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Alexei Parakhonyak, Anton Sobolev

NON-RESERVATION PRICE EQUILIBRIA AND SEARCH WITHOUT PRIORS

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Alexei Parakhonyak [1] *and Anton Sobolev* [2]

NON-RESERVATION PRICE EQUILIBRIA AND SEARCH WITHOUT PRIORS[3]

In this paper we analyse a model of oligopolistic competition in which consumers search without priors. Consumers do not have prior beliefs about the distribution of prices charged by firms and thus try to use a robust search procedure: they minimise the loss relative to the searcher, who knows the price distribution, in the worst case scenario. We derive the optimal stopping rule and show that it does not possess the reservation price property. This means that for a range of prices for which consumers stop searching with a probability strictly between zero and one. We show that for any distribution of search costs there is a unique market equilibrium characterised by price dispersion. Therefore search without priors helps resolve the famous Diamond (1971) paradox. We show that although listed prices approach the monopoly price as the number of firms increases, the effective price paid by consumers does not depend on the number of firms. We show that prices in our model are lower than those in a model where consumers know the distribution of prices. The reason is that consumers actively search in equilibrium, and this pushes prices down. This effect is so strong that the price decrease more than compensates consumers for their extra search costs.

Keywords: consumer search, search without priors, robust search, Diamond paradox, non-reservation price equilibrium.

JEL-Codes: D83, D43, L11

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[1]National Research University Higher School of Economics, Moscow, Russia. E-mail: parakhonyak@yandex.ru.

[2]University of Vienna, Austria.

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1 Introduction

Uncertainty lies at the heart of any optimal search problem. Most of the search literature reduces uncertainty to risk, by assuming that options are sampled from a known distribution. In the consumer search literature this risk comes from mixed pricing strategies employed by firms (e.g. Stahl (1989), Janssen et al. (2005)), or from a random component in the utility (e.g. Wolinsky (1986), Anderson and Renault (1999), Armstrong and Zhou (2010)). In the recent literature a more sophisticated approach is often employed: consumers are unaware of some parameters of the model and "estimate" them in a Bayesian way using the observed prices (e.g. Dana (1994), Janssen et al. (2011)). All these models require consumers to have certain "priors" – distributional assumptions over prices, utilities or other parameters of the model.

As even economists have not reached a consensus on the "correct" market model it is hard to believe that consumers can do so. In an environment where consumers do not know prices, it is natural to assume that they also no little about how many firms operate in the market, and what those firms' costs and market shares might be. Therefore in this paper we propose a robust search procedure, where the solution of the optimal stopping problem does not rely either on priors about parameters of the model, or on the market model structure itself. As consumers are agnostic about the distribution of options from which they sample, they cannot base their decisions on traditional expected utility. In our paper consumers use the optimal stopping rule which minimises the loss in comparison with an (imaginary) informed searcher (who knows the distribution of the prices) in the *worst case scenario*. By doing so consumers are guaranteed that the expected difference between their utility and utility obtained under ex ante optimal behaviour is below a certain bound.

In our paper we look at a market with homogenous good in which consumers search for the best price, although our results can be applied to a wide variety of settings. Following Salop and Stiglitz (1977) and Dana (1994) we use the so-called "newspaper search protocol": after observing the first price for free, consumers can learn at some cost all the prices in the market by visiting a price aggregator website (in previous times by buying a newspaper with advertisements). A traditional sequential search approach is very complicated in the setting of search without priors and is prone to a dynamic inconsistency problem as pointed out by Hayashi (2009): consumers might want to change their search strategy as they gather information about prices. As the first search is costless in our paper, there is effectively one point in time where consumers make a search decision, and, hence the dynamic inconsistency issue vanishes. This approach also allows us to consider equilibrium settings with more than two firms. Newspaper search protocol has its limitations: in markets with physical transportation costs sequential search is a more realistic model. Both protocols coincide in duopoly markets.

It turns out that the optimal stopping rule is stochastic: for sufficiently high prices consumers randomise between stopping and continuing to search. Once a consumer has observed a price, she suffers from both *greed* (a concern that by not searching, she may miss out on a lower price) and *fear* (a concern that searching may not yield a lower price, and thus be wasteful). In principle the consumer has to think about all possible distributions, then figure out what an informed searcher would do for each distribution and estimate the maximum loss. However, we show that there are only two worst case distributions: one associated with greed and another associated with fear. This considerably simplifies the analysis. As the minimax regret approach is equivalent to playing a fictitious strictly competitive game against malicious nature (which maximises consumer's regret by its choice of price distribution), it results in a mixed strategy search behaviour. An important feature of our approach is that the search behaviour is robust to model specification and distributional assumptions over parameters. As such it can be applied to various market models without any change.

We then look at the consequences of the minimax optimal stopping rule for market outcomes. In a market model, firms are strategic players who can correctly anticipate the behaviour of consumers and maximise their profit, while consumers are agnostic about the market primitives and minimise maximum regret. We allow for a general distribution of search costs and solve for the market equilibrium. We find that for all parameter values and search costs distributions there is a unique pricing equilibrium, which is characterised by price dispersion. An intriguing feature of this result is that price dispersion arises in equilibrium even if all consumers have strictly positive costs, i.e. our model avoids the Diamond (1971) paradox. We find that the average *listed* price increases with the number of firms and approaches the monopoly price as the number of firms grows without bound. However the average price *paid* by consumers stays constant. As the number of firms grows, more consumers decide to search and, conditional on deciding to search, they observe more price quotations. As under the newspaper search protocol the price paid is the order statistic, consumers end up paying the same prices although the listed prices go up.

Finally we provide an analytical characterisation of equilibrium in a duopoly market, when some consumers are shoppers (with zero search costs) and others are searchers (with the same positive search costs). We compare our results with Stahl (1989) model with informed searchers, i.e. consumers who hold a correct belief about firms' pricing strategies. It turns out that average price is lower when searchers do not have a prior about the price distribution. The reason is that in our model consumers actively search, whereas in Stahl's model they do not. Active search leads to more price comparisons, stronger competition and hence lower prices. Even when the search costs are explicitly taken into account, searchers have lower cost of acquiring the good than in Stahl (1989), provided that the number of searchers is sufficiently large. We also run a numerical experiment in order to assess the equilibrium loss (difference in utilities) in two environments. Firstly, we put an atomistic searcher who does not know the price distribution in Stahl's equilibrium model. Secondly, we introduce an atomistic rational searcher in our equilibrium model. In both cases the maximum loss does not exceed a few percentage points of the valuation of the good.

The minimax approach was axiomatised by Hayashi (2008) and Stoye (2011). It has several applications to various decision making problems under information uncertainty.

The closest article to ours is Bergemann and Schlag (2011a). Although we use a similar minimax regret approach to search problem, our paper differs from theirs in several ways. We follow the consumer search literature by studying an environment with both search costs and perfect recall. Bergemann and Schlag (2011a) on the other hand consider a setup more common in the labour search literature, without search costs and recall, and only briefly discuss costly search. We use the so-called "newspaper" search protocol, when consumers can obtain information about all other options via a price aggregator. This allows us to look at environments with more than two alternatives. Another important difference is that we do not restrict attention to just the optimal stopping problem, but we also consider its consequences for oligopolistic competition. Riedel (2009) considers the optimal stopping problem with multiple priors. The objective of the searcher is to maximise the (ex ante) expected reward in the worst case scenario, which is close to our approach. Hayashi (2009) also considers sequential search with multiple priors and points out that there is a dynamic inconsistency problem: consumers might revise their search strategy after observing certain prices. In order to avoid this problem he considers strategies with commitment. As we use newspaper search protocol with free first search this problem does not arise in our model. Nashimura and Ozaki (2004) look at the problem of job search under Knightian uncertainty: a worker faces an ε-contamination of the original distribution, i.e. the distribution is not known for sure, but it belongs to a restricted set of distribution functions. Unlike in our paper, the optimal stopping rule in this situation has the reservation price property. Chou and Talmain (1993), followed by Parakhonyak (2014), consider an alternative axiomatic approach to search without priors, when consumer behaviour satisfies certain consistency requirements, which results in beliefs with maximum entropy and non-stationary reservation prices. The literature on decision making with minimax regret includes, among others, research on optimal monopoly pricing (Bergemann and Schlag (2008), Bergemann and Schlag (2011b)) and treatment with missing data (Manski (2007), Stoye (2012)).

Our paper makes several contributions to the literature on consumer search. First, we provide a tractable framework that allows for an arbitrary search cost distribution and that can be applied to various market structure specifications.

Second, this paper does not differentiate between incomplete information (e.g., about production costs or some other parameters of the model) and strategic uncertainty which arises from equilibrium mixed strategies in complete information games. This significantly simplifies the analysis of models with asymmetric information about production costs (see e.g., Dana (1994), Tappata (2009), Janssen et al. (2011)). Models with unknown production costs are characterised by reservation price search strategies, but when the heterogeneity in production costs is sufficiently large a pricing equilibrium does not exist. Janssen et al. (2014) propose a non-reservation price equilibria in order to avoid this problem. Our paper also uses non-reservation price search strategies, though derived from different considerations. The pricing distribution in Janssen et al. (2014) is similar to the one we derive in Section 5 of this paper. The departure from the rationality paradigm brings with it the advantage that we can study a more general model than Janssen et al. (2014): we allow for multiple firms, while they consider only duopoly; we allow for general search cost distributions while they look only at binary distribution of search costs (zero and positive). Moreover, the equilibrium in our model is unique, while they have multiple equilibria.

Third, we propose a new way to avoid the Diamond (1971) paradox. Stahl (1996) pointed out that the necessary condition to rule out monopoly pricing is to have a positive measure of consumers with zero search costs. We have a market equilibrium with price dispersion for an arbitrary search cost distribution (provided that search costs for some consumers are positive and less than the valuation of the good). Stiglitz (1987) also considered a model in which all consumers have positive search costs. He assumed that if one of the firms deviates from the equilibrium, then it is observed by consumers, but consumers cannot figure out which firm has deviated. Recently Rhodes (2014) proposed an elegant way to avoid the Diamond paradox by considering multiproduct firms.

Fourth, our model has an interesting comparative statics. Textbook oligopoly models suggest that an increase in the number of firms should lead to lower prices. In the search literature this result is often reversed. Stahl (1989) showed that as the number of firms increases, listed prices approach the monopoly price, and the effective price paid by all consumers first increases (when the search cost binds) and then stays stable (when the valuation binds). In our model listed prices also increase and approach the monopoly price as the number of firms approaches infinity. However, the average price is constant regardless of the level of search costs. The reason is that although the listed prices are higher, consumers search more and each search pays off better because of our choice of search protocol.

Arguably our model has more appealing empirical properties than models of sequential consumer search which the reservation price property. In those models, consumers stop searching after the first search, which is not true in reality (see, de los Santos et al. (2012)). Our model explains why consumers actively search in markets for homogeneous goods. Moreover, the empirical prediction of our model is that for the same search history consumers can make different decisions, depending on the realization of their mixed strategy. This result comes from the absence of priors and loss aversion of consumers, rather than from some fundamental irrationality.

