



NATIONAL RESEARCH UNIVERSITY
HIGHER SCHOOL OF ECONOMICS

Nikolay Y. Ushakov

**WHAT IS THE ROLE OF HIGHER
WAGE FLEXIBILITY OF NEW
HIRES FOR OPTIMAL MONETARY
POLICY?**

BASIC RESEARCH PROGRAM

WORKING PAPERS

SERIES: ECONOMICS
WP BRP 44/EC/2013

This Working Paper is an output of a research project implemented at the National Research University Higher School of Economics (HSE). Any opinions or claims contained in this Working Paper do not necessarily reflect the views of HSE.

WHAT IS THE ROLE OF HIGHER WAGE FLEXIBILITY OF NEW HIRES FOR OPTIMAL MONETARY POLICY?²

Higher wage flexibility of new hires is introduced as an extension of the baseline model in Gali (2010), combining the New Keynesian monetary analysis framework with labor market frictions. It was shown that the possibility of higher wage flexibility of new hires has an implication for crucial labor market decisions made by households and firms, as well as on the form of social welfare loss function that is used to evaluate alternative monetary policies. Obtained extension allows one to conduct normative monetary policy analysis for different scenarios of degrees of higher wage flexibility for new hires. Optimal monetary policy in the presence of higher wage flexibility of new hires is characterized by a higher incentive to make inflation more stable and by less incentive to facilitate adjustment of real wages in response to real shocks. Thus, the possibility of higher wage flexibility of new hires provides support toward more strict inflation targeting in the presence of nominal price and wage rigidities.

JEL Classification: E32, E52.

Keywords: Relative wage flexibility of new hires, optimal monetary policy, New Keynesian framework, search and matching in the labor market, unemployment.

¹National Research University Higher School of Economics. Laboratory for Macroeconomic Analysis. Junior Research Fellow; E-mail: nickushak@mail.ru

²The author is grateful to Lev Lubimov, Sergey Pekarski, Kirill Sosounov, Sergey Merzlyakov, Andrei Dementiev, Oxana Malakhovskaia and Eren Arbatli (National Research University Higher School of Economics) for support and discussion of the present project during the macroeconomics scientific seminar held at the Higher School of Economics on April 17, 2013 in Moscow. The author is grateful to Jordi Gali (CREI, University of Pompeu Fabra) for discussing the project during the macroeconomics summer school "Monetary policy and Labor markets" in Barcelona in 2013. The study was implemented in the framework of the Basic Research Program at the National Research University Higher School of Economics in 2013.

1. Introduction

What are the normative implications of higher wage flexibility of new hires? How should central banks account for the possibility that new hires may have a higher probability of wage negotiations? What are the implications of such a wage-setting heterogeneity between new workers and existing workers for an economy's reaction to macroeconomic shocks, and for the costs and benefits of alternative monetary policies? How does an optimal monetary policy change in response to different degrees of higher wage flexibility for new hires?

Conventional monetary policy prescriptions in the presence of both price and wage stickiness was first formulated in Erceg et al (2000), and may also be found in Woodford (2003) and Gali (2008). The standard result of that literature states that in response to real shocks it is optimal for the central bank to allow to adjust both prices and wages to some extent, thereby facilitating proper real wage adjustment. The degree of optimal wage and inflation volatility will depend on the relative degree of wage and price rigidity. Such a prescription became known in the literature as flexible price inflation targeting, as opposed to strict inflation targeting. Moreover, Erceg et al (2000) have shown that strict wage inflation targeting is a significantly better policy than strict price inflation targeting. It would be interesting to know how the relative wage flexibility of new hires may change the proposed policy and be considered as a benchmark policy design principle.

The first steps toward analyzing optimal monetary policy in a model containing both nominal price and wage rigidities, along with the search and matching process in the labor market, and explicit analysis of the role and consequences for unemployment were made in Thomas (2008) and Gali (2010). Both authors come to a similar conclusion to that of Erceg et al (2000), namely that flexible inflation targeting is an optimal policy. Gali (2010) proposes a baseline theoretical framework that includes the necessary features required for introducing labor market frictions into the New Keynesian framework that could be used for both normative and positive analysis of monetary policy. Gali's (2010) framework is different from Thomas's (2008) model in that it ignores capital accumulation and considers the diminishing returns of intermediate firms as a natural source of inefficiencies from staggering wages. Another distinguishing feature of Gali's (2010) framework is that it includes the endogenous labor participation decisions of households, a property that is not usually considered in the search and matching literature and that is very natural for the New Keynesian and business-cycle literature, where the labor supply is endogenous. As a result, unemployment is determined not only by the hiring decisions of firms, but also by decisions of households regarding labor market

participation. The search and matching framework in the labor market assumes that there is a constant need of hiring of new workers by firms due to exogenous job destruction.

As was emphasized in Pissaridis (2009), if new hires could negotiate their wage freely at the time when they are hired, then the existence of long spells with unchanged wages of incumbent workers (but remaining in the bargaining sets) would have no direct effect on hiring decisions and, as a result, on output and employment. Still, the empirical evidence on the wage flexibility of new hires remains controversial. One group of authors (Haefke, Sontag, and van Rens, 2008, and references in Pissaridis, 2009) provide evidence that confirm the hypothesis regarding the wage flexibility of new hires, while other authors, such as Gertler and Trigari (2009) and Galuscak et al (2008), reject the hypothesis of a significant difference in the frequencies of wage setting for new and existing workers.

Despite a lack of consensus on the degree of relative wage flexibility of new hires, it would be useful to introduce such a possibility in the standard framework and study both positive and normative implications.

The first attempt to introduce relative wage flexibility for new hires was made in Bodart et al (2006) for the purpose of positive analysis in a medium size monetary model. The proposed model lacks the microstructure necessary for normative analysis purposes and considers labor supply to be exogenous.

The goal of this paper is to study implications of relative wage flexibility of new hires in Gali's baseline model with unemployment. The main reason for considering Gali's model as a benchmark is that this model allows us to conduct both a positive and normative analysis in the presence of labor market frictions, nominal rigidities, and endogenous labor participation of households.

The paper is organized as follows. Section 2 formulates the baseline model combining nominal rigidities and labor market frictions. Section 3 presents calibration of the model and equilibrium reaction of macroeconomic variables in response to monetary and real shocks for different degrees of relative wage flexibility for new hires, given the same exogenous Taylor rule with respect to the wage flexibility of new hires. In Section 4 the optimal allocation of social planner is found, welfare loss function in the presence of relative wage flexibility of new hires is derived, and a comparative analysis of optimal policy is conducted under different degrees of relative wage flexibility. Section 5 offers a conclusion.

2. A model with nominal rigidities and labor market frictions

I follow the proposed baseline set-up in Galí (2010), which combines nominal rigidities and search and matching frictions in the labor market within the New Keynesian monetary framework, and extend the baseline model to explore implications for a higher relative wage flexibility of new hires.

2.1 Households

It is assumed that the economy consists of a large number of identical households. Each household consists of a continuum of members on a unit interval and maximizes the expected utility of the form

$$E_t \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \quad (1)$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$ is an index of consumed final goods, ε is the constant elasticity of substitution between any final goods, and L_t is an index of time spent by the household members on work and job searching. More specifically, L_t is defined as:

$$L_t = N_t + \psi U_t \quad (2)$$

where N_t is the number of employed members of the household and U_t is the number of involuntary unemployed members that are searching for a job. Coefficient $\psi \leq 1$ measures the relative weight of disutility from searching for a job as compared to the disutility from work in the index of labor market effort L_t .

Period utility function is the following

$$U(C_t, L_t) \equiv \log C_t - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \quad (3)$$

Parameters φ and χ determine labor supply, $\frac{1}{\varphi}$ is the Frisch labor supply elasticity, and χ is a scaling parameter relating utility from consumption and disutility from labor market effort.

The log specification of consumption utility function is taken because it is consistent with the balanced growth path. Note, in contrast to the standard business cycle literature, where the disutility from employment is only considered, here the labor market effort of both employed and unemployed members of the household are taken into account in the utility function.

For this specification of utility function, it is implicitly assumed that all members of the household (participating or not participating in the labor market) consume the same amount of final goods. This is possible because there is a transfer of income from employed members to members that are unemployed and not participating in the labor market. In the context of the search and matching model, such perfect risk sharing was firstly introduced by Merz (1995).

Another implicit property of the above utility function is that non-participation in the labor market does not bring disutility to a household. Any nonparticipating member has an advantage that the member does not spend time on searching for a job and has a disadvantage of not earning wages for the household. Thus, this trade-off creates a problem of optimal allocation of household members between participation and non-participation in the labor market.

Aggregate employment evolves according to

$$N_t = (1 - \delta)N_{t-1} + x_t U_t^0 \quad (4)$$

where δ is an exogenous job destruction rate, showing a constant share of job positions that are closed every period. x_t is a job finding rate – the probability of finding a job for an unemployed worker at the beginning of any quarter. At the end of any quarter there are $U_t = U_t^0 - x_t U_t^0$ unemployed workers left. Note, this timing assumption is necessary to make employment a non-predetermined variable, which would be consistent with most of the business cycle literature and which differs from the search and matching literature, where each worker becomes a part of productive employment only in the next period after he or she was hired. Such a timing assumption was first introduced by Blanchard and Gali (2010). For a quarterly frequency, it is quite reasonable to assume that the worker becomes productive in the period of finding a job and would be irrelevant to assume this for a monthly frequency.

A household maximizes utility (1) in subject to the sequence of budget constraints

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j) N_t(j) dj + \Pi_t$$

where $P_t(i)$ is the price of final good i , $W_t(j)$ is the wage paid to the worker employed in firm j , which produces the intermediate good, B_t is the quantity of one-quarter bonds that are bought by the household in the beginning of the quarter for price $Q_t < 1$ and sold back at the end of the quarter for the price of 1, thus bringing a quarter interest rate of $\frac{1-Q_t}{Q_t}$, Π_t is the lump sum component (e.g. dividends or taxes). Another constraint prevents households from engaging in Ponzi-type schemes.

The resulting demand for individual final good i

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t \quad (5)$$

The familiar optimal intertemporal choice condition is represented by the Euler equation

$$Q_t = \beta E_t \left\{ \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\} \quad (6)$$

2.2 Firms

In order to introduce both nominal price rigidities and search and matching frictions in one model, but while treating these features separately, a two-sector structure of the economy is assumed: a final goods sector and an intermediate goods sector. Firms in the final goods sector are subject to nominal rigidities and use only an intermediate good as an input for their production. Thus they do not directly face a search and matching problem in the labor market. In contrast, firms in the intermediary sector fully face flexible competitive prices for their produced homogenous goods and do face a search and matching problem of hiring workers that are needed for producing intermediate goods and who engage in the process of wage bargaining with firms. A combination of nominal price rigidities and search and matching frictions in such a way is a standard approach for this literature. This approach was originally proposed by Walsh (2005) and it avoids difficulties that appear when both pricing and hiring decisions are made by the same firm, creating room for the interdependence of pricing and hiring decisions, which is not a focus of this paper.

There is a continuum on the unit interval of firms in the final goods sector in the economy. Each firm produces a differentiated good and sells it to households on a monopolistically competitive market. Every differentiated good is produced using the same technology:

$$Y_t(i) = X_t(i)$$

where $Y_t(i)$ is the quantity of a differentiated good produced by firm i and $X_t(i)$ is the quantity of the homogeneous intermediate good demanded by firm i on a competitive intermediate goods market.

