2} & \frac{1}{2}A_1(D,C)^{-1/2}A_1(C,D)^{1/2} \\ 0 & 1 \end{pmatrix}$$
$$\times \begin{pmatrix} -(A_2(D,C)A_2(C,D))^{1/2} & (A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & A_1(D,C) \end{pmatrix}^{-1}$$
$$\times \begin{pmatrix} 0 \\ (A_1(D,C))(A_2(D,C)+A_2(C,D)) - A_2(C,D)(A_1(D,C)+A_1(C,D)) \end{pmatrix} dI_1$$
$$+ \begin{pmatrix} -(A_2(D,C)A_2(C,D))^{1/2} & (A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & A_1(D,C) \end{pmatrix}^{-1}$$
$$\times \begin{pmatrix} 0 \\ (A_2(D,C)+A_2(C,D)) - A_1(D,C)\frac{(A_2(D,C))}{(A_1(D,C))} - A_2(C,D) \end{pmatrix} dI_1$$

or more compactly, defining

$$M \equiv \begin{pmatrix} -(A_2(D,C)A_2(C,D))^{1/2} & (A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & A_1(D,C) \end{pmatrix},$$

$$\begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix}$$
$$= -M^{-1}\begin{pmatrix} \frac{1}{2}\frac{(A_2(D,C))}{(A_1(D,C))}A_2(D,C)^{-1/2}A_2(C,D)^{1/2} & \frac{1}{2}A_1(D,C)^{-1/2}A_1(C,D)^{1/2} \\ 0 & 1 \end{pmatrix} M^{-1}$$
$$\times \begin{pmatrix} 0 \\ (A_1(D,C))(A_2(D,C)+A_2(C,D)) - A_2(C,D)(A_1(D,C)+A_1(C,D)) \end{pmatrix} dI_1$$
$$+ M^{-1}\begin{pmatrix} 0 \\ (A_2(D,C)+A_2(C,D)) - A_1(D,C)\frac{(A_2(D,C))}{(A_1(D,C))} - A_2(C,D) \end{pmatrix} dI_1$$

The agenda now is simply to show that either $dx_1$ or $dx_2$ is zero. First of all notice that the final term does simplify to zero, so we have

$$\begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix}$$
$$= -M^{-1}\begin{pmatrix} \frac{1}{2}\frac{(A_2(D,C))}{(A_1(D,C))}A_2(D,C)^{-1/2}A_2(C,D)^{1/2} & \frac{1}{2}A_1(D,C)^{-1/2}A_1(C,D)^{1/2} \\ 0 & 1 \end{pmatrix} M^{-1}$$
$$\times \begin{pmatrix} 0 \\ (A_1(D,C))(A_2(D,C)+A_2(C,D)) - A_2(C,D)(A_1(D,C)+A_1(C,D)) \end{pmatrix} dI_1$$

Now the rightmost matrix also simplifies a bit:

$$\begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix}$$

$$= -M^{-1} \begin{pmatrix} \frac{1}{2} \frac{(A_2(D,C))}{(A_1(D,C))} A_2(D,C)^{-1/2} A_2(C,D)^{1/2} & \frac{1}{2} A_1(D,C)^{-1/2} A_1(C,D)^{1/2} \\ 0 & 1 \end{pmatrix} M^{-1}$$

$$\times \begin{pmatrix} 0 \\ A_1(D,C)A_2(D,C) - A_2(C,D)A_1(C,D) \end{pmatrix} dI_1$$

Now use the inverse of $M$ explicitly:

$$M^{-1} \equiv \frac{1}{\Delta} \begin{pmatrix} A_1(D,C) & -(A_1(D,C)A_1(C,D))^{1/2} \\ +A_2(C,D) & -(A_2(D,C)A_2(C,D))^{1/2} \end{pmatrix}$$

with

$$\Delta = -A_1(D,C)(A_2(D,C)A_2(C,D))^{1/2} + A_2(C,D)(A_1(D,C)A_1(C,D))^{1/2}$$

We can do a bit of manipulation with the middle matrix:

$$\begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix}$$

$$= -M^{-1} \begin{pmatrix} \frac{1}{2} \frac{1}{(A_1(D,C))} A_2(D,C)^{1/2} A_2(C,D)^{1/2} & \frac{1}{2} \frac{1}{(A_1(D,C))} A_1(D,C)^{1/2} A_1(C,D)^{1/2} \\ 0 & 1 \end{pmatrix} M^{-1}$$

$$\times \begin{pmatrix} 0 \\ A_1(D,C)A_2(D,C) - A_2(C,D)A_1(C,D) \end{pmatrix} dI_1$$