Finally, our paper is related to Haan and Siekman (2013) who also consider the search problem faced by loss-averse consumers. This paper differs from ours in several ways. Firstly, consumers know the distribution of search options in their model, so there is no ambiguity. Secondly, they consider a model with differentiated products similar to Anderson and Renault (1999). Thirdly, their interpretation of loss aversion is different. Our consumers care about making correct decisions on average, i.e. decisions close to those of an informed searcher. Thus, they care about making ex ante correct decisions. The consumers in Haan and Siekman (2013) care about losses incurred during the search process compared to their current option, i.e. they are ex post loss averse and regret not being lucky.[4]

[4] If we draw a parallel with financial markets, our agents hate to be wrong in making fundamental decisions, such as the choice of fractions of stocks and bonds in the portfolio made by professional

The rest of the paper is organised as follows. Section 2 presents the model setup, including the formal statement of the objective function. In Section 3 we derive the optimal stopping rule. Section 4 considers the pricing equilibrium in a general model and studies its main properties. Section 5 provides a detailed comparison with the Stahl (1989) model for the case of duopoly. Section 6 provides a discussion of the robustness of our results and briefly concludes. Technical proofs are presented in the Appendix.

2 Model

There are $N \geq 2$ firms in the industry. Firms produce a homogeneous good at marginal cost $m > 0$ and compete in prices. There is a unit mass of consumers. Each consumer has a unit demand for the good, and values it at v which we normalise to 1. These consumers are split evenly among the firms and know the price at their firm but not at the others.[5] After observing the price at their firm, consumers can pay a cost c and get access to all the prices in the market via a price aggregator. Search costs are distributed on $[\underline{c}, \overline{c}]$ according to distribution function $\mu(c)$. We assume that $\underline{c} < 1$. Consumers can return to the previously sampled option without incurring any additional costs[6]. A consumer is agnostic about the underlying mechanisms of the market: she is unaware both of the market structure and possible parameter distribution. She holds a belief about the support of the price distribution $[\underline{q}, \overline{q}]$, but does not know the actual shape of the distribution function for any of the firms. The usual way to model beliefs is to assume that the consumer knows the true boundaries of the support of the theoretical distribution (see, e.g. Bergemann and Schlag (2011a)). This assumption is not plausible in a consumer search context, since the derivation of these boundaries requires knowledge of all the parameters of the model. In this paper we derive the optimal stopping rule for an arbitrary belief $[\underline{q}, \overline{q}]$, assuming that it is not possible to observe "surprising" prices,

managers. However they do not regret passing up the opportunity to invest in a no-name stock, which ends up giving high returns. The latter is the source of regret in Haan and Siekman (2013).

[5]Janssen et al. (2005) relax the assumption that the first observation is free.

[6]Janssen and Parakhonyak (2014) relax this assumption and explicitly model return costs.

i.e. $p < \underline{q}$ or $p > \overline{q}$. Then, for the equilibrium analysis we make the assumption $\underline{q} = 0$ and $\overline{q} = v$, i.e. consumers only rule out the prices at which a firm cannot make any profit. Although we are going to look at a symmetric equilibrium of the model, from uninformed searcher perspective there is no reason to assume that the distributions are the same, i.e. that the firms are ex ante symmetric or play a symmetric equilibrium. It is also possible that an uniformed searcher does not know the number of firms in the market. We assume that an uninformed searcher holds the belief that there are up to M firms operating in the market, and that this belief is correct, i.e. $N \leq M$.

Note that as consumers do not know the shape of the distribution, the objective function of consumers cannot take the form of an expected utility function, simply because the expectation can only be defined when the distribution function is known. We assume that consumers are regret minimisers, in the sense that they want to minimise their maximum loss compared to what they would get if they knew the true distribution. More formally, let mapping $s(p) : [0, \infty] \to [0, 1]$ be a stopping rule which indicates the probability of accepting a price p[7]. Let $\mathbb{F} = \{F_i\}_{i=1}^{M}$ be a collection of probability distributions over the prices of all M firms which can be potentially active in the market. We require that the price distributiona of truly existing firms $F_i(\cdot)$, $i = \overline{1, N}$ have supports consistent with the belief $[\underline{q}, \overline{q}]$, i.e. with $F_i(\underline{q} - \varepsilon) = 0$ and $F_i(\overline{q} + \varepsilon) = 1$ for any $\varepsilon > 0$. This implies that there is no "surprise" price discovery. We denote this collection of probability distributions by $\mathbb{F}_N = \{F_i\}_{i=1}^{N}$. For the purpose of characterising the optimal stopping rule, we model the remaining $M - N$ that are not actually present in the market as artificial active firms which charge prices above the valuation: for all $M \geq j > N$ $F_j(p) = \mathbb{I}_{p \geq p_j}$, $p_j > v$. These artificial "firms" do not affect the behaviour of informed searchers, which stays the same both under collection $\mathbb{F}$ and $\mathbb{F}_N$. However when uninformed searchers make their decision, they take into account the possibility that these $M - N$ firms can play non-degenerate strategies. Suppose an uninformed searcher observes the price in store j and has to decide whether to continue searching. Denote $\mathbb{F}_{-j} = \{F_i\}_{i \neq j}$. The loss

[7] Standard reservation price optimal stopping rules take the form $s(p) = 0$ for all prices above the reservation price, and $s(p) = 1$ for all prices less or equal to the reservation price.

function is defined by

$$L(s, F_j, \mathbb{F}_{-j}) = \begin{array}{ll} & -\int_{\underline{q}}^{\rho_{\mathbb{F}_{-j}}} p_j dF_j(p_j) - \int_{\rho_{\mathbb{F}_{-j}}}^{\overline{q}} \int_{\underline{q}}^{\overline{q}} \min\{p_j + c, y + c\} dF_j(p_j) dH_{\mathbb{F}_{-j}}(y) + \\ + & \int_{\underline{q}}^{\overline{q}} \int_{\underline{q}}^{\overline{q}} \left[s(p_j)p_j + (1 - s(p_j)) \min\{p_j + c, y + c\}\right] dF_j(p_j) dH_{\mathbb{F}_{-j}}(y) \end{array} \tag{1}$$

where $y = \min(p_1, \ldots, p_{j-1}, p_{j+1}, \ldots, p_M)$ and $H_{\mathbb{F}_{-j}}(y) = 1 - \prod_{i \neq j, i \leq M}(1 - F_i(y))$, and $\rho_{\mathbb{F}_{-j}}$ is a reservation price if consumers knew the distribution function which is defined by:

$$\rho_{\mathbb{F}_{-j}} = c + H_{\mathbb{F}_{-j}}(\rho_{\mathbb{F}_{-j}}) \mathbb{E}_{H_{\mathbb{F}_{-j}}}(y | y \leq \rho_{\mathbb{F}_{-j}}) + (1 - H_{\mathbb{F}_{-j}}(\rho_{\mathbb{F}_{-j}})) \rho_{\mathbb{F}_{-j}}$$

i.e. $\rho_{\mathbb{F}_{-j}}$ is the price at which consumers are indifferent between buying now and searching.

Note, that the first two integrals in (1) define the expected price paid in the informed consumer case: what a consumer pays if the first price is less than $\rho_{\mathbb{F}_{-j}}$, and what she pays if it is greater and one more search is conducted. The third integral defines the expected price paid by an uninformed consumer who uses stopping rule $s(p)$. By rearranging terms in (1) we get

$$L(s, F_j, \mathbb{F}_{-j}) = -\int_{\underline{q}}^{\overline{q}} \int_{\underline{q}}^{\overline{q}} \Big[\mathbb{I}_{p \leq \rho_{\mathbb{F}_{-j}}} (1 - s(p))(p - \min(p, y) - c) + \\ + \mathbb{I}_{p > \rho_{\mathbb{F}_{-j}}} s(p)(\min(p, y) + c - p)\Big] dH_{\mathbb{F}_{-j}}(y) dF_j(p)$$

Denote $M_{\mathbb{F}_{-j}}(p) = \mathbb{E}_{H_{\mathbb{F}_{-j}}} \min(p, y) + c$. Then,

$$L(s, F_j, \mathbb{F}_{-j}) = \int_{\underline{q}}^{\overline{q}} \Big[\mathbb{I}_{p \leq \rho_{\mathbb{F}_{-j}}} (1 - s(p))(M_{\mathbb{F}_{-j}}(p) - p) + \mathbb{I}_{p > \rho_{\mathbb{F}_{-j}}} s(p)(p - M_{\mathbb{F}_{-j}}(p))\Big] dF_j(p) \tag{2}$$

The first summand represents the loss from too much search – if the distributions were known it would be better to stop. The second term represents the loss from stopping too early when it is in fact optimal to continue searching. In the full information case, we have $\rho_{\mathbb{F}_{-j}} = M(\rho_{\mathbb{F}_{-j}})$. It is straightforward to verify that this solution always exists and is unique.

Let $\mathcal{S}$ be the set of all stopping rules $s(p)$ and let $\mathcal{F}$ be a set of all possible collections of probability distributions $\mathbb{F} = \{F_j, \mathbb{F}_{-j}\}$. The objective of a consumer is to find the optimal stopping rule which minimises the loss function in the worst case scenario (over all possible collections of probability distributions):

$$s \in \operatorname*{Arg\,min}_{s \in \mathcal{S}} \max_{\mathbb{F}=\{F_j, \mathbb{F}_{-j}\} \in \mathcal{F}} L(s, F_j, \mathbb{F}_{-j})$$

We are going to look at equilibrium pricing in our model under the assumption that firms are strategic. We do this even though consumers are not strategic, as they minimise their regret in a situation of ambiguity, and therefore they do not necessarily play best-responses to firms' strategies in a normal game-theoretical sense. More formally, we define equilibrium in the following way. Let $F(p), G(p) : [0, \infty) \to [0, 1]$ be pricing strategies (possibly degenerate distributions over prices) and s be a search strategy.

Definition 1. *An equilibrium of the model consists of pricing strategies $F(p)$ and search strategies s such that:*

1. *A firm's profit is maximised by $F(p)$ given the strategies of consumers and other firms: $\pi(F_i(p), F_{-i}(p), s) \geq \pi(G_i(p), F_{-i}(p), s)$.*
2. *A consumer starts at a random firm j and follows the strategy which minimises regret: $s \in \operatorname{Arg\,min}_{s \in \mathcal{S}} \max_{\mathbb{F} \in \mathcal{F}} L(s, F_j, \mathbb{F}_{-j})$.*

3 Optimal Stopping Rule

We start our analysis by deriving the optimal stopping rule. Later we use this optimal stopping rule for the equilibrium analysis. We define the maximum loss as

$$\overline{L} = \min_{s \in \mathcal{S}} \max_{\mathbb{F} \in \mathcal{F}} L(s, F_j, \mathbb{F}_{-j})$$

The following Theorem defines the optimal stopping rule.

Theorem 1. *The optimal stopping rule*

$$s(p; c, \underline{q}) = \begin{cases} 1 & \text{if} \quad p \leq \underline{q} + c \\ \frac{c}{p - \underline{q}} & \text{if} \quad p \in (\underline{q} + c, \min(\overline{q} + c, v)] \\ 0 & \text{if} \quad p > \min(\overline{q} + c, v) \end{cases} \tag{3}$$

minimises the maximum loss and guarantees that $\overline{L} = \left(1 - \frac{c}{\overline{q} - \underline{q}}\right) c$.

