Victor V. Gorbatov

ONTΟLOGICAL ARGUMENT: TWO-DIMENSIONAL VERSION

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The ontological argument has been reintroduced into the area of analytical metaphysics in 20th century due to the development of modal logic and possible worlds semantics. However, there are few attempts to approach this argument with two-dimensional possible worlds framework. The present paper provides a new two-dimensional interpretation of Anselm's proof in terms of modal logic with an actuality operator (AML). It is argued that the standard modal explications of Anselm’s enigmatic concept “greater” and its connection with the concept of “actuality” have some essential shortcomings compared with the two-dimensional approach.

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1. Introduction

The ontological argument (OA) has always been a stumbling block for a variety of philosophical and theological doctrines. The history of this argument and the related controversy demonstrates, on the one hand, an inherent desire of metaphysicians to find a rationally acceptable way of transcending the phenomenal world. On the other hand, it shows that even a slightest mistake in this way can cause a great confusion. Moreover, the fundamental question of the very existence of such a way still remains open.

The main purpose of this paper is to clarify the original Anselms intentions and to demonstrate a peculiar feature of his argument: how its logical validity is coupled with its semantic vagueness. In the beginning, we consider a common understanding of OA and outline the logical evolution from non-modal interpretation to modal one. We go on with observing that standard modal explications of Anselm’s argument have some essential shortcomings compared with the two-dimensional approach. In the last chapters the basic ideas of two-dimensional interpretation of OA are represented together with its new formalization.

2. Proslogion: “single argument” or “a series of arguments”?

Proslogion notoriously contains several versions of OA, the most important of them can be found in Chapters II, III, and XV. Commentators have no consensus on where the main proof is located and how all these versions relate to each other. Some authors [e.g., Charlesworth 1965, Barnes 1972, Anscombe 1993] claim that the main proof is in Proslogion II, and that the rest of the work draws out corollaries of it. Most of them believe that the argument in Proslogion II can be interpreted in terms of theory of reference and therefore they use primarily the “referential vocabulary” – names, descriptions, quantified noun phrases, indexicals, etc.

Other commentators [e.g., Malcolm 1960, Hartshorne 1961, Plantinga 1974] believe that the main argument is in Proslogion III, and that the proof in Proslogion II is just a preliminary analysis of the issue. They consider Anselm’s argument as a modal proof and use mainly the “modal vocabulary” – necessity, possibility, contingency, essential properties, etc.

Yet another approach is advocated by authors who take seriously Anselm’s idea of God being beyond the limits of human thought [e.g., Priest 1995, Jacquette 1997]. They prefer the “intensional vocabulary” – conceivability, scrutability, apriority, comprehension, etc. and thus indicate the primary role of Proslogion XV.

3 In English translation of the Proslogion term “argument” in a number of cases stands for Anselm’s “argumentatio” rather than “argumentum”, and it complicates the whole picture.
There are also interpreters who take *Proslogion II and Proslogion III* to be a single argument [e.g., La Croix 1972]. As Étienne Gilson put it, “the *Proslogion*’s proof, even though it reaches its object in chapter II, has its full intelligibility in chapter III. (…) This is why chapter III of the *Proslogion* must not, under any pretext, be considered as separable from chapter II, nor the inverse.” [Gilson 1934, p.13, trans. Sadler 2006] Moreover, some authors maintain that nearly the whole text, including *Proslogion XV* and even *Prooemium*, should be treated as a single argument [see Campbell 1976; Herrera 1979]. This point of view is supported by those commentators who consider it necessary to take into account the implications of Anselm’s larger corpus for the argument and proof [see Sadler 2006].