It is well-known that under flexible prices, when all firms may change their prices every period, the optimal price in the monopolistically competitive market is

$$P_t(i) = M^p (1 - \tau) P_t^I$$

i.e. the optimal price is a markup M^p over the current period's nominal marginal costs $(1 - \tau)P_t^I$, where τ is a subsidy for the purchase of an intermediate good at the price P_t^I .

As all firms under flexible prices will face an identical problem, they will all set the same price equal to the average price for final goods equal to

$$P_t = M^p(1 - \tau)P_t^I$$

It is assumed that the prices are sticky in a way proposed by Calvo (1983). Every firm has the same probability θ_p to reset the price and this probability is independent across the firms and time that has passed after the last price readjustment. The following log-linearized optimal price-setting rule can be derived (details of derivation are shown in Chapter 3 of Galí, 2008)

$$p_t^* = \mu^p + (1 - \beta\theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k (E_t\{p_{t+k}^I\} - \tau) \quad (7)$$

where small letters denote logarithms of original variables, e.g. $p_t^* = \log P_t^*$, $\mu^p = \log M^p$, etc. The condition for optimal price setting states that, under pricing restrictions a la Calvo, it is optimal to choose the price that is a desired mark-up (mark-up in an environment of flexible prices) over the expected discounted average nominal current and future marginal costs, with the discounting factor determined by the time preference discounting factor β^k and the probability that the set price will remain effective in future quarters, θ_p^k .

Calvo pricing implies the following approximate formula for log aggregate price level

$$p_t = \theta_p p_{t-1} + (1 - \theta_p) p_t^* \quad (8)$$

Formula (8) is a result of the assumption that in every period a share of $(1 - \theta_p)$ of all firms face the same price-setting problem and choose the same price P_t^* . As firms that reset their prices in the current quarter are chosen randomly, therefore all left firms that did not reset their price in the current quarter have an average price equal to the previous quarter's average price level for all firms.

The combination of equation for optimal price (7) with equation for the law of motion of aggregate price level (8) results in familiar price inflation in the New Keynesian Phillips Curve:

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p \hat{\mu}_t^p \quad (9)$$

where $\pi_t^p = p_t - p_{t-1}$, $\lambda_p \equiv \frac{(1 - \theta_p)(1 - \beta\theta_p)}{\theta_p}$ and $\hat{\mu}_t^p = \mu_t^p - \mu^p = p_t - (p_t^I - \tau) - \mu^p$. The last formula states that inflation is driven only by current and future deviations of actual mark-up

μ_t^p from desired mark-up under an environment of flexible prices or steady state mark-up. As the actual mark-up is determined by the price of the intermediate goods sector dealing with the search and matching and bargaining problem, then the dynamics of inflation will reflect all frictions present in the intermediate goods sector and affecting the price inflation of final goods through the mark-up channel.

There is a continuum on the unit interval of intermediate firms producing a homogeneous good and which have access to the same production technology with diminishing returns for labor input

$$Y_t^I(j) = A_t N_t(j)^{1-\alpha}$$

where $Y_t^I(j)$ is the quantity of produced intermediate goods by firm j using $N_t(j)$ employees. A_t represents a technological parameter that is common and exogenous for all intermediate firms and described by the autoregressive process for $a_t \equiv \log A_t$ with autoregressive coefficient ρ_a and the variance σ_a^2 .

Employment dynamics within an intermediate firm j is described by equation

$$N_t(j) = (1 - \delta)N_{t-1}(j) + H_t(j) \quad (10)$$

where δ is an exogenous and common for intermediate firms rate of job destruction per quarter and $H_t(j)$ is the level of hiring by firm j . Assumptions underlying the timing of this equation were described in the section for the household problem.

As stated earlier, intermediate firms engage in the labor market characterized by search and matching frictions, meaning that the hiring process requires costs on the side of the firm to employ new workers and that the wage is not given by a Walrasian auction, but instead is bargained individually with every employee at some point in time. An important feature of this market is that each employee signs a contract specifying that the employee will receive a wage according to individual bargaining between the employee and firm with an exogenously given probability of renegotiations in the future, and that the employee may lose his or her job every quarter with an exogenously given probability of δ .

Labor market frictions are introduced in the form of the cost per hire G_t , as was proposed in Blanchard and Gali (2010). The cost per hire G_t is the amount of resources that a firm should spend to hire one worker. It is assumed that G_t for each individual firm is exogenous and, thus, does not depend on an individual firm's hiring level. It is commonly assumed that for the economy as a whole the cost per hire may depend on aggregate factors. One natural factor

determining cost per hire is the level of labor market tightness (the ratio of vacancies to unemployed workers), which can be approximated by the job finding rate $x_t \equiv \frac{H_t}{U_t^0}$ (the ratio of the aggregate level of hiring during the quarter to the number of unemployed workers at the beginning of the quarter). Specifically,

$$G_t = G_t(x_t) = \Gamma x_t^\gamma$$

It was shown in Gali (2010) that the proposed approach for introduction of labor market frictions is equivalent to the search and matching function approach developed by Diamond, Mortensen, and Pissarides, and described in Pissaridis (2000). In short, one can derive that $G_t = \frac{V_t \Gamma}{M(V_t, U_t^0)}$, where V_t is the number of aggregate vacancies posted in the economy, Γ is the unit cost of posting a vacancy, $M(V_t, U_t^0)$ is the matching function, measuring the aggregate hiring per quarter as a function of the number of aggregate vacancies and unemployed workers at the beginning of the quarter. It is straightforward to show that, if the matching function represented by the Cobb-Douglas function with first-degree homogeneity, $M(V_t, U_t^0) = V_t^\zeta U_t^{0^{1-\zeta}}$, then the hiring cost is $G_t = \Gamma x_t^{\frac{1-\zeta}{\zeta}}$, which coincides with the proposed formula for G_t above, if $\gamma = \frac{1-\zeta}{\zeta}$.

2.2.1 Introduction of higher wage flexibility of new hires for employment decisions

I extend a model developed by Gali (2010) to incorporate the higher wage flexibility of new hires. The original model, proposed by Gali (2010), contained a hiring condition for firms producing intermediate goods

$$MRPN_t(j) = \frac{w_t(j)}{p_t} + G_t - (1 - \delta) E_t \{ \Lambda_{t,t+1} G_{t+1} \} \quad (11)$$

where $MRPN_t(j) \equiv \frac{p_t^I}{p_t} (1 - \alpha) A_t (N_t(j))^{-\alpha}$ is the real marginal revenue product of the intermediate good produced by firm j . Condition (11) states that an intermediary firm j should have employment level $N_t(j)$ – and thus hiring level $H_t(j)$ – such that the real marginal revenue of the marginal worker is equalized with the marginal costs associated with hiring a marginal worker, and the latter costs are the real wage and the cost per hire minus the next quarter's expected benefit from hiring a new worker today, as there will be no need to hire an additional worker in case the position will not be destroyed with probability $(1 - \delta)$.

The above formula shows that, at the stage of hiring, an intermediate firm takes the following variables as exogenously given: the wage that the new workers will get in case of hiring ($W_t(j)$), the aggregate price level of final goods (P_t), intermediate goods price (P_t^I), cost per hire (G_t), and the level of technology (A_t). From all listed determinants, only the nominal wage $W_t(j)$ that the new hired workers will get is firmspecific, as different firms may have wage bargaining in different quarters. Hence the optimal employment level is also firm specific and thus different for firms with different wages that are paid to hired or new employees in the current quarter.

Relative wage flexibility of new hires with respect to all employees is introduced in the sense that the new hires do not necessarily sign a contract with a wage equal to the wage of existing workers hired in the previous quarter. Thus, it is assumed that a portion κ of newly hired workers get an average wage of existing contracts of the previous period hires. That average wage is determined or “inherits” both the wage that was set for workers who worked at the time of employee negotiations, and wages of workers newly hired after company bargaining. The other portion of newly hired workers, $(1 - \kappa)$, have the opportunity to conduct exclusive negotiations during the period of hiring. Thus, in general, the new hires have a higher probability of wage negotiations than existing employees.

First, it should be emphasized that under a higher wage flexibility of new hires, the wages of new and previously hired employees may differ within the same company and the hiring decision is based on the wages of new hires and not on previously hired workers. Thus, it is suggested to index the wage in the hiring condition (11) with a subscript *new*, stressing that the wage for new hires is considered by the firm when the hiring decision is made. Second, it is assumed that when deciding how many workers to hire the firm will consider that the wage of an additional employee is a weighted average of the wage of existing employees hired in the previous period and newly negotiated wage with weights κ and $(1 - \kappa)$, respectively. Specifically, optimal hiring condition in the extended version of the model in Galí (2010) takes the form

$$MRPN_t(j) = \frac{\bar{W}_t^{new}(j)}{P_t} + G_t - (1 - \delta)E_t\{\Lambda_{t,t+1}G_{t+1}\} \quad (12)$$

$$\bar{W}_t^{new}(j) = (1 - \kappa)W_t^* + \kappa \bar{W}_{t-1}^{new}(j)$$

where $\bar{W}_t^{new}(j)$ is an average wage of new hires, W_t^* is a currently negotiated wage exclusively for new workers, $\bar{W}_{t-1}^{new}(j)$ is the average wage of new workers hired in the previous quarter.

Forward iteration of the previous equation results in

$$G_t = E_t \left\{ \sum_{k=0}^{\infty} \Lambda_{t,t+1} (1 - \delta)^k \left(MRPN_{t+k}(j) - \frac{\bar{W}_{t,t+k}^{new}(j)}{P_{t+k}} \right) \right\}$$

where $\bar{W}_{t,t+k}^{new}(j)$ is a notation denoting estimated wage that currently hired workers (in the quarter t) are going to get in the future quarter $t + k$. The last equation states that under optimal hiring the cost per hire should be equal to the expected discounted value of the current and future differences in the real marginal revenue product and the average wage of new hires at t (or surpluses from new hires in the event that they are not destroyed in the future, which is accounted for by multiplying the probability of survival for currently newly occupied positions in the future, $(1 - \delta)^k$).

It is convenient to define net hiring costs $B_t \equiv G_t - (1 - \delta)E_t\{\Lambda_{t,t+1}G_{t+1}\}$ and simplify the hiring condition as follows

$$MRPN_t(j) = \frac{\bar{W}_t^{new}(j)}{P_t} + B_t \quad (13)$$

Note that the above equation implicitly contains a relation between actual markup $\left(\frac{P_t}{(1-\tau)P_t^I}\right)$ in the final goods sector, average real wages of new hires, hiring costs, technology parameter and optimal level of employment according to the formula of $MRPN_t(j)$. Log-linearization of (13) around steady state, expression in log-deviations, and integration over all intermediate firms gives

$$\hat{\mu}_t^p = (a_t - \alpha \hat{n}_t) - [(1 - \Phi)\hat{w}_t + \Phi \hat{b}_t] \quad (14)$$

where $\Phi \equiv \frac{B}{(W/P)+B}$, $\hat{n}_t \cong \int_0^1 \hat{n}_t(j) dj$, $w_t \cong \int_0^1 w_t(j) dj$, $\hat{w}_t \cong \int_0^1 \hat{w}_t(j) dj$. Variables with caps denote log-deviations from steady state values, i.e. $\hat{x}_t \equiv x_t - x$.