Now multiply the first two matrices (ignoring the determinant):

$$\begin{pmatrix} A_1(D,C) & -(A_1(D,C)A_1(C,D))^{1/2} \\ -A_2(C,D) & -(A_2(D,C)A_2(C,D))^{1/2} \end{pmatrix}$$

$$\times \begin{pmatrix} \frac{1}{2} \frac{1}{(A_1(D,C))} A_2(D,C)^{1/2} A_2(C,D)^{1/2} & \frac{1}{2} \frac{1}{(A_1(D,C))} A_1(D,C)^{1/2} A_1(C,D)^{1/2} \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} A_2(D,C)^{1/2} A_2(C,D)^{1/2} & \begin{matrix} \frac{1}{2} A_1(D,C)^{1/2} A_1(C,D)^{1/2} \\ -(A_1(D,C)A_1(C,D))^{1/2} \end{matrix} \\ -\frac{1}{2} \frac{A_2(C,D)}{A_1(D,C)} A_2(D,C)^{1/2} A_2(C,D)^{1/2} & \begin{matrix} -\frac{1}{2} \frac{A_2(C,D)}{A_1(D,C)} A_1(D,C)^{1/2} A_1(C,D)^{1/2} \\ -(A_2(D,C)A_2(C,D))^{1/2} \end{matrix} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} A_2(D,C)^{1/2} A_2(C,D)^{1/2} & -\frac{1}{2} A_1(D,C)^{1/2} A_1(C,D)^{1/2} \\ -\frac{1}{2} \frac{A_2(C,D)}{A_1(D,C)} A_2(D,C)^{1/2} A_2(C,D)^{1/2} & \begin{matrix} -\frac{1}{2} \frac{A_2(C,D)}{A_1(D,C)} A_1(D,C)^{1/2} A_1(C,D)^{1/2} \\ -(A_2(D,C)A_2(C,D))^{1/2} \end{matrix} \end{pmatrix}$$

Now multiply this times $M^{-1}$ on the right, yielding

$$\begin{pmatrix} \frac{1}{2} A_2(D,C)^{1/2} A_2(C,D)^{1/2} & -\frac{1}{2} A_1(D,C)^{1/2} A_1(C,D)^{1/2} \\ -\frac{1}{2} \frac{A_2(C,D)}{A_1(D,C)} A_2(D,C)^{1/2} A_2(C,D)^{1/2} & \begin{matrix} -\frac{1}{2} \frac{A_2(C,D)}{A_1(D,C)} A_1(D,C)^{1/2} A_1(C,D)^{1/2} \\ -(A_2(D,C)A_2(C,D))^{1/2} \end{matrix} \end{pmatrix}$$

$$\times \begin{pmatrix} A_1(D,C) & -(A_1(D,C)A_1(C,D))^{1/2} \\ A_2(C,D) & -(A_2(D,C)A_2(C,D))^{1/2} \end{pmatrix}$$

$$= \begin{pmatrix} W_1 & W_2 \\ Z_1 & Z_2 \end{pmatrix}$$

with

$$W_1 \equiv \frac{1}{2}A_2(D,C)^{1/2}A_2(C,D)^{1/2}A_1(D,C) + \frac{1}{2}A_1(D,C)^{1/2}A_1(C,D)^{1/2}A_2(C,D)$$

$$W_2 \equiv -\frac{1}{2}A_2(D,C)^{1/2}A_2(C,D)^{1/2}(A_1(D,C)A_1(C,D))^{1/2} + \frac{1}{2}A_1(D,C)^{1/2}A_1(C,D)^{1/2}(A_2(D,C)A_2(C,D))^{1/2}$$

The upper right hand corner term is

$$-\frac{1}{2}A_2(D,C)^{1/2}A_2(C,D)^{1/2}(A_1(D,C)A_1(C,D))^{1/2} + \frac{1}{2}A_1(D,C)^{1/2}A_1(C,D)^{1/2}(A_2(D,C)A_2(C,D))^{1/2} = 0$$

and when multiplied times the final matrix we obtain $dx_1 = 0$, thus completing the proof. □

**Proof:** (of Corollary 3.2) Point (ii) follows by the rectangle property: one firm moves along the rectangle and has zero change in its payoffs whilst the other has improved payoffs. □

## A.1 Characterizing the formation rectangle

We have seen that the formation rectangle is invariant with respect to investment that preserves the parallelogram structure. We have also seen that once formation has occurred, then if there are no costs of investment, the equilibrium is at the apex of the formation rectangle. Thus, if we can characterize the formation rectangle, we can immediately characterize the equilibrium. This is useful if the initial game frontier has a mixture of properties: asymmetric bargaining power and asymmetric leverage.

