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PRICING INNOVATION IN THE PRESENCE OF WORD OF MOUTH COMMUNICATION

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The paper studies the optimal pricing strategy of innovator in the presence of word of mouth communication. In the model, an innovator develops and sells a new product to initially uninformed population of consumers. Consumers are engaged in word of mouth communication and can learn about the product and its quality directly from an advertisement or from their neighbors, who have acquired the product. The innovator knows statistical properties of the network and chooses pricing strategy to maximize profits. We find that the optimal price is a non-monotonic function of the product quality, which first increases and then decreases in the product quality. We also show that targeting highly connected consumers by offering bonuses for a neighbor's purchase with simultaneous increase in the price is an optimal strategy.

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1 Introduction

The importance of word of mouth communication as one of the essential marketing tools is well appreciated by both business and academic communities[1]. In some situations word of mouth communication is actually the only effective mechanism of spreading information about a product among consumers. This is especially the case for innovative products for which traditional means of advertisement are not efficient. Indeed, it is quite hard to explain a consumer in 10 seconds TV advertisement, what this product is aimed for, why a consumer may need it, and most importantly, why consumer has to spend a sizeable amount of money on its purchase. In contrast, sharing consumer experience with friends about new products or services is everyday experience.

One of the examples of such products is an innovative service by Dropbox company founded in 2007 by two MIT students. The service offers users storage space in the internet. Despite the fact that the company provides consumers with some space for free, in the beginning it had very few customers. One of the reasons was that consumers did not understand well why do they need to store their data in the internet, instead of having it on a flash drive with them. Advertisement using adwords with search engines like Google did not help much either. Consumers were not looking for combinations of such words, while using simpler patterns resulted in a much low targeting efficiency. The company decided to rely on the positive word of mouth communication among consumers to promote new service. The strategy turned out to be successful. Since official lunch in 2008 by 2012 the company had reached 100 million registered users.

The main focus of this article is to study the optimal pricing strategy for innovative goods in the presence of word of mouth communication. In particular, we study how the optimal price depends on the product quality, and network characteristics such as average connectivity and spreading efficiency of the network[2]. We also consider when it is optimal to give the product for free, to offer bonuses and to use freemium business model that assumes that part of the services consumers get for free.

We use the following modeling assumptions. An innovator creates a new product and sells it to a continuum of consumers. Due to the innovative nature of the product its quality is realized when development process already took place and that is why producer treats quality as given exogenously. Later on in the analysis we relax this assumption and endogenize quality choice. Consumers are embedded into a social network, which is represented by a random graph. Each consumer has an outside option distributed according to a uniform distribution and buys the product if a utility from purchase is higher than the outside option.

[1]See for instance Campbell (2009), Galeotti and Goyal (2007), Leskovec et al. (2007), Iribarren and Moro (2011), Lopez-Pintado and Watts (2008).

[2]A spreading efficiency is the ratio of the expected number of second neighbors (neighbors of neighbors) to the expected number of first neighbors. This ratio shows how many consumers become aware of the product if a consumer tells about it to one of her neighbors who buys it.

Initially, consumers are not aware of the product and to induce sales the innovator advertises it to a finite set of consumers directly. The rest of the population can learn about the product and its quality from their neighbors who have already acquired the product. In the case of innovative products informing consumers about the product and its features is costly activity and that is why the producer marketing campaign mostly relies on the word of mouth communication. In the model there is no asymmetry of information about the product quality and everyone who becomes aware of the product knows immediately its quality. The innovator knows statistical properties of a consumer network and chooses a pricing strategy to maximize profits.

We show that, in general, the optimal price is a non-monotonic function of the product quality. At first the price increases with the product quality, but after some threshold, it is optimal for the innovator to decrease the price. When the product quality is sufficiently low the diffusion is mostly limited to the first consumer, which gets a direct advertisement from the producer. In this case the optimal price is close to the monopolist price when all consumers are aware of the product and rises with the product quality. However, as the product becomes of a sufficiently high quality, longer chains of connected consumers buy it and a perimeter of diffusion wave increases. By perimeter we understand all those consumers who became aware of the product, but find its price too high. When the innovator decreases the price some share of consumers on the perimeter start to buy the product and generate further information flows. Hence, at some point the informational gains of cutting the price outweigh losses, and the optimal price decreases.