Log-linearization and expression in log-deviations from steady states for net hiring costs results in

$$\hat{b}_t = \frac{1}{1-\beta(1-\delta)} \hat{g}_t - \frac{\beta(1-\delta)}{1-\beta(1-\delta)} (E_t\{\hat{g}_{t+1}\} - \hat{r}_t) \quad (15)$$

where the last expression in brackets uses the Euler equation for optimal intertemporal choice of consumption by the household.

From the optimal hiring condition it is straightforward to derive a relation between the relative employment of firm j with respect to the average employment in the economy and the relative

average wage of new hires in a firm j with respect to the average wage of new hires in the economy

$$\alpha(n_t(j) - n_t) = -(1 - \Phi)(\bar{\omega}_t^{new}(j) - \bar{\omega}_t^{new}) \quad (16)$$

2.3 Monetary policy

For the purpose of positive analysis, the actually conducted monetary policy is described by the Taylor type rule

$$i_t = \rho + \phi_\pi \pi_t^p + \phi_y \hat{y}_t + v_t \quad (17)$$

where $i_t \equiv -\log Q_t$ represents the return for a one-quarter riskless bond and $\rho \equiv -\log \beta$ is a household's time discount rate of the next quarter utility, π_t^p is the quarter-to-quarter change in the price level, \hat{y}_t is the deviation of actual quarterly output from the steady state quarterly output, v_t is an exogenous policy shifter described by an AR process with autoregressive coefficient ρ_v and variance σ_v^2 .

2.4 Labor market frictions and wage determination under higher wage flexibility of new hires

Two alternative assumptions regarding wage setting are considered: cases of flexible wages and sticky wages. The case of flexible wages is defined as a situation when all workers bargain with firms over wages in every quarter. A case of sticky wages is defined as a situation when workers negotiate over wages in not every quarter. Moreover, I consider an extension to the sticky wages case proposed in Galí (2010) in which a portion of new hires has the opportunity to exclusively bargain with the firm, whereas the rest of the existing and some of the newly hired workers do not perform negotiations.

Under flexible wages, an employed worker accrues the following marginal value from his or her labor market participation

$$V_t^N(j) = \frac{W_t(j)}{P_t} - MRS_t + E_t\{\Lambda_{t,t+1}((1 - \delta)V_{t+1}^N(j) + \delta V_{t+1}^U)\}$$

This value is measured in terms of real goods consumption and takes account for the benefit of the real wage, disutility from time spent on work ($MRS_t \equiv \chi C_t L_t^\varphi$), and the expected value of the employment relation for the next quarter. It is expected that with probability $1 - \delta$ the

household member will continue work in the next quarter and with probability δ he or she will be unemployed and search for a new job from the beginning of the next quarter.

The marginal value from sending an additional household member to search for a job is represented by

$$V_t^U = x_t \int_0^1 \frac{H_t(z)}{H_t} V_t^N(z) dz + (1 - x_t) (-\psi MRS_t + E_t \{ \Lambda_{t,t+1} V_{t+1}^U \})$$

This value takes account for the expected value of being hired during the current quarter (this expectation is based on the chances to be hired by firms with potentially different levels of hiring). On the other hand, in case a household member does not find the job in the current quarter, he or she creates disutility for the household due to spending time searching for a job. The final term is the expectation of the value of remaining unemployed at the beginning of the next quarter.

In the section describing the household problem it was assumed that the household makes an optimal participation decision in the labor market. A household should choose the number of members that should participate in the labor market, taking into account that searching for a job brings disutility and at the same time a perspective to be hired and to receive a certain level of wages, while non-participation does not bring any disutility for searching, but also does not provide any perspective to be hired in current or future quarters.

For the sake of convenience, the value of non-participation is normalized to zero $V_t^{NP} = 0$. Assuming that a household has a positive number of both participating members and non-participating members, meaning that the solution is interior, it should be indifferent between an alternative to send an additional worker to the labor market or to ask the member to stay at home. That implied, the optimal participation condition is satisfied when $V_t^U = V_t^{NP} = 0$. Applying this condition for the marginal value for an unemployed worker gives the following relation

$$\psi MRS_t = \frac{x_t}{1-x_t} \int_0^1 \frac{H_t(z)}{H_t} V_t^N(z) dz \quad (18)$$

The relation means that the household will send members to search for a job to the point when the marginal disutility from the job search would be equalized with the expected benefit from employment. Therefore, the surplus accruing to the household (defined as the difference between the marginal values of employed and unemployed workers) is

$$S_t^H(j) = \frac{W_t(j)}{P_t} - MRS_t + (1 - \delta) E_t \{ \Lambda_{t,t+1} (S_{t+1}^H(j)) \} \quad (19)$$

where $S_t^H(j) \equiv V_t^N - V_t^U = V_t^N$.

Similarly, the surplus for a firm that shows the marginal value accruing to the firm from marginal employees is

$$S_t^F(j) = MRPN_t(j) - \frac{W_t(j)}{P_t} + (1 - \delta)E_t\{\Lambda_{t,t+1}(S_{t+1}^F(j))\} \quad (20)$$

The firm's surplus takes account of the benefit of the real marginal revenue product that is created by the marginal employee and the cost of paying the real wage to the worker and expected future surpluses if the position will not be destroyed with probability $(1 - \delta)$. Note that firm's surplus does not include hiring costs, as the surplus is calculated for existing positions, when the worker is already hired and thus the cost per hire has already been paid.

The minimum wage when employment is beneficial ($S_t^H(j) = 0$) for the household is

$$\Omega_t^H(j) = MRS_t - (1 - \delta)E_t\{\Lambda_{t,t+1}(S_{t+1}^H(j))\}$$

The maximum wage when hiring is beneficial is

$$\Omega_t^F(j) = MRPN_t(j) + (1 - \delta)E_t\{\Lambda_{t,t+1}(S_{t+1}^F(j))\}$$

The difference between the maximum possible wage and minimum possible wage represents the sum of surpluses accruing to the household and the firm.

$$\Omega_t^F(j) - \Omega_t^H(j) = S_t^F(j) + S_t^H(j) \geq G_t$$

It is assumed that firms and household members engage in the Nash bargaining and the resulting bargained wage is a solution for the following problem

$$\max_{W_t(j)} S_t^H(j)^{1-\xi} S_t^F(j)^\xi$$

where ξ is the bargaining power of the firm and $1 - \xi$ is the bargaining power of the household member.

The optimal sharing rule is

$$\xi S_t^H(j) = (1 - \xi) S_t^F(j)$$

and the Nash bargained wage is

$$\frac{W_t(j)}{P_t} = \xi \Omega_t^H(j) + (1 - \xi) \Omega_t^F(j) = \xi MRS_t + (1 - \xi) MRPN_t(j) \quad (21)$$

The above expression states that bargained wages represent a weighted average of the current quarter marginal rate of substitution and the marginal revenue product from additional hiring, with weights corresponding to bargaining powers.

As all employees and firms face the same Nash bargaining problem in the same quarter that the wages are negotiated every quarter by all employees, all negotiations will result in the same wage ($\frac{W_t(j)}{P_t} = \frac{W_t}{P_t}$) if the real marginal products are identical for all firms. Given that the wages are identical, the hiring decision rule implies the same level of employment for all firms and thus the same level of marginal revenue product ($MRPN_t(j) = MRPN_t$). Therefore, satisfaction of both the optimal hiring condition and Nash bargaining condition results in

$$\frac{W_t}{P_t} = \xi MRS_t + (1 - \xi)MRPN_t \quad (22)$$

Combining the previous Nash bargaining wage condition with the optimal hiring condition gives

$$G_t - (1 - \delta)E_t\{\Lambda_{t,t+1}G_{t+1}\} = \xi(MRPN_t - MRS_t) \quad (23)$$

Combining the Nash bargaining condition with the optimal participation condition for households and optimal hiring condition gives

$$\xi\psi MRS_t = (1 - \xi)\frac{x_t}{1-x_t}G_t \quad (24)$$

Note that the optimal hiring condition defines relation for G_t coinciding with the relation for $S_t^F(j) = S_t^F$. Thus it is always the case that the surplus the firm gets from hiring is equal to the cost per hire, that is common for all firms. Using the sharing rule to express the household's surplus as a function of the firm's surplus from hiring gives the results in (24).

An extension of the sticky wages model proposed by Gali (2010) is considered below. As in Gali (2010) I assume sticky wages for *existing* workers or workers hired in previous quarters a la Calvo, when for every existing employment relation there is a probability of $1 - \theta_w$ for wage negotiations in every quarter, which is independent across employment contracts and time passed after the last negotiation. The relative wage flexibility of new hires is introduced by the assumption that a portion of $1 - \kappa$ of new employees can renegotiate their wage even if it happened such that existing employees did not bargain the wage in the current quarter.

The marginal value of an employed member of a household is

$$\begin{aligned}
V_{t+k|t}^N &= \frac{W_t^*}{P_{t+k}} - MRS_{t+k} \\
&+ E_{t+k} \{ \Lambda_{t+k,t+k+1} [(1-\delta)(\theta_w V_{t+k+1|t}^N + (1-\theta_w) V_{t+k+1|t+k+1}^N) + \delta V_{t+k+1}^U] \}
\end{aligned} \tag{25}$$

where $V_{t+k|t}^N$ is the marginal value of an additional employed member of the household in quarter $t+k$, given that the last negotiations happened k quarters ago in quarter t .

The marginal value of an unemployed member of the household in the beginning of quarter t is

$$V_t^U = x_t \int_0^1 \frac{H_t(z)}{H_t} V_t^N(z) dz + (1-x_t)(-\psi MRS_t + E_t \{ \Lambda_{t,t+1} V_{t+1}^U \})$$

As previously under condition of optimal participation ($V_t^U = V_t^{NP} = 0$), the household's surplus for quarter $t+k$ given that the last wage negotiation took place k quarters ago is

$$S_{t+k|t}^H = \frac{W_t^*}{P_{t+k}} - MRS_{t+k} + E_{t+k} \{ \Lambda_{t+k,t+k+1} [(1-\delta)(\theta_w S_{t+k+1|t}^H + (1-\theta_w) S_{t+k+1|t+k+1}^H)] \} \tag{26}$$

The marginal value for the unemployed worker under optimal participation implies

$$\psi MRS_t = \frac{x_t}{1-x_t} \int_0^1 \frac{H_t(z)}{H_t} S_t^H(z) dz \tag{27}$$

It is straightforward to get a solution to (26) by forward iteration for the case when negotiations happen in the same quarter ($k=0$)

$$\begin{aligned}
S_{t|t}^H &= E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k} \left(\frac{W_t^*}{P_{t+k}} - MRS_{t+k} \right) \right\} \\
&+ (1-\theta_w)(1-\delta) E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k+1} S_{t+k+1|t+k+1}^H \right\}
\end{aligned} \tag{28}$$

A firm's surplus from a marginal employee is

$$S_{t+k|t}^F = MRPN_{t+k|t} - \frac{W_t^*}{P_t} + (1-\delta) E_t \{ \Lambda_{t,t+1} (\theta_w S_{t+k+1|t}^F + (1-\theta_w) S_{t+k+1|t+k+1}^F) \} \tag{29}$$

The expectations of continuation value are based on the probabilities of overall employee renegotiations in the next quarter.

