NATIONAL RESEARCH UNIVERSITY
HIGHER SCHOOL OF ECONOMICS

Maarten Janssen, Alexei Parakhonyak

PRICE MATCHING GUARANTEES AND CONSUMER SEARCH

BASIC RESEARCH PROGRAM

WORKING PAPERS

SERIES: ECONOMICS
WP BRP 03/EC/2011

Maarten Janssen[1], *Alexei Parakhonyak*[2]

PRICE MATCHING GUARANTEES AND CONSUMER SEARCH[3]

This paper examines the effect of price matching guarantees (PMGs) in a *sequential* search model. PMGs are simultaneously chosen with prices and some consumers (shoppers) know the firms' decisions before buying, while others (non-shoppers) enter a shop first before observing a firm's price and whether or not the firm has a PMG. In such an environment, PMGs increase the value of buying the good and therefore increase consumers' reservation prices. This increase is so large that even after accounting for the possible execution of PMGs, firms profits are larger under PMGs than without. We also consider the incentives of firms to choose PMGs and show that an equilibrium where all firms offer PMGs does not exist because of a free-riding problem. PMGs can only be an equilibrium phenomenon in an equilibrium where some firms do and others do not offer these guarantees.

JEL Classification: D40; D83; L13.
Keywords: Sequential Search, Price Matching Guarantees, Welfare Analysis.

---

[1]Department of Economics, University of Vienna and State University Higher School of Economics (Moscow). Email: maarten.janssen@univie.ac.at

[2]Center for Advanced Studies, State University Higher School of Economics. Email: parakhonyak@yandex.ru

[3]This paper has benefitted from presentations at Higher School of Economics (Moscow), the Royal Economic Society 2011 Meetings, The II Workshop on Search and Switching Costs (Groningen), the European Economic Association Meeting 2011 (Oslo) and EARIE 2011 (Stockholm). We especially are grateful to Heski Bar-Isaac, Marco Haan, Andrew Rhodes, Karl Schlag and Sandro Shelegia for comments and helpful suggestions on an earlier version of the paper.

1 Introduction

It is well known that price matching guarantees (PMGs) of one sort or the other are found in many sectors and industries. In retail markets, these guarantees often take the form that sellers offer consumers who buy their products to match any other price a competitor charges for identical products provided that they have a proof that an identical product is sold by a competitor at a nearby shop within a well-defined time period. Alternative forms of low price guarantees offer to give back (100+x)% of the price difference (so called price beating guarantees, PBGs) or offer a “free lunch” in addition to matching prices (see, e.g., IKEA stores).[1] Most firms give the price difference only to consumers who provide evidence of lower prices elsewhere and do not commit to change list prices.[2]

The effect of PMGs on the (pricing) behavior of competitors has been discussed in the economics as well as in the business and the law literature. The main conclusion from these literatures is that despite the appearance of creating additional competitive pressure on the pricing behaviour of firms, PMGs are in fact highly anticompetitive by greatly reducing the incentives of rival firms to undercut. In two empirical papers, Arbatskaya et al. (2004, 2006) consider important differences in the use and strategic effects of PMGs and PBGs. By having access to price data in the latter paper, the authors are able to consider and implement a new methodology to study the effects of PMGs and PBGs. For any pair of firms in their dataset where one firm does not have a low price guarantee and the other firm either has a PMG or a PBG, they have registered which firm has the lowest price. If the discouraging price undercutting theory is correct, then in a pairwise comparison test, both the firms with PMGs and PBGs should have (weakly) lower prices. Arbatskaya et al. (2006) find, however, that this is only true for the PBG firms and not for the PMG firms. Thus, they find that in the same market, firms offering PMGs have weakly higher advertised prices than rival firms without low price guarantees.

This paper aims at contributing to a better understanding of the fact that firms offering PMGs tend to set higher prices by casting this marketing instrument in a consumer search perspective. We argue that PMGs have an important effect on the search behaviour of consumers. This effect has largely been neglected in the previous literature. From a consumer search perspective, the main decision consumers have to make concerns their stopping rule: at which price do they continue to search and when do they stop searching and buy. This decision is usually characterized by a reservation price, i.e., by a maximal price at which consumers will buy instead of continue to search for lower prices. A PMG increases this reservation price as consumers do not only buy the commodity under consideration, but in addition also buy *an option* that if they are later informed

[1] The biggest supermarket in the Netherlands, Albert Heijn, introduced in spring 2009 a policy that gave customers a free apple pie in addition to ”all your money back policy” in case customers could show that other shops had lower prices for identical products.

[2] There are, however, some firms that commit to lowering list prices if competitors offer lower prices (see, e.g., Comet Services at comet.co.uk).

of lower prices, they get the price difference back. Consumers value this option and this increases their reservation prices. Higher reservation prices, in turn, give a firm that offers a PMG the opportunity to raise their list prices, thereby increasing their profits. There is, however, also an important indirect strategic effect on the prices charged by others firms in that they react to the higher prices of firms with PMGs by raising their prices themselves.

To study this search perspective on PMGs, we use a conventional sequential search setup a la Stahl (1989), where a fraction of consumers, the shoppers, is fully informed about all prices, while others (the nonshoppers) have to pay a search cost for each additional firm they visit. As PMGs stipulate that goods have to be identical for the guarantee to become effective, the model of Stahl (1989) is the most appropriate search model to use as it is by now the standard search model for markets with homogeneous goods. We add to the Stahl model that after the purchase and with a certain probability a consumer will be informed about another price quotation. This probability represents the level of information communication among consumers (as in Galeotti (2010)).

On the different strategic effects of PMGs we have the following results. First, the support of the equilibrium price distribution of a firm that provides a PMG is always above the support of the distribution of a firm without PMG. This explains the second finding of Arbatskaya et al. (2006) that firms setting PMGs have prices that are not below the prices of rival firms without a PMG. To understand this result that lower prices cannot be set by a firm offering PMGs, note first that pricing behaviour in any equilibrium is characterized by price dispersion for the same reason as there is price dispersion in Stahl (1989). Price dispersion equilibrates the incentive to lower prices to capture the shoppers and the incentives to raise prices to increase revenues over nonshoppers. Offering PMGs and a price in the interior of the price distribution, then implies that there is a positive probability that another firm has a lower price, that a nonshopper who already bought gets informed about this price and that this lower price has to be matched to that consumer. Without offering PMGs, this consumer would continue to buy from the firm, but at a higher expected effective price. In addition, whether or not a firm is able to attract shoppers is independent of whether PMGs are offered as shoppers always buy at the lowest price. Thus, deviating to not offering PMGs gives the same sales at a higher expected effective price.

Second, when we consider the interaction between firms in a market where firms can either choose to offer PMGs, or not to offer them, two types of symmetric equilibria exist: one where firms do not set PMGs at all, and one where firms set PMGs with a certain positive probability, which is strictly smaller than one. This explains an implicit finding of Arbatskaya et al. (2006), namely that in markets where firms do offer PMGs, there are likely to be other firms that do not. An equilibrium where all firms offer PMGs does not exist.

Third, to understand the proper effect of PMGs empirically, our paper suggest that one should not just compare prices in stores with and without PMGs, but one also should inquire whether the prices in stores without PMGs are shifted upwards. We show that in the equilibrium where PMGs are offered, even the firms that do not offer them charge higher expected prices than in the equilibrium where no PMGs are offered. Despite the fact that consumers can execute their PMG if they are informed of lower prices, consumers are strictly worse off when PMGs are offered as the price increasing effect on list prices dominates the fact that they may buy at a lower price. Moreover, the more consumers communicate with each other (the more dense their network is), the higher the equilibrium prices in the equilibrium where PMGs are offered with positive probability and the higher the prices consumers expect to pay even taking the probability into account that consumers can execute the PMG. That is, consumers are being "punished" for being better informed through friends.

Finally, we have also some results on asymmetric equilibria. Asymmetric equilibria are inherently difficult to characterize as consumers' reservation prices are nonstationary. We show that in case of duopoly asymmetric equilibria do not exist, while with three firms they do exist. In the asymmetric equilibrium we characterize one firm does not have PMGs, one firm offers PMGs for sure and one firm randomizes between offering and not offering PMGs. The three qualitative symmetric equilibrium properties mentioned above, continue to hold in this asymmetric equilibrium. Interestingly, our numerical analysis shows that for given exogenous parameters, multiple asymmetric equilibria exist and that profits can be as much as ten times higher in an equilibrium where some firms offer PMGs compared to the equilibrium without PMGs.

There is now a reasonably large literature on the effect of low price guarantees on the (pricing) behavior of competitors. As said before, the main conclusion that arises from this literature is that PMGs are anticompetitive, as they are in the environment we study. One argument that has been made (cf., Salop (1986)) is that PMGs facilitate collusion as they remove the incentives to undercut when firms engage a market interaction. PMGs, so it is argued, do not just contain information for consumers, but in fact convey the information to competitors that any attempt to undercut will be automatically followed, i.e., PMGs work as a trigger strategy that helps firms to collude. Moreover, PMGs are an extremely cheap way of doing so as firms do not have to spend any resources on monitoring competitor's behaviour. Although some PMGs take the form that firms ex ante commit to change their list prices if they are informed that a competitor has a lower price (see above), most PMGs restrict the PMG to the client that has informed the firm of a lower price elsewhere, i.e., list prices are unaffected. This means that most PMGs actually are dissimilar to trigger strategies and it is therefore unclear whether they really support collusive practices.