The question of wherever or not Anselm regarded himself as offering several different proofs remains open. On the one hand, in the main chapters of *Proslogion*, he introduces a number of formulations which seem to be quite different in their logical form. On the other hand, in the *Prooemium*, he clearly expressed his aspiration to replace a number of interconnected arguments from his previous and much longer work, the *Monologion*, with a “single argument” (*unum argumentum*):

> When I considered that this work [the *Monologion*] was put together by the interweaving of a great number of arguments, I began to ask myself whether there might not perhaps be found some one argument which should have no need of any other argument beside itself to prove it, and might suffice by itself to demonstrate that God really exists and is the Supreme Good, which needeth nothing beside itself to give it being or well-being, but without which nothing else can have either the one or the other; and whereof all other things are true which we believe concerning the divine essence. [*Proslogion, prooemium*4]

This passage indicates that God’s existence and God’s attributes, in Anselm’s thought, were *intimately connected*. That is why he intended to present a single reasoning, where the proof of the existence of God would come along with an explication of various features about his nature. As Sadler rightly remarks, “the proof and its linchpin Q [*quو majus cogitari neguit*] are only given their content through the unfolding or unpacking of Q in the rest of the argument.” [Sadler 2006, pp. 8-9]. It follows that in spite of the variety of formulations we should find an approach to the argument that will enable us to combine *tools of proving* God’s existence with *tools of conceiving* him.

At the same time, the required approach should combine all the three vocabularies, which were used to clarify OA: referential, modal and intensional. Now we are going to trace the

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4 All translations of Anselm are taken from *The Devotions of St. Anselm* (1903), by Clement C. J. Webb.
development of this argument from non-modal interpretation to modal one, and show that a natural consequence of this development should be an appeal to the two-dimensional modal logic and semantics.

3. From OA to MOA

Anselm starts with distinguishing two modes of existence – in mind (in intellectu) and in reality (in re). Assuming that the latter is “greater” than the former, Anselm consequently points out that the specific object “id quo majus cogitari neguit” must have existence not only of the first, but also of the second kind.

Thus even the fool is certain that something exists, at least in his understanding, than which nothing greater can be conceived; because, when he hears this mentioned, he understands it, and whatsoever is understood, exists in the understanding. And surely that than which no greater can be conceived cannot exist only in the understanding. For if it exist indeed in the understanding only, it can be thought to exist also in reality; and real existence is more than existence in the understanding only. If then that than which no greater can be conceived exists in the understanding only, then that than which no greater can be conceived is something a greater than which can be conceived: but this is impossible. Therefore it is certain that something than which no greater can be conceived exists both in the understanding and also in reality. [Proslogion II]

This proof had been repeatedly debated by medieval scholastics, as well as by great Modern metaphysicians such as Descartes, Spinoza, and Leibniz. Kant coined the name for this argument: he called it the “ontological argument” (OA) because of its construction in a priori ontological terms. He also proposed a substantial objection to OA based on the claim that existence is not a real predicate and that it adds nothing to the content of a concept. Moreover, he declared that no a priori proof of God’s existence could be constructed at all. However, a number of authors [Hartshorne 1961, Malcolm 1960, Plantinga 1974], have argued that Kant’s criticism of the argument is quite misleading, since the question is not whether existence is a predicate or whether “God” is a proper name. They noted that “referential” vocabulary is not enough to deal with the OA and proposed a new version of it allegedly derived from Proslogion III.

God is that, than which nothing greater can be conceived…. And [God] assuredly exists so truly, that it cannot be conceived not to exist. For, it is possible to conceive of a being
which cannot be conceived not to exist; and this is greater than one which can be conceived not to exist. Hence, if that, than which nothing greater can be conceived, can be conceived not to exist, it is not that, than which nothing greater can be conceived. But this is an irreconcilable contradiction. There is, then, so truly a being than which nothing greater can be conceived to exist, that it cannot even be conceived not to exist; and this being thou art, O Lord, our God. [Proslogion III]

Charles Hartshorne and Norman Malcolm were probably the first philosophers who have noted that Anselm’s argument has a precise modal structure and used the language of modal logic to propose a formalization for it. Their account inspired Alvin Plantinga to reformulate OA\(^5\) in terms of the possible worlds semantics. This semantics, associated with Kripke’s seminal works, clarified the standards of validity for modal reasoning by reference to possible worlds and to possible things therein.