This phenomenon may explain aggressive pricing strategies of innovator firms that struggle to create new markets. For example, Google company, despite having quite established brand, prices what they call chromebooks below the average level that would be charged for a hardware of such quality on the standard notebooks market. Our paper shows that in the presence of word of mouth communication such strategy is not only efficient in terms of increasing market size, but is also optimal if the only aim of the company is to maximize revenues.

Some producers find it optimal to give a product for free to a set of consumers to fuel word of mouth communication[3]. In this way a firm by sacrificing profits from purchases of the first consumers, ensures that all their neighbors become aware of the product. In the paper we study the condition under which such free sampling policy is optimal. It turns out that the product quality has a different effect on the optimality of the free sampling, which depends on the network characteristics. When the average connectivity is lower than 1 and spreading efficiency is higher than 1 a sufficiently high product quality is a necessary condition for the optimality of free sampling. In contrast, when the opposite is true, a sufficiently low product quality is a sufficient condition for the optimality of free sampling.

[3]For example web-sites like www.bzzagent.com and houseparty.com specialize on offering products of different producers to consumers for free to generate word of mouth communication.

In the first case when the product quality rises, the probability that the diffusion reaches neighbors of the first consumer becomes sufficiently high, making it unnecessary to give the product for free. The condition is reversed in the second case, when spreading efficiency is higher than 1. An increase in the product quality increases exponentially the payoff from further sales, making it optimal to sacrifice profits from selling the product to the first consumer in order to ensure that the further diffusion will occur.

Another popular method to fuel word of mouth communication among consumers is to offer bonuses for recommendations that lead to a product purchase[4]. In contrast to price discounts, which apply to all buyers, bonuses allow a seller to treat consumers with different connectivity differently. We study the optimality of bonuses strategy and its impact on the optimal price using classical random graph model. We show that it is always optimal to offer bonuses to fuel the propagation of word of mouth. Moreover, the higher is the bonus that consumers receive, the higher is the optimal price. This policy allows the innovator to subsidize consumption of highly connected consumers at the same time extracting high profits per purchase from consumers with few links. The diffusion of the product in this case mostly takes place on the core of the network that consists of highly connected consumers that are interlinked among themselves.

One of the sales models that recently gains popularity in the business community especially among software producers is a freemium model[5]. Under the freemium business model a consumer may choose between two options - freemium and premium. The freemium option assumes limited services for free, while the premium option includes all services at some price. We show that even when there is no cost of providing services, if the valuation of the premium option is a threshold function (consumers are satisfied with the amount of services or not) then the freemium business model is never optimal. In contrast, when the valuation of the premium option linearly increases in the amount of services that a consumer uses then it is always optimal to provide consumers a free option.

There is a recent stream of network literature that studies strategic diffusion of information (see, for instance, Campbell, 2010; Galeotti and Goyal, 2009; Galeotti and Mattozzi, 2008; Chuhay, 2012). Campbell (2010) studies the optimal pricing and advertising strategy of a monopolist in the presence of word of mouth communication treating the quality as given. Our paper focuses on the impact of product quality and network characteristics on the optimal pricing. In the extension of the model we augment innovator's pricing strategy by possibilities of free sampling of the product, freemium option and use of bonuses.

[4]This strategy is often employed by branches of ing-direct banking in different countries. Okabashi, the largest US manufacturer of sandals and flip-flops, and Dropbox are companies that are known for most efficient use of this strategy. Leskovec et al (2007) also studies usage of referrals on the books market.

[5]One of the prominent examples of such policy is Dropbox company. The majority of smartphone games producers and anti-virus software offer limited versions for free, while to get the full version a user should pay.

A paper Galeotti and Goyal (2009) studies the model of strategic diffusion of information, where authors allow for network externalities in adoption decision. In the paper the authors limit diffusion only to immediate neighbors of a consumer. We model the diffusion process in explicit way, which allows us to study the effect of such network properties as average connectivity and spreading efficiency on the propagation of information. In addition the main focus of our paper is on the optimal pricing strategy, which is absent in Galeotti and Goyal (2009).

A paper Candogan et al (2010) studies the optimal pricing problem of the monopolist from a different perspective. Authors assume that the monopolist knows complete structure of the network and may decide how much each consumer should pay for the product. Knowledge of whole network of consumers requires enormous amounts of information. Moreover, as authors show the problem of the monopolist which may select consumers who get discounted price is a NP-hard problem. In contrast, we assume that the monopolist knows only stochastic properties of the network, which boil down to at most three moments of the degree distribution. Moreover, we show that by offering bonuses the innovator efficiently differentiates consumers with respect to their connectivity without knowing precise number of connections of each consumer.

The rest of the paper is organized as follows. In Section 2 we present the model. Section 3 presents the main properties of the demand function and optimal pricing strategy. In Section 4 we relax assumption about fixed quality and endogenize it as a part of the innovator's problem. Section 5 examines the optimality of the free sampling strategy. Section 6 presents results for the use of bonuses. Section 7 studies optimality of use of freemium business model and finally Section 7 concludes.

2 Model

There is an innovator that creates a new product, for which there are no close substitutes and that is why the producer acts as a monopolist. We assume that due to the innovative nature of the product its quality v is realized only after production process took place. The innovator chooses price P to maximize profits and advertises the product to a finite set of consumers.

There is a continuum of consumers that are embedded into a social network, which is represented by a random graph with degree distribution $\{p(k)\}_{k=0}^{\infty}$. We assume that consumers initially are not aware about the existence of the product and may receive information about the product and its quality either directly from producer, or from neighbors who already have bought the product. All consumers have an outside option γ_i with valuation distributed according to uniform distribution $U[0, \gamma]$. Without loss of generality we assume that $\gamma = 1$. A consumer i buys the product if $v - P > \gamma_i$. Thus a randomly selected consumer buys the product with the probability $q = v - P$. We assume that there is no asymmetry of information regarding the product quality and when a consumer buys the product all her neighbors become aware about it and its quality.

3 Main Results

In this section we first derive the demand function and study its properties. Then we study the effect of product quality and network characteristics on the optimal pricing strategy.

3.1 Demand

We begin our analysis by presenting an intuitive derivation of the demand function and studying its properties. The more rigorous derivation that rely on the use of generating functions' approach based on Newman et al. (2001) can be found in the Appendix of the paper (see Lemma 1)

A degree distribution of neighbor of randomly selected consumer plays an important role in the further analysis. Note that it is not the same as the degree distribution of a randomly selected consumer, since the more links a consumer has the greater is the probability that she is someone's neighbor. A consumer with k links has k-times higher probability to be a neighbor of randomly selected consumer than a consumer with just one link. Therefore, the probability to have a neighbor with k links is proportional to $kp(k)$. After normalization we obtain a degree distribution of neighboring consumer $\xi(k)$, which is the following:

$$\xi(k) = \frac{kp(k)}{\sum_{j=1}^{\infty} jp(j)} = \frac{kp(k)}{z_1},$$

where a normalizing factor z_1 is the average number of links that a randomly chosen consumer possesses. Using the degree distribution of neighboring consumer, we can find the expected number of second neighbors z_2, which is a number of consumers that are situated 2 links away from the current consumer. Each neighbor with degree k has $k-1$ additional links and a consumer has on average z_1 neighbors, thus $z_2 = z_1 \sum_{k=1}^{\infty}(k-1)\xi(k)$. Hence, if a consumer tells about the product to one of her neighbors and she relays the information further then on average $\frac{z_2}{z_1}$ consumers become aware of the product. We call this measure as a network spreading efficiency.

Lets calculate the expected number of consumers that become aware of the product I and that buy it N if we advertise the product to a randomly chosen consumer. Independently of the way a consumer becomes informed about the product proportion q of them buys it and thus $I = \frac{N}{q}$. A consumer that receives an advertisement becomes aware of the product, its quality and buys it with probability q, in which case all her z_1 neighbors also find out about the product. If one of these neighbors buys the product she tells about it to all her neighbors and $\tilde{z} = \frac{z_2}{z_1}$ consumers become aware of the product. The first consumer buys the product with probability q and thus if N consumers buy the product

$N - q$ of them receive the information from neighbors. The number of consumers who become aware of the product is $1 + qz_1 + \frac{z_2}{z_1}(N - q)$. Taking into account that it also should be equal to $\frac{N}{q}$ we get the following self-consistency condition:

$$1 + qz_1 + \tilde{z}(N - q) = \frac{N}{q}$$

Solving equation for N and substituting $q = v - P$ we can obtain an expression for the demand function. The following lemma formalizes the result and has more formal derivation with the use of generating functions approach.

Lemma 1 *The demand function in the case of no giant cascade of sales condition is the following:*

$$D(v, P) = q\left(1 + \frac{qz_1}{1 - q\tilde{z}}\right),$$

where $q = v - P$

Proof See Appendix □

Using the demand function we can calculate the price elasticity of demand, which is the following expression:

$$\varepsilon_P = P\left(\frac{1}{v - P} + \frac{z_1}{(\gamma + (v - P)(z_1 - \tilde{z}))(1 - (v - P)\tilde{z})}\right)$$

The price elasticity in the presence of word of mouth communication is higher than in the case of full information, which is $\frac{P}{v-P}$.

Proposition 1 *When consumers are engaged in the word of mouth communication the following holds:*

(i) The demand function increases in v.

(ii) The demand function increases in $\tilde{z}$.

(iii) The price elasticity of demand is increasing in $\tilde{z}$.

Proof See Appendix □

The first statement of the proposition implies that the higher is the product quality v the higher is the demand. This is obviously true, since probability q that a consumer buys the product is increasing in its quality and thus more consumers buy the product when they become aware of it. The second statement in particular implies that if we consider two networks, one of which is obtained by applying mean preserving spread to the degree distribution of another than the demand is higher in the latter network. The mean preserving spread increases heterogeneity in the number of links, which results in a higher expected connectivity of a neighboring consumer. This increases the spreading efficiency of the network and more consumers become aware of the product when a new consumer buys it. The same logic applies to the price elasticity. The higher is $\tilde{z}$ the more efficient is the network in the spreading of information about the product existence and thus the higher is the proportion of consumers' whose decision is affected by a price change.

One of the important characteristics of the diffusion process is a perimeter. The perimeter is the number of potential buyers who are aware of the product, but do not buy it because of high outside option. The perimeter can be obtained by multiplying the demand function by $\frac{1-q}{q}$, which is the following expression:

$$H(v, P) = (1 - (v - P)) \left(1 + \frac{(v - P)z_1}{1 - (v - P)\tilde{z}}\right)$$

Proposition 1 implies that the demand always increases in the valuation of the product, since more consumers want to buy it. However, this not always the case for the number of potential buyers. The following proposition summarizes the result:

Proposition 2 *If valuation of the product increases then the number of potential buyers:*

- *(i) Increases if both z_1 and $\tilde{z}$ are greater than 1.*
- *(ii) Decreases if both z_1 and $\tilde{z}$ are less than 1.*
- *(iii) First decreases and then increases if $z_1 < 1 < \tilde{z}$.*
- *(iv) First increases and then decreases if $\tilde{z} < 1 < z_1$.*

Proof See Appendix □

To understand better the proposition lets think about the number of new consumers that become aware of the product when the innovator sells it to the first consumer or to any other consumer situated further in the chain of buyers. If the firm sells to the first consumer then in expected terms z_1 neighbors of this consumer become aware of the product. If, however, the innovator sells the product to any consumer after the first one, we have seen that the number of consumer that become aware of the product is $\tilde{z}$. In the first two cases, described in the proposition, the firm by selling the product to any consumer, independently of her position in the chain, gets access to more or to less than one consumer and thus $H(\cdot)$ increases and decreases correspondingly. In the third case the average connectivity is smaller than one, but the network spreading efficiency is higher than one and thus the number of potential buyers first decreases and then increases. In the forth case the same logic applies and result is reversed.

3.2 Optimal Pricing

Lets turn to the optimal pricing strategy. We assume that the product is innovative and the firm ex-ante does not know the eventual quality of the product. Thus the innovator maximizes profits by choosing price and takes product quality v as given.

Proposition 3 *The optimal price decreases in $\tilde{z}$ and is lower than the optimal price in the case of full information $P_{FI}^{*} = \frac{v}{2}$.*

Proof See Appendix □

In the case of full information all consumers are aware of the product and a price variation affects the decision of the fixed number of consumers that is all population. However, when consumers initially are not aware of the product and information spreads through the word of mouth communication, a price variation affects also the number of consumers that become aware of the product. This effect leads to a lower optimal price than in the case of full information.

The decreasing behavior of the optimal price with respect to $\tilde{z}$ follows immediately from the fact that the price elasticity is increasing in $\tilde{z}$. The network spreading efficiency $\tilde{z}$ shows how many consumers become aware if consumer tells about the product to a neighbor. The higher is the spreading efficiency, the higher is the number of potential buyers whose decision to buy is affected by a price variation. This in turn leads to a lower optimal price.

Proposition 4 *There exists $\tilde{z}_c < 1$ such that if $\tilde{z}_c < \tilde{z} < 2z_1(2+\sqrt{2})$ then the optimal price is non-monotonic function in product quality v. In particular, the optimal price first increases in v and then decreases.*

Proof See Appendix □

The intuition behind the proposition is the following. If the product quality is low enough then even with price equal to zero the diffusion is mostly limited to the first consumer. In this case the innovator sets the price close to the monopolistic price $\frac{v}{2}$, since independently of the price, the first consumer is informed about the product by the firm. Thus in the beginning the optimal price rises with the quality of the product. However, as the quality rises further the optimal price decreases. By Proposition 2 we know that when $\tilde{z}$ is higher than 1 and v is sufficiently high the number of potential buyers is growing in the product quality. Thus gains of a higher product awareness overweigh losses due to lower profits from each purchase, and the optimal price decreases in the product quality.

The result holds for all $\tilde{z}$ higher than $\tilde{z}_c$, but lower than $2z_1(2+\sqrt{2})$. The lower bound appears because when the spreading efficiency is too low the component of connected consumers upon which information spreads is too small. In this case there is no sense to lower the price since majority of consumers are aware of the product and gain in new consumers does not compensate for losses in revenues from all consumers that would buy the product at a higher price. The upper bound comes from the fact that in our model we assume that the diffusion is limited and thus in the case when $\tilde{z} > 1$ the maximal product value is limited by $\frac{1}{\tilde{z}}$. An increase in the $\tilde{z}$ beyond the $2z_1(2+\sqrt{2})$ makes maximal product value to be very small and as we know the price is lower than $\frac{v}{2}$. Thus the price is already too small and increase in awareness of consumers does not compensate for the fall in revenues from existing consumers.

To study the effect of average connectivity on the optimal price level we should first make an assumption regarding the degree distribution. The reason is that average connectivity z_1 and spreading efficiency of the network $\tilde{z}$ are not independent and their relationship depends on the assumed degree distribution. In the analysis that follows we consider the case of classical random graph. Connectivity of nodes in classical random graph follows Poisson distribution and arises in the network where each node has a uniform probability to create a link to any other node. In the case of the classical random graph $\tilde{z} = z_1$ and average connectivity fully characterizes the network.

Proposition 5 *In the case of classical random graph the optimal price always decreases in the average connectivity of the network.*

Proof See Appendix □

When z_1 equals zero there is only one consumer which knows about the product from advertisement and thus the optimal price coincides with the full information price P^*_{FI}. When the average connectivity increases, the number of uninformed consumers that can become aware of the product only from neighbors who bought the product rises. This effectively lowers the optimal price, since the only way to reach these consumers is to make the product attractive enough for long chains of connected consumers.

4 Endogenous Product Quality

In previous analysis we considered the optimal pricing strategy assuming that the product quality is given exogenously. In this section we assume that the innovator can choose product quality paying some cost $c(v)$. We study how network characteristics, such as average connectivity and spreading efficiency, affect the choice of optimal quality and price. In particular, we want to check wether non-monotonic behavior of the optimal price is robust to the assumption about endogenous product quality, which is natural in many situations.

We assume that the cost of the product is associated with the development stage and its further provision to consumers is costless. This formulation, for example, fits well the case of IT industry, where designing the product is costly process, but making program copies is costless. Thus the optimization problem of the innovator becomes:

$$\max_{v,P} PD(v,P) - c(v)$$

We allow for a general form of the cost function that satisfies the following properties $c(0) = 0$, $c'(v) > 0$, $c''(v) > 0$. We start the analysis by considering the impact of average connectivity and mean preserving spread on the optimal quality of the product. The following proposition formalizes the result.

Proposition 6 *The optimal quality v^* always increases in network spreading efficiency $\tilde{z}$. It also increases in average connectivity z_1 if the transformation of degree distribution that leads to a higher average connectivity simultaneously increases $\tilde{z}$.*

Proof See Appendix □

A higher spreading efficiency of the network assumes more channels for information to spread and thus increases the marginal effect of the product quality on the quantity demanded (cross-derivative $\frac{\partial^2 D}{\partial v \partial \tilde{z}} > 0$). This in turn leads to a higher optimal quality level.

To study the effect of the average connectivity on the optimal strategy we should make an assumption about the way in which a degree distribution is transformed. Note that adding links affects not only the average connectivity but also the expected number of second neighbors and thus the spreading efficiency may rise or fall depending on the precise form of the transformation.

The second part of the proposition states that if a degree distribution transformation simultaneously increases both the average connectivity and spreading efficiency then the optimal quality rises. An increase in the average connectivity, when network spreading efficiency is fixed, increases the number of neighbors of the first consumer. The optimal product quality increases as the innovator tries to ensure that the first consumer buys the product and additional information channels are used. Obviously, the effect is even higher when the network spreading efficiency increases too.

As we have seen the result regarding the average connectivity holds if a transformation simultaneously increases both the average connectivity and spreading efficiency. For example, it is easy to show that transformation that increases the connectivity of nodes with k links by factor β with $\beta > 1$ simultaneously increases the spreading efficiency:

$$\tilde{z}' = \frac{\sum_{k=1}^{\infty} \beta k(\beta k - 1)p(k)}{\left(\sum_{j=1}^{\infty} \beta j p(j)\right)^2} = \frac{\sum_{k=1}^{\infty} k\left(k - \frac{1}{\beta}\right)p(k)}{\left(\sum_{j=1}^{\infty} j p(j)\right)^2} > \frac{\sum_{k=1}^{\infty} k(k-1)p(k)}{\left(\sum_{j=1}^{\infty} j p(j)\right)^2} = \tilde{z}$$

Another example of transformation that satisfies the property is the following. We consider a transformation that adds one link to share α of consumers with $\hat{k}$ connections. The new network spreading efficiency is given by the following expression:

$$\tilde{z}' = \frac{\sum_{k=1}^{\infty} k(k-1)p(k) - \alpha\hat{k}(\hat{k}-1) + \alpha\hat{k}(\hat{k}+1)}{\left(\sum_{j=1}^{\infty} jp(j) - \alpha\hat{k} + \alpha(\hat{k}+1)\right)^2} = \frac{z_2 z_1 + 2\alpha\hat{k}}{(z_1+\alpha)^2}$$

By construction we know that the transformation increases the average connectivity. Moreover, it is easy to show that $\tilde{z}'$ is higher than $\tilde{z}$ when $\hat{k} > \tilde{z} + \alpha\frac{\tilde{z}}{2z_1}$. In particular, the last result implies that if a transformation increases the connectivity of sufficient number of well connected nodes then its application increases both the average connectivity and spreading efficiency.

Now we turn to the analysis of the optimal pricing strategy when the product quality is endogenous. The following proposition formalizes the result regarding the impact of average connectivity on the pricing strategy for the case of classical random graph.

Proposition 7 *In the case of classical random graph, the optimal quality $v^*(z_1)$ is the solution to the following equation $c'(v) = \frac{1}{z_1}\left(\frac{1}{\sqrt{1-v}} - 1\right)$. Moreover, if $c''(v^*(0)) < 2$, $c''(v^*(1)) > \frac{2}{\sqrt{1-v^*(1)}}$ and function $c''(v) - 2(c'(v)+1)$ is monotonically increasing on the interval $[0,1]$ then the optimal price first increases and than decreases in z_1.*

Proof See Appendix □

Proposition 7 states that at least in the case of classical random graph, when the quality level is endogenous the effect of z_1 on the optimal price is, in general, non-monotonic. Recall that when product quality is fixed according to Proposition 5 an increase in the average connectivity leads to a lower optimal price, independently of the quality level. This indicates that an increase in the price for small z_1 should be attributed to the increasing quality level.

Indeed, when z_1 is close to zero the network essentially consists of one node. When the average connectivity increases the innovator increases the product quality along with simultaneous increase in the price. This allows the firm to increase sales while simultaneous increasing revenue from each purchase. However, at some level of z_1, a further increase in the product quality becomes too expensive, while gains in terms of increasing sales a from higher v are rising at a higher pace, $\frac{\partial^3 D}{\partial v \partial^2 \tilde{z}} > 0$. Thus when the product quality is sufficiently high the firm continues to increase quality, but at the same time lowers the price.

One of the examples of cost function that satisfies all the properties outlined in Proposition 7 is $\frac{1}{25(1-v)}$. If a cost function does not satisfy monotonicity property then non-monotonicity in price behavior, potentially, can be even higher. If, instead, the boundary conditions are violated then the optimal price may be increasing or decreasing on the whole interval.

5 Free Sampling Strategy

In the previous analysis we assumed that the innovator advertises the product to the first consumer, which decides whether to buy it. However, sometimes firms find it optimal to give the product for free to some number of consumers. In this section we try to capture such possibility by assuming that the innovator gives the product for free to a consumer who gets a direct advertisement of the product. We call this strategy free sampling.

Note that the profit function in the case of free sampling is different from the standard one in two respects. First, we should subtract the profit that firm makes on the first consumer, since it gives the product for free. Second, we should divide what is left by the probability that the first consumer buys the product, since she always accepts the gift. After modifications we obtain the following profit function:

$$\hat{\pi}(P) = P\frac{(v-P)z_1}{1-(v-P)\tilde{z}}$$

In the case of free sampling, the innovator does not profit directly from the first consumer, but ensures that her neighbors become aware of the product. The following two proposition characterizes the optimal price in the case of free sampling.

Proposition 8 *If $\tilde{z} > \frac{3}{4}$ then the optimal price in the case of free sampling $\hat{P}^*$ is non-monotonic function in v, which first increases and then decreases. Moreover, $\hat{P}^*$ is higher than the optimal price in the standard case if $z_1 > \tilde{z}$ and is lower otherwise.*

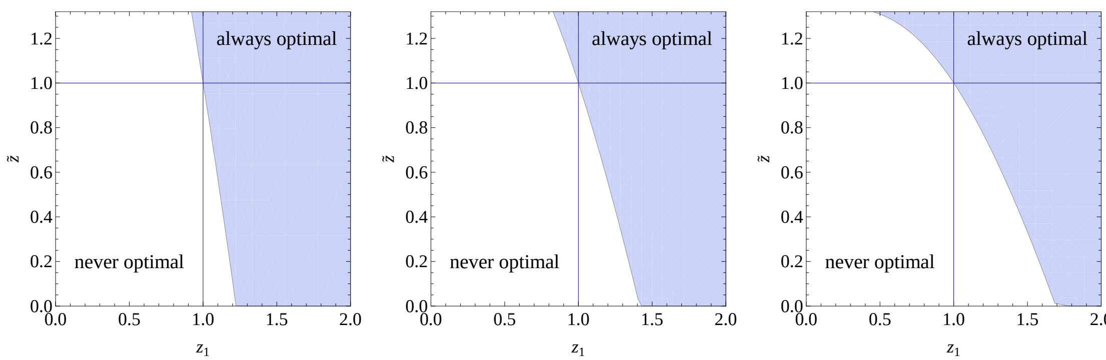
Proof See Appendix □

The intuition behind the non-monotonic behavior of the optimal price in v is exactly the same as in the standard case. When v is too small the diffusion is mostly limited to z_1 neighbors of the first consumer and the price increases in v as in the full information case. When $\tilde{z} > \frac{3}{4}$ and v is sufficiently high informational gains of spreading outweigh losses of lowering the price.

Note that in the case of free sampling the average connectivity enters into the optimization problem for the optimal price only as a factor by which profit is multiplied. Thus in the case of free sampling the optimal price does not depend on the average connectivity. Actually, one can think about the free sampling problem as a standard one where z_1 equals to $\tilde{z}$. Potential losses of profit when the first consumer does not buy the product are higher for the case when $z_1 > \tilde{z}$ as compared to $z_1 < \tilde{z}$. Thus the optimal price in the standard case is lower than $\hat{P}^*$ if $z_1 > \tilde{z}$ and is higher otherwise.

The following two propositions characterize a condition under which free sampling is optimal the optimal strategy.

Proposition 9 *The free sampling strategy gives a higher payoff than the standard one if and only if the following condition holds:*



(a) $v = 0.35$ (b) $v = 0.55$ (c) $v = 0.75$

Figure 1: Showing the area where it is optimal to use the free sampling strategy for different values of product quality.

$$(2p^3 - 9pq + 27r)^2 + 4(3q - p^2)^3 > 0,$$

$$\textit{where } p = -2v - \frac{1}{z_1 - \tilde{z}},\ q = v^2 + \frac{(v - R\tilde{z})