As in the case of flexible wages, the definition of $S_{t+k|t}^F$ above coincides with the expression for G_{t+k} under optimal hiring conditions, thus under optimal hiring

$$S_{t+k|t}^F = G_{t+k}$$

Forward iteration of (29) when the wage is set in the same quarter t gives the solution

$$S_{t|t}^F = E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k} \left(MRPN_{t+k|t} - \frac{W_t^*}{P_{t+k}} \right) \right\} \\ + (1-\theta_w)(1-\delta)E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k+1} S_{t+k+1|t+k+1}^F \right\} \quad (30)$$

The Nash bargaining problem under sticky wages is the following

$$\max_{W_t^*} S_{t|t}^H {}^{1-\xi} S_{t|t}^F {}^\xi$$

Optimal sharing rule

$$\xi S_{t|t}^H = (1-\xi) S_{t|t}^F \quad (30)$$

Substituting expressions for surpluses into optimal sharing condition gives

$$E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k} \left(\frac{W_t^*}{P_{t+k}} - \Omega_{t+k|t}^{tar} \right) \right\} = 0 \quad (31)$$

where

$$\Omega_{t+k|t}^{tar} \equiv \xi MRS_{t+k} + (1-\xi) MRPN_{t+k|t} \quad (32)$$

Log-linearization of optimal sharing rule (30) results in

$$w_t^* = (1-\beta(1-\delta)\theta_w) \sum_{k=0}^{\infty} (\beta(1-\delta)\theta_w)^k E_t \{ \omega_{t+k|t}^{tar} + p_{t+k} \} \quad (33)$$

In contrast to the case of flexible wages where the Nash bargained wage should always be equal to the current period target wage, in the case of sticky wages the Nash bargained wage is an average of the current and future quarter target wages with the discount factor being a function of the next quarter utility discount factor β , the probability that the job contract will not be exogenously destroyed is $1-\delta$ and the probability that the current bargained wage will be effective next quarter is θ_w .

Log-linearization of (32) gives

$$\widehat{\omega}_{t+k|t}^{tar} = (1 - \Upsilon)(\widehat{c}_{t+k} + \varphi \widehat{l}_{t+k}) + \Upsilon(-\widehat{\mu}_{t+k}^p + a_{t+k} - \alpha \widehat{n}_{t+k|t}) \quad (34)$$

where $\Upsilon \equiv \frac{(1-\xi)MRPN}{W/P}$. Note that the target wage is calculated for every period for each employment relation, but it does not mean that the employee negotiates a wage for every period. For example, $\widehat{\omega}_{t+k|t}^{tar}$ is the target wage in quarter $t+k$ and the last negotiations for this employment contract was k quarters ago in t . The above equation states that the targeted wage is different for two employment contracts if their last negotiations happened in different periods and should be the same if the previous negotiations happened in the same period.

It is possible to define an average target wage for the economy as a whole by integrating (34) over all intermediate firms. According to (34), such an average level of target wage corresponds to the targeted wage for a firm with an average level of employment in the economy

$$\widehat{\omega}_t^{tar} = (1 - \Upsilon)(\widehat{c}_t + \varphi \widehat{l}_t) + \Upsilon(-\widehat{\mu}_t^p + a_t - \alpha \widehat{n}_t) \quad (35)$$

Subtracting (35) from (34) gives a relation of the relative target wage with respect to the average target wage in the economy to relative employment with respect to average employment in the economy

$$\widehat{\omega}_{t+k|t}^{tar} = \widehat{\omega}_{t+k}^{tar} - \alpha \Upsilon (\widehat{n}_{t+k|t} - \widehat{n}_{t+k})$$

Combining previous relation and relative employment demand (16), a relation of the relative target wage with respect to average target wage in the economy to relative average wage of new hires with respect to average wage of new hires in the economy is obtained

$$\widehat{\omega}_{t+k|t}^{tar} = \widehat{\omega}_{t+k}^{tar} + \Upsilon(1 - \Phi)(\bar{\omega}_{t+k|t}^{new} - \bar{\omega}_{t+k}^{new}) \quad (36)$$

where

$$\bar{\omega}_{t+k|t}^{new} = \kappa^k \omega_t^* + (1 - \kappa) \sum_{q=1}^k \kappa^{k-q} \omega_{t+q}^*$$

The derivation of the last equation is made in Appendix A.

Note that for an extension of Galí (2010) in the form of introducing relative wage flexibility of new hires, it is stressed that relative employment is determined not by the average wage of *all* workers in a firm, but by the average wage of *new hired workers* in a firm.

By substituting the targeted wage equation (36) into the Nash optimal sharing rule wage equation (33), and after several algebraic manipulations, the following expression is derived for the Nash bargained wage. (Sketches of derivations are shown in Appendix B.)

$$w_t^* = A_1 B_1 E_t \{w_{t+1}^*\} + A_1 (1 - \beta(1 - \delta)\theta_w)(\omega_t^{tar} + p_t - (1 - \Upsilon)\Phi \bar{w}_t^{new}) \quad (37)$$

where

$$A_1 \equiv \frac{(1 - \beta(1 - \delta)\kappa\theta_w)}{1 - \beta(1 - \delta)\kappa\theta_w - (1 - \beta(1 - \delta)\theta_w)(1 - \Upsilon)\Phi}$$

$$B_1 \equiv \left((1 - \beta(1 - \delta)\theta_w)(1 - \Upsilon)\Phi(1 - \kappa) - \frac{(1 - \beta(1 - \delta)\theta_w)(1 - \Upsilon)\Phi}{1 - \beta(1 - \delta)\kappa\theta_w} + 1 \right) \beta(1 - \delta)\theta_w$$

As shown in Appendix C, the average wage of new hires for the economy as a whole will evolve according to

$$\bar{w}_t^{new} = \kappa\theta_w \bar{w}_{t-1}^{new} + (1 - \kappa\theta_w)w_t^* \quad (38)$$

Combining equations for the Nash bargained wage (37) and for the average wage of new hires in the economy (38) gives wage inflation for the New Keynesian Phillips curve derived in the environment of a search and matching model with sticky nominal wages allowing for heterogeneity in the wage flexibility of newly hired and existing employees.

$$\pi_t^{w,new} = \frac{A_1 B_1}{\kappa\theta_w} E_t \{\pi_{t+1}^{w,new}\} - \frac{(1 - \kappa\theta_w)}{\theta_w} A_1 (1 - \beta(1 - \delta)\theta_w)(\hat{\omega}_t^{new} - \hat{\omega}_t^{tar}) \quad (39)$$

where $\pi_t^{w,new} \equiv \bar{w}_t^{new} - \bar{w}_{t-1}^{new}$ measures the inflation of wages for new hires and $\hat{\omega}_t^{new} \equiv \bar{w}_t^{new} - p_t$ measures the real average wages of new hires.

Participation condition of the household expressed in log-deviations from a steady state is

$$\hat{c}_t + \varphi \hat{l}_t = \frac{1}{1-x} \hat{x}_t + \hat{g}_t - \Xi \pi_t^{w,new} \quad (40)$$

where $\Xi \equiv \frac{\xi(W/P)}{(1-\xi)G} \frac{\kappa\theta_w}{(1-\kappa\theta_w)(1-\beta(1-\delta)\theta_w)}$. The derivation of participation conditions assuming a

higher wage flexibility of new hires is shown in Appendix D.

2.5 Aggregate demand and output

The hiring costs are in the form of a bundle of final goods with elasticity of substitution, coinciding with a consumer's elasticity of substitution ε . Thus, the production of each final good

is not only consumed by the households, but also by the firms in order to hire new workers. In this case, the demand for final good i is $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} (C_t + G_t H_t)$, where $H_t \equiv \int_0^1 H_t(j) dj$.

The aggregate output is defined as $Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$, which results in

$$Y_t = C_t + G_t H_t \quad (41)$$

Note that aggregate demand evolves according to the Euler equation for consumption and the hiring decisions of firms and the cost per hire.

On the supply side, one can determine the following relation between aggregate intermediate good production and aggregate final good output

$$X_t \equiv \int_0^1 X_t(i) di = Y_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di \quad (42)$$

where $D_t^p \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di \geq 1$ reflects efficiency losses resulting from unequal consumption and production of final goods.

The supply of intermediate goods represents the integrated supply of individual firms

$$X_t = \int_0^1 Y_t^l(j) dj = A_t N_t^{1-\alpha} \int_0^1 \left(\frac{N_t(j)}{N_t}\right)^{1-\alpha} dj \quad (43)$$

where $D_t^1 \equiv 1 / \int_0^1 \left(\frac{N_t(j)}{N_t}\right)^{1-\alpha} dj \geq 1$, capturing efficiency losses from the unequal production of intermediate goods under decreasing returns technology.

Note that the variation in prices and wages is a result of a staggering assumption of the Calvo price and wage setting. In the case where the prices may be adjusted in a synchronized way, technology and preferences imply equal prices goods and wages of all employees.

Combination of previous conditions (42) and (43) allows us to derive the following approximate up to a first-order relation

$$Y_t = A_t N_t^{1-\alpha} \quad (44)$$

3. Equilibrium dynamics: The effects of monetary policy and technology shocks

3.1 Steady state and calibration

I follow the strategy of calibration proposed by Gali (2010). It is assumed that there is no secular growth and zero inflation in the steady state. Also, the steady state is independent of the degree of price and wage stickiness and monetary policy, meaning that the hiring conditions under flexible and sticky nominal wages are the same. The calibrated parameters are summarized in Table 1.

Goods market clearing condition in the steady state

$$N^{1-\alpha} = C + \delta N \Gamma x^\gamma \quad (45)$$

Optimal hiring condition in the steady state

$$(1 - \beta(1 - \delta)) \Gamma x^\gamma = \xi \left(\frac{1-\alpha}{M^p(1-\tau)} N^{-\alpha} - \chi C L^\varphi \right) \quad (46)$$

A household's optimal participation condition in the steady state

$$(1 - x) \xi \psi \chi C L^\varphi = (1 - \xi) \Gamma x^{1+\gamma} \quad (47)$$

Definition of job finding rate and time spent on labor market activities

$$xU = (1 - x) \delta N \quad (48)$$

$$L = N + \psi U \quad (49)$$

Table 1. Calibration of parameters

Parameter	Value
A	1
D^p	1
D^w	1
N	0.59
F	0.62
U	0.03
UR (Unemployment rate)	0.048

x , quarterly	0.7
δ	0.12
α	1/3
β	0.99
φ	5
θ_p	0.75
θ_w	0.75
κ	[1; 0.85; 0.5; 0.01]
γ	1
$M^p(1 - \tau)$	1
G	0.045(W/P)
$\theta = \frac{\delta NG}{Y} = (0.045)\delta S^n$	0.0014
S^n (labor income share)	2/3
Γ	0.02
ξ	0.5
ψ	0.041
χ	15.5
ϕ_π	1.5
ϕ_y	0.5/4

Source: Gali (2010)

As the main purpose of the paper is normative analysis, I consider the baseline calibration that is consistent with efficient steady state, in as much as for that case the quadratic approximation of welfare losses is a reliable technique for optimal monetary policy analysis.

3.2 The effects of monetary policy and technology shocks

In this section the effects of the monetary and technological shocks are studied under different degrees of relative wage flexibility of new hires with parameter κ taking 4 different values, indicating a baseline case of no relative wage flexibility for new hires ($\kappa = 1$), which corresponds completely to the baseline model of Gali (2010), a moderate and high degree of relative wage flexibility for new hires ($\kappa = 0.85$ and $\kappa = 0.5$, accordingly) and case of completely flexible wages for new hires as compared to the wages of existing workers ($\kappa = 0$).