**Proof:** (of Proposition 3.3) Examining formula (15), and noting that in the subsequent development the construction is such that $dx_1 = 0$, the formula then determines $x_1$. By reversing the roles of firm 1 and firm 2, we can then determine $x_2$ as well. Combining this with the formula for $x_1$ then determines the locus of the apex.

Let us examine the formula for $x_1$ in a little more detail, and in particular express the formula for $x_1$ more explicitly in terms of bargaining power and leverage. Starting with formula (15), we can first decompose the left hand matrix as follows:

$$\begin{pmatrix} -A_1(D,C)^{1/2}(A_1(C,D))^{1/2} & 0 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} -\frac{A_2(D,C)^{1/2}A_2(C,D)^{1/2}}{A_1(D,C)^{1/2}(A_1(C,D))^{1/2}} & -1 \\ -A_2(C,D) & A_1(D,C) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ (A_1(D,C))(A_2(D,C)+A_2(C,D)) - A_2(C,D)(A_1(D,C)+A_1(C,D)) \end{pmatrix}$$

We can then drop the leading matrix without effect:

$$\begin{pmatrix} -\frac{A_2(D,C)^{1/2}A_2(C,D)^{1/2}}{A_1(D,C)^{1/2}(A_1(C,D))^{1/2}} & -1 \\ -A_2(C,D) & A_1(D,C) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ (A_1(D,C))(A_2(D,C)+A_2(C,D)) - A_2(C,D)(A_1(D,C)+A_1(C,D)) \end{pmatrix}$$

We can also cancel a couple of terms on the right hand side:

$$\begin{pmatrix} -\frac{A_2(D,C)^{1/2}A_2(C,D)^{1/2}}{A_1(D,C)^{1/2}A_1(C,D)^{1/2}} & -1 \\ -A_2(C,D) & A_1(D,C) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ A_1(D,C)A_2(D,C) - A_2(C,D)A_1(C,D) \end{pmatrix}$$

Now recall formulas (5) and (8) from TK,

$$\frac{x_2}{x_1} = \sqrt{\frac{\beta_2}{\beta_1}} = \left(\frac{A_2(D,C)A_2(C,D)}{A_1(D,C)A_1(C,D)}\right)^{1/2}$$

We can express the formula for $x_1$ in terms of these ratios. In particular, we can write (15) as

$$\begin{pmatrix} -\frac{\beta_2^{1/2}}{\beta_1^{1/2}} & -1 \\ -A_2(C,D) & A_1(D,C) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ A_1(D,C)A_2(D,C) - A_2(C,D)A_1(C,D) \end{pmatrix}$$

with solution

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{-A_1(D,C)\frac{\beta_2^{1/2}}{\beta_1^{1/2}} - A_2(C,D)} \begin{pmatrix} A_1(D,C) & 1 \\ A_2(C,D) & -\frac{\beta_2^{1/2}}{\beta_1^{1/2}} \end{pmatrix}$$
$$\times \begin{pmatrix} 0 \\ A_1(D,C)A_2(D,C) - A_2(C,D)A_1(C,D) \end{pmatrix}$$

The solution for $x_1$ is then

$$\frac{1}{-A_1(D,C)\frac{\beta_2^{1/2}}{\beta_1^{1/2}} - A_2(C,D)} \Big( A_1(D,C)A_2(D,C) - A_2(C,D)A_1(C,D) \Big)$$

Expressing this formula in terms of ratios yields

$$\frac{-1}{A_2(C,D)\left(\frac{A_1(D,C)}{A_2(C,D)}\frac{\beta_2^{1/2}}{\beta_1^{1/2}} + 1\right)} A_2(C,D)\Big(\frac{A_1(D,C)}{A_2(C,D)}A_2(D,C) - A_1(C,D)\Big)$$

with the cancellation yielding

$$\frac{-1}{\left(\frac{A_1(D,C)}{A_2(C,D)}\frac{\beta_2^{1/2}}{\beta_1^{1/2}} + 1\right)} \Big(\frac{A_1(D,C)}{A_2(C,D)}A_2(D,C) - A_1(C,D)\Big)$$

Now focus on the denominator term. Multiplying out, we have

$$\frac{A_1(D,C)}{A_2(C,D)}\frac{\beta_2^{1/2}}{\beta_1^{1/2}} = \frac{A_1(D,C)}{A_2(C,D)} \left(\frac{A_2(D,C)A_2(C,D)}{A_1(D,C)A_1(C,D)}\right)^{1/2}$$
$$= \frac{A_1(D,C)^{1/2}}{A_2(C,D)^{1/2}} \left(\frac{A_2(D,C)}{A_1(C,D)}\right)^{1/2} = \frac{1}{1-\delta}$$