Note, that the optimal stopping rule does not possess the standard reservation price property: for sufficiently high prices, consumers stop with certain probability. This means that unlike in many search models with homogeneous goods[8] (based on Stahl (1989)) there is active search in equilibrium. To understand the reasons behind the stochastic optimal stopping rule, suppose that the consumer faces price p. In making her decision, she considers two worst case scenarios, which for expositional purposes we associate with two basic emotions:

- *Greed*: if the consumer decides to stop, she is concerned that the supports of (some of) other price distributions include only the the lower bound $\underline{q}$, and it was optimal to continue;

- *Fear*: if the consumer decides to search, she is concerned that the support of other price distributions contain only the current price p (and possibly prices above p), so the search is wasteful.

The higher the probability of stopping, the larger is the regret in the first scenario. Likewise the lower the probability of stopping, the higher is the regret in the second scenario.

[8]See, for example, Dana (1994), Janssen et al. (2011), Janssen and Parakhonyak (2013), Parakhonyak (2014)

The maximum of these two regrets is minimised by some intermediate probability of stopping, for which both regrets are equal.

Another notable feature of the optimal stoping rule is that it depends (when it is between 0 and 1) only on the lower bound of the support of the distribution, while the upper bound plays a passive role. This is also the case in Parakhonyak (2014). In both cases the reason is the same: given the assumption of perfect recall, when determining whether to stop or not, only the distribution of prices below the current price matters. This implies that if the consumer does not have any prior, only the lower bound of the distribution (together with search costs) determines the optimal stopping rule. Note that the maximum loss is defined by $(1 - s(\overline{q}; c, \underline{q}))c$, which is strictly better than the naive strategy "always to search", for which the maximum loss equals c. Note, that whether consumers know the number of firms $(N = M)$ or not $(N < M)$ does not affect the optimal stopping rule.

The following proposition summarises the properties of the optimal stopping rule:

Proposition 1. *For all $p \in [\underline{q} + c, \overline{q}]$ the optimal stopping probability $s(p; c, \underline{q})$:*

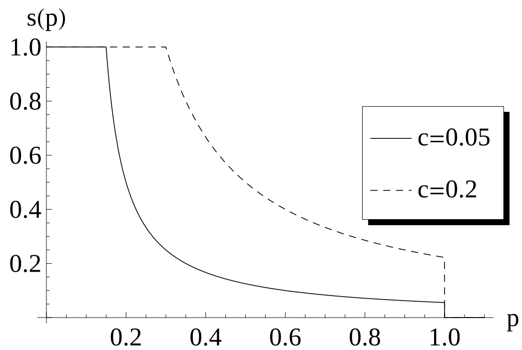
1. *decreases in price p;*
2. *increases in search costs c;*
3. *increases in the belief about the lower bound of the support $\underline{q}$.*

Thus, consumers are more likely to stop if the search cost is high, or if price is low in comparison with the lower bound. For the rest of the paper we will omit the list of parameters wherever it does not affect understanding and just use notation $s(p)$. Figure 1 shows what the optimal stopping rule looks like for different sets of parameters.

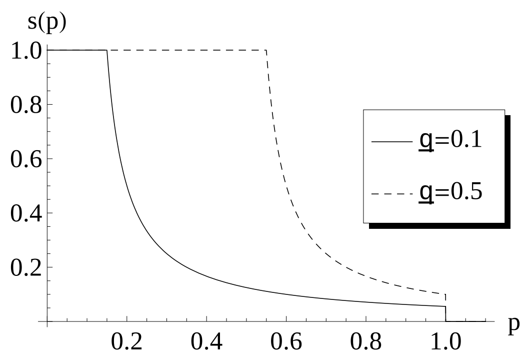
4 Equilibrium Analysis

In this section we use the optimal stopping rule derived in Theorem 1 to solve for market equilibrium. In order to put more structure on the model, in this section we are going to

Figure 1: $s(p)$ for different parameter values



(a) $\underline{q} = 0.1$, $c \in \{0.05, 0.2\}$



(b) $c = 0.05$, $\underline{q} \in \{0.1, 0.5\}$

impose that $\underline{q} = 0$ and $\overline{q} = 1$, i.e. consumers rule out only those prices that are clearly irrational (strictly dominated) from the firms' perspective. Recall, that consumers do not know the marginal cost of production m and therefore can assume that there might be prices lower than the actual value of the marginal cost. As long as $\underline{q}$ is an exogenous parameter, this assumption is not critical for our results, but allows us to reduce the dimensionality of the parameter space and make our results more comprehensive. In our analysis we are going to concentrate on symmetric market equilibria.

In the case of $\underline{q} = 0$ and $\overline{q} = 1$ the optimal stopping probability can be rewritten as

$$s(p;c) = \begin{cases} 1 & \text{if} \quad p \leq c \\ \frac{c}{p} & \text{if} \quad p \in (c, 1] \\ 0 & \text{if} \quad p > 1 \end{cases} \tag{4}$$

Let us define

$$\sigma(p) = \int_{\underline{c}}^{\overline{c}} s(p;c)d\mu(c) \tag{5}$$

$\sigma(p)$ is the probability that a randomly chosen consumer decides to stop after she observes a price p. Note, that for all $p \leq \underline{c}$ we have $\sigma(p) = 1$ and for all prices $p \in (\underline{c}, 1]$ we have $\sigma(p) \in (0,1)$. Before we are ready to write down the profit function, we need to establish a few preliminary results: (i) there is no pure strategy pricing equilibria, (ii) the equilibrium distribution of prices is continuous, and (iii) the support of the equilibrium distribution is a convex set. The following Lemma shows that any symmetric pricing equilibrium is characterised by price dispersion.

Lemma 1. *There is no symmetric pure strategy pricing equilibria in the game.*

The result of Lemma 1 is different from the classical results in the consumer search literature. Diamond (1971) showed that if all consumers have the same positive search cost, and if the first search is for free, there is a unique equilibrium in which firms charge the monopoly price. Stahl (1996) showed that this result holds for an arbitrary distribution of search costs, unless there is a strictly positive mass of consumers with zero search costs. Note that Lemma 1 requires neither $\underline{c}$ to be equal to zero, nor $\mu(c)$ to have an atom at $\underline{c}$. Therefore, search without priors is a new way to avoid Diamond's paradox. Stiglitz (1987) avoided it by assuming that consumers know the empirical distribution of prices in the market, but do not know which firm charges which price. This approach implies that consumers possess much more information than in the original Diamond (1971) model, while our paper assumes the opposite: consumers are less informed about the primitives of the model. Rhodes (2014) avoided the paradox by considering multi-product firms.

We have established that there is no pure strategy equilibria and now we are are going to look at the properties of mixed strategy equilibria. We denote the equilibrium mixed strategy distribution as $F(p)$. We denote the lower (upper) bound of the support of the mixed strategy of the firms as $\underline{p}$ ($\overline{p}$). The following Lemma establishes that $F(p)$ does not have atoms, i.e. there are no prices which are played with positive probability.

Lemma 2. *$F(p)$ is continuous on its support.*

Finally we show that the support of $F(p)$ does not have gaps.

Lemma 3. *The support of $F(p)$ is a convex set.*

Now we are ready to write down the profit function of a firm.

$$\pi(p) = \frac{p}{N}\Bigg(\sigma(p) + (1-\sigma(p))(1-F(p))^{N-1} \\ + (N-1)(1-F(p))^{N-2}\int_p^{\overline{p}}(1-\sigma(x))dF(x)\Bigg) \quad (6)$$

There is a fraction $1/N$ of consumers who start searching at our firm which charges a price p. $\sigma(p)$ of these consumers decide to stop and buy the good. $(1-\sigma(p))$ decide to search and they buy from our firm only if all other prices are higher. Finally, there are consumers who start their search from each of the remaining $N-1$ firms. After observing price x at some other firm, they decide to search with probability $(1-\sigma(x))$, and buy from our firm if both x and all other prices are higher than our price. Now we can proceed with the characterisation of the support of the equilibrium price distribution.

Lemma 4. *The upper bound of the equilibrium price distribution equals* 1.

This Lemma illustrates a very interesting choice faced by the firms. Note that at the upper bound consumers who leave the firm after their first search never come back. Thus, if a firm faces only fresh consumers, the threat of losing them is not strong enough to charge a price which is lower than the monopoly price. The motivation to charge lower prices is sufficient only when a firm takes into account those consumers who first visited other stores. Lemma 4 allows us to pin down the equilibrium level of profits:

$$\overline{\pi} = \frac{\sigma(1)}{N}(1-m)$$

Now we are ready to establish the existence and uniqueness of equilibrium.

Theorem 2. *For any number of firms N and any search cost distribution $\mu(c)$ there is a unique symmetric equilibrium.*

Although search models with homogeneous goods under complete information usually have a unique symmetric equilibrium (see, e.g. Stahl (1989), Janssen and Parakhonyak (2014)) this is not typical for models with incomplete information. For example, models with common production costs which are unknown to consumers (e.g., Dana (1994), Tappata (2009), Janssen et al. (2011)) suffer from non-existence of equilibria whenever there is large dispersion in production costs. An attempt to solve this issue by considering non-reservation price equilibria by Janssen et al. (2014) encounters the opposite problem – multiplicity of equilibria. Our paper gives a framework which can be directly used for studying this type of problem: as the demand side of the model is not affected by production cost uncertainty, the fact that the marginal cost of production is not known to consumers does not change (6). Theorem 2 is still applicable, with the interpretation that m is a realisation from some distribution of production costs and the model still has a unique equilibrium which always exists.

For the rest of this Section we concentrate on the properties of the equilibrium. We start with comparative statics with respect to N. Note that as we have a unit mass of consumers and unit demand, the expected price paid by consumers equals the total profit collected by the firms plus total production cost: $\mathbb{E}p = N\overline{\pi} + m = (1-m)\sigma(1) + m$. As $\sigma(1) < 1$ the expected price is increasing in the marginal cost of production. As $\sigma(1)$ does not depend on N the average price paid by consumers is constant in the number of firms *for any distribution of search costs*. This result is different from comparative statics of the Stahl (1989) model, where average price paid by all consumers first increases with the number of firms, and then remains constant once the reservation price hits the valuation. In our model the upper bound of the support of the equilibrium price distribution equals 1 for any distribution of search costs and thus total industry profit (and the average price) is constant in N. The distribution in Stahl's model approaches $\mathbb{I}_{p\geq 1}$ as the number of firms approaches infinity, thus consumers with positive search costs tend to pay higher prices as the number of firms goes up. As shown in the following proposition, our price distribution has a similar limit property.

Proposition 2. *As the number of firms becomes larger, the probability mass concentrates in a neighbourhood around 1:*

$$\lim_{N \to \infty} F(p) = \mathbb{I}_{p \geq 1}$$

However even though the listed prices are higher, the average price paid by consumers is constant. The reason is that consumers who decided not to search pay higher prices, but consumers who decided to search pay lower prices as they compare more prices.