Png and Hirschleifer (1987) argue that PMGs are an effective way to price discriminate between shoppers and non-shoppers. In the absence of PMGs, the activity of shoppers forces firms to reduce prices market- wide. Shoppers provide a positive externality to non-shoppers and force firms to set more competitive prices. With PMGs, however, the effect of the disciplining power of shoppers is limited to these shoppers themselves according to Png and Hirschleifer and act as a price discrimination mechanism for firms that can set high list prices and provide shoppers with discounts (see, also, Edlin (1997)). Png and Hirschleifer do not model the search behavior of non-shoppers explicitly, however, and they do not consider the possibility that nonshoppers are informed about prices via friends.[3]

Recently, Moorthy and Winter (2006) have argued that PMGs may actually have a pro-competitive effect in case products are horizontally differentiated and firms have different production costs. In such a context PMGs may signal to consumers that the firm under consideration really has a lower price. The lower price that is charged generates sufficient additional demand to compensate the firm for the lower profit per unit. High cost firms may find it too expensive to imitate the low pricing behavior of low cost firms, thereby allowing PMGs to work as a signalling device. Moorthy and Winter's model nicely illustrates how PMGs may work in markets with product heterogeneity. Most PMGs clauses, however, stipulate that the guarantee only comes into effect if prices of identical products at nearby shops are compared. This means that Moorthy and Winter's analysis is restricted to markets where geographical differentiation is important and transportation costs are high.

We study the incentives firms have to set PMGs and the impact they have on prices and consumer welfare in an environment where prices and PMGs are chosen simultaneously. This is also the set-up of other theoretical contributions studying the effects of low price guarantees such as Corts (1995, 1997) and Kaplan (2000). In this set-up, the shoppers are fully informed about prices and whether or not firms offer low price guarantees, while the nonshoppers are uninformed about all aspects of the firms' strategies until they arrive at their shop. There are two different ways to interpret this environment. First, one can think of firms advertising prices and whether they have PMGs or not. In this interpretation, the advertisement only reaches a certain fraction of the consumers (the shoppers); cf., Varian (1980). This setting where information about PMGs is advertised simultaneously with prices fits major consumer markets, where PMGs are advertised.[4] Second, one may also think of a very different environment where PMGs and prices are not announced at all. Shoppers shop around and buy at the lowest price and non-shoppers visit a store and only discover there whether or not a firm has a PMG and what price it charges. This interpretation best fits markets where firms (e.g., supermarkets) often put a label "low price guarantee" on some of their price labels, but not on their whole assortment. Moreover, at different points in time these firms have different products to which the PMG applies.

There is a recent paper by Yankelevich (2010) that also studies the search

[3]Chen et al. (2001) also show that price matching policies may have pro-competitive effects in case they are pre-announced and there are consumers who prefer to shop at a particular store but are mindful of saving opportunities. In this paper the "search" behaviour of consumers is also exogenously given as in Png and Hirschleifer (1987) and Varian (1980).

[4]For example, Dixons is an electronic store in the Netherlands, which provides price-matching guarantees. It advertises these guarantees together with the prices using post. However, a lot of people either do not get the advertisement because they opted not to get ads to their post-box, or because they do simply do not read ads coming to their house. This category of people will realize that Dixons offers PMGs only when they enter the shop and look at price labels."

theoretic implications of PMGs. Yankelevich studies a different environment where firms first advertise PMGs before they engage in price setting. This implies that the rival firm and all consumers are aware of which firm has a PMG and which firm does not and consumers may therefore direct their search activity to firms that do or do not offer a PMG. The main effect of PMGs in his model comes from the fact shoppers are also assumed to search sequentially and that some of them prefer to go back to a previously visited firm if it has a lower price, while others prefer to activate their PMG. Thus, in the model of Yankelevich (2010) a firm may still sell to shoppers even if it does not have the lowest price. He obtains multiple (asymmetric) equilibria in the pricing subgames, which complicates his analysis considerably. Some of his numerical results show that at least one firm charges PMGs in every industry.[5]

The structure of the paper is as follows. In the next section we present the setup of the model. Section 3 contains the analysis of symmetric equilibria, while Section 4 contains the welfare analysis of these equilibria. Section 5 provides some results on asymmetric equilibria. Section 6 concludes with a discussion that includes the possible roles of PBGs in this framework. Formal proofs can be found in the appendix.

2 The Model

Consider a market where N firms produce a homogeneous good and have identical production costs, which we normalize to zero. Firms set prices and decide whether or not to provide price matching guarantees (PMGs). By providing a PMG, a firm commits to compensate the difference between its price and the price of a competitor, if the consumer who has bought the product from the firm provides evidence that a lower price exists.

Like in the model of Stahl (1989) there are two types of consumers. A fraction $\lambda \in (0,1)$ of all consumers are "shoppers", i.e. these consumers like shopping or have zero search costs for other reasons. We assume that these consumers know all prices in the market as well as whether some of the firms set PMGs. The remaining fraction $1-\lambda$ of consumers is uninformed. These consumers engage in sequential search and get their first price quotation for free, but a subsequent price quotation comes at a search cost c.[6] We assume perfect recall. All consumers have identical valuation for the good denoted by v and $v > c$. We assume that v is non-binding in the model, i.e. it is sufficiently large not to influence the consumers' decisions. Whether a firm provides PMGs or not is revealed simultaneously with observing the price quotation of that firm. After the consumer has bought the

[5] An early paper by Lin (1988) investigated an unorthodox consumer search model with price-matching guarantees with three distinctive features: search costs are increasing (to infinity on the third round), there are two types of consumers with different valuations of a good and there are three types of firms which differ in marginal costs.

[6] We follow here the standard assumption in the literature on consumer search. An alternative specification where consumers also have to pay for obtaining a first price quote has been analyzed by Janssen et al. (2005)

good there is an exogenous probability $\mu \in (0,1)$ that she observes (costlessly) the price of one randomly chosen other firm that is different from the currently visited firm, but that may have been observed already at a previously visited firm.[7] This information can come either from friends (as in Galeotti (2010)) or just accidentally because she noticed the price in another store. For many fashionable electronic products such as the i-Pad or the i-Phone people do talk to each other quite a bit about where they bought the product and for which price.

The timing in the model is as follows. First, firms simultaneously decide on their prices and whether to provide PMGs. Firm i decides to set PMGs with probability α^i, and then sets prices with a probability distribution $F_0^i(p)$ if it chooses not to offer a PMG, and with $F_1^i(p)$ if it provides a PMG. Thus, the strategy of firm i is a tuple $\{\alpha^i, F_0^i(p), F_1^i(p)\}$. We denote by $\underline{p}_j$ and $\overline{p}_j$ the lower and upper bounds of $F_j(p)$, $j = 0, 1$ and $\underline{p} = \min\{\underline{p}_0, \underline{p}_1\}$. Moreover, we define $F(p) = (1-\alpha)F_0(p) + \alpha F_1(p)$ to be the weighted average of the two equilibrium price distributions. Second, consumers decide after the outcome of the possible mixed strategies has been realized. Shoppers choose to buy at the lowest price in the market,[8] while uninformed consumers first randomly go to one of the shops and then, after observing their first price (and possibly PMG offer), optimally decide whether to buy at that firm or to continue to search. If the consumer continues to search, she observes a second price and again optimally decides whether to buy or to continue to search. At each stage t of the search process, the optimal behavior of consumers is characterized by two reservation prices: one in case the consumer happens to visit a store with a PMG, denoted by $r_1(t)$ and one in case the store does not offer PMGs, denoted by $r_0(t)$. Note that these reservation prices need not be stationary. For easy reference, we denote the reservation prices at the first stage simply by r_1 and r_0. Importantly, as the uninformed consumers do not know whether a firm has a PMG or not before searching, they cannot condition their search behaviour on this information. After all purchasing decisions have been made, there is a probability μ of being informed about one price quotation of a firm the consumer did not buy from. If this price is smaller than the purchase price, and the purchase was made in a firm providing a PMG, the consumer obtains the price difference back from the firm at which she has bought the product.

An equilibrium in this game is a set of strategies for the firms and consumers,

[7] Obviously in the absence of recall costs, consumers do not want to engage in costly search after the purchase (and before hearing from a friend). This action is weakly dominated by continue to search before purchasing. Formally, if a consumer would have the option to continue searching after she knows that she will not hear from a friend anymore, then she may want to do so if the price quote is relatively high and she would have this option. We assume, however, that such a moment never arises. This is a realistic approximation of reality where one never knows exactly when information arrives and whether one's own search activities result in a new price quote.

[8] In principle, if one of the firms charges a price below its competitor, while the competitor offers a PMG, shoppers are indifferent between buying from any of the two firms. We take as a tie-breaking rule that shoppers buy at the firm with the lowest price. One can rationalize this assumption by considering there is an infinitely small cost of claiming PMGs.

such that consumers' search rule is sequentially rational given the strategies of the firms and the strategies of the firms are optimal given the strategies of the other firms and of consumers.

3 Analysis of Symmetric Equilibria

We start our analysis by demonstrating the pivotal role of r_0 in the prices that can be set by firms offering PMGs and by those firms that do not. As our first Proposition shows, prices below r_0 will be set only by firms not offering a PMG, while prices above r_0, if at all, will only be set by firms offering PMGs. This result provides a first explanation for the empirical evidence provided by Arbatskaya et al. (2006) showing that firms offering PMGs tend to set higher prices than those that do not. To prove this Proposition we first need, however, to prove a few auxiliary results.

First, the reservation prices $r_1(t)$ and $r_0(t)$ are monotonic in the sense that if consumers are indifferent to buy at a certain price in search round t, then they weakly prefer to buy at the same price in the next search round $t+1$.

Lemma 3.1. *For both types of reservation prices* $r_i(t), i = 0, 1,$*in a symmetric equilibrium* $r_i(t-1) \leq r_i(t)$, *for all* $t \leq N-1$.

Note that certainly in the last search round (i.e., when there is still one firm's price and PMG behaviour that is not observed), the reservation prices $r_i(N-1)$ are strictly larger than those in the previous round $r_i(N-2)$. This follows from the fact that if a consumer continues to search after round $N-1$ he effectively knows all the prices and it does not matter anymore whether the last firm offers a PMG or not. This is not true for search round $N-2$ as there if a consumer continues to search he may then stop at a firm offering PMG in the hope he will still be informed about the price of the last firm through her communications with friends.

The next result then says that firms will never charge prices above the reservation prices in the first search round. The argument underlying this lemma is very similar in nature to the argument in Stahl (1989) that no one searches beyond the first firm.