(See TK p. 454 formula (4).) Substituting into the formula for $x_1$

$$\frac{-1}{\left(\frac{1}{1-\delta}+1\right)}\Big(\frac{A_1(D,C)}{A_2(C,D)}A_2(D,C)-A_1(C,D)\Big)$$

Again grouping terms yields

$$\frac{-1}{\left(\frac{1}{1-\delta}+1\right)}A_1(C,D)\Big(\frac{A_1(D,C)A_2(D,C)}{A_2(C,D)A_1(C,D)}-1\Big)$$
$$=\frac{1}{\left(\frac{1}{1-\delta}+1\right)}\Big(\frac{1}{(1-\delta)^2}-1\Big)A_1(C,D)$$

We can write this as

$$\frac{-1}{\left(\frac{1}{1-\delta}+1\right)}\Big(\frac{1}{1-\delta}-1\Big)\Big(\frac{1}{1-\delta}+1\Big)A_1(C,D)=-\Big(\frac{1}{1-\delta}-1\Big)A_1(C,D)$$

We know from our examples and from fundamental reasoning that this quantity should be positive and increasing in $\delta$ under symmetry. The leading coefficient, $\frac{\delta}{1-\delta}$, is in fact increasing in $\delta$, and $-A_1(C,D)$ is positive. We can view $A_1(C,D)$ as a scaling factor.

Repeating the exercise for $x_2$ yields

$$-\Big(\frac{1}{1-\delta}-1\Big)A_1(C,D)$$

Notice that this would be invariant with respect to investment by firm 1. The economics does make sense: a relatively large value of $A_1(C,D)$ in absolute value means greater bargaining power.

Thus, we can locate the apex of the formation rectangle easily:

$$\left(\frac{1}{1-\delta}-1\right)\Big(-A_2(C,D),-A_1(C,D)\Big) \tag{18}$$

thus completing the derivation. □

# B Leverage

We can develop a formula for leverage. We define leverage as the sensitivity of the rival firm's bargaining power to a firm's investment. Denoting a firm's leverage by $\lambda_i$, firm 1's leverage is

$$
\begin{aligned}
\lambda_1 \equiv \left| \frac{d}{dI_1} \frac{A_2(D,C) - A_2(C,C)}{A_1(C,C) + (-A_1(C,D))} \right| &= \left| \frac{d}{dA_1(C,C)} \frac{A_2(D,C) - A_2(C,C)}{A_1(C,C) - A_1(C,D)} \right| \\
&= \left| -\frac{A_2(D,C) - A_2(C,C)}{(A_1(C,C) - A_1(C,D))^2} \right| \\
&= \left| -\beta_2 \frac{1}{A_1(C,C) - A_1(C,D)} \right| \\
&= \left| -\beta_2 \frac{1}{A_1(D,C)} \right| \\
&= \beta_2 \frac{1}{A_1(D,C)}
\end{aligned}
$$

The first equality follows because firm 1's investment is equivalent to shifting out the cooperation value $A_1(C,C)$. Thus, the higher firm 1's defection value, holding firm 2's bargaining power fixed, the lower is firm 1's leverage in absolute value.

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Мы рассматриваем контрактные альянсы между компаниями, желающими получить преимущество за счет взаимодополнения сторон в рамках конкретных проектов. Примерами таких ситуаций являются альянсы между биотехнологическими и фармацевтическими компаниями для создания перспективных молекул. Мы разделяем работу таких альянсов на две стадии: на первой стадии каждая фирма инвестирует в проект, на второй происходят действия, необходимые для реализации проекта. Мы называем эти стадии «стадия инвестирования» и «стадия исполнения» соответственно.

Фирмы, вступающие в такие альянсы, имеют расходящиеся интересы и альтернативные проекты, что может привести к недоинвестированию на первой стадии и уклонению от обязанностей по проекту на стадии исполнения. Мы показываем, как взаимодействие этих сил влияет на возникновение равновесного контракта.

Наша теория дает количественное воплощение способности каждой фирмы извлекать ренту из контракта на стадии исполнения; мы называем это переговорной силой. Мы показываем, что избыточная переговорная сила может помешать возникновению альянса из-за дополнительной склонности к фрирайдерству. Но инвестиции компаний могут ослабить переговорную силу в достаточной для преодоления данной инициативы мере, что позволяет заключать альянсы. Мы количественно оцениваем такие контракты в терминах инвестиций, инвестиционных издержек, платежей и чистой прибыли. Все эти показатели могут быть асимметрично распределены между фирмами.

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