Now we consider comparative statics with respect to search costs. As we work with general cost distributions, rather than common search costs, we formulate our result in terms of first order stochastic dominance.

Proposition 3. *Consider two possible distributions of search costs: $\mu_L(c)$ and $\mu_H(c)$. Suppose $\mu_H(c) \leq \mu_L(c)$ for all c. If $F_L(p)$ and $F_H(p)$ are equilibrium price distributions for $\mu_L(c)$ and $\mu_H(c)$ correspondingly, then $F_H(p)$ stochastically dominates $F_L(p)$.*

Following an increase in search costs consumers search less, there is less competition and firms charge higher prices. This result is common to most search models. Unlike higher N, higher search costs reduce search activity and do not affect the benefits of search, except via changes in the price distribution. Consequently higher search costs lead to higher expected prices in the sense of first order stochastic dominance. Finally, we define $\mu_\alpha(c)$ to be a distribution of search costs on $[\alpha\underline{c}, \alpha\overline{c}]$ such that $\mu_\alpha(\alpha c) = \mu(c)$. As $\alpha \to 0$ the aggregate stopping probability $\sigma(1) \to 0$ and $F(p) \to \mathbb{I}_{p \geq m}$, i.e. as search costs approach zero the market approaches perfect competition.

5 Comparison with Stahl (1989)

In this section we compare our results with a seminal paper by Stahl (1989). Stahl considers a model with two types of consumers: a fraction λ are shoppers (with zero search cost) whilst the remaining fraction are searchers (with positive search cost c_0). Thus,

$$\mu_{\text{Stahl}}(c) = \begin{cases} \lambda & \text{if} \quad c \in [0, c_0) \\ 1 & \text{if} \quad c \geq c_0 \end{cases} \tag{7}$$

Stopping probabilities are then defined in the following way:

$$\sigma_{\text{Stahl}}(p) = \begin{cases} 1 - \lambda & \text{if} \quad p \in [0, c_0) \\ (1 - \lambda)s(p; c_0) & \text{if} \quad p \in [c_0, 1] \end{cases} \tag{8}$$

For this section we concentrate on the case of $N = 2$, as in this case the equilibrium price distribution can be explicitly characterised both in our model and in Stahl's. Note that for $N = 2$ the price-aggregator search protocol coincides with standard sequential search, which makes a direct comparison with Stahl (1989) possible. We have discussed the comparative statics of the two models with respect to N in the previous section: price distributions in both models tend to degenerate at 1 as the number of firms approaches infinity, while the total industry profit (and expected price paid) in Stahl's model first increases for sufficiently small search costs, while in our in our model it does not depend on N.

Notice that the results of Lemma 4 and Theorem 2 are directly applicable here. Therefore there is unique symmetric mixed strategy equilibrium with the upper bound of the support of price distribution equal to 1. The equilibrium level of profits is defined by:

$$\overline{\pi}(\lambda, c_0) = \frac{1 - \lambda}{2} s(1; c_0)(1 - m) \tag{9}$$

Let us define the following function[9]:

[9]Actually, it is possible to derive an explicit expression for ψ, but it is not particularly useful due to

$$\psi(p;\lambda,c_0,m)=\int_p^1 \frac{dx}{(x-m)^2\sqrt{1-(1-\lambda)s(x;c_0)}}$$

It is easy to see that $\frac{\partial\psi}{\partial p}<0$, $\frac{\partial\psi}{\partial\lambda}<0$, $\frac{\partial\psi}{\partial c}>0$, $\frac{\partial\psi}{\partial m}>0$. Now we are ready derive the equilibrium price distribution.

Proposition 4. *Suppose $N=2$. Then for any $c_0<1$ there is a unique $\lambda^*(c_0)$ such that*

- *For any $\lambda<\lambda^*(c_0)$ there is a unique equilibrium with $\underline{p}>c_0$ and the equilibrium price distribution is defined by*

$$F(p)=\frac{\sqrt{1-(1-\lambda)s(1;c_0)}-\overline{\pi}(\lambda,c_0)\psi(p;,\lambda,c_0,m)}{\sqrt{1-(1-\lambda)s(p;c_0)}} \tag{10}$$

- *For any $\lambda>\lambda^*(c_0)$ there is a unique equilibrium with $\underline{p}<c_0$ and the equilibrium distribution is defined by*

$$F(p)=1-\frac{\overline{\pi}(\lambda,c_0)}{\lambda(p-m)}-\frac{1-\lambda}{2\lambda}\left(1+\int_{c_0}^1(1-s(x))dF(x)\right) \tag{11}$$

  for all $\underline{p}\le p\le c_0$ and by (10) for all $c_0<p\le 1$.

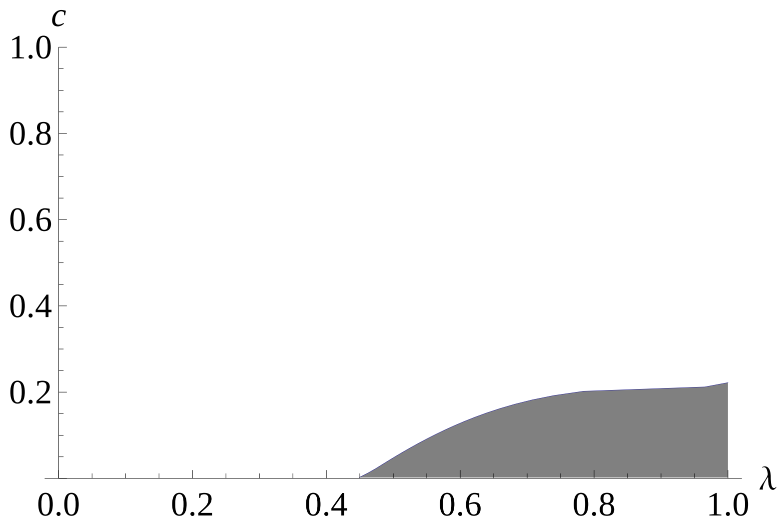
Unlike in Stahl (1989), in our model the upper bound of the support always equals the valuation and is not affected by search costs. This pushes the expected price upwards in comparison with the reference models. However active search in our model makes it less attractive to charge higher prices, therefore they are charged with lower probability, which pushes the expected price down. The following proposition establishes the interrelation between these two forces.

Proposition 5. *For any $(c_0,\lambda)\in(0,1)^2$ the expected price in the model of search without priors is lower that in the model with informed searchers.*

The average price is always lower in our model than in the reference model by Stahl (1989). However in his model consumers do not search, while in ours they do and therefore

its complexity.

Figure 2: Total Cost of Buying in Comparison with Stahl (1989), $m = 0.2$



they incur extra search costs. Figure 2 illustrates the total expected cost of acquiring the good, which includes price and search cost. In the gray area non-shoppers end up paying more for acquiring the good in our model than in Stahl's. Thus if the number of shoppers is sufficiently small, excessive search puts downward pressure on prices and this is enough to compensate consumers for their search cost (regardless how large that search cost might be). If the number of shoppers is sufficiently large the upper bound of the support of the price distribution in Stahl (1989) is far from the valuation and consumers end up paying less in his model.

Note that Proposition 3 formulated for a general model is applicable in this particular case. It implies that higher search costs c_0 or lower λ lead to higher prices in the model. This result is in line with the model with informed searchers.

Now we are going to look at the cost of being agnostic about the pricing distribution. We make two types of comparisons. First, we look at how much an (atomistic) informed searcher would be better off if she were put in our equilibrium environment. Second, we look how much an (atomistic) uninformed searcher would be worse off if she is put in an environment with informed searchers, i.e. the Stahl (1989) model. The results are represented in Figure 3: the left panel is for an informed searcher in our model, the right one is for an uninformed searcher is Stahl's model. It turns out that the loss is not dramatic: in all scenarios it does not exceed a few percentage points. As the fraction of shoppers grows, prices tend to concentrate near the lower bound of the support, uninformed searchers search very rear (while informed do not search at all) and the loss decreases. As the marginal cost of production increases, the price distribution shifts upwards and uninformed searchers search suboptimally often. In Stahl (1989) is it optimal not to search, so it is natural that the loss is increasing in search cost. In our model although consumers search a lot for small search cost, this cost is small and the loss is small. As c grows the loss increases. For large c both informed and uninformed searchers reduce their search activity, thus the loss decreases.

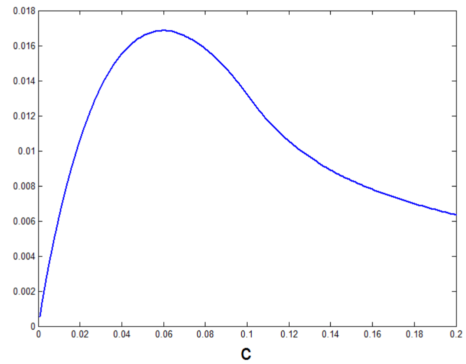
6 Discussion and Conclusions

In this paper we proposed a framework for search without priors: consumers do not have information about the distribution from which they sample and try to minimise the maximum regret. The optimal stopping rule does not possess the reservation price property, i.e. consumers stop searching with certain probability. This implies that unlike in standard search models with homogeneous goods there is active search in equilibrium. This has two important implications for empirical research: consumers search beyond the first option (which coincides with findings of de los Santos et al. (2012)) and might have different choices for the same price histories.

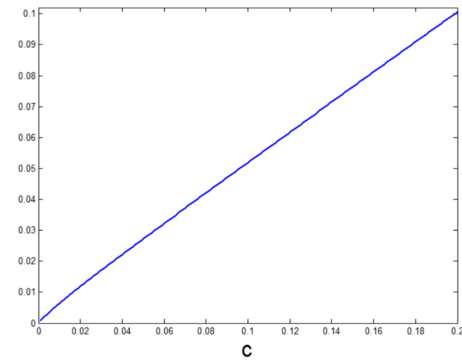
As the optimal stopping behaviour does not require any information about the underlying market model and its parameters, our optimal search strategy is applicable to a wide variety of settings. Moreover, the optimal search strategy is invariant to the choice of a market model. This approach allows us to solve the non-existence problem in a consumer search model with production cost information asymmetry.