The main idea of modal ontological argument (MOA) is that the esse in intellectu / esse in re opposition is very close to the modal one: namely, to the opposition of pure possibility (existence in some possible world) and its actualization (existence in the actual world). According to this approach, “…that than which nothing greater can be conceived” (id quo majus cogitari neguit) is (probably the only) such object that has a coincidence between possibility and reality. In short, this argument is as follows: it is possible that God exists; God is not a contingent being, i.e., either it is necessary that God exists, or it is necessary that he does not; hence, it is necessary that God exists. Advocates of MOA argue that the necessary existence, unlike mere existence, can be considered as a “real predicate”. Obviously this point makes MOA less vulnerable to Kantian criticism.

4. The challenge of modal realism

However, a substantial critique of MOA was presented by David Lewis [Lewis 1970]. He drew attention to the ambiguity of its key terms (possibility, conceivability, and actuality) and considered four possible non-modal translations of the main premise in Proslogion II: “Something exists in the understanding, than which nothing greater can be conceived” (these translations are formulated in metalinguistic terms of possible worlds semantics). It turned out that all four versions of the argument are either invalid or question-begging. In particular, the notorious Plantinga’s formulation, although being valid, appeared to be based on a false premise.

\(^5\) Plantinga himself found this formulation so impregnable that he called it the “Victorious Argument” [Plantinga 1974].
Lewis’s analysis exposed the fact that Plantinga and other advocates of MOA were committed to a kind of “metaphysical egocentrism”. They presume that the actual world is the only “real” while all others are “merely possible”. Lewis’s own approach, widely known as modal realism, denies that presumption and considers the concept of “actual” as an indexical: “actual”, in his thought, refers at any world $w$ to the world $w$. That is why he claims: “It is true that our world alone is actual; but that does not make our world special, radically different from all other worlds.” His conclusion sounds categorically: “The world an ontological arguer calls actual is special only in that the ontological arguer resides there – and it is no great distinction for a world to harbor an ontological arguer” [Lewis 1970, p. 184].

Now the main issues are:

(i) Even if we manage to prove God’s existence in the actual world, does it imply any metaphysical necessity?

(ii) Is there any other kind of necessity that could be associated with claims, which are valid only in the actual world?

(iii) If so, what are the correct logic and semantics for such arguments?

Indeed, what the actual world is like seems logically contingent, and so the answer to the first question has to be negative. Actual world should not be considered as a metaphysically fixed point, because it is just “this” world, *primus inter pares*. However, the other two issues remain open.

Taking seriously the indexicality of the “actual world”, we have to turn it from a fixed point into a “floating” one. This decentralization forces us to adopt a more flexible interpretation of intensions, because they are sensitive to the choice of the “actual world”. That, in turn, breaks the usually accepted connection between the modality and intensionality and generates an “intensional challenge” for the MOA. To cope with this challenge, we need our semantics to become two-dimensional.

5. Two-dimensional semantics: the basics

Two-dimensional (2D) semantics is a formal framework that can be applied both to linguistic expressions and to thought contents. The epistemic version of two-dimensionalism, developed by David Chalmers [Chalmers 2004], provides a broadly rationalist account of meaning that aims to restore the so-called “golden triangle” of necessary constitutive relations between meaning, apriority, and necessity. As Chalmers put it, “Two-dimensional semantics promises to restore the
golden triangle. While acknowledging the aspects of meaning and modality that derive from Kripke, it promises to explicate further aspects of meaning and modality that are more closely tied to the rational domain.” [Chalmers 2006]

The notions of necessity and apriority can roughly be paraphrased by saying that necessary is something that could not have failed to be the case, and a priori is something opposite to what could not have been thought. The first is usually called metaphysical necessity, and the second earned the name of epistemic necessity. Chalmers argues that the 2D framework can be used to isolate an aspect of meaning that meets our philosophical intuitions about the epistemic necessity and to show its priority to the metaphysical one. In this framework, the truth values of statements, relative to a given possible world, are determined in two different ways: they depend both on what the facts are and on what the sentences mean.