Figure 1 demonstrates the effect of a 0.25-percentage-point increase in the quarterly interest rate (which is equivalent to a 1-percentage-point increase of an annualized interest rate) on six macro variables. The monetary policy shock v is described by an autoregressive process with a degree of persistence $\rho_v = 0.5$ and standard deviation of 0.0025.

Figure 1 indicates that both the unemployment rate and employment volatility – and thus labor force volatility – is reduced with the introduction of a higher wage flexibility for new hires. The intuitive explanation is that higher wage flexibility in the case of a contractionary monetary policy shock leads to a quicker downward adjustment of wages, which discourages participation in the labor market and thus reduces labor market effort and unemployment. Employment reduction becomes slightly less, as with a higher wage flexibility of new hires comes a higher share of employment contracts, which is characterized by renegotiated wages at level that is lower than the level before the contractionary shock. Note that inflation volatility increases slightly in the short-run, which is explained by a higher volatility of the price markup caused by stronger reaction of more flexible wages for new hires, as was demonstrated in (14).

Figure 1. The effects of a contractionary monetary policy shock

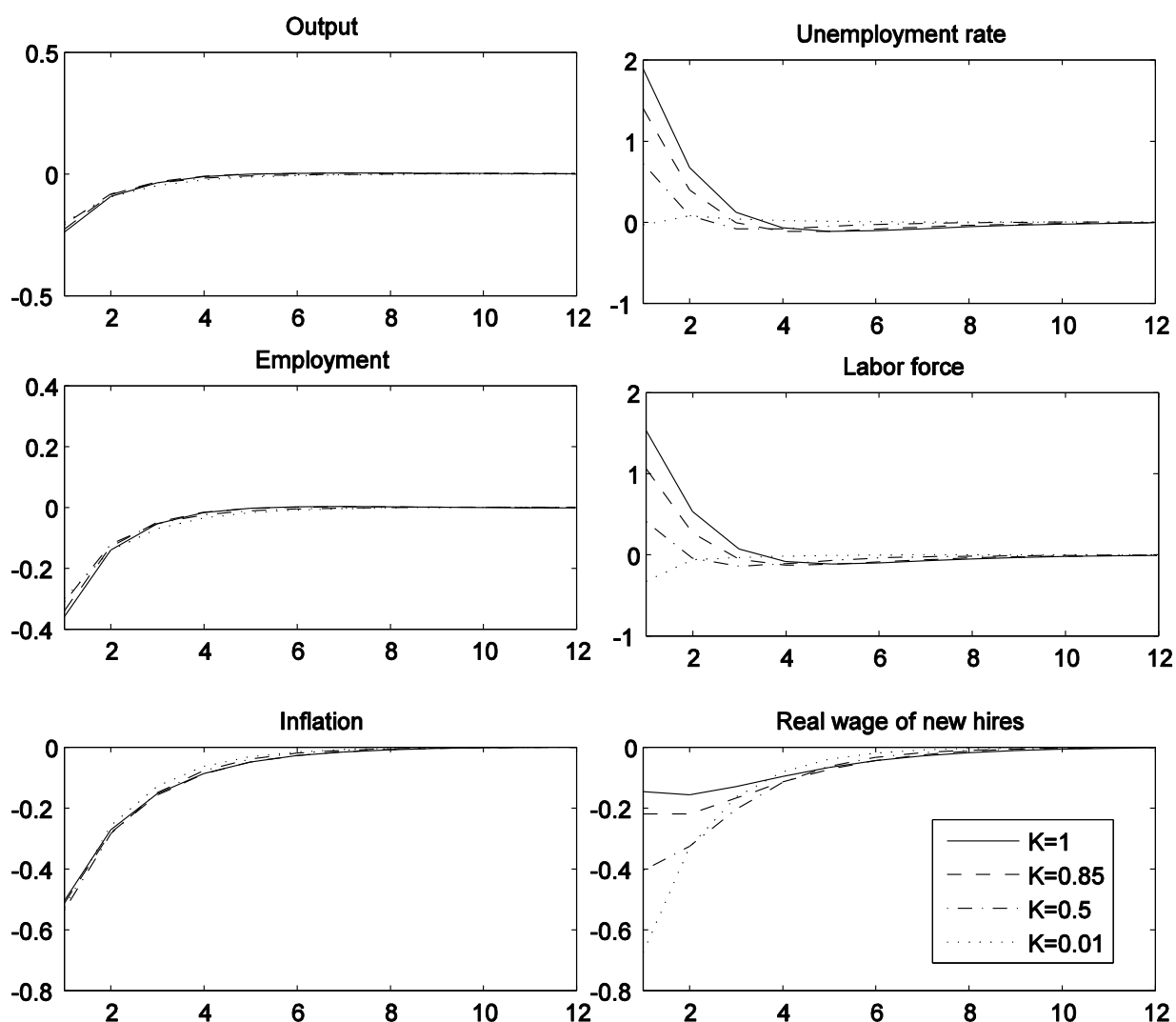


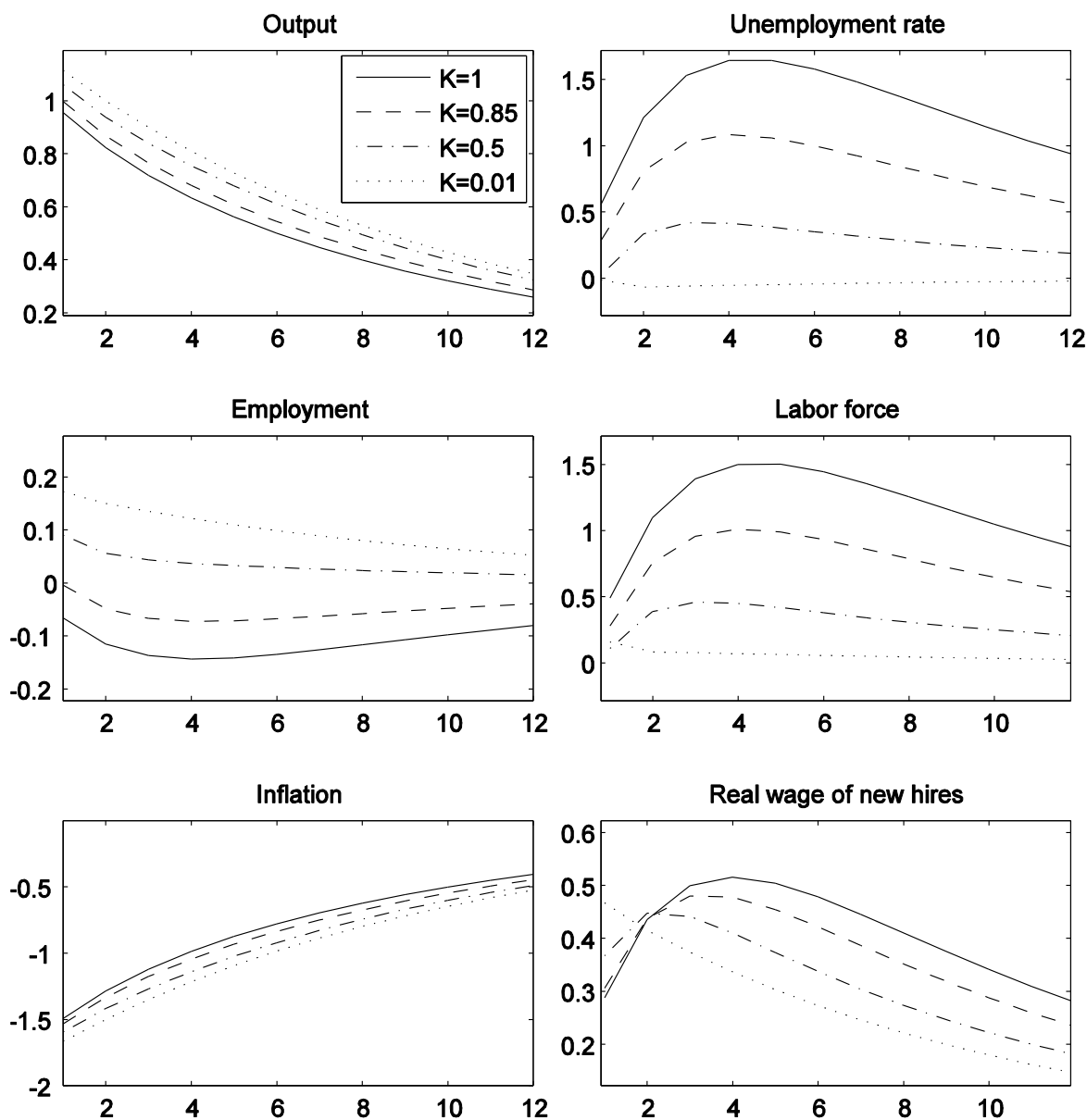
Figure 2 shows the response of the six macro-variables to an increase in the technology parameter by one percent. Technology parameter a_t is described by an autoregressive process with a degree of persistence $\rho_a = 0.9$ and a standard deviation equal to 0.01.

In this case, introducing a higher wage flexibility of new hires reduces the unemployment rate volatility, while the effect on employment volatility depends on the degree of wage flexibility of new hires. In case of relatively low degree of relative wage flexibility of new hires ($\kappa = 0.85$) the employment volatility reduces and then increases as the degree of relative wage flexibility is higher (for $\kappa = 0.5$ and $\kappa = 0$). The labor force volatility is reduced unambiguously with the introduction of higher wage flexibility.

The intuition of obtained results is as follows. As the wage of new hires becomes relatively more flexible in the short-run, wages becomes higher than in the more baseline case ($\kappa = 1$), but, at the

same time, it becomes less persistent. As the agents are forward looking, it means that a short-run increase in wages is not enough to increase labor market participation, given that in the medium and long-run the wage is systematically lower than for the baseline case ($\kappa = 1$). As for the hiring decision, as it is seen in the Figure 2, firms expect lower wages in the medium- and long-run, when the wages are more flexible, and hire more as compared to the baseline case of $\kappa = 1$.

Figure 2. The effects of expansionary technology shocks



Note that in the exercises above the implications of changes in the degree of wage flexibility of new hires are examined for the macroeconomic equilibrium reaction to the shocks *all else being equal*, including the same monetary policy rule (the Taylor type rule describe earlier). In later

sections it will be shown that for each level of relative wage flexibility of new hires, the central bank follows optimal monetary policy that itself depends on the relative wage flexibility of new hires.

4. Labor market frictions, nominal rigidities, and monetary policy design

4.1 The social planner's problem

The following problem is faced by the social planner who maximizes a representative household's utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \right)$$

subject to the resource constraint

$$C_t + \Gamma x_t^\gamma H_t = A_t N_t^{1-\alpha}$$

and definitions of time spent on labor activities, hiring, and the job-finding rate

$$L_t \equiv N_t + \psi U_t$$

$$H_t \equiv N_t - (1 - \delta)N_{t-1}$$

$$x_t \equiv \frac{H_t}{U_t/(1 - x_t)}$$

Note that the social planner is able to internalize the impact of hiring participation decisions on the job finding rate and, as a consequence, on the cost per hire. Firms and households, who considered the probability of hiring as being fully exogenous, ignored that effect.