In this paper we considered a model with heterogeneous searchers, which is quite often a difficult task in models with homogeneous goods (see, e.g. Stahl (1996)). We

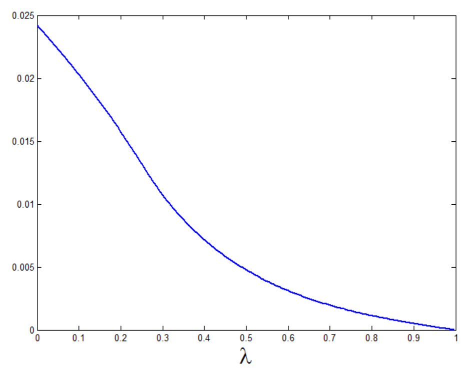
Figure 3: Equilibrium Loss



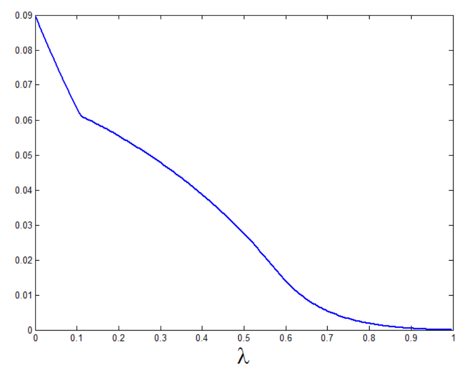
(a) Uninformed Model, $\lambda = 0.25$, $m = 0.05$



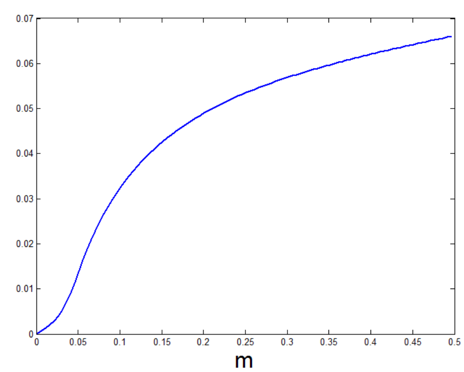
(b) Informed Model, $\lambda = 0.25$, $m = 0.05$



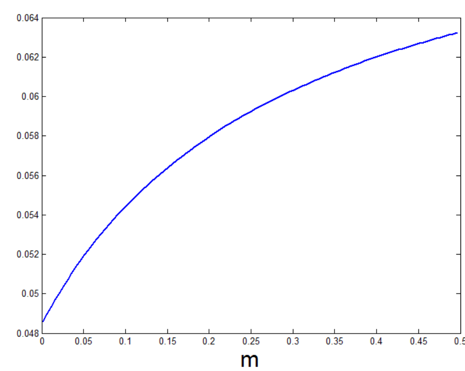
(c) Uninformed Model, $c = 0.1$, $m = 0.05$



(d) Informed Model, $c = 0.1$, $m = 0.05$



(e) Uninformed Model, $c = 0.1$, $\lambda = 0.25$



(f) Informed Model, $c = 0.1$, $\lambda = 0.25$

showed that for all parameter values there is a unique equilibrium in the model. Average prices paid by consumers in this equilibrium are lower than in Stahl (1989) (for the same parameter values). This equilibrium exhibits interesting comparative statics in the number of firms: listed prices increase and effective prices remain constant as the number of firms grows.

Throughout the paper we have made several assumptions, which we are going to discuss briefly here. Firstly, we assumed that consumers do not know the number of firms in the market. This assumption can be relaxed by setting $M = N$ and none of our results change. Secondly, in the equilibrium analysis we assumed that firms have a fixed common marginal cost. Many papers (see, e.g. Dana (1994), Tappata (2009), Janssen et al. (2011), Janssen et al. (2014)) consider a model with common stochastic marginal cost which is not known to consumers. The standard outcome of this literature (except Janssen et al. (2014)) is that consumers are worse off than in the model with complete information about the costs. Our model can be easily applied to this setting as the demand function remains the same. It turns out that it is possible (numerically) to find such a combination of model parameters that consumers are better off under cost uncertainty. Thirdly, one might ask the question: what happens if consumers know the boundaries of the support of the distribution? In this case $\underline{q}$ would depend on firms' equilibrium strategies. It turns out that the results from Section 5 do not change considerably: now it is the case that the upper bound of the support can be lower than the valuation, but the equilibrium still exists, is unique and has a distribution shape similar to the one described in Proposition 4. Finally, we assumed that uninformed searchers do not impose any symmetry assumptions on equilibrium pricing distributions. We find this assumption reasonable in most situations. In order to relax this assumption the family of the least favourable distributions has to be altered: the distribution associated with fear remains the same, while the support of the distribution associated with greed now must contain the observed price. Although it is clear that the optimal stopping rule remains stochastic (as the searcher plays a strictly competitive game against malicious nature) we were not able to obtain a closed form solution for the stopping probability.

We think that our approach to search without priors opens up a broad research agenda. It creates new possibilities for empirical research: as the optimal stopping rule is invariant to the market model choice, the model allows for direct comparison of search costs in different markets. From a theoretical perspective it would be very interesting to have a fully robust model of a search market, in which consumers do not know pricing strategies of the firms and firms do not know distribution of search costs (or, like in Bergemann and Schlag (2008) valuations) of consumers.

Appendix

Theorem 1. *The optimal stopping rule*

$$s(p;c,\underline{q}) = \begin{cases} 1 & \textit{if} \;\; p \leq \underline{q}+c \\ \frac{c}{p-\underline{q}} & \textit{if} \;\; p \in (\underline{q}+c, \min(\overline{q}+c, v)] \\ 0 & \textit{if} \;\; p > \min(\overline{q}+c, v) \end{cases} \tag{12}$$

minimises the maximum loss and guarantees that $\overline{L} = \left(1 - \frac{c}{\overline{q}-\underline{q}}\right) c.$

Proof. We consider the non-trivial case where search costs are sufficiently small such that $\underline{q}+c < v$.

If $p < \underline{q}+c$ the consumer obviously wants to stop: the best price she can get is $\underline{q}$, but she also has to pay search costs of c.

If $p > \min(\overline{q}+c, v)$ the consumer is either guaranteed to get a better offer, or rejects it, because the price is higher than the valuation of the good. In either case no purchase is made.

Now, we consider the case of $p \in [\underline{q}+c, \min(\overline{q}+c, v)]$. Note, that if $p > (\leq)\rho_{\mathbb{F}_{-j}}$ then $M_{\mathbb{F}_{-j}}(p) < (\geq)p$.

Let's consider the degenerate distribution functions $\underline{F} = \{\underline{q}\}$ and $\overline{F}_p = \{p\}$ that can help us to compute the loss function. Let's denote $\underline{\mathbb{F}}_{-j} = \{F_i : i \neq j, \exists k : F_k = \underline{F}\}$ and

$\overline{\mathbb{F}}_{-j,p} = \{F_i : \forall i \neq j F_i = \overline{F}_p\}$. Clearly $M_{\underline{\mathbb{F}}_{-j}}(q) \leq M_{\mathbb{F}_{-j}}(q) \leq M_{\overline{\mathbb{F}}_{j,q}}(q)$ for any $q \in [\underline{q}, \overline{q}]$. It is easy to verify that $\rho_{\underline{\mathbb{F}}_{-j}} \leq \rho_{\mathbb{F}_{-j}}$ and $\rho_{\overline{\mathbb{F}}_{-j,p}} = p + c$.

Since $M_{\mathbb{F}_{-j}}(p) \leq M_{\overline{\mathbb{F}}_{-j,p}}(p)$ we have that $(1-s(p))(M_{\mathbb{F}_{-j}}(p)-p) \leq (1-s(p))(M_{\overline{\mathbb{F}}_{-j,p}}(p)-p)$ for any p. Thus, the loss from excessive search is higher for the worst possible distribution for searching $\overline{\mathbb{F}}_{-j,p}$ rather then for $\mathbb{F}_{-j}$. Similarly the loss from stopping too early is maximized by $\underline{\mathbb{F}}_{-j}$. Therefore we get that

$$L(s, F_j, \mathbb{F}_{-j}) \leq \int_{\underline{q}}^{\overline{q}} \left[\mathbb{I}_{p \leq \rho_{\mathbb{F}_{-j}}}(1 - s(p))(M_{\overline{\mathbb{F}}_{-j,p}}(p) - p) + \mathbb{I}_{p > \rho_{\mathbb{F}_{-j}}} s(p)(p - M_{\underline{\mathbb{F}_{-j}}}(p))\right] dF_j(p)$$

Notice that from the definition of loss function for any $p > \rho_{\mathbb{F}_{-j}}$ (and therefore for $p > \rho_{\underline{\mathbb{F}}_{-j}}$) we have that $L(s, \mathbb{I}_{q \geq p}, \underline{\mathbb{F}}_{-j}) = s(p)(p - M_{\underline{\mathbb{F}}_{-j}}(p))$. For any $p \leq \rho_{\overline{\mathbb{F}}_{-j,p}}$, including those $p \leq \rho_{\mathbb{F}_{-j}}$, we obtain $L(s, \mathbb{I}_{q \geq p}, \overline{\mathbb{F}}_{-j,p}) = (1 - s(p))(M_{\overline{\mathbb{F}}_{-j,p}}(p) - p)$. Finally we derive the upper bound for the loss function:

$$\begin{aligned}
L(s, F_j, \mathbb{F}_{-j}) &\leq \int_{\underline{q}}^{\overline{q}} \left[\mathbb{I}_{p \leq \rho_{\mathbb{F}_{-j}}}(1 - s(p))(M_{\overline{\mathbb{F}}_{-j,p}}(p) - p) + \mathbb{I}_{p > \rho_{\mathbb{F}_{-j}}} s(p)(p - M_{\underline{\mathbb{F}}_{-j}}(p))\right] dF_j(p) = \\
&= \int_{\underline{q}}^{\overline{q}} \left[\mathbb{I}_{p \leq \rho_{\mathbb{F}_{-j}}} L(s, \mathbb{I}_{q \geq p}, \overline{\mathbb{F}}_{-j,p}) + \mathbb{I}_{p > \rho_{\mathbb{F}_{-j}}} L(s, \mathbb{I}_{q \geq p}, \underline{\mathbb{F}}_{-j})\right] dF_j(p) \leq \\
&\leq \int_{\underline{q}}^{\overline{q}} \max_p \left[\mathbb{I}_{p \leq \rho_{\mathbb{F}_{-j}}} L(s, \mathbb{I}_{q \geq p}, \overline{\mathbb{F}}_{-j,p}) + \mathbb{I}_{p > \rho_{\mathbb{F}_{-j}}} L(s, \mathbb{I}_{q \geq p}, \underline{\mathbb{F}}_{-j})\right] dF_1(p) = \\
&= \max_p \{\max\{L(s, \mathbb{I}_{q \geq p}, \overline{\mathbb{F}}_{-j,p}), L(s, \mathbb{I}_{q \geq p}, \underline{\mathbb{F}}_{-j})\}\}
\end{aligned}$$

This holds for any F_j and $\mathbb{F}_{-j}$ therefore

$$\max_{F_j, \mathbb{F}_{-j}} L(s, F_j, \mathbb{F}_{-j}) \leq \max_p \max_{\mathbb{G} \in \{\underline{\mathbb{F}}_{-j}, \overline{\mathbb{F}}_{-j,p}\}} L(s, \mathbb{I}_{q \geq p}, \mathbb{G})$$

Denote $\mathcal{F}$ as the set of all possible combination of any M distribution functions on $[\underline{q}, \overline{q}]$. As $\{\{\mathbb{I}_{q \geq p}, \underline{\mathbb{F}}_{-j}\}, \{\mathbb{I}_{q \geq p}, \overline{\mathbb{F}}_{-j,p}\}\} \subset \mathcal{F}$ for any p we have that

$$\max_{\mathbb{F} \in \mathcal{F}} L(s, F_j, \mathbb{F}_{-j}) \geq \max_p \max_{\mathbb{G} \in \{\underline{\mathbb{F}}_{-j}, \overline{\mathbb{F}}_{-j,p}\}} L(s, \mathbb{I}_{q \geq p}, \mathbb{G})$$

.