This two-fold dependence of truth on fact and meaning can be modeled by assigning to each expression not just one (ordinary) intension, but two intensions which are different in respect to considering the possible worlds. Indeed, possible worlds in 2D-semantics play two distinct roles: they serve as contexts of use (WA: worlds considered as actual), and as circumstances of evaluation (WC: worlds considered as counterfactual). In this framework every expression has several intensions: (1) its primary intension is a function f: WA → E from actual worlds to extensions; (2) its secondary intension is a function f: WC → E from counterfactuals worlds to extensions; (3) its two-dimensional intension assigns it for any actual world a secondary intension, determining a function f: WA → (WC → E) that portrays the connection between the two previously mentioned intensions.

In order to restore the “golden triangle”, Chalmers proposes a special account of modal epistemology, widely known as modal rationalism (MR). This account establishes a secure link between a priori conceivability and metaphysical possibility:

- (MR) Every epistemically possible scenario – a complete description of what the world might be like together with the speaker’s location within that world – describes a genuine metaphysically possible (centered) world.

“Given the association relation between scenarios and worlds, – Chalmers argues, – one can define the diagonal intension of a sentence’s two-dimensional intension. This will be a mapping from scenarios to truth-values. (…) The diagonal intension of a sentence will straightforwardly be equivalent to its epistemic intension. One can therefore reconstruct an expression's epistemic intension from its two-dimensional intension by diagonalizing, just as one can reconstruct its subjunctive intension by holding fixed the actualized scenario.” [Chalmers 2006] This means that, contrary to Kripke’s examples, the so-called “strong necessities” are to be excluded: any
statement that is conceivably true, even if it is not \textit{prima facie} possibly true, still expresses a genuine possibility which can be properly obtained by diagonalization.

\textbf{6. Actuality, diagonalization, and hybridization}

There are differing views on the relationship between two-dimensional semantics and two-dimensional modal logic. Chalmers believes that the later is “merely an optional means of representation” for some basic ideas of the former [Chalmers 2006]. However, a number of authors dispute this assumption and consider two-dimensional semantics as essentially grounded in two-dimensional modal logic [e.g. Humberstone 2004, Kuhn 2012, Wehmeier 2013]. The most famous type of 2D modal logic is AML – modal logic with actuality operator.

From a semantic point of view diagonalization of intension is based on our ability to consider any arbitrary world both as a counterfactual and as the actual. From a syntactic point of view, diagonalization can be expressed with the operator “A” meaning “it is actually the case that”. Regardless of its place in the formula, this operator severs the scope of any existing therein modal operators and redirects the reference of target expression to the actual world’s domain. Thus, A-operator expands the expressive power of modal logic formalism because it allows one to represents claims such as “It is possible for everything which is in fact P to be Q”, as “\(\forall x (AP(x) \supset Q(x))\)” [Davies & Humberstone 1980].

Let us consider models \(<W, @, V>\), in which \(W\) is nonempty set of possible worlds, \(@ \in W\) is the actual world, and \(V\) assigns subsets of \(W\) to propositional variables. “\(M \models_w \phi\)” stands for “the formula \(\phi\) is true at \(w\) in \(M\)”. Then truth conditions for “\(\Box\)” and “\(A\)” are described by the following clauses:

\[
M \models_w \Box \phi \text{ iff for all } u \in W, M \models_u \phi \\
M \models_w A \phi \text{ iff } M \models_\emptyset \phi
\]

The simplest logic with the actuality operator can be obtained from S5 by adding axioms which grant the distributivity of A relative to truth-functional connectives, as well as two additional axioms, linking it with the alethic modal operators: \(\Box p \rightarrow Ap\) and \(Ap \rightarrow \Box Ap\) (the resulting system is called S5A).