The optimal hiring (or employment) condition according to the social planner's problem

$$MRS_t = MPN_t - (1 - \gamma)(G_t - (1 - \delta)E_t\{\Lambda_{t,t+1}G_{t+1}\}) \quad (50)$$

Optimal participation (or the level of unemployment) according to the social planner's problem

$$\psi MRS_t = \gamma \frac{x_t}{1-x_t} G_t \quad (51)$$

Two previous optimality conditions when considered at the steady state give an optimal steady state for hiring and participation levels

$$(1 + \gamma)(1 - \beta(1 - \delta))\Gamma x^\gamma = (1 - \alpha)N^{-\alpha} - \chi CL^\varphi \quad (52)$$

$$(1 - x)\psi\chi CL^\varphi = \gamma\Gamma x^{1+\gamma} \quad (53)$$

Note that the optimal hiring condition (46) and participation condition (47) in the decentralized market equilibrium when considered at steady state coincide with the planner's solutions (50) and (51) under the following conditions

$$M^p(1 - \tau) = 1 \quad (54)$$

This relation states that in an efficient steady state the market power of firms in the final good sector should be completely offset by the subsidy on purchases of intermediate goods.

$$\xi(1 + \gamma) = 1 \quad (55)$$

The above relation is called a Hosios condition and is presented in the same form as in Blanchard and Gali (2010), stating that the efficient steady state is characterized by a negative relation between the bargaining power of firm ξ and the elasticity of the hiring function γ (note, as shown earlier, in the matching function interpretation of the hiring costs $\gamma = \frac{1-\zeta}{\zeta}$ represents matching function's ($M(V_t, U_t^0) = V_t^\zeta U_t^{0^{1-\zeta}}$) relative elasticity of unemployment in the beginning of quarter $(1 - \zeta)$ with regard to the creation of a matching function elasticity of vacancies ζ). The relative bargaining power of a firm with respect to the bargaining power of a worker should maximize the efficiency of the matching process or minimizing the hiring costs and households' disutility from unemployment for the given level of hiring. The condition implies that the higher the elasticity of vacancy creation is in the matching function, the higher the bargaining power of firms should be, as in that case firms would bargain lower wages and will have a higher incentive to create new vacancies and households would have less incentive to send its members to the labor market. Such an allocation would avoid so-called congestion externality in the labor market that results from creation of excessive vacancies or excessive pool of unemployed individuals searching for a work.

4.2 Optimal monetary policy under higher wage flexibility of new hires

Extending Gali's framework by introducing the relative wage flexibility of new hires has an implication for the consumer welfare loss function, derived for the baseline model in Gali

(2010). Appendix E presents a sketch of the derivation of the extended welfare loss function, allowing for a higher wage flexibility of new hires. The resulting welfare loss function is the following

$$\mathbb{L} \equiv \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\varepsilon}{\lambda_p} (\pi_t^p)^2 + \frac{(1-\Phi)^2(1-\alpha)}{\alpha \lambda_w^*} (\pi_t^{new,w})^2 + \frac{(1+\varphi)(1-\Omega)N}{(1-\alpha)L} \left(\tilde{y}_t + \frac{(1-\alpha)\psi U}{N} \tilde{u}_t \right)^2 \right)$$

$$\lambda_w^* = \frac{(1-\kappa\theta_w)^2(1-\theta_w\kappa^2\beta)}{\theta_w\kappa^2(1-\theta_w)}$$

where \tilde{y}_t and \tilde{u}_t denote gaps of output and unemployment with regard to constrained efficiency levels or natural levels, $\Omega = 1 - \frac{MRS}{MPN} = \frac{B(1+\gamma)}{MPN}$ measures the gap between a steady state marginal rate of substitution and a marginal product of labor.

The main difference as compared to Galí's baseline model is in stressing that the efficiency losses appear in the model as a result of the staggering in new hire's wages and not average wages and, second, that the coefficient of inflation for wages of new hires wage in the loss function depends on the degree to which the wages of new hires are more flexible than the wages of existing workers κ . In Galí's model the average wages of new hires and average wages of all employees were equivalent, as the new hires completely inherited the stickiness of the existing employees. An extension proposed here allows higher wage flexibility of new hires, thus reducing the effect of transferring the wage stickiness of existing workers to that of new workers.

Figure 3. Equilibrium reaction of variables to expansionary technology shocks under optimal monetary policy

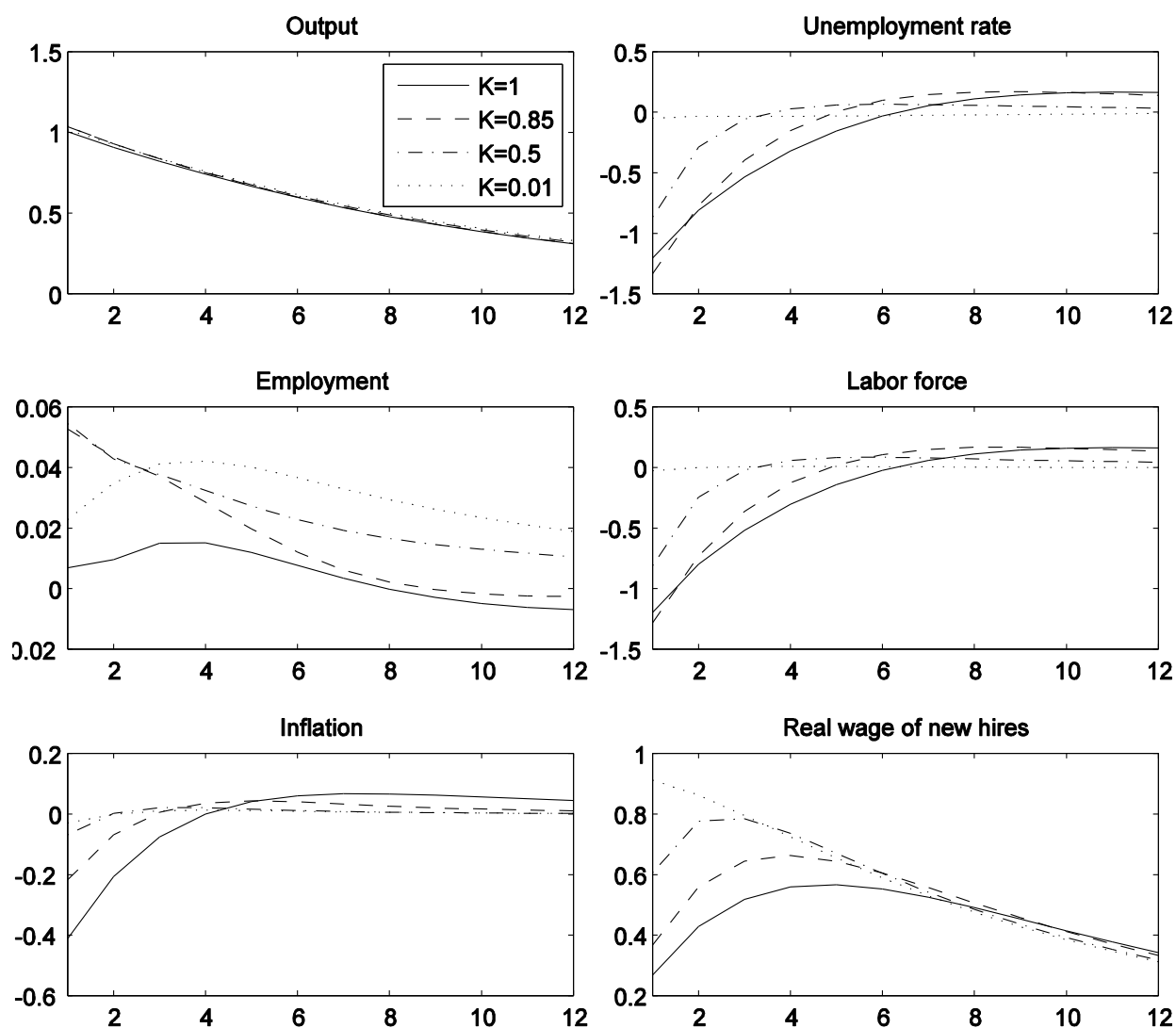


Figure 3 illustrates the equilibrium reactions of macroeconomic variables under optimal monetary policy and for different levels of relative wage flexibility of new hires κ . A higher wage flexibility of new hires modifies the optimal monetary policy in a way that decreases the volatility of price inflation at the cost of higher volatility of wage inflation. Such a modification in optimal policy results from the fact that, under higher flexibility of new wages, the losses from wage inflation become lower, while the losses from inflation volatility remain the same, which motivates the central bank to reduce the volatility of price inflation by increasing the volatility of wage inflation in such a way that is beneficial for households.

Note that under an extreme case of flexible wages of new hires, or $\kappa = 0$, optimal policy in response to a real shock is a full stabilization of price inflation, which is usually called a strict

inflation targeting policy. Additionally, Figure 3 illustrates that even a moderately higher wage flexibility of new hires for $\kappa = 0.85$, which is consistent with some survey-based empirical studies (e.g. ECB final report of wage dynamics network, 2009), reduces the optimal volatility of price inflation twofold.

5. Conclusions

A higher wage flexibility for new hires was introduced as an extension of the baseline model in Gali (2010), combining the New Keynesian monetary analysis framework with labor market frictions. It was shown that the possibility of higher wage flexibility of new hires has an implication on crucial labor market decisions made by households and firms, and also on the form of the social welfare loss function that is used to evaluate alternative monetary policies.

In an extended model, the wages within a firm are no longer constant and, thus, the hiring decision of firms are based on the expected average wage of new hires in each firm, taking into account the fact that a certain share κ of new hires conduct wage negotiations in the current quarter, while the rest of the firms $(1 - \kappa)$ get the wage determined by past negotiations in previous quarters. A higher wage flexibility of new hires plays a role for the hiring decisions of firms as the average wage of new hires adjusts to macroeconomic shocks quicker than the wage of existing workers.

A higher wage flexibility of new hires also has an implication for optimal labor market participation decisions made by those households that expect a higher probability of receiving a renegotiated wage and change their labor participation decisions.

A New Keynesian Wage Phillips Curve derived by Gali (2010) was reformulated in terms of the wage inflation of new hires, as the wage of new hires is crucial in determining firm and household decisions and for welfare analysis. A higher wage flexibility of new hires makes the reaction of wage inflation for new hires quicker in response to macroeconomic shocks. Note that the introduction of higher wage flexibility of new hires did not change the planning horizon for the bargaining process that is determined by the probability of overall employee negotiations and the exogenous probability of job destruction. Although the planning horizon of negotiations remained the same as in the baseline model of Gali (2010), the present value of the future marginal revenue of products (that determines the expected targeted wages for negotiations) has changed as a result of the fact that some part of future new employees will be hired at a renegotiated wage, while existing workers may not renegotiate their wages for long time.

It was emphasized that the wages of new hires, as opposed to wages of all existing workers, determine the welfare losses from staggered adjustment of wages, and the welfare loss function was derived taking into account the degree of the relative wage flexibility of new hires, calibrated with parameter κ .

The optimal monetary policy in the presence of a higher wage flexibility of new hires is modified to capture a higher incentive to make inflation more stable, thereby ignoring the incentive to facilitate an adjustment of real wages in response to real shocks. Note that even under conservative estimates of the possible wage flexibility of new hires, $\kappa = 0.85$ (when 15% of new hires renegotiate their wage in quarters when there are no overall firm negotiations), the optimal price volatility reduces twofold. Therefore, the possibility of higher wage flexibility of new hires provides support toward propositions of more strict inflation targeting, as opposed to flexible inflation targeting.