Lemma 3.2. *Whether or not a firm offers a PMG, it will always choose a price that is immediately accepted by uninformed consumers, i.e.,* $\overline{p}_i \leq r_i, \ i = 0, 1.$

Finally, it is clear that in a symmetric equilibrium the reservation prices satisfy $r_1 > r_0$ as a firm offering a PMG, not only offers the product for a certain price, but also offers the option of receiving a lower price if the consumer is later informed that a lower price exists in the market. As in a symmetric equilibrium consumers do not learn anything from the current observation about the likelihood of firms offering PMGs along their future search path, the only relevant difference between a firm offering PMGs and a firm that does not, is that the former offers an additional option value. The lemma is therefore stated without formal proof.

Lemma 3.3. *In any symmetric equilibrium the reservation prices satisfy* $r_1 > r_0$.

We are now then ready to state and prove, the first important result of this paper.

Proposition 3.4. *(i) In any symmetric equilibrium, any price below* r_0 *will not be set by a firm offering PMGs. (ii) Moreover, in any symmetric equilibrium a price* $r_1 \geq p > r_0$ *will only be offered by a firm offering PMGs.*

The Proposition exploits the following basic facts. First, whether or not a firm offers PMGs does not effect the behaviour of shoppers: they simply go to the firm with the lowest price in the market. The success rate of attracting these consumers is thus only dependent on the price a firm sets. Two firms with the same price, one offering PMGs, while the other does not, have the same probability of attracting the shoppers. Second, the uninformed consumers that come to the firm under consideration will always buy at prices below r_0. The only difference offering PMGs makes is that these consumers may effectively pay a lower price if the firm offers a PMG. As with PMGs the firm has effectively a lower margin, while demand remains the same, it will never choose to combine these low prices with offering PMGs.

The second part of the Proposition shows the reverse, namely that prices above r_0 will not be charged by a firm without offering PMGs as uninformed consumers will continue to search, while they will buy in the hope of being informed of a lower price later, in case the firm offers a PMG. Thus, given a price above r_0 a PMG will lead to higher demand, although possibly yielding a lower revenue per unit on these additional sales as the consumers may execute their PMG. As only the uninformed consumers may pay at lower prices later (as the informed consumers anyway only buy if the firm has the lowest price in the market), PMGs yield higher profits at prices larger than r_0.

It immediately follows that an equilibrium where all firms offer PMGs for sure cannot exist:

Corollary 3.5. *An equilibrium where all firms choose PMGs does not exist.*

The idea behind this Corollary is the same as before. From the definition of r_0 it follows that some prices that are charged in the equilibrium are smaller than r_0. If all firms charge PMGs, it thus must be that some of the prices these firms charge with positive probability are below r_0. The previous Proposition then says that a firm can profitably deviate by setting a price equal to r_0, but cancel the PMG. Thus, any individual firm can free ride on the PMGs offered by the other firms.

To further explain the empirical evidence offered by Arbatskaya et al. (2006) we need to characterize the possible equilibria further and show that indeed there exist equilibria where some firms may offer PMGs and others that do not. Two types of these equilibria come to mind. There may exist asymmetric equilibria according to which some firms offer PMGs for sure, while other do not. In these equilibria, firms may still randomize their pricing decisions as is common in

Stahl type search models. On the other hand, there may be symmetric equilibria where firms choose to offer PMGs with some positive probability strictly smaller than one. In the latter case, we also do observe with strictly positive probability markets where both behaviours co-exist. In this section we will focus on the second type of equilibrium, while the Section 5 discusses asymmetric equilibria.

Unfortunately, symmetric mixed strategy equilibria are difficult to characterize for general N as we will show and therefore we only characterize the equilibrium for the case where $N = 2$. To do that, we start by investigating the optimal search behaviour of uninformed consumers.

Lemma 3.6. *Consider the case where $N = 2$. Uninformed consumers accept all prices at or below r_0 at a firm that does not provide a PMG, and continue to search otherwise; they accept all the prices at or below r_1 at a firm with a PMG, and continue to search otherwise, where r_0 and r_1 are defined by*

$$\begin{aligned} \int_{\underline{p}}^{r_0} F(p)dp &= c \\ \int_{\underline{p}}^{r_1} F(p)dp &= \frac{c}{1-\mu} \end{aligned} \tag{1}$$

We already know that $r_1 > r_0$, i.e., a consumer is willing to buy at a higher price if the firm happens to provide a PMG. Now, we clearly see, however, that if μ is close to one, consumers visiting a firm with a PMG clause prefer to stop searching in the PMG store, even if she observed a very high price. This is because she almost surely pays the minimum of the two prices in the market, while if she decides to proceed to search then she has to pay c and again buys at the minimum of the two prices that are set. Thus, for high values of μ the option of continuing to search after visiting a PMG store is not attractive.

Note also that the characterization of both reservation prices r_0 and r_1 in Lemma 3.6 is not valid for general N, but for different reasons. To see that first consider the continuation pay-off of continuing to search if r_0 was observed in a firm not offering PMGs. If the consumer continues to search and now finds a slightly higher price at a firm offering PMGs, she may rationally decide to buy from that firm in the hope that she will later be informed about a third price. The continuation pay-off depends then on the probabilities that such slightly higher prices will be observed. With $N = 2$ if a consumer decides to continue to search, she will always want to buy at the lowest price as she will be completely informed after having performed another search. Thus, it is obvious that the characterization of the reservation price r_0 for the case $N > 2$ is affected in a nontrivial way by the possibility of being informed about yet another price after having bought at a firm offering a PMG. Next consider the continuation pay-off of continuing to search if r_1 was observed in a firm offering PMGs. If the consumer continues to search and finds a lower price at another firm offering PMGs, she not only gets the lower price for sure, but with $N > 2$ may still be informed about yet another price which is still lower. Moreover, if she would observe in the second search round a lower price in a firm not offering PMGs, she may now decide not to buy from that firm in the hope she will be informed through friends about lower prices if she stays with the firm offering PMGs. Thus, the continuation pay-off depends again on the probabilities that these events happen and the prices observed in these cases. This complicates the characterization of r_1 considerably in case $N > 2$ and we will not analyze this further in any depth.

Given the characterization of the reservation prices when $N = 2$, the following Proposition shows that indeed, for certain parameter values, a symmetric equilibrium exists where under duopoly some firms may offer PMGs, while others do not.

Proposition 3.7. *Consider $N = 2$. An equilibrium where firms offer PMGs with a strictly positive probability that is smaller than one, i.e. where $\alpha \in (0, 1)$, exists if and only if*

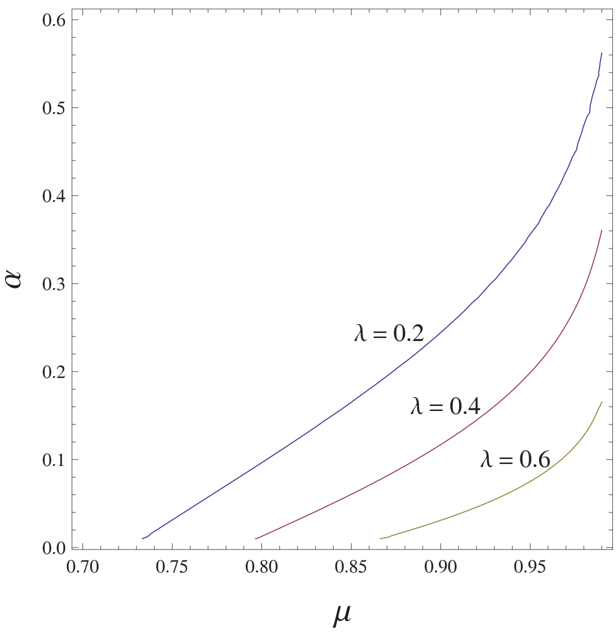
$$1 > \mu > \frac{4\lambda^2}{(1-\lambda)^2 \ln \frac{1-\lambda}{1+\lambda} + 2\lambda(1+\lambda)} > \frac{2}{3} \qquad (2)$$

This result may seem to be somewhat counterintuitive: firms offers (with some probability) PMGs only if there is a sufficiently large probability that consumers would exercise them. The explanation, of course, is that if μ is sufficiently large, consumers would accept much higher prices at the store offering PMGs. This increase in prices set by firms offering PMGs, more than offsets the adverse effect of consumers exercising PMGs on firms' profits.

Figure 1 represents the relationship between the equilibrium probability of observing PMGs and the remaining exogenous parameters of the model in case $N = 2$. Equation (2) shows that the probability μ of being informed about the price set by the other firm should be relatively large. Figure 1 depicts the relation between the equilibrium probability of firms offering PMGs and the probability with which consumers observe another price quotation. The figure shows that this relationship is positive: α is increasing with μ. Although high values of μ imply that *ex post* most of the consumers are informed, uninformed consumers are willing to buy at higher prices for higher μ. If μ is close to one, consumers are willing to accept virtually any price. Therefore firms are more likely to set PMGs when μ is large. Not surprisingly, the larger the fraction λ of shoppers, the lower the probability with which firms offer PMGs. The precise numbers of the parameters for which the equilibrium does exist should not be taken too literally as for one, these parameter values are only valid for the case of duopoly, and second, the model assumes that there are no hassle costs of exercising PMGs. There is nothing in our equilibrium analysis for the case of $N = 2$ that makes that this type of symmetric equilibrium does not exist when $N > 2$. In Section 5 we show that for $N = 3$ asymmetric equilibria exist for much lower values of μ.

The Propositions presented so far show that in our model, for certain parameter values an equilibrium exists where some firms may choose to offer PMGs, while others do not. Moreover in this equilibrium the firms offering PMGs set higher prices. These equilibrium fails to exist, however, when μ is close to 0 as

Figure 1: How equilibrium probability of PMG depends on μ



in that case the option value the PMG represents is not high. The question thus arises whether other equilibria exist, and if so, how they can be characterized.