An additional operator F (“fixedly”) can be defined with clause:
M ⊨_w Fφ iff for all models M’ = <W, @’, V> we have M’ ⊨_w φ

F-operator is very important for 2D-approach due to the fact that prefixed to φ, the compound operator “FA” gives the claim that φ ends up true at the actual world of a model no matter which world is designated as actual. This captures an alternative sense of necessity, which is ineffable in the standard modal systems with “□”. It is the so-called “deep necessity”, or “apriority”, as opposed to “superficial” necessity (conveyed by “□”). The resulting system is called S5AF.

As shown in [Humberstone 2004]^6, the following schemas are valid in S5AF:

(1) φ → (□(φ ↔ (φ ↔ Aφ)) & FA(φ ↔ Aφ))
(2) ¬φ → (□(¬φ ↔ (¬φ ↔ A¬φ)) & FA(¬φ ↔ A¬φ))

This means that formal properties of “A” and “FA” allow picking up two “hybrid” counterparts for any contingent a posteriori statement. In particular, for a contingent a posteriori true φ, we have necessarily true Aφ, and for a contingent a posteriori false φ, we have necessarily false Aφ (note that in both cases, Aφ is a priori equivalent to φ). Similarly, for a contingent a posteriori true φ, we have a priori true φ ↔ Aφ; for a contingent a posteriori false φ, we have a priori false φ ↔ ¬Aφ (note that these formulas is necessarily equivalent to φ in the first and the second case respectively).

Thus, A-operator can be regarded as the operator forming a priori ontologically determined (necessarily true or necessarily false) counterpart for an arbitrary sentence.

(3) FA (φ ↔ Aφ) & (□Aφ ∨ □¬Aφ)

Moreover, we can introduce the dual C-operator, which forms epistemically determined (a priori true or a priori false) counterpart for an arbitrary sentence. It comes with the following definition:

Def 1 (“apriorization”) Cφ =^def \[
\begin{align*}
φ & \leftrightarrow Aφ, \text{ if } φ \\
φ & \leftrightarrow ¬Aφ, \text{ if } ¬φ
\end{align*}
\]

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^6 Propositions (3.6)-(3.7). Notation is slightly changed: we use “□” for Humberstone’s “□_n” and “FA” for “□_{ap}”
This obviously implies:

\[(4) \square(\varphi \leftrightarrow C\varphi) \& FAC\varphi\]

It is worth observing that despite the apriority of $C\varphi$ its secondary intension may be radically opaque due to Def 1. This peculiarity of $C$-operator reflects the fact that has been discussed since first Kripke’s examples of the contingent \textit{a priori} [Kripke 1980]: we may know \textit{a priori} that what is expressed is a truth, but we do not know \textit{a priori} the truth so expressed [e.g., Donnellan 1977, Tharp 1989, Humberstone 2004]. The philosophical interpretation of this feature depends on whether we call what is expressed a proposition (set of possible worlds), or a propositional concept (function from worlds to propositions).

What is the relationship between “hybridization” operators and MR-thesis? Roughly speaking, proposition (3) provides a logical ground for MR, whereas proposition (4) provides a logical ground for the conversion of MR:

- (MR’) Every metaphysically possible world can be transformed into an epistemically possible scenario

These scenarios-to-worlds (and vice versa) transformations enable us to cope with Kripke’s examples and restore the “golden triangle” by appropriate hybridization schemas. Thus, Chalmers’s 2D semantics together with the appropriate 2D modal logic can serve as a tool of analysis of \textit{Anselm’s argument}.