So we can simplify our problem by narrowing the set of the candidates to be the worst distributions:

$$\max_{\mathbb{F}\in\mathcal{F}} L(s, F_j, \mathbb{F}_{-j}) = \max_p \max_{\mathbb{G}\in\{\underline{\mathbb{F}}_{-j}, \overline{\mathbb{F}}_{-j,p}\}} L(s, \mathbb{I}_{q\geq p}, \mathbb{G})$$

Now we solve the problem $\min_s \max_{\mathbb{G}\in\{\underline{\mathbb{F}}_{-j}, \overline{\mathbb{F}}_{-j,p}\}} L(s, \mathbb{I}_{q\geq p}, \mathbb{G})$ for a given p. Note, that our problem is equivalent to a zero-sum game between a consumer and malicious nature: a searcher tries to minimize the loss function, while the nature maximises it. The consumer chooses whether to stop or not, while nature chooses distribution $\underline{\mathbb{F}}_{-j}$ and $\overline{\mathbb{F}}_{-j,p}$ and possibly mixture of them. Then,

$$L(s, \mathbb{I}_{q\geq p}, \underline{\mathbb{F}}_{-j}) = s(p - \underline{q} - c) \tag{13}$$

as the consumer gets a price p with probability 1, then stops with probability s and pays p instead of the expected minimal price $\underline{q}$ plus the search cost c. If the distribution is $\overline{\mathbb{F}}_{-j,p}$, then

$$L(s, \mathbb{I}_{q\geq p}, \overline{\mathbb{F}}_{-j,p}) = (1 - s)c \tag{14}$$

as the consumer incurs losses only if she faces price p and decides to continue searching, when it is optimal to stop. Note that the zero-sum game we consider admits only a mixed-strategy equilibrium. Therefore, the consumer must choose the optimal stopping probability in such a way, that the nature is indifferent between playing $\underline{\mathbb{F}}_{-j}$ and $\overline{\mathbb{F}}_{-j,p}$:

$$L(s, \mathbb{I}_{q\geq p}, \underline{\mathbb{F}}_{-j}) = L(s, \mathbb{I}_{q\geq p}, \overline{\mathbb{F}}_{-j,p})$$

which gives

$$s(p - \underline{q} - c) = (1 - s)c$$

Solving for s gives

$$s = \frac{c}{p - \underline{q}}$$

The saddle point probability μ of playing $\{\mathbb{I}_{q \geq p}, \underline{\mathbb{F}}_{-j}\}$ can be obtained from the fact that consumer is indifferent between stopping and continuing to search:

$$p = c + (1 - \mu)p + \mu \underline{q}$$

which gives

$$\mu = s = \frac{c}{p - \underline{q}}.$$

Clearly the maximum loss is achieved by choosing $F_j = \mathbb{I}_{q \leq \overline{q}}$ and equals to

$$\overline{L} = (1 - s(\overline{q}))c = \left(1 - \frac{c}{\overline{q} - \underline{q}}\right) c$$

.

□

Proposition 1. *For all $p \in [\underline{q} + c, \overline{q}]$ the optimal stopping probability $s(p; c, \underline{q})$:*

1. *decreases in price p;*
2. *increases in search costs c;*
3. *increases in the belief about the lower bound of the support $\underline{q}$.*

Proof. By taking partial derivatives we get:

$$\frac{\partial s}{\partial p} = -\frac{c}{(p - \underline{q})^2} < 0$$

$$\frac{\partial s}{\partial \underline{q}} = \frac{c}{(p - \underline{q})^2} > 0$$

$$\frac{\partial s}{\partial c} = \frac{1}{p - \underline{q}} > 0$$

□

Lemma 1. *There is no symmetric pure strategy pricing equilibria in the game.*

Proof. Suppose, that the equilibrium price is $p^* > \underline{c}$. Each firm earns $\pi_0 = (p^* - m)/N$ in equilibrium. Note, that $1 - \sigma(p^*)$ consumers search in equilibrium. Thus, a firm deviating from p^* to $p = p^* - \varepsilon$ earns $\pi_1 = (1/N + (1 - \sigma(p^*))(N-1)/N)\,(p^* - \varepsilon - m)$. Then $\pi_1 - \pi_0 = (1 - \sigma(p^*))(N-1)/N(p^* - \varepsilon - m) - \varepsilon/N$, which is clearly positive for ε small enough. Thus, there is a profitable deviation and $p^* > \underline{c}$ cannot be an equilibrium.

Now, suppose $p^* < \underline{c}$. In this case consumers do not search, and a firm can gain by deviating to $p = \underline{c}$.

Finally, suppose that $p^* = \underline{c}$. Consider a deviation to $p = 1$. Then a firm earns

$$\pi(1) = \frac{1-m}{N}\sigma(1) = \frac{1-m}{N}\int_{\underline{c}}^{\overline{c}} s(1,c)d\mu(c) \geq \frac{1-m}{N}s(1,\underline{c}) = \frac{1-m}{N}\underline{c} \geq \frac{\underline{c}-m}{N} = \pi(\underline{c})$$

The first inequality holds due to Proposition 1. □

Lemma 2. *$F(p)$ is continuous on its support.*

Proof. It makes sense to consider the case where $F(\underline{c}) < 1$. Otherwise, if $F(\underline{c}) = 1$ all consumers buy immediately and there is no reason to play mixed strategy. Such a candidate would be a pure strategy and according to Lemma 1 there is no pure strategy equilibrium.

Suppose that there is a point mass at price p. We show that there is an optimal deviation to a slightly lower price $p - \epsilon$ which allows the firm to avoid a tie at p. Consider the difference in profits before and after the deviation:

$$\Delta\Pi = Pr(p_i > p-\epsilon, p_i \neq p, \forall i)(p-\epsilon-m)\left(1/N + (N-1)/N \int_{p-\epsilon}^{\overline{p}} (1-\sigma(q))dF(q)\right) -$$

$$- Pr(p_i > p)(p-m)\left(1/N + (N-1)/N \int_{p}^{\overline{p}} (1-\sigma(q))dF(q)\right) +$$

$$+ Pr(p_j < p-\epsilon \text{ for some } j)(p-\epsilon-m)\sigma(p-\epsilon)/N - Pr(p_j < p \text{ for some } j)p\sigma(p)/N +$$

$$+\sum_{k=2}^{N} Pr(p_i \geq p-\epsilon, p_i = p \text{ for } k \text{ stores})(p-\epsilon-m)\left(1/N + (N-1)/N \int_{p-\epsilon}^{\overline{p}} (1-\sigma(q))dF(q)\right) -$$

$$-\sum_{k=2}^{N} Pr(p_i \geq p, p_i = p \text{ for } k \text{ stores})(p-m)\left(1/N + (N-k)/N \int_{p-\epsilon}^{\overline{p}} (1-\sigma(q))dF(q)\right)$$

Clearly, when ϵ approaches 0 the sum of the four first terms approaches 0, but the sum of the two last terms remains positive. □

Lemma 3. *The support of $F(p)$ is a convex set.*

Proof. Suppose there is a gap, i.e. there is an interval of prices $[p_1, p_2]$, such that $F(p_1) = F(p_2)$. Consider a price $p' \in (p_1, p_2)$. Then

$$N(\pi(p') - \pi(p_2)) = (\sigma(p') - \sigma(p_2))\left[1 - (1-F(p_2))^{N-1}\right](p'-m) > 0$$

as $\sigma(p)$ is a decreasing function. Thus the deviation to p' is profitable. □

Lemma 4. *The upper bound of the equilibrium price distribution equals* 1.

Proof. Note that

$$\pi(\overline{p}) = \frac{\overline{p}-m}{N}\sigma(\overline{p}) = \frac{1}{N}\int_{\underline{c}}^{\overline{c}} (\overline{p}-m)s(\overline{p};c)d\mu(c)$$

As for any c and $\overline{p} < 1$ we have

$$\frac{\partial[(\overline{p}-m)s(\overline{p};c)]}{\partial\overline{p}} > 0$$

we obtain $\partial\pi/\partial\overline{p} > 0$. Thus, it is optimal to set the upper bound to be equal to 1. □

Theorem 2. *For any number of firms N and any search cost distribution $\mu(c)$ there is a unique symmetric equilibrium.*

Proof. We establish existence of equilibrium by proving that equation (6) together with the boundary condition $\overline{\pi} \equiv \pi(1) = \frac{1-m}{N}\sigma(1)$ has a unique solution, which gives a well-defined distribution function.

Denote $\Phi(p) = (1 - F(p))^{N-2}$. Then (6) together with the boundary condition can be rewritten as

$$\frac{\overline{\pi}N}{(p-m)\Phi(p)} = \frac{\sigma(p)}{\Phi(p)} + (1-\sigma(p))\Phi(p)^{\frac{1}{N-2}} - (N-1)\int_p^1 (1-\sigma(x))d\Phi(x)^{\frac{1}{N-2}} \tag{15}$$

Denote $\phi(p) \equiv \frac{d\Phi(p)}{dp}$ and $\xi(p) \equiv \frac{d\Phi(p)^{\frac{1}{N-2}}}{dp} = \frac{\Phi(p)^{\frac{1}{N-2}}\phi(p)}{(N-2)\Phi(p)}$. Then by differentiating (15) we obtain:

$$-\frac{\overline{\pi}N(\Phi(p) + (p-m)\phi(p))}{(p-m)^2\Phi(p)^2} = \left(\frac{\sigma'(p)}{\Phi(p)} - \frac{\sigma(p)\phi(p)}{\Phi(p)^2}\right) + N(1-\sigma(p))\xi(p) - \sigma'(p)\Phi^{\frac{1}{N-2}} \tag{16}$$

By plugging in the expression for $\xi(p)$ and rearranging terms we obtain:

$$\phi(p) = \Phi(p)\frac{(p-m)^2\sigma'(p) + N\overline{\pi} - \sigma'(p)\Phi(p)^{\frac{N-1}{N-2}}}{(p-m)^2\sigma(p) - N(p-m)\overline{\pi} - \frac{N}{N-2}(1-\sigma(p))\Phi(p)^{\frac{N-1}{N-2}}} \tag{17}$$

Now we have a boundary value problem (17) with the initial condition $\Phi(1) = 0$. Both the numerator and denominator are differentiable almost everywhere on $p \in (0, 1]$. As $\sigma'(p) < 0$ the numerator of (17) is positive. Note, that

$$(p-m)^2\sigma(p) - N(p-m)\overline{\pi} = N(p-m)\left(\frac{(p-m)\sigma(p)}{N} - \overline{\pi}\right)$$

The term in brackets is negative (see the proof of Lemma 4), thus the numerator of (17) is negative. As the numerator of (17) is positive and the denominator is negative for all $p < 1$ if a solution to (17) exists, then $H(p)$ is decreasing, so $F(p)$ is a well-defined distribution function.