Appendix A. Derivation of the average wage of new hires when the last overall employee negotiation was k periods ago

A firm's average wage of new hires in quarter $t + k$, where the last overall employee negotiations for all workers happened in quarter t is

$$\bar{\omega}_{t+k|t}^{new} = \kappa \bar{\omega}_{t+k-1|t}^{new} + (1 - \kappa) \omega_{t+k}^*$$

Backward iteration of the above equation results in

$$\bar{\omega}_{t+k|t}^{new} = \kappa^k \omega_t^* + (1 - \kappa) \sum_{q=1}^k \kappa^{k-q} \omega_{t+q}^*$$

Appendix B. Derivation of the New Keynesian Wage Phillips curve for the case of higher wage flexibility of new hires

Substituting targeted wage equation (36) into Nash optimal sharing rule wage equation (33) gives

$$\begin{aligned} \hat{w}_t^* = & (1 - \beta(1 - \delta)\theta_w) E_t \sum_{k=0}^{\infty} (\beta(1 - \delta)\theta_w)^k \left\{ \hat{\omega}_{t+k}^{tar} \right. \\ & \left. + (1 - \Upsilon)(1 - \Phi) \left(\kappa^k \hat{w}_t^* + (1 - \kappa) \sum_{q=1}^k \kappa^{k-q} \hat{w}_{t+q}^* - \bar{w}_{t+k}^{new} \right) + \hat{p}_{t+k} \right\} \end{aligned}$$

$$\begin{aligned}
&= (1 - \beta(1 - \delta)\theta_w)E_t \sum_{k=0}^{\infty} (\beta(1 - \delta)\theta_w)^k \{\widehat{\omega}_{t+k}^{tar} + \hat{p}_{t+k} - (1 - \Upsilon)(1 - \Phi)\bar{w}_{t+k}^{new}\} \\
&\quad + (1 - \beta(1 - \delta)\theta_w)(1 - \Upsilon)(1 - \Phi) \frac{\widehat{w}_t^*}{1 - \beta(1 - \delta)\kappa\theta_w} \\
&\quad + (1 - \beta(1 - \delta)\theta_w)E_t \sum_{k=1}^{\infty} (\beta(1 - \delta)\theta_w)^k \{(1 - \Upsilon)(1 - \Phi)(1 - \kappa)\widehat{w}_{t+k}^*\}
\end{aligned}$$

The above expression is a solution to the following difference equation with expectations

$$\begin{aligned}
\widehat{w}_t^* &= \left(\frac{1 - \beta(1 - \delta)\kappa\theta_w}{1 - \beta(1 - \delta)\kappa\theta_w - (1 - \beta(1 - \delta)\theta_w)(1 - \Upsilon)(1 - \Phi)} \right) \\
&\quad (1 - \beta(1 - \delta)\theta_w)(\widehat{\omega}_t^{tar} + \hat{p}_t - (1 - \Upsilon)(1 - \Phi)\bar{w}_t^{new}) \\
&\quad + \left(\frac{1 - \beta(1 - \delta)\kappa\theta_w}{1 - \beta(1 - \delta)\kappa\theta_w - (1 - \beta(1 - \delta)\theta_w)(1 - \Upsilon)(1 - \Phi)} \right) \\
&\quad \left((1 - \beta(1 - \delta)\theta_w)\{(1 - \Upsilon)(1 - \Phi)(1 - \kappa)\} - \frac{(1 - \beta(1 - \delta)\theta_w)(1 - \Upsilon)(1 - \Phi)}{1 - \beta(1 - \delta)\kappa\theta_w} + 1 \right) \beta(1 \\
&\quad - \delta)\theta_w E_t \widehat{w}_{t+1}^*
\end{aligned}$$

Additional algebraic manipulations allow us to write the above equation in terms of differences of the type $\widehat{w}_{t+k}^* - \bar{w}_{t+k-1}^{new}$ for $k = 0, 1$

$$\begin{aligned}
\widehat{w}_t^* - \bar{w}_{t-1}^{new} &= \left(\frac{1 - \beta(1 - \delta)\kappa\theta_w}{1 - \beta(1 - \delta)\kappa\theta_w - (1 - \beta(1 - \delta)\theta_w)(1 - \Upsilon)(1 - \Phi)} \right) \\
&\quad (1 - \beta(1 - \delta)\theta_w)(\widehat{\omega}_t^{tar} + \hat{p}_t - \bar{w}_t^{new} + \bar{w}_t^{new} - (1 - \Upsilon)(1 - \Phi)\bar{w}_t^{new}) \\
&\quad + \left(\frac{1 - \beta(1 - \delta)\kappa\theta_w}{1 - \beta(1 - \delta)\kappa\theta_w - (1 - \beta(1 - \delta)\theta_w)(1 - \Upsilon)(1 - \Phi)} \right) \\
&\quad \left((1 - \beta(1 - \delta)\theta_w)\{(1 - \Upsilon)(1 - \Phi)(1 - \kappa)\} - \frac{(1 - \beta(1 - \delta)\theta_w)(1 - \Upsilon)(1 - \Phi)}{1 - \beta(1 - \delta)\kappa\theta_w} + 1 \right) \beta(1 \\
&\quad - \delta)\theta_w E_t (\widehat{w}_{t+1}^* - \bar{w}_t^{new} + \bar{w}_t^{new}) - \bar{w}_{t-1}^{new}
\end{aligned}$$

Using the relation between wage inflation and average wages $\widehat{w}_t^* - \bar{w}_{t-1}^{new} = \frac{\pi_t^w}{(1 - \kappa\theta_w)}$, which is a reformulated version of (38) in terms of the wage inflation rate, the following modified New Keynesian Wage Phillips curve is derived

$$\pi_t^w = \frac{A_1 B_1}{\kappa \theta_w} E_t(\pi_{t+1}^w) + \frac{(1 - \kappa \theta_w)}{\kappa \theta_w} A_1 (1 - \beta(1 - \delta) \theta_w) (\bar{\omega}_t^{tar} - \bar{\omega}_t^{new}) + \frac{(1 - \kappa \theta_w)}{\kappa \theta_w} C_1 \bar{\omega}_t^{new}$$

$$A_1 = \frac{(1 - \beta(1 - \delta) \kappa \theta_w)}{1 - \beta(1 - \delta) \kappa \theta_w - (1 - \beta(1 - \delta) \theta_w) (1 - \gamma) (1 - \phi)}$$

$$B_1 = \left((1 - \beta(1 - \delta) \theta_w) (1 - \gamma) (1 - \phi) (1 - \kappa) - \frac{(1 - \beta(1 - \delta) \theta_w) (1 - \gamma) (1 - \phi)}{1 - \beta(1 - \delta) \kappa \theta_w} + 1 \right) \beta(1 - \delta) \theta_w$$

$$C_1 = \frac{(1 - \beta(1 - \delta) \theta_w) (1 - \gamma) (1 - \phi) ((1 - \kappa) \kappa \beta(1 - \delta) \theta_w \beta(1 - \delta) \theta_w)}{1 - \beta(1 - \delta) \kappa \theta_w - (1 - \beta(1 - \delta) \theta_w) (1 - \gamma) (1 - \phi)}$$

The obtained equation contains nominal wage term $\bar{\omega}_t^{new}$, which does not have a constant steady state value and thus an equation with this term could not be used to solve the model using standard programs such as Dynare. Thus, I impose a restriction of $C_1=0$ when solving the model in Dynare. Note that under the proposed calibration C_1 is very close to zero and the model reduces to the standard New Keynesian Wage Phillips, derived by Galí (2010) in the absence of a higher wage flexibility of new hires (for $\kappa = 1$).

Appendix C. Derivation of the economy's average wage of new hires

By analogy with the standard Calvo model, the evolution of the distribution of wages in a new setting with a higher flexibility of new wages is derived here.

For each period after a shock (when all wages are not equal and do not represent a steady state atom value) the distribution $\{\bar{\omega}_t^{new}(j)\}$ may be described by some pdf function $f_t(\bar{\omega}^{new})$.

Then the share of firms $1 - \theta_w$ represents an atom in $f_t(\bar{\omega}^{new})$, with all firms that readjust the wage set it equal value of $\bar{\omega}^{new} = w_t^*$. There is a share of $1 - \theta_w$ of such firms in the entire distribution of firms.

Then the remaining share of firms θ_w fix the share κ of new wages at a level that was set during the last their overall employee negotiations and conduct new negotiations with $(1 - \kappa)$ new workers. Thus, the mean value of the wage of new hires is

$$\bar{\omega}_t^{new} \equiv E_j \bar{\omega}_t^{new}(j)$$

and the average wage for new hires changes (or wage inflation of new hires)

$$\begin{aligned}
\bar{\omega}_t^{new} - \bar{\omega}_{t-1}^{new} &= E_j(\bar{\omega}_t^{new}(j) - \bar{\omega}_{t-1}^{new}) \\
&= \theta_w E_j(\kappa \bar{\omega}_{t-1}^{new}(j) + (1 - \kappa)w_t^* - \bar{\omega}_{t-1}^{new}) + (1 - \theta_w)E_j(w_t^* - \bar{\omega}_{t-1}^{new}) = \\
&= \theta_w(\kappa E_j \bar{\omega}_{t-1}^{new}(j) + (1 - \kappa)w_t^* - \bar{\omega}_{t-1}^{new}) + (1 - \theta_w)(w_t^* - \bar{\omega}_{t-1}^{new}) = \\
&= \theta_w(\kappa \bar{\omega}_{t-1}^{new} + (1 - \kappa)w_t^* - \bar{\omega}_{t-1}^{new}) + (1 - \theta_w)(w_t^* - \bar{\omega}_{t-1}^{new}) = \\
&= \theta_w(1 - \kappa)(w_t^* - \bar{\omega}_{t-1}^{new}) + (1 - \theta_w)(w_t^* - \bar{\omega}_{t-1}^{new}) = \\
&= (\theta_w - \kappa\theta_w + 1 - \theta_w)(w_t^* - \bar{\omega}_{t-1}^{new}) = \\
&= (1 - \kappa\theta_w)(w_t^* - \bar{\omega}_{t-1}^{new})
\end{aligned}$$

which is equivalent to (38)

$$\bar{w}_t^{new} = \kappa\theta_w \bar{w}_{t-1}^{new} + (1 - \kappa\theta_w)w_t^*$$

Appendix D. Participation condition under higher wage flexibility of new hires

The log-linearization of (27) requires the log-linearization of $Q_t \equiv \int_0^1 \frac{H_t(z)}{H_t} S_t^H(z) dz$, which is up to first-order approximation and gives

$$\begin{aligned}
Q_t &\cong \int_0^1 S_t^H(z) dz \\
&= (1 - \theta_w) \bar{S}_{t|t}^H \\
&\quad + \theta_w \left((1 - \theta_w) \bar{S}_{t|t-1}^H + \theta_w \left((1 - \theta_w) \bar{S}_{t|t-2}^H + \theta_w \left((1 - \theta_w) \bar{S}_{t|t-3}^H + \dots \right) \right) \right) \\
&= (1 - \theta_w) \bar{S}_{t|t}^H + \theta_w (1 - \theta_w) (\bar{S}_{t|t-1}^H + \theta_w \bar{S}_{t|t-2}^H + \theta_w^2 \bar{S}_{t|t-3}^H + \dots)
\end{aligned}$$

where

$$\bar{S}_{t|t-k}^H = \frac{\bar{W}_{t+k|t}^{new}}{P_t} - MRS_t + (1 - \delta) E_t \left\{ \Lambda_{t,t+1} \left((1 - \theta_w) \bar{S}_{t+1|t+1}^H + \theta_w \bar{S}_{t+1|t}^H \right) \right\}$$