7. \textit{Two-dimensional version} of \textit{Anselm’s argument}

Let us consider the most enigmatic concept in Anselm’s argument, namely the concept of being “greater” (\textit{majus}). It is obvious that “greater” cannot be interpreted here as an empirical relationship. The correct interpretation must be \textit{a priori} and we should associate it with logical validity. Taking into account the plurality of logical systems, it seems natural to choose S5AF as a logic governing the kind of apriority that is required for our analysis. We shall say that $\varphi$ is greater than $\psi$, if and only if the former implies the later, but not vice versa:

\[
\text{Df2 ("majus")}. \quad \varphi > \psi \leftrightarrow_{df} \varphi \models_{S5AF} \psi \text{ and } \psi \nmid S5AF \varphi
\]

Since $C\varphi \models_{S5AF} \varphi$ and $\varphi \nmid S5AF C\varphi$, this definition allows us to claim that

\[(5) C\varphi > \varphi\]
Taking into account that $C\phi$ is the epistemically determined equivalent of $\phi$, this definition may be also treated as claiming that epistemically determined equivalent of $\phi$ is greater than $\phi$. Therefore, “$C\phi \leftrightarrow \phi$” should be read as “a priori known $\phi$ is greater than mere $\phi$”. That, in turn, naturally leads us to the definition:\footnote{“$\phi \leftrightarrow_{ap} \psi$” is the abbreviation for “FA($\phi \leftrightarrow \psi$)”}

\[
Df3 \text{ (“God’s existence”)} \quad g \leftrightarrow_{ap} (g \leftrightarrow Cg)
\]

Thereby God is defined as the object whose existence is greater than itself. Starting with this definition we can construct a deduction (“PL” stands for the rules of classical propositional logic):

1. $g \leftrightarrow_{ap} (g \leftrightarrow Cg)$  \hspace{1cm} Df3 (“God’s existence”)
2. $\neg g$  \hspace{1cm} premise
3. $\neg g \leftrightarrow Cg$  \hspace{1cm} 1, 2, \emph{a priori} equivalence
4. $Cg \leftrightarrow_{ap} (g \leftrightarrow \neg Ag)$  \hspace{1cm} 2, Def 2 (“rectification”)
5. $\neg g \leftrightarrow (g \leftrightarrow \neg Ag)$  \hspace{1cm} 3, 4, \emph{a priori} equivalence
6. $g \leftrightarrow (g \leftrightarrow Ag)$  \hspace{1cm} 5, PL
7. $(g \leftrightarrow g) \leftrightarrow Ag$  \hspace{1cm} 6, PL
8. $Ag$  \hspace{1cm} 8, PL
9. $g$  \hspace{1cm} 9, S5AF
10. $\neg \neg g$  \hspace{1cm} 2, 9, by contradiction
11. $g$  \hspace{1cm} 10, PL

Of course, this argument (let us call it OA2D) is a somewhat simplified version of Anselm’s proof and it needs further elaboration. Nevertheless, it much more accurately conveys Anselm’s basic vision, than MOA.

What does a person state exactly when she says “iqmcn exists” and at the same time realizes that she has got no precise concept of “iqmcn”? I argue that her statement in effect means: “[In all possible worlds] there is something that I labeled as “iqmcn”, however there may be some other possible where my term “iqmcn” has a very different denotation (or even does not have it at all)”. Sometimes we have no conceptual tools to carry out the cross-identification of object involved (its essential properties are completely unknown to us), but we can still maintain its existence.
Thus we can see that a new two-dimensional interpretation of Anselm's proof in terms of AML demonstrates its core feature: we may know \textit{a priori} that sentence “God exists” is true, but we do not know \textit{a priori} what exactly this sentence means in our world. Howbeit, it is better than nothing, and so, according to Anselm, it is greater than nothing.

\textbf{References}


Victor V. Gorbatov
National Research University Higher School of Economics. Faculty of Philosophy, Department of Ontology, Logic and Theory of Knowledge: Senior Lecturer;
E-mail: vgorbatov@hse.ru; Tel. +7 (495) 772.95.90*2698

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