Now we proceed with the existence result. Denote the denominator of 17 as $w(p)$, so

$$w(p) = (p-m)^2\sigma(p) - N(p-m)\overline{\pi} - \frac{N}{N-2}(1-\sigma(p))\Phi(p)^{\frac{N-1}{N-2}}$$

Then,

$$\begin{aligned}
w'(p) = 2(p-m)\sigma(p) + (p-m)^2\sigma'(p) - N\overline{\pi} - \\
\frac{N}{N-2}\left[(1-\sigma(p))\frac{N-1}{N-2}\Phi(p)^{\frac{1}{N-2}}\phi(p) - \sigma'(p)\Phi(p)^{\frac{N-1}{N-2}}\right] = \\
= 2(p-m)\sigma(p) + (p-m)^2\sigma'(p) - N\overline{\pi} + \sigma'(p)\frac{w(p)-(p-m)^2\sigma(p)+N(p-m)\overline{\pi}}{(1-\sigma(p))} + \\
\frac{N-1}{N-2}(w(p)-(p-m)^2\sigma(p)+N(p-m)\overline{\pi})\frac{(p-m)^2\sigma'(p)+N\overline{\pi}-\sigma'(p)\frac{w(p)-(p-m)^2\sigma(p)+N(p-m)\overline{\pi}}{\frac{N}{N-2}(1-\sigma(p))}}{w(p)} \equiv \\
\alpha(p) + \beta(p)w(p) + \gamma(p)\frac{1}{w(p)} \qquad (18)
\end{aligned}$$

where $\alpha(p), \beta(p), \gamma(p)$ are corresponding coefficients, which are all are bounded continuously differentiable almost everywhere on $[0, 1]$. Now let $\omega(p) = w^2(p)$, then using $\omega' = 2ww'$ we can rewrite our equation as:

$$\omega' = 2\alpha(p)\sqrt{\omega} + 2\beta(p)\omega + 2\gamma(p) \qquad (19)$$

The boundary condition is $\omega_1 = 0$. The right hand side of (19) is continuous both in p and ω, thus, by Peano's theorem there exists a solution to differential equation (19), and, therefore, for (17).

Now we proceed with establishing uniqueness of the solution. Let's prove the result by contradiction. We know that there is at least one solution to this equation. Assume that there is more than one solution to this problem and consider two such functions $F_1(p), F_2(p)$ satisfying (6) and terminal condition $F_1(1) = F_2(1) = 1$. Suppose $F_1(p) \neq$

$F_2(p)$ for all p. Without loss of generality, let price $\tilde{p}$ be such that $F_2(\tilde{p}) > F_1(\tilde{p})$ and for all $p > \tilde{p}$ we have $F_2(p) \geq F_1(p)$. Obviously such a $\tilde{p}$ exists.

Consider the case where $\tilde{p} \geq \underline{c}$, so $\sigma(\tilde{p}) < 1$. Define $\Delta(\tilde{p}) = (\pi(\tilde{p}, F_1) - \pi(\tilde{p}, F_2))/(\tilde{p} - m)$. As $\pi(1, F_1) = \pi(1, F_2)$ it must be the case that $\Delta(\tilde{p}) = 0$. Now we write down the expression for $\Delta(\tilde{p}) = 0$:

$$\Delta(\tilde{p}) = \frac{1}{N}(1 - \sigma(\tilde{p}))\left[(1 - F_1(\tilde{p}))^{N-1} - (1 - F_2(\tilde{p}))^{N-1}\right] +$$
$$\frac{N-1}{N}\left[(1 - F_1(\tilde{p}))^{N-2}\int_{\tilde{p}}^{1}(1 - \sigma(x))dF_1(x) - (1 - F_2(\tilde{p}))^{N-2}\int_{\tilde{p}}^{1}(1 - \sigma(x))dF_2(x)\right]$$

Note, that the first summand of $\Delta(\tilde{p})$ is clearly positive. Now, consider the second one:

$$\begin{aligned}
(1 - F_1(\tilde{p}))^{N-2}\int_{\tilde{p}}^{\overline{p}}(1 - \sigma(x))dF_1(x) - (1 - F_2(\tilde{p}))^{N-2}\int_{\tilde{p}}^{\overline{p}}(1 - \sigma(x))dF_2(x) = \\
\left[(1 - F_1(\tilde{p}))^{N-2} - (1 - F_2(\tilde{p}))^{N-2}\right]\int_{\tilde{p}}^{1}(1 - \sigma(x))dF_1(x) + \\
(1 - F_2(\tilde{p}))^{N-2}\int_{\tilde{p}}^{1}(1 - \sigma(x))d(F_1(x) - F_2(x)) = \\
\left[(1 - F_1(\tilde{p}))^{N-2} - (1 - F_2(\tilde{p}))^{N-2}\right]\int_{\tilde{p}}^{1}(1 - \sigma(x))dF_1(x) + \\
(1 - F_2(\tilde{p}))^{N-2}\left[(F_2(\tilde{p}) - F_1(\tilde{p}))(1 - \sigma(\tilde{p})) + \int_{\tilde{p}}^{1}\sigma'(x)(F_1(x) - F_2(x))dx\right] > 0
\end{aligned}$$

as $F_2(\tilde{p}) > F_1(\tilde{p})$, for all $p > \tilde{p}$ $F_2(p) > F_1(p)$ and $\sigma'(p) < 0$. Thus, $\Delta(\tilde{p}) > 0$ and we arrived at contradiction.

Now consider the case of $\tilde{p} < \underline{c}$. In this case we have

$$\Delta(\tilde{p}) = \frac{N-1}{N}\left[(1 - F_1(\tilde{p}))^{N-2}\int_{\tilde{p}}^{1}(1 - \sigma(x))dF_1(x) - (1 - F_2(\tilde{p}))^{N-2}\int_{\tilde{p}}^{1}(1 - \sigma(x))dF_2(x)\right]$$

which is positive, as we have shown before. Therefore solution to (6) is unique.

$\square$

Proposition 2. *As the number of firms becomes larger, the probability mass concentrates in a neighbourhood around 1:*

$$\lim_{N\to\infty} F(p) = \mathbb{I}_{p\geq 1}$$

Proof. Define

$$G(p, F(\cdot), N) = \sigma(p) + (1-\sigma(p))(1-F(p))^{N-1} + \\ (N-1)(1-F(p))^{N-2}\int_p^1 (1-\sigma(x))dF(x) - \frac{N\overline{\pi}}{p-m} \quad (20)$$

Let $F_N(p)$ be the equilibrium price distribution, i.e. $G(p, F_N(\cdot), N) = 0$.

Now define

$$\tilde{G}(p, F(\cdot), N) = \sigma(p) + N(1-\sigma(1))(1-F(p))^{N-1} - \frac{N\overline{\pi}}{p-m}$$

It is straightforward to verify that $\tilde{G}(p, F(\cdot), N) \geq G(p, F(\cdot), N)$. Let $\tilde{F}_N(p)$ be a price distribution, which solves $\tilde{G}(p, \tilde{F}_N(\cdot), N) = 0$. Note that if $F_1(p) \geq F_2(p)$ for all p, then $\tilde{G}(p, F_1(\cdot), N) \leq \tilde{G}(p, F_2(\cdot), N)$.

Now we show that for all p $F_N(p) \leq \tilde{F}_N(p)$. Suppose this is not true and there is $\hat{p}$ such that $F_N(\hat{p}) > \tilde{F}_N(\hat{p})$. Then,

$$\tilde{G}(\hat{p}, F_N(\hat{p}), N) < \tilde{G}(\hat{p}, \tilde{F}_N(\hat{p}), N) = 0 = G(\hat{p}, F_N(.), N)$$

which contradicts the fact that $\tilde{G}(p, F(\cdot), N) \geq G(p, F(\cdot), N)$. Thus for all p $F_N(p) \leq \tilde{F}_N(p)$. Note that

$$\tilde{F}_N(p) = 1 - \left(\frac{(1-m)\sigma(1) - p\sigma(p)}{N(1-\sigma(1))}\right)^{\frac{1}{N-1}}$$

As $\sigma(1) < 1$ and $(1-m)\sigma(1) - p\sigma(p) > 0$ (see the proof of Lemma 4 for any $p < 1$

$\lim_{N\to\infty} \tilde{F}_N(p) = 0$, thus the same holds for $F_N(p)$. □

Proposition 3. *Consider two possible distributions of search costs: $\mu_L(c)$ and $\mu_H(c)$. Suppose $\mu_H(c) \leq \mu_L(c)$ for all c. If $F_L(p)$ and $F_H(p)$ are equilibrium price distributions for $\mu_L(c)$ and $\mu_H(c)$ correspondingly, then $F_H(p)$ stochastically dominates $F_L(p)$.*

Before we prove the proposition we need the following technical Lemma.

Lemma A.1. *Suppose that the random variables L and H are respectively distributed according to $\mu_L(c)$ and $\mu_H(c)$ with supports $[\underline{c}_L, \overline{c}_L]$ and $[\underline{c}_H, \overline{c}_H]$. Suppose μ_H first order stochastically dominates μ_L. Let $\gamma : [\underline{c}_L, \overline{c}_H] \to R$ is an increasing and differentiable function. Then $E[\gamma(H)] \geq E[\gamma(L)]$.*

Proof. Assume that the difference of distribution functions $\theta(c) = \mu_H(c) - \mu_L(c)$ has M points of discontinuity c_i, $i = 1, \ldots, M$ on $(\underline{c}_L, \overline{c}_H]$. We will consider $\theta(c)$ on each of the interval $[c_i, c_{i+1}]$, where there is only one point of discontinuity at c_{i+1} in each of them. Then the part of the difference in expected values taken for i-th interval is give by:

$$\xi([c_i, c_{i+1}]) = \int_{c_i}^{c_{i+1}} \gamma(c) d\theta(c) + \Big[\gamma(c)\theta(c)\Big]_{c_{i+1}-0}^{c_{i+1}}$$

Using integration by parts for piecewise smooth function we get:

$$\begin{aligned}\xi([c_i, c_{i+1}]) = \Big[\gamma(c)\theta(c)\Big]_{c_{i+1}-0}^{c_{i+1}} + \Big[\gamma(c)\theta(c)\Big]_{c_i}^{c_{i+1}-0} - \int_{c_i}^{c_{i+1}} \frac{d\gamma(c)}{dc}\theta(c)dc = \\ = \Big[\gamma(c)\theta(c)\Big]_{c_i}^{c_{i+1}} - \int_{c_i}^{c_{i+1}} \frac{d\gamma(c)}{dc}\theta(c)dc\end{aligned}$$

Taking the sum over all intervals, where $c_0 = \underline{c}_L$ and $c_{M+1} = \overline{c}_H$ we get:

$$\xi([\underline{c}_L, \overline{c}_H]) = \sum_{i=0}^{M} \Big[\gamma(c)\theta(c)\Big]_{c_i}^{c_{i+1}} - \int_{\underline{c}}^{\overline{c}} \frac{d\gamma(c)}{dc}\theta(c)dc = -\gamma(\underline{c})\theta(\underline{c}) - \int_{\underline{c}}^{\overline{c}} \frac{d\gamma(c)}{dc}\theta(c)dc$$

Since $\theta(c) \leq 0$ and γ is increasing, we get that $\xi([\underline{c}_L, \overline{c}_H]) \geq 0$ or equivalently $E[\gamma(H)] \geq E[\gamma(L)]$. □

Now we proceed with the proof of Proposition 3.