From the discussion above, one may get the impression that for low values of μ an equilibrium exists where none of the firms offer PMGs as when μ is small the option value of a PMG for consumers is small and firms cannot charge much higher prices by offering them. The next Proposition shows that this is indeed the case. Moreover, the Stahl type of equilibrium that emerges always exists and is thus robust to firms having the option of offering PMGs.

Proposition 3.8. *For all values of the parameters an equilibrium exists where all firms choose $\alpha = 0$. The equilibrium price distribution in this case is*

$$F_0(p) = 1 - \left(\frac{1-\lambda}{\lambda N}\frac{r_0 - p}{p}\right)^{\frac{1}{N-1}}, \quad p \in [\frac{1-\lambda}{1+\lambda(N-1)}r_0, r_0] \tag{3}$$

where r_0 is defined as $\int_{\underline{p}}^{r_0} F_0(p)dp = c$.

This equilibrium is identical to the equilibrium found in Stahl (1989). The equilibrium price distribution balances the fact that at higher prices, a firm has higher margins, but lower demand. The proof (in the Appendix) is, however, somewhat more involved as it still has to checked that a firm does not find it optimal to deviate and offer a PMG, while charging higher prices.

4 Welfare Analysis of Symmetric Equilibria

In the previous Section we have shown that there is a region of the parameter space where multiple symmetric equilibria exist: the Stahl equilibrium where firms do not offer PMGs and an equilibrium where some firms offer PMGs with a strictly positive probability and charge higher prices when they do offer PMGs. For this region where multiple equilibria exist, the following Proposition compares expected prices paid by consumers and expected profits by firms across the two equilibria.

Proposition 4.1. *Expected profits for firms in the equilibrium where PMGs are offered with positive probability are higher than the expected profits in the equilibrium without PMGs. As a consequence, in the equilibrium where PMGs are offered with positive probability consumers pay higher expected prices (after a possible execution of their PMG) than in the equilibrium without PMGs.*

Proposition 4.1 shows the “anticompetitive” effect of PMGs in a search environment in the sense that in the equilibrium with PMGs the expected price is higher than in the equilibrium where PMGs are not offered. The source of the anticompetitive effect is, however, different from that so far studied in the literature. It is not the case here that there is some type of collusive behaviour between the firms where PMGs play the role of a monitoring device and list prices will be automatically adjusted downwards if firms undercut a rival with a PMG. In our case the result is fully driven by consumers’ search behaviour, namely by the willingness of consumers to accept higher prices when firms do offer PMGs. Another interesting observation is that the higher expected price paid in the equilibrium with PMGs comes from two sources. The *direct* effect is that a firm charging a PMG can set a higher prices on average because of the higher reservation price at firms offering a PMG. The *indirect* effect shows that firms without PMG react to these *possibly* higher prices by setting higher prices themselves. In other words, in the equilibrium where PMGs are played with positive probability, but where the realization is such that none of the firms actually do offer them, the expected prices are still higher than in the equilibrium where PMGs are not offered at all (described in Proposition 3.8). Table 1 shows how a firm’s expected profit depends on μ in the equilibrium where PMGs are offered with some probability and the equilibrium profit without PMGs. In the latter case, profits are, of course, a constant, whereas they are exponentially increasing in μ whenever this equilibrium exists. In this sense, consumers are ”punished” for being better informed.

5 Asymmetric Equilibria

We next consider the possible existence of asymmetric equilibria where some firms offer PMGs for sure and other firms do not offer them at all. Asymmetric equilibria are difficult to analyze in full as the search behaviour of consumers is

Table 1: Total industry profit depending on λ and μ ($c = 1$)

μ	0.75	0.85	0.95	Stahl
$\lambda = 0.05$	37.1	55.7	113.9	29.0
$\lambda = 0.10$	16.9	25.1	49.8	13.9
$\lambda = 0.20$	6.9	10.1	18.9	6.3

non-stationary and reservation prices can be non-monotone. To see that reservation prices are non-monotone consider a potential asymmetric equilibrium where k out of N firms do not offer PMGs and the remaining firms do and that the k firms without PMG have lower prices than the firms offering PMGs. Suppose that somehow we are able to define a first round reservation price r_1 at a firm offering PMGs. To characterize this reservation price we should know the pay-off the consumer receives if she continues to search. However, after observing the price at a firm offering PMGs, the consumer knows (given the equilibrium strategies of the firms) that out of the remaining $N-1$ firms k do have lower prices, so that the probability of observing a lower price is larger than before visiting the first firm. Thus, if the consumer was indifferent between buying and continuing to search in the first search round after observing r_1, she would definitely prefer to continue searching if she would observe again r_1 at a firm offering PMGs at the second search round. Thus, the lemmas characterizing the reservation prices at the beginning of Section 3 fail to hold in case of asymmetric equilibria.

Despite these complications, we can show that asymmetric equilibria do not exist for the case where $N = 2$, while we can characterize an asymmetric equilibrium for $N = 3$ even for low values of μ. Let us first consider the case where $N = 2$. If an asymmetric equilibrium would exist in this case, then it would be true that one firm offers a PMG and the other does not. Taken together, the following observations demonstrate that an asymmetric equilibrium in case $N = 2$ does not exist. First, it should be the case that after observing the reservation price r_0 the consumer is indifferent between buying and continuing to search the PMG firm, implying that the firm offering a PMG should set prices strictly below r_0 with some strictly positive probability. But like in the case of symmetric equilibria, the firm offering PMGs would be better off dropping the PMG clause and setting the same price. This would not affect the chances of attracting consumers, but it effectively means selling only at the list price, rather than at a lower effective expected price. If the firm offering a PMG would not set prices below r_0, there cannot be such a reservation price. Prices equal to the willingness to pay v can also not be an equilibrium, as firms would then have an incentive to undercut, because of the presence of shoppers. Thus, we can conclude the following.

Proposition 5.1. *There does not exist an asymmetric equilibrium for $N = 2$ where one firm offers PMGs and the other does not.*

We next characterize an asymmetric with three firms, where firm 1 does not offer PMG, firm 2 offers PMG with probability α and firm 3 offers PMG for sure. As firm 1 and 2 have to randomize over the same interval $[\underline{p}_0, r_0)$, it is clear firm 1 has to have a mass point at r_0 (with mass α). Moreover, as there will always be a firm charging prices below r_0 (and shoppers buy at the lowest prices, rather than from a firm offering PMG) it must be the case that the firms with PMGs charge a pure strategy price at r_1 (as they only sell to nonshoppers).

The equilibrium construction then requires to specify r_0, r_1 and α. If we can find values for these variables that satisfy the relevant restrictions, then such an asymmetric equilibrium exists. Let us therefore consider the rules determining the different variables. First, consider r_0. It follows from Bayesian updating of beliefs that upon observing r_0 a consumer "knows" (believes with probability 1) he is visiting firm 1. To determine r_0 we also need to determine the continuation pay-off of searching. To this end, let us hypothesize the consumer stops searching after he finds a price r_1 at that stage of the search process. We will now check under which conditions this is indeed the optimal search rule. If the consumer stops (S) and buys at a firm offering a PMG, then he receives a pay-off of[9]

$$V_S = (1-\mu)r_0 + \mu\left(\frac{r_0}{2} + \frac{1}{2}\left(\frac{\alpha r_0}{1+\alpha} + \frac{(1-\alpha)\mathbb{E}p + \alpha r_0}{1+\alpha}\right)\right),$$

where $\mathbb{E}p$ is a short-cut notation for the expected price conditional upon it being smaller than r_0. The expression can be understood as follows. With probability $1-\mu$ the consumer does not receive another price quote from a friend and then can buy at a price r_0, while if he receives a price quote from a friend, it is either (with probability half) from the firm he already has visited himself (and offered r_0) or it is from the unvisited firm. In the latter case, there is a probability $\alpha/(1+\alpha)$ that the consumer is at firm 2 (in which case he observes r_1 for sure upon continuing to search) and there is a probability $1/(1+\alpha)$ that the consumer is at firm 3 (in which case if he continues searching observes a price r_1 with probability α (in which case the consumer buys at r_0) and a random price below r_0 with probability $1-\alpha$).

Following the same logic, if the consumer continues (C) to search after first observing r_0 and then r_1 his expected payoff is

$$V_C = c + \frac{\alpha r_0}{1+\alpha} + \frac{(1-\alpha)\mathbb{E}p + \alpha r_0}{1+\alpha}.$$

Thus, it is optimal to stop if

$$r_0 \geq \mathbb{E}p + \frac{2(1+\alpha)c}{(1-\alpha)(2-\mu)}. \tag{4}$$

In what follows, we consider that this is the case and check for which parameter values this condition holds. The equilibrium construction then proceeds as

[9]Note that after first observing a price r_0 and then observing a PMG firm offering r_1 the consumer is always certain not to pay more than r_0.

follows. First, using (4) and the conditions defining the optimal stopping rules, we derive r_0 and r_1 as functions of α, expected price and expected minimum price. Second, we use the equal profit condition for mixed strategy equilibria to substitute away the reservation prices and construct a relation between α, expected price and expected minimum price that have to hold in equilibrium. Third, we employ the distribution functions and expression for r_0 to derive expressions for the expected price and the expected minimum price. Fourth, we substitute these expressions into the condition we derived in the second step and solve for α. Finally, we check that (4) is satisfied.

If it is optimal to stop after observing r_0 and then r_1 then the reservation price is defined by the following expression:

$$r_0 = c + \frac{1}{2}(1-\alpha)\mathbb{E}p + \frac{1}{2}(1+\alpha)V_S,$$

i.e., after observing r_0 the consumer "knows" it is at firm 1 and therefore, if he continues to search, there is a probability of $\frac{1}{2}(1-\alpha)$ that he encounters firm 2 and a lower price there and with the remaining probability he encounters a price r_1 and then gets the pay-off of stopping there. This simplifies to

$$r_0 = \mathbb{E}p + \frac{4c}{(1-\alpha)(2+\mu)} \tag{5}$$

It immediately follows that (4) is satisfied if and only if

$$\alpha < \frac{2-3\mu}{2+\mu} \tag{6}$$

which implies that a necessary condition is that $\mu < \frac{2}{3}$. Note that this condition is independent of λ and c.

Now we proceed by identifying r_1. If the consumer stops after observing r_1 then his expected payoff is

$$\begin{aligned} W_S = \quad & r_1(1-\mu) + \frac{\alpha\left(\frac{r_1}{2} + \frac{1}{2}(\mathbb{E}p(1-\alpha) + r_0\alpha)\right)\mu}{1+\alpha} \\ + \quad & \frac{\left(\frac{1}{2}(\mathbb{E}p(1-\alpha) + r_0\alpha) + \frac{1}{2}(\mathbb{E}p(1-\alpha) + r_1\alpha)\right)\mu}{1+\alpha} \end{aligned}$$

where the first term is for the case when no other price will be observed, and the latter two terms for the cases when the consumer will be informed about some other price. The second term represents the case where the consumer believes he is at firm 2 offering a PMG. In that case there is an equal probability he is informed about a price set by firm 1 or firm 3. The third terms reflects the updated belief of $1/(1+\alpha)$ the consumer is at firm 3 in which case he can be informed about prices at firm 1 and 2 with equal probability. Now if the consumer decides to continue to search at r_1 then the expected payoff equals

$$
\begin{aligned}
W_C = c + &\frac{\alpha\left(\frac{1}{2}(\mathbb{E}p(1-\alpha)+r_0\alpha)+\frac{1}{2}(s+\mathbb{E}p(1-\alpha)+r_0\alpha)\right)}{1+\alpha} \\
&+ \frac{1}{1+\alpha}\left(\frac{1}{2}(\alpha(s+\mathbb{E}p(1-\alpha)+r_0\alpha)+(1-\alpha)(\mathbb{E}p(1-\mu)\right. \\
&+ ((1-\alpha)\mathbb{E}\min(p_1,p_2)+\alpha\mathbb{E}p)\mu))+\frac{1}{2}((1-\alpha)(\mathbb{E}p(1-\mu) \\
&+ ((1-\alpha)\mathbb{E}\min(p_1,p_2)+\alpha\mathbb{E}p)\mu)+\alpha(r_0(1-\mu)+(\mathbb{E}p(1-\alpha)+r_0\alpha)\mu)))
\end{aligned}
$$

This value is the sum of (i) search costs, (ii) the pay-off the consumer is at firm 2 (in which case on the next search step he either faces firm 1 and stops, or faces firm 3 and then continues to search for firm 1), (iii) the pay-off in case the consumer on the first search step is in firm 3 (in which case if he observes firm 2 on the next search step one of two cases can happen: first, if this firm offers r_1 then search continues and, second, if this firm offers some price below r_0, then the consumer buys from a firm which sets PMG for sure, and then either claims this price, or, if he gets informed about a better price (with probability μ) claims that price. Roughly the same logic applies for the case when the consumer faces firm 1 at the second search round with the only difference that he immediately stops if he observes r_0).

Solving for $W_S = W_C$ and substituting the expression for r_0 as given by (5) yields the expression for r_1:

$$
\begin{aligned}
r_1 = &\frac{2\mathbb{E}p(1+\alpha-(2+(-2+\alpha)\alpha)\mu)}{2(1+\alpha-\mu)} + \\
&\frac{4s(1+\alpha)^2+2s(1-\alpha(3+2\alpha))\mu-2\mathbb{E}\min(p_1,p_2)(1-\alpha)^3\mu(2+\mu)}{2(1+\alpha-\mu)(1-\alpha)(2+\mu)}.
\end{aligned} \tag{7}
$$

This finishes the first step of our plan and we proceed with writing down the profits evaluated at r_0 and r_1:

$$
\pi(r_0) = \frac{1}{3}(1-\lambda)r_0 + \alpha\lambda r_0;
$$

$$
\pi(r_1) = \frac{1}{3}(1-\lambda)\left((1-\mu)r_1 + \left(\frac{r_1}{2}+\frac{1}{2}(1-\alpha)\mathbb{E}p+\frac{\alpha r_0}{2}\right)\mu\right).
$$

As firm 2 is indifferent between offering and not-offering PMGs, α is determined by equating these two profit expressions. By substituting the expressions for r_0 and r_1 as given in (5) and (7) and equating the profits we get a complicated expression linking expected price with the expected minimum of two prices and α.[10]

The fourth step is to compute $\mathbb{E}p$ and $\mathbb{E}\min(p_1,p_2)$ directly from the distribution function:

[10] Details are available upon request.

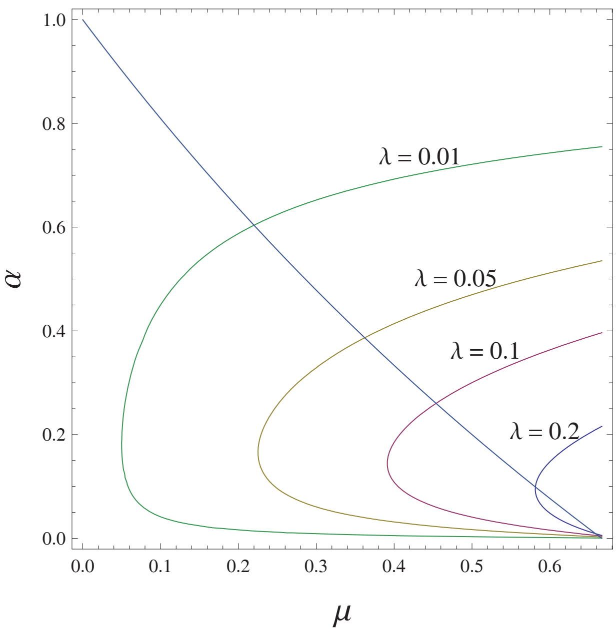
$$\mathbb{E}p = \int_{\underline{p}}^{r_0} pf(p)dp;$$

$$\mathbb{E}\min(p_1, p_2) = 2\left(\mathbb{E}p - \int_{\underline{p}}^{r_0} F(p)pf(p)dp\right).$$

Substituting into the complicated expression linking expected price, expected minimum of two prices and α gives an implicit expression for α which we denote in the form $\phi(\alpha, \lambda, \mu) = 0$.[11] Note that c plays the role of scaling parameter scaling up and down reservation prices and expected price, but not influencing the value of α. It is clear that it is too complicated to derive analytic expressions for α, and therefore we resort to a numerical analysis. Whenever condition (6) is satisfied, α represents the equilibrium mixing probability in the model.

The following graph illustrates our numerical findings.

Figure 2: How α depends on μ for $\lambda = 0.01,\ 0.05,\ 0.1,\ 0.2$.



The decreasing function in Figure 2 represents condition 6: only combinations (μ, α) below this curve satisfy the condition that the search rule is optimal. The four remaining curves represent solutions of the expression $\phi(\alpha, \lambda, \mu) = 0$ for different levels of λ. It is immediately clear that for some parameter values of λ and μ there exist two equilibria in the model: one with a high and one with a low value of α. Some of the values of α are presented in the table below. When in a cell of the table two values are printed below one another, then these two values represent the two different equilibrium values of α. Note that for a given value of

[11] The expression is too large to be presented here. The details are available upon request.

λ, the equilibria exist for relatively low (but, not too low) values of μ. Moreover, the lower the value of λ, the larger the rage of values of μ for which at least one such an asymmetric equilibrium exists. In Figure 2 we have depicted values of λ smaller than 0.2. This does not mean that this type of equilibrium does not exist for larger values of λ. However, the range of μ values for which this type of equilibrium exists for larger values of λ becomes very small.

Table 2: How α depends on λ and μ

μ	0.1	0.2	0.3	0.4	0.5	0.6
$\lambda = 0.01$	0.042	0.016	0.009	0.005	0.003	0.002
	0.450	0.588	–	–	–	–
$\lambda = 0.05$	–	–	0.061	0.032	0.017	0.007
	–	–	0.329	–	–	–
$\lambda = 0.10$	–	–	–	0.108	0.041	0.016
	–	–	–	0.185	–	–
$\lambda = 0.20$	–	–	–	–	–	0.048
	–	–	–	–	–	–

Tables 3 and 4 represent market outcomes. First, we present the reservation prices. Then, we compute industry profits, which is effectively the weighted price paid by all the consumers. The last column of each of the tables provides the outcomes of Stahl's model (or the equilibrium without PMGs being offered) for ease of comparison.

Table 3: Reservation prices $(r_0; r_1)$ depending on λ and μ $(c = 1)$

μ	0.1	0.2	0.3	0.4	0.5	0.6	Stahl
$\lambda = 0.01$	145; 146	128; 129	120; 121	114; 115	108; 109	104; 105	67.3
	776; 787	1760; 1795	–	–	–	–	
$\lambda = 0.05$	–	–	29.4; 30.1	25.7; 26.3	23.6; 24.2	22.0; 22.7	14.0
	–	–	82.0 ; 87.2	–	–	–	
$\lambda = 0.10$	–	–	–	17.3; 18.5	13.3; 14.1	11.8; 12.6	7.30
	–	–	–	22.9; 25.1	–	–	
$\lambda = 0.20$	–	–	–	–	–	7.10; 8.13	3.94
	–	–	–	–	–	–	

We can derive the following interesting observations from the tables. Firstly, reservation prices for firms with PMGs are higher than without. This finding is in line with our findings for the symmetric equilibrium. Secondly, reservation prices and industry profits are higher than in (the Stahl) equilibrium without PMGs. This is also in line with our results for the symmetric case. Thirdly, for some parameter values there are two equilibria and those with higher values of

Table 4: Total industry profit depending on λ and μ ($c = 1$)

μ	0.1	0.2	0.3	0.4	0.5	0.6	Stahl
$\lambda = 0.01$	144	127	119	113	108	103	66.7
	779	1773	–	–	–	–	
$\lambda = 0.05$	–	–	28.2	24.5	22.5	20.9	13.3
	–	–	81.9	–	–	–	
$\lambda = 0.10$	–	–	–	16.2	12.1	10.7	6.57
	–	–	–	21.9	–	–	
$\lambda = 0.20$	–	–	–	–	–	5.89	3.15
	–	–	–	–	–	–	

α are characterized by (much) higher prices. Profits and reservation prices are approximately two times higher in the asymmetric equilibrium with low values of α compared to the equilibrium without PMGs, whereas this ratio can go up to 10 and more for the equilibrium with a high value of α. In the high α equilibrium, it is more likely that firms offer PMGs and as in the analysis of symmetric equilibria this has a direct effect (PMG firms offer higher prices) and an indirect effect (the no PMG firm reacts by increasing prices-as the chance he has to compete for the shoppers decreases in α). Depending on which part of the α-μ curve we are at α (and prices) can be either increasing or decreasing function of μ. The latter is a new feature of asymmetric equilibria that does not arise with symmetric equilibria.

6 Discussion and Conclusion

This paper has analyzed the effect of firms offering PMGs in a consumer search model where reservation prices are endogenously determined. We have analyzed markets where prices and PMGs are chosen simultaneously and some consumers are informed, while others are uninformed about the strategies firms adopt until they arrive at their shop. We have shown that there are two types of equilibria in this model. In one type of equilibrium some firms do offer PMGs, while others do not, while in the second type of equilibrium no one firm offers PMGs. The first type of equilibrium is either symmetric in nature with all firms randomizing their decision whether or not to offer PMGs, or asymmetric in nature and then some firms do offer PMGs, while some others do not. In the first type of equilibrium, firms offering PMGs have higher list prices than those that do not and also the expected effective price they receive for their product is higher than in a firm without PMGs. Comparing across equilibria, we have shown that the expected price consumers pay at firms that do not offer PMGs are higher in the equilibrium where some firms may offer PMGs than in the equilibrium where no firm offers PMGs. Thus, PMGs soften price competition.

This paper has not considered the strategic effects of price beating strategies (PBGs). In principle, PBGs could be incorporated into our framework, and if we would do so, their strategic effect will be very different from those that we considered in this paper for PMGs. Allowing for PBGs, will yield that all equilibria have firms choosing PBGs with strictly positive probability and all firms choose price equal to the monopoly price. There is a continuum of these equilibria and they only differ in the fraction of firms setting PBGs, PMGs and no low price guarantees at all. This could partially explain why Arbatskaya et al. (2006) find that the strategic effects of PBGs are quite different from those of PMGs and that empirically it seems that firms offering PBGs tend to have not higher prices than other firms in the market. The main reason for this result, is that with arbitrarily low hassle costs, shoppers will have an incentive to buy at a price beating firm if it has higher prices as this gives them an effective purchase price lower than any of the list prices, and this makes that rival firms do not want to undercut a firm offering PBGs.

These results for the case firms can offer PBGs critically depend, however, on the assumption that there are no hassle costs. If hassle costs are non-negligible, shoppers would continue to buy at the lowest price firm even if another firm offers a PBG and has a somewhat higher price. This complicates the formal analysis considerably. Our results in case of PMGs given in the paper do not critically depend on the assumption of no hassle costs, as the shoppers in our model buy from the lowest price firm in the market independent of whether or not this firm offers a PMG. This will continue to be the case if we would allow for hassle costs. The above mentioned result for PBGs also critically depends on whether a PMG is written on the basis of observed prices or on the basis of effective purchasing prices (cf., Kaplan (2000)), an issue not covered in this paper. Also, another downside of offering a PBG is that it is a risky strategy in the sense that rival firms could willingly set price equal to marginal costs to force rival firms offering PBGs to sell below cost. Hassle costs a la Hviid and Shaffer (1999), PMGs based on list prices or effective purchasing prices and the risk of being forced by rival firms to price below cost should be considered in a full explanation of why firms do or do not offer PBGs. This is a topic beyond the scope of the current paper, but could be taken up in future research.

Appendix: Proofs

Lemma 3.1. *For both types of reservation prices* $r_i(t), i = 0, 1, r_i(t-1) \leq r_i(t)$, *for all* $t \leq N - 1$.

Proof. In a symmetric equilibrium, consumers cannot get information about the likelihood that other firms offer or do not offer PMGs from the fact that they happen to encounter a particular firm with or without PMGs now. Moreover, in search round t consumers can always imitate the optimal continuation search behaviour in search round $t + 1$ and therefore, the continuation cost of searching in round t should not be larger than the continuation cost of searching in round

$t + 1$. As the benefits of buying at a certain price are independent of the search round, it should be the case that reservation prices cannot be strictly lower in later search rounds. □

Lemma 3.2. *Whether or not a firm offers a PMG, it will always choose a price that is immediately accepted by uninformed consumers, i.e.,* $\overline{p}_i \leq r_i, i = 0, 1$.

Proof. First of all, note that in a symmetric equilibrium both $F_0(p)$ and $F_1(p)$ are atomless. If some would be charged with strictly positive probability, then a standard argument can be used to demonstrate that each firm would have an incentive to deviate and charge a slightly lower price instead.[12] The rest of the proof is by induction, starting at the last search round. It cannot be that $\overline{p}_i \leq r_i(N-1)$. If this were the case, then after observing this price, a consumer would never come back to that price and after all prices are observed, it has certainly observed a lower price and buys from a firm there.

Suppose then that we have shown that for a certain $t, \overline{p}_i \leq r_i(t+1)$. we will now show that it then also has to be the case that $\overline{p}_i \leq r_i(t)$. Suppose not. Then a firm charging $\overline{p}_0$ will not get any consumers before search t. A consumer who observes this price in search round t will continue to search and will either encounter a lower price p_0 from a firm not offering a PMG, or it will observe a price $p_1 \leq r_1(t+1)$ from a firm offering PMG and buy there. thus, Thus, this consumer will also not buy at this price. A similar argument holds true for a firm charging $\overline{p}_1$.

□

Proposition 3.4. *(i) In any symmetric equilibrium, any price below* r_0 *will not be set by a firm offering PMGs. (ii) Moreover, in any symmetric equilibrium a price* $r_1 \geq p > r_0$ *will only be offered by a firm offering PMGs.*

Proof. From the previous lemmas we know that $\overline{p}_i \leq r_i$ and that $r_1 > r_0$. To prove then the first part of the Proposition, consider then a firm with a price $\underline{p}_0 < p \leq r_0$ not offering PMGs. As in any equilibrium all prices set will be immediately accepted, its profits are given by

$$\left[(1 - F(p))^{N-1} + \frac{1-\lambda}{N}\right] p.$$

If the firm would offer an PMG while keeping the same price p, its profits will be equal to

$$(1 - F(p))^{N-1} p + \frac{1-\lambda}{N} \left[(1-\mu)p + \mu \int_{\underline{p}}^{p} q f(q) dq\right],$$

which is strictly lower than the expected profits without offering PMGs as by offering PMGs there is a strictly positive probability that the firm has to give a

[12] See, for example, Stahl (1989) for a detailed discussion.

lower effective price to the uninformed consumer. (The latter follows from the fact that no price is charged with strictly positive probability in any symmetric equilibrium and the fact that from the definition of r_0 it follows that in any equilibrium with some positive probability some prices have to be lower than r_0).

The second part of the proposition follows immediately from the lemmas as no firm that does not offer PMGs will set a price larger than r_0. □

Corollary 3.5. *An equilibrium where all firms choose PMGs does not exist.*

Proof. If all firms choose $\alpha = 1$ then each one of them has a profitable deviation by choosing $\alpha = 0$ and price at r_0 which (as $F(p) = F_1(p)$ in this case) is defined by $\int_{\underline{p}_1}^{r_0} F_1(p)dp = c$. Indeed, since it has to be the case that $\underline{p}_1 < r_0 < r_1$, r_0 lies in the support of $F_1(p)$ and we get

$$\pi(r_0) = \lambda(1 - F(r_0))r_0 + \frac{1-\lambda}{N}r_0 >$$
$$> \lambda(1 - F(r_0))r_0 + \frac{1-\lambda}{N}((1-\mu)r_0 + \mu\mathbb{E}(\min(p, r_0))) = \pi_1.$$

Therefore, there is a profitable deviation. □

Lemma 3.6. *Consider $N = 2$. Uninformed consumers accept all prices at or below r_0 at a firm that does not provide a PMG, and continue to search otherwise; they accept all the prices at or below r_1 at a firm with a PMG, and continue to search otherwise, where $\{r_0, r_1\}$ are defined by*

$$\begin{aligned} \int_{\underline{p}}^{r_0} F(p)dp &= c \\ \int_{\underline{p}}^{r_1} F(p)dp &= \frac{c}{1-\mu} \end{aligned} \tag{8}$$

Proof. After observing price r_0 at a firm without PMGs, a consumer has to be indifferent between buying now and continuing to search. If the consumer continues to search, she proceeds to the next firm. The next firm does not have an PMG with probability $1 - \alpha$, and in this case the consumer can choose the smallest price of r_0 and a random price p that is distributed according to F_0. A similar expression holds in case she continues to search and happens to visit a store with an PMG, which occurs with probability α. Therefore, the reservation price should satisfy the following equation:

$$\begin{aligned} r_0 &= c + (1-\alpha)\left(F_0(r_0)\mathbb{E}_0(p|p < r_0) + (1 - F_0(r_0))r_0\right) \\ &+ \alpha\left(F_1(r_0)\mathbb{E}_1(p|p < r_0) + (1 - F_1(r_0))r_0\right) \end{aligned}.$$

Using integration by parts, this expression can be simplified to the usual rule determining reservation prices,

$$\int_{\underline{p}}^{r_0} F(p)dp = c.$$

Now consider the case where the consumer finds herself at a shop that provides an PMG. In this case if she accepts the price there is a probability μ that later she observes another price. this new price is either set by a no-PMG firm (with probability $1-\alpha$) or from an PMG firm (with probability α). If she decides to continue searching, the situation is similar to the case described above. Therefore, the reservation price is defined by

$$\begin{aligned}(1-\mu)r_1 &+ \mu[(1-\alpha)\left(F_0(r_1)\mathbb{E}_0(p|p<r_1)+(1-F_0(r_1))r_1\right)\\ &+ \alpha\left(F_1(r_1)\mathbb{E}_1(p|p<r_1)+(1-F_1(r_1))r_1\right)] =\\ &= c+(1-\alpha)\left(F_0(r_1)\mathbb{E}_0(p|p<r_1)+(1-F_0(r_1))r_1\right)\\ &+ \alpha\left(F_1(r_1)\mathbb{E}_1(p|p<r_1)+(1-F_1(r_1))r_1\right)\end{aligned},$$

which, after integrating by parts, simplifies to

$$\int_{\underline{p}}^{r_1} F(p)dp = \frac{c}{1-\mu}.$$

□

Proposition 3.7. *Consider $N=2$. An equilibrium where firms offer PMGs with a strictly positive probability that is smaller than one, i.e. where $\alpha \in (0,1)$, exists if and only if*

$$1 > \mu > \frac{4\lambda^2}{(1-\lambda)^2 \ln\frac{1-\lambda}{1+\lambda} + 2\lambda(1+\lambda)} > \frac{2}{3} \qquad (9)$$

Proof. To prove the proposition we explicitly construct an equilibrium and then show that there is such a value of α that all the equilibrium conditions are satisfied.

Equilibrium price distribution support contains two parts: $[\underline{p}, r_0] \cup [\underline{p}_1, r_1]$.

The lower part of the support. The lower part of the support is defined by three equations:

$$\pi = \lambda(1-F(p))p + \frac{1-\lambda}{2}p$$

$$F(\underline{p}) = 0$$

$$F(r_0) = 1-\alpha.$$

These three equations allow us to write the relevant endogenous parameters as a function of (r_0, α) as

$$\pi(r_0,\alpha) = \frac{2\alpha\lambda+1-\lambda}{2}r_0 \qquad (10)$$

$$\underline{p}(r_0,\alpha) = \frac{2\alpha\lambda+1-\lambda}{1+\lambda}r_0$$

$$F(p; r_0, \alpha) = \frac{(1+\lambda)p - (1-\lambda+2\alpha\lambda)r_0}{2\lambda p}.$$

Combining this with the optimal search rule

$$\int_{\underline{p}}^{r_0} F(p)dp = c$$

gives an expression for r_0:

$$r_0 = \frac{2\lambda c}{2(1-\alpha)\lambda + (1-\lambda+2\alpha\lambda)\ln\left(\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}. \tag{11}$$

Thus, if the value of α is known the probability distribution on the lower part of the support is fully described.

The upper part of the support. Let us then consider the upper part. The profit function is here defined by:

$$\pi = \lambda(1-F(p))p + \frac{1-\lambda}{2}\left(p - \mu\int_{\underline{p}}^{p} F(q)dq\right). \tag{12}$$

Since $\int_{\underline{p}}^{r_1} F(p)dp = \frac{c}{1-\mu}$ we get

$$\pi = \frac{1-\lambda}{2}\left(r_1 - \frac{\mu c}{1-\mu}\right).$$

Similarly, since $\int_{\underline{p}}^{\underline{p}_1} F(p)dp = c + (1-\alpha)(\underline{p}_1 - r_0)$ we get

$$\pi = \alpha\lambda\underline{p}_1 + \frac{1-\lambda}{2}\left(\underline{p}_1 - \mu(c + (1-\alpha)(\underline{p}_1 - r_0))\right).$$

Now, using equations (10) and (11) we can get expressions for r_1 and $\underline{p}_1$ as functions of α.

$$r_1 = \frac{2\lambda(1-\lambda(1-(2-\mu)\alpha)+\alpha\mu) + (1-\lambda)(1-\lambda+2\alpha\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}}{(1-\lambda)(1-\mu)\left(2(1-\alpha)\lambda + (1-\lambda+2\alpha\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}c$$

and

$$\underline{p}_1 = \frac{(1-\lambda+2\alpha\lambda)\left(2\lambda + (1-\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}{(1-\lambda(1-(2-\mu)\alpha-\mu)-\mu(1-\alpha))\left(2(1-\alpha)\lambda + (1-\lambda+2\alpha\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}c.$$

Note, that $\underline{p}_1 \geq r_0$ since $\mu\int_{\underline{p}}^{\underline{p}_1} F(p)dp \geq 0$.

Determination of α. To determine the value of α we use the following approach. We solve for the probability distribution on the upper part of the support using differential equation (12). The solution requires the determination of a constant, say Q, using a boundary condition. as we have two boundary conditions, namely $F(\underline{p}_1) = 1-\alpha$ and $F(r_1) = 1$, this gives us two values of the constant Q_1 and Q_2. Since the solution must satisfy both boundary conditions, it should be that $Q_1 = Q_2$ which gives us the equation determining α. Note, that we do not calculate the optimal search integral here, since it is already incorporated in the determination of r_1[13].

We start with the following differential equation:

$$Ay(x) + Bxy'(x) + Cx + D = 0. \tag{13}$$

The solution of this equation is

$$y(x) = Qx^{A/B} - \frac{Cx}{A+B} - \frac{D}{A}.$$

Now, if we compare (13) with (12) we observe that it is the same equation with $x = p$, $y(x) = \int_{\underline{p}}^{p} F(p)dp$, $y'(x) = F(p)$, $A = -\frac{1-\lambda}{2}\mu$, $B = -\lambda$, $C = \frac{1+\lambda}{2}$, $D = \pi$.

Thus, the equilibrium price distribution is defined by

$$F(p) = \frac{1+\lambda}{2\lambda + (1-\lambda)\mu} - Q\frac{(1-\lambda)\mu}{2\lambda}p^{-\frac{2\lambda+(1-\lambda)\mu}{2\lambda}}$$

where Q is determined by the initial conditions $F(r_1) = 1$ and $F(\underline{p}_1) = 1-\alpha$. These two values Q_1 and Q_2 have to be equal.

$$\begin{aligned} Q_1 &= -2\lambda(1-\lambda)\mu\left(1 - \frac{1+\lambda}{2\lambda+(1-\lambda)\mu}\right) \\ &\left(\frac{c\left(2\lambda(1-\lambda(1-\alpha(2-\mu)-\mu)-\mu(1-\alpha)) - (1-\lambda)(1-(1-2\alpha)\lambda)\mu \ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}{(1-\lambda)(1-\mu)\left(2(1-\alpha)\lambda + (1-(1-2\alpha)\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}\right)^{1+\frac{(1-\lambda)\mu}{2\lambda}} \end{aligned}$$

$$\begin{aligned} Q_2 &= -2\lambda(1-\lambda)\mu\left(1 - \alpha - \frac{1+\lambda}{2\lambda+(1-\lambda)\mu}\right) \\ &\left(\frac{c(1-(1-2\alpha)\lambda)\left(2\lambda+(1-\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}{(1-\lambda(1-\alpha(2-\mu)-\mu)-\mu(1-\alpha))\left(2(1-\alpha)\lambda+(1-(1-2\alpha)\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}\right)^{1+\frac{(1-\lambda)\mu}{2\lambda}}. \end{aligned}$$

Equation $Q_1 = Q_2$[14] can be reduced to:

[13] Another, may be more natural approach, is to use just one boundary condition and then explicitly calculate the search integral to get the equation for r_0 and α, as we did for the lower part of the support. However this approach results in more analytical complications, so we use the one presented in the text.

[14] Note, that Q_1 is always greater than 0, so the distribution function is increasing

$$\left(\frac{(1-\lambda)(1-\mu)}{1-\lambda(1-\alpha(2-\mu)-\mu)-(1-\alpha)\mu}\right)^{\frac{(1-\lambda)\mu}{2\lambda}} =$$

$$\left(\frac{2\lambda(1-\lambda+2\alpha\lambda-\alpha(1+\lambda)\mu)+(1-\lambda)(1-\lambda+2\alpha\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}}{(1-\lambda+2\alpha\lambda)\left(2\lambda+(1-\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}\right)^{\frac{2\lambda+(1-\lambda)\mu}{2\lambda}} \tag{14}$$

First, we evaluate (14) at $\alpha = 0$. It is easy to see that both LHS and RHS take the values of 1 for all (λ, μ). Second, we claim that the LHS of equation (14) evaluated at $\alpha = 1$ is larger than the RHS. Indeed, after canceling some terms the equation can be rewritten as $\left(\frac{1-\lambda}{1+\lambda}\right)^{\frac{(1-\lambda)\mu}{2\lambda}} = (1-\mu)$. Thus, the LHS is increasing in λ and as $\lambda \to 0$ it goes to e^{μ} which is larger than $(1-\mu)$. Finally, we examine the behaviour both of LHS and RHS around $\alpha = 0$. Obviously, if LHS decreases faster than the RHS, there must be an intersection point at $\alpha \in (0, 1)$. The derivative of the LHS with respect to α evaluated at $\alpha = 0$ equals to $-\frac{\mu(\lambda(2-\mu)+\mu)}{2\lambda(1-\mu)}$. The derivative of RHS evaluated at $\alpha = 0$ equals to $-\frac{(1+\lambda)(\lambda(2-\mu)+\mu)\mu}{(1-\lambda)\left(2\lambda+(1-\lambda)\mu\ln\frac{1-\lambda}{1+\lambda}\right)}$. Solving

$$-\frac{\mu(\lambda(2-\mu)+\mu)}{2\lambda(1-\mu)} < -\frac{(1+\lambda)(\lambda(2-\mu)+\mu)\mu}{(1-\lambda)\left(2\lambda+(1-\lambda)\mu\ln\frac{1-\lambda}{1+\lambda}\right)}$$

gives $\mu \in (-\frac{2\lambda}{1-\lambda}, 0) \cup (\frac{4\lambda^2}{(1-\lambda)^2\ln\frac{1-\lambda}{1+\lambda}+2\lambda(1+\lambda)}, \infty)$. Given that μ is between 0 and 1 we get (2).

Now we show that $f(\lambda) \equiv \frac{4\lambda^2}{(1-\lambda)^2\ln\frac{1-\lambda}{1+\lambda}+2\lambda(1+\lambda)} > 2/3$. First, this expression is increasing in λ with $f(1) = 1$. Second, we take a limit $\lim_{\lambda\to 0} f(\lambda)$. By applying l'Hopital's rule twice we get:

$$\lim_{\lambda\to 0} f(\lambda) = \frac{8}{4\frac{3+\lambda(3+\lambda)}{(1+\lambda)^2} + 2\ln\frac{1-\lambda}{1+\lambda}} = \frac{2}{3}.$$

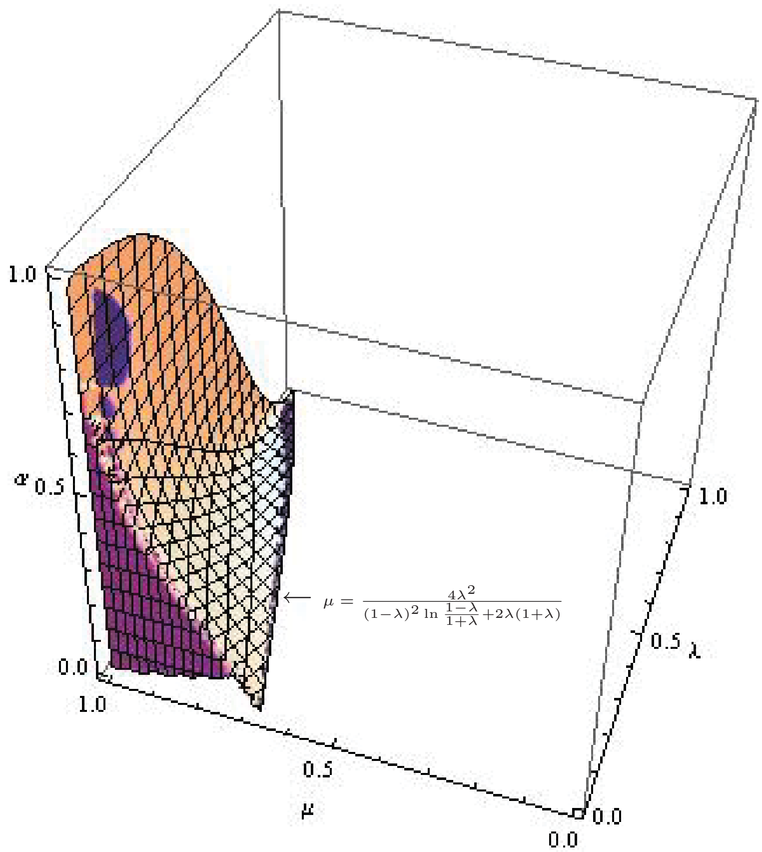
To prove that (2) is also a necessary condition we show that if the derivative of LHS of (14) is larger than the derivative of the RHS at $\alpha = 0$ than the LHS is larger than the RHS for any other α. That implies that there is no such a value of α which can equate both sides of the equation. Note, that both LHS and RHS of (14) are smooth functions in α, λ, μ. Therefore, we can directly verify the result on a grid for $(\alpha, \lambda, \mu) \in (0, 1)^3$. Numerical verification shows that (2) is indeed not only sufficient, but a necessary condition as well. The structure of the solution is presented in Figure 4. The intersection of the solution surface and surface $\alpha = 0$ is exactly the equality $\mu = \frac{4\lambda^2}{(1-\lambda)^2\ln\frac{1-\lambda}{1+\lambda}+2\lambda(1+\lambda)}$.

□

Proposition 3.8. *For all values of the parameters an equilibrium exists where all firms choose $\alpha = 0$. The equilibrium price distribution in this case is*

$$F_0(p) = 1 - \left(\frac{1-\lambda}{\lambda N}\frac{r_0 - p}{p}\right)^{\frac{1}{N-1}}, \quad p \in [\frac{1-\lambda}{1+\lambda(N-1)}r_0, r_0] \tag{15}$$

Figure 3: Equilibrium α as a function of (λ, μ)



.

where r_0 is defined as $\int_{\underline{p}}^{r_0} F_0(p)dp = c$.

Proof. As consumers expect all firms not to offer MPGs, we have that $F(p) = F_0(p)$. As no firm offers MPGs in equilibrium, we can define r_1 implicitly by

$$\begin{aligned}(1-\mu)r_1 \quad &+ \quad \mu[(F_0(r_1)\mathbb{E}_0(p|p < r_1) + (1 - F_0(r_1))r_1)] = \\ &= \quad c + (F_0(r_1)\mathbb{E}_0(p|p < r_1) + (1 - F_0(r_1))r_1)\end{aligned} .$$

Integrating by parts, it follows that the two reservation prices are given by

$$\int_{\underline{p}}^{r_0} F_0(p)dp = c$$
$$\int_{\underline{p}}^{r_1} F_0(p)dp = \frac{c}{1-\mu}.$$

In equilibrium each firm gets a profit of $\pi_0 = \frac{1-\lambda}{N} r_0$. Assume, one firm deviates and offers an MPG. Then the highest possible profit that can be obtained is by charging $p = r_1$. Indeed, it is clear that a firm only benefits from the deviation if $p > r_0$, but in that case the shoppers would not buy from this firm anyway, so the firm has to extract maximum profits from the uninformed consumers, which is attained by charging $p = r_1$. Then

$$\pi_1 = \frac{1-\lambda}{N}\left((1-\mu)r_1 + \mu\mathbb{E}(p|p < r_1)\right) = \frac{1-\lambda}{N}\left((1-\mu)r_1 + \mu(r_0 - c)\right)$$

so that

$$\pi_1 > \pi_0 \Leftrightarrow r_1 - r_0 > \frac{\mu c}{1-\mu}.$$

But we have

$$r_1 - r_0 = \int_{r_0}^{r_1} 1dp = \int_{r_0}^{r_1} F_0(p)dp = \int_{\underline{p}}^{r_1} F_0(p)dp - \int_{\underline{p}}^{r_0} F_0(p)dp = \frac{c}{1-\mu} - c = \frac{\mu c}{1-\mu}.$$

Thus, the best possible deviation gives the same payoff and a firm cannot strictly benefit from deviating. □

Proposition 4.1. *Expected profits for firms in the equilibrium where PMGs are offered with positive probability are higher than the expected profits in the equilibrium without PMGs. As a consequence, in the equilibrium where PMGs are offered consumers pay higher expected prices (after a possible execution of their PMGs) than in the equilibrium without PMGs.*

Proof. In fact, the equilibrium without MPG described by the same formulas as the equilibrium with MPG when α is set to be zero. The level of equilibrium profits for the equilibrium with MPG is

$$\pi(\alpha) = \frac{\lambda(1-\lambda+2\alpha\lambda)}{2(1-\alpha)\lambda + (1-\lambda+2\alpha\lambda)\ln\left(\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}c$$

Then

$$\frac{\partial\pi}{\partial\alpha} = \frac{4(1-\alpha)\lambda^3}{\left(2(1-\alpha)\lambda + (1-\lambda+2\alpha\lambda)\ln\left(\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)\right)^2}c > 0$$

Thus, profits are strictly increasing in α, so the lowest level of profits is attained when there are no MPGs ($\alpha = 0$). Since we have unit demand and full participation of consumers in the market, the same result holds for prices. □

References

M. Arbatskaya, M. Hviid, and G. Shaffer. On the incidence and variety of low-price guarantees. *Journal of Law and Economics*, 47:307–332, 2004.

M. Arbatskaya, M. Hviid, and G. Shaffer. On the use of low-price guarantees to discourage price cutting. *International Journal of Industiral Organization*, 24: 1139–1156, 2006.

Y. Chen, C.Narasiman, and Z.J. Zhang. Consumer heterogeneity and competitive price mathcing guarantees. *Marketing Sience*, 20(3):300–314, 2001.

K. Corts. On the robustness of the argument that price-matching is anti-competitive. *Economics Letters*, 47:417–421, 1995.

K. Corts. On the competitive effects of price-matching policies. *International Journal of Industiral Organization*, 15:283–299, 1997.

A. Edlin. Do guaranteed-low-price policies guarantee high prices and can anbtitrust rise to the challenge? *Harvard Law Review*, 11:529–575, 1997.

A. Galeotti. Searching, talking and pricing. *International Economic Review*, 51 (4):1151–1174, 2010.

M. Hviid and G. Shaffer. Hassle costs: The achilles heel of price matching guarantees. *Journal Economics and Management Strategy*, 8(4):489–521, 1999.

M. C. W. Janssen, J. L. Moraga-González, and M. Wildenbeest. Truly costly sequential search and oligopolistic pricing. *International Journal of Industiral Organization*, 23:451–466, 2005.

T. Kaplan. Effective price-matching: a comment. *International Journal of Industiral Organization*, 18:1291–1294, 2000.

Y.J. Lin. Price matching in a model of equilibrium price dispersion. *Southern Economic Journal*, 55(1):57–69, 1988.

S. Moorthy and R. Winter. Price matching guarantees. *RAND Journal of Economics*, 37(2):449–465, 2006.

I. D. Png and Hirschleifer. Price matching through offers to match price. *Journal of Business*, 60:365–383, 1987.

S. Salop. Practices that credibly facilitate oligopoly coordination. In *New Developments in the Analysis of Market Structure (J. Stiglitz and F. Mathewson, 1986)*. MIT Press, 1986.

D.O. Stahl. Oligopolistic pricing with sequential consumer search. *American Economic Review*, 79:700–712, 1989.

H. Varian. A model of sales. *American Economic Review*, 70(4):651–659, 1980.

A. Yankelevich. Price-matching in a sequential search duopoly. mimeo, 2010.

Alexei N. Parakhonyak
Higher School of Economics (Moscow, Russia). Center for Advanced Studies.
Researcher;
Email: parakhonyak@yandex.ru