As was shown in Appendix A:

$$\bar{\omega}_{t+k|t}^{new} = \kappa^k \omega_t^* + (1 - \kappa) \sum_{q=1}^k \kappa^{k-q} \omega_{t+q}^*$$

For average wage calculation, it is more convenient to use a different notation showing the average wage of new hires at t given that the last overall employee negotiation occurred at $t - k$

$$\bar{\omega}_{t|t-k}^{new} = \kappa^k \omega_{t-k}^* + (1 - \kappa) \sum_{q=1}^k \kappa^{k-q} \omega_{t-k+q}^*$$

Thus

$$\bar{S}_{t|t-k}^H = \kappa^k S_{t|t-k}^H + (1 - \kappa) \sum_{q=1}^k \kappa^{k-q} S_{t|t-k+q}^H$$

Then

$$Q_t \cong (1 - \kappa\theta_w) \sum_{q=0}^{\infty} (\kappa\theta_w)^q S_{t|t-q}^H$$

$$Q_t \cong (1 - \kappa\theta_w) \sum_{q=0}^{\infty} (\kappa\theta_w)^q (S_{t|t-q}^H - S_{t|t}^H + S_{t|t}^H)$$

Applying optimal hiring condition (11) and optimal sharing rule (31) results in

$$Q_t \cong \frac{(1 - \xi)}{\xi} G_t + (1 - \kappa\theta_w) \sum_{q=0}^{\infty} (\kappa\theta_w)^q (S_{t|t-q}^H - S_{t|t}^H)$$

Using the definitions $S_{t|t-q}^H$ and $S_{t|t}^H$ results in

$$S_{t|t-q}^H - S_{t|t}^H = \frac{W_{t-q}^*}{P_t} - \frac{W_t^*}{P_t} + \theta_w(1 - \delta) E_t \{ \Lambda_{t,t+k} (S_{t+1|t-q}^H - S_{t+1|t}^H) \}$$

Applying the standard forward-looking solution results in

$$S_{t|t-q}^H - S_{t|t}^H = \frac{W_{t-q}^* - W_t^*}{P_t} E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} (\theta_w(1 - \delta))^k \frac{P_t}{P_{t+k}}$$

Substituting the expression obtained above into Q_t gives

$$Q_t \cong \frac{(1 - \xi)}{\xi} G_t + \frac{(\bar{W}_t^{new} - W_t^*)}{P_t} E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} (\theta_w(1 - \delta))^k \frac{P_t}{P_{t+k}}$$

A relation between the average wage and wage inflation is derived:

$$\bar{w}_t^{new} = (1 - \kappa\theta_w)w_t^* + \kappa\theta_w\bar{w}_{t-1}^{new}$$

$$\pi_t^w \equiv \bar{w}_t^{new} - \bar{w}_{t-1}^{new} = (1 - \kappa\theta_w)(w_t^* - \bar{w}_{t-1}^{new})$$

$$\bar{w}_t^{new} - w_t^* = (1 - \kappa\theta_w)w_t^* - w_t^* + \kappa\theta_w\bar{w}_{t-1}^{new} = \kappa\theta_w(\bar{w}_{t-1}^{new} - w_t^*)$$

Thus,

$$\bar{w}_t^{new} - w_t^* = -\frac{\kappa\theta_w}{(1 - \kappa\theta_w)}\pi_t^w$$

Approximate relation holds

$$\left(\frac{\bar{W}_t^{new} - W_t^*}{W}\right) \cong -\frac{\kappa\theta_w}{(1 - \kappa\theta_w)}\pi_t^{new,w}$$

As a result

$$Q_t \cong \frac{(1 - \xi)}{\xi}G_t - \frac{\kappa\theta_w}{(1 - \kappa\theta_w)}\pi_t^{new,w}\frac{W}{P_t}E_t\sum_{k=0}^{\infty}\Lambda_{t,t+k}(\theta_w(1 - \delta))^k\frac{P_t}{P_{t+k}}$$

Finally, the first-order approximation of the above expression gives

$$\hat{q}_t = \hat{g}_t - \frac{\xi}{(1 - \xi)G}\left(\frac{1}{1 - \beta\theta_w(1 - \delta)}\right)\left(\frac{W}{P}\right)\frac{\kappa\theta_w}{(1 - \kappa\theta_w)}\pi_t^{new,w}$$

Thus, the log-linearized optimal participation condition is

$$\hat{c}_t + \varphi\hat{l}_t = \frac{1}{1 - x}\hat{x}_t + \hat{g}_t - \frac{\xi}{(1 - \xi)G}\left(\frac{1}{1 - \beta\theta_w(1 - \delta)}\right)\left(\frac{W}{P}\right)\frac{\kappa\theta_w}{(1 - \kappa\theta_w)}\pi_t^{new,w}$$

Appendix E. Derivation of the welfare loss function under higher wage flexibility of new hires

The following derivation is based on Appendix 4 of Gali (2010). The loss function is derived by combining a second-order expansion of the utility function of the representative household around the constrained-efficient allocation – meaning that any variable x_t , \tilde{x}_t is a gap of the form $\tilde{x}_t \equiv x_t - x_t^{constr.efficient}$ – and the resource constraint. Note that the constraint efficiency allocation is equivalent to flexible wages and prices equilibrium under an efficient steady state or that is usually referred to as natural equilibrium.

$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{U}_t \cong -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\Theta} (d_t^p + d_t^w) + \frac{1}{2} (1+\varphi) \chi L^{1+\varphi} \hat{l}_t^2 \right)$$

where

$$d_t^p \cong \frac{\varepsilon}{2} \text{var}_i(p_t(i))$$

$$d_t^w \cong \frac{(1-\Phi)^2(1-\alpha)}{2\alpha} \text{var}_j\{\bar{\omega}_t^{new}(j)\}$$

The Calvo pricing environment implies

$$\sum_{t=0}^{\infty} \beta^t \text{var}_i(p_t(i)) = \frac{\theta_p}{(1-\theta_p)(1-\beta\theta_p)} \sum_{t=0}^{\infty} \beta^t (\pi_t^p)^2$$

Modified for the case of higher wage flexibility of new hires, the Calvo wage setting environment implies

$$\sum_{t=0}^{\infty} \beta^t \text{var}_j\{\bar{\omega}_t^{new}(j)\} = t.i.p. + \frac{\theta_w \kappa^2 (1-\theta_w)}{(1-\kappa\theta_w)^2 (1-\theta_w \kappa^2 \beta)} \sum_{t=0}^{\infty} \beta^t (\pi_t^w)^2$$

The relationship above is derived by analogy of the derivation of the relationship between the present value of current and future variances in wage inflation to the present value of current and future wage inflations, as in Woodford (2003, Chapter 6) for a classical Calvo price setting. The derivation was modified to account for the higher wage flexibility of new hires.

Combining the previous results and letting $\mathbb{L} \equiv -E_0 \sum_{t=0}^{\infty} \beta^t \tilde{U}_t \frac{C}{Y}$ denote the utility losses expressed as a share of steady-state GDP results in

$$\begin{aligned} \mathbb{L} \equiv \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t & \left(\frac{\varepsilon}{\lambda_p} (\pi_t^p)^2 + \frac{(1-\Phi)^2(1-\alpha)}{\alpha \lambda_w^*} (\pi_t^{new,w})^2 \right. \\ & \left. + \frac{(1+\varphi)(1-\Omega)N}{L(1-\alpha)} \left(\tilde{y}_t + \frac{(1-\alpha)\psi U}{N} \tilde{u}_t \right)^2 \right) \end{aligned}$$

where

$$\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$$

$$\lambda_w^* = \frac{(1 - \kappa\theta_w)^2(1 - \theta_w\kappa^2\beta)}{\theta_w\kappa^2(1 - \theta_w)}$$

$$1 - \Omega = \frac{MRS}{MPN} = 1 - \frac{B(1 + \gamma)}{MPN}$$

References

- Blanchard, Olivier & Jordi Galí, 2010. "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics*, American Economic Association, vol. 2(2), pages 1-30, April.
- Calvo, Guillermo A., 1983. "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, Elsevier, vol. 12(3), pages 383-398, September.
- Erceg, Christopher J. & Henderson, Dale W. & Levin, Andrew T., 2000. "Optimal monetary policy with staggered wage and price contracts," *Journal of Monetary Economics*, Elsevier, vol. 46(2), pages 281-313, October.
- European Central Bank, 2009. Wage dynamics in Europe: Final report of the wage dynamics network. http://www.ecb.europa.eu/home/pdf/wdn_finalreport_dec2009.pdf
- Galí, Jordi, 2010. "Monetary Policy and Unemployment," *Handbook of Monetary Economics*, in: Benjamin M. Friedman & Michael Woodford (ed.), *Handbook of Monetary Economics*, edition 1, volume 3, chapter 10, pages 487-546 Elsevier.
- Gertler, Mark & Antonella Trigari, 2009. "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Journal of Political Economy*, University of Chicago Press, vol. 117(1), pages 38-86, 02.
- Galuscak, Kamil & Keeney, Mary & Nicolitsas, Daphne & Smets, Frank & Strzelecki, Pawel & Vodopivec, Matija, 2012. "The determination of wages of newly hired employees: Survey evidence on internal versus external factors," *Labour Economics*, Elsevier, vol. 19(5), pages 802-812.
- Haefke, Christian & Sonntag, Marcus & van Rens, Thijs, 2008. "Wage Rigidity and Job Creation," *IZA Discussion Papers 3714*, Institute for the Study of Labor (IZA).
- Krueger, Alan B. & Mueller, Andreas I., 2008. "The Lot of the Unemployed: A Time Use Perspective," *IZA Discussion Papers 3490*, Institute for the Study of Labor (IZA).

Merz, Monika, 1995. "Search in the labor market and the real business cycle," *Journal of Monetary Economics*, Elsevier, vol. 36(2), pages 269-300, November.

Pissarides, Christopher A., 2000. "Equilibrium Unemployment Theory, 2nd Edition," MIT Press Books, The MIT Press, edition 1, volume 1, number 0262161877, October.

Pissarides, Christopher A. 2009. "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?," *Econometrica*, Econometric Society, vol. 77(5), pages 1339-1369, 09.

Thomas, Carlos, 2008. "Search and matching frictions and optimal monetary policy," *Journal of Monetary Economics*, Elsevier, vol. 55(5), pages 936-956, July.

Walsh, Carl E., 2005. "Labor Market Search, Sticky Prices, and Interest Rate Policies," *Review of Economic Dynamics*, Elsevier for the Society for Economic Dynamics, vol. 8(4), pages 829-849, October.

Woodford (2003), *Interest and Prices: Foundations of Theory of Monetary Policy*, Princeton University Press.

Nikolay Y. Ushakov

National Research University Higher School of Economics (Moscow, Russia). Laboratory for Macroeconomics Analysis. Junior Research Fellow;

E-mail: nickushak@mail.ru, Tel. +7 (968) 941-05-32

Any opinions or claims contained in this Working Paper do not necessarily reflect the views of HSE.

© Ushakov, 2013