Proof. All prices in the equilibrium support must be higher then m. Consider the case $p > \underline{c}$. Notice that $s(p,c)$ increases in c for $p > \underline{c}$. Therefore using Lemma A.1 for $\gamma(c) = s(p,c)$ it is easy to see that for stochastically dominant distribution of costs the aggregated probability to stop is not smaller:

$$\Delta\sigma(p) = \sigma_H(p) - \sigma_L(p) \geq 0$$

Let's denote rearranging profit function as G_i for $i = L, H$:

$$G_i(p,F) = \frac{1}{N}(p-m)(1-\sigma_i(p))(1-F(p))^{N-1} +$$
$$\frac{N-1}{N}(p-m)(1-F(p))^{N-2}\int_p^{\overline{p}}(1-\sigma_i(q))dF(q) - \frac{1}{N}\left(\sigma_i(\overline{p})(\overline{p}-m) - \sigma_i(p)(p-m)\right)$$

Consider the following difference for arbitrary distribution function F:

$$\Delta G(p,F) = G_L(p,F) - G_H(p,F) =$$
$$\frac{1}{N}\Delta\sigma(p)(p-m)(1-F(p))^{N-1} + \frac{N-1}{N}(p-m)\int_p^{\overline{p}}\Delta\sigma(p)dF(q) + \psi(p) \quad (21)$$

where $\psi(p) = (\sigma_H(\overline{p})(\overline{p}-m) - \sigma_H(p)(p-m)) - (\sigma_L(\overline{p})(\overline{p}-m) - \sigma_L(p)(p-m))$. Let's show that the sign of $\psi(p)$ is not negative:

$$\psi(p) = (\sigma_H(\overline{p})(\overline{p}-m) - \sigma_L(\overline{p})(\overline{p}-m)) - (\sigma_H(p)(p-m) - \sigma_L(p)(p-m)) =$$
$$= [(\sigma_H(x)(x-m))' - (\sigma_L(x)(x-m))'](\overline{p}-p) = [E[\tilde{\gamma}(H)] - E[\tilde{\gamma}(L)]](\overline{p}-p)$$

where $\tilde{\gamma}(c) = [s(x,c)(x-m)]'_x$ and $p \leq x \leq \overline{p}$. Since for $x > \underline{c}$ we have that $\frac{\partial}{\partial c}\tilde{\gamma}(c) = \frac{m}{x^2} > 0$ and therefore $\tilde{\gamma}(c)$ is increasing function. Using Lemma A.1 we get that $E[\tilde{\gamma}(H)] \geq E[\tilde{\gamma}(L)]$ and therefore $\psi(p) \geq 0$ for any $p > \underline{c}$. Consequently we can deduce that $G_L(p,F) \geq G_H(p,F)$ for any $p > \underline{c}$. Now assume that there is $\hat{p} > \max\{\underline{c}, m\}$ such that $F_H(\hat{p}) > F_L(\hat{p})$. Then

$$G_L(\hat{p}, F_L) - G_L(\hat{p}, F_H) = \frac{1}{N}(1-\sigma_L(\hat{p}))(\hat{p}-m)[(1-F_L(\hat{p}))^{N-1} - (1-F_H(\hat{p}))^{N-1}] +$$
$$+ \frac{N-1}{N}(\hat{p}-m)\left((1-F_L(\hat{p}))^{N-2}\int_{\hat{p}}^{\overline{p}}(1-\sigma_L(q))dF_L(q)\right.$$
$$\left. - (1-F_H(\hat{p}))^{N-2}\int_{\hat{p}}^{\overline{p}}(1-\sigma_L(q))dF_H(q)\right) >$$
$$> \frac{N-1}{N}(\hat{p}-m)(1-F_H(\hat{p}))^{N-2}\int_{\hat{p}}^{\overline{p}}(1-\sigma_L(q))(f_L(q)-f_H(q))dq >$$
$$> \frac{N-1}{N}(\hat{p}-m)(1-F_H(\hat{p}))^{N-2}(1-\sigma_L(\hat{p}))(F_H(\hat{p})-F_L(\hat{p})) > 0$$

Since F_L is equilibrium distribution for $\mu_L(c)$ then $G_L(\hat{p}, F_L) = 0$. Therefore

$$G_L(\hat{p}, F_H) < G_L(\hat{p}, F_L) = 0 = G_H(\hat{p}, F_H)$$

That contradicts the fact that $\Delta G(\hat{p}, F_H) \geq 0$. Thus $F_H(p) \leq F_L(p)$ for any $p > \underline{c}$.

Now consider the case where $m < p \leq \underline{c}$ or $\sigma(p) = 1$. In this case the function $G_i(p, F)$ will take the form:

$$G_i(p, F) = \frac{N-1}{N}(p-m)(1-F(p))^{N-2}\int_{\underline{c}}^{\overline{p}}(1-\sigma_i(q))dF(q) - \frac{1}{N}\left(\sigma_i(\overline{p})(\overline{p}-m) - (p-m)\right)$$

Consider the difference:

$$\Delta G(p, F) = G_L(p, F) - G_H(p, F) = \frac{N-1}{N}(p-m)\int_c^{\overline{p}}\Delta\sigma(p)dF(q) + \psi_1,$$

where $\psi_1 = (\sigma_H(\overline{p})(\overline{p}-m) - \sigma_L(\overline{p})(\overline{p}-m)) \geq 0$. Thus we get that $\Delta G(p, F) \geq 0$. Now assume that there is $\hat{p} \leq \underline{c}$ such that $F_H(\hat{p}) > F_L(\hat{p})$.Using the previous inequalities we can simply obtain:

$$G_L(\hat{p}, F_L) - G_L(\hat{p}, F_H) > \frac{N-1}{N}(p-m)(1-F_H(p))^{N-2}(1-\sigma_L(c))(F_H(\hat{p}) - F_L(\hat{p})) = 0$$

Therefore we get the same contradiction $G_L(\hat{p}, F_H) < G_H(\hat{p}, F_H)$. $\square$

Proposition 4. *Suppose $N = 2$. Then for any $c_0 < 1$ there is a unique $\lambda^*(c_0)$ such that*

- *For any $\lambda < \lambda^*(c_0)$ there is a unique equilibrium with $\underline{p} > c_0$ and the equilibrium price distribution is defined by*

$$F(p) = \frac{\sqrt{1-(1-\lambda)s(1;c_0)} - \overline{\pi}(\lambda, c_0)\psi(p;, \lambda, c_0, m)}{\sqrt{1-(1-\lambda)s(p;c_0)}} \tag{22}$$

- *For any $\lambda > \lambda^*(c_0)$ there is a unique equilibrium with $\underline{p} < c_0$ and the equilibrium distribution is defined by*

$$F(p) = 1 - \frac{\overline{\pi}(\lambda, c_0)}{\lambda(p-m)} - \frac{1-\lambda}{2\lambda}\left(1 + \int_{c_0}^{1}(1-s(x))dF(x)\right) \tag{23}$$

for all $\underline{p} \leq p \leq c_0$ and by (10) *for all $c_0 < p \leq 1$.*

Proof. Set $N = 2$. Note, that for $\sigma(p) = \sigma_{\text{Stahl}}$ the equilibrium profit (6) can be rewritten as (we omit c_0 argument everywhere):

$$\pi(p) = \lambda(1-F(p))(p-m) + \frac{1-\lambda}{2}\Bigg(s(p) + (1-s(p))(1-F(p)) + \\ \int_{p}^{\overline{p}}(1-s(x))dF(x)\Bigg)(p-m) \tag{24}$$

For derivation of the distribution function for $p > c_0$, we divide both sides of (24) by $(p-m)$ and take the derivative with respect to p:

$$-\frac{\overline{\pi}}{(p-m)^2} = -\lambda f(p) + \frac{1-\lambda}{2}\left(s'(p)F(p) - 2(1-s(p))f(p)\right)$$

Homogeneous equation can be rewritten in the form:

$$\frac{f(p)}{F(p)} = \frac{1-\lambda}{2}\frac{s'(p)}{\lambda + (1-\lambda)(1-s(p))}$$

Thus,

$$F(p) = \frac{C(p)}{\sqrt{\frac{1}{1-\lambda} - s(p)}}$$

Then by plugging it into the original equation we get

$$C'(p) = \frac{\overline{\pi}}{(1-\lambda)(p-m)^2\sqrt{\frac{1}{1-\lambda} - s(p)}}$$

which gives us the expression in the proposition. Uniqueness follows from the uniqueness of the solution of the differential equation.

Suppose $\underline{p} < c_0$. For $p > c_0$ the equal profit condition is the same as before, and thus the equilibrium distribution is the same. For prices $p \leq c_0$ we have $s(p) = 1$, which means that consumers who visit the firm in the first search round, stop with probability one. Moreover, if a consumer comes from the competing firm, we know that she has observed a price higher than c_0 there. Therefore this consumer also stops with probability one. Thus, we have

$$\overline{\pi} = \lambda(1 - F(p))p + \frac{1-\lambda}{2}\left(1 + \int_c^1 (1 - s(q))dF(q)\right)p$$

which gives the first part of the distribution stated in the proposition.

Finally, we need to establish that there is $\lambda^*(c_0)$ such that for $\lambda \leq \lambda^*(c_0)$ for $\underline{p} > c_0$ and vice versa. Define $G(\lambda, c_0)$ to be the numerator of (10) evaluated at $p = c_0$. If $G(\lambda, c_0) < 0$, then the lower bound is naturally defined by $F(p) = 0$ and is higher than c. If $G(\lambda, c_0) > 0$, then $F(c_0) > 0$, which implies that $\underline{p} < c_0$. Now, note that (i) $G(1, c_0) = 1 > 0$, (ii) $\lim_{\lambda \to 0} = -\infty$ and $\frac{\partial G}{\partial \lambda} > 0$ since $\sqrt{1 - (1-\lambda)s(1)}$ is increasing and both $\overline{\pi}(\lambda, c_0)$ and $\psi(c_0; \lambda, c_0)$ are decreasing in λ. Therefore, there is a unique $\lambda^*(c_0)$ such that $G(\lambda^*, c_0) = 0$. $\square$

Proposition 5. *For any $(c_0, \lambda) \in (0, 1)^2$ the expected price in the model of search without priors is lower that in the model with informed searchers.*

Proof. Note that the expected profit in Stahl's model is defined by $2 * \pi^S + m = (1 - \lambda)(\rho - m) + m$, where ρ is the reservation price. Thus, the expected price (and profit) is our model is lower if and only if $\rho > c_0(1-m)+m$. Note, that $\rho = \min(1, c_0/M(\lambda)+m)$ where

$$M(\lambda) = 1 - \frac{(1-\lambda)\ln\frac{1+\lambda}{1-\lambda}}{2\lambda}$$

As $M(0) = 0, M(1) = 1$ and $M'(\lambda) > 0$ we obtain $\rho > c_0 + m > c_0(1-m) + m$ which completes the proof. □

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Alexei N. Parakhonyak

National Research University Higher School of Economics, International College of Economics and Finance and Department of Economics, Moscow, Russia.

Email: parakhonyak@yandex.ru

Anton Sobolev

University of Vienna, Vienna Graduate School of Economics, Vienna, Austria.

Email: sobolevanton.56@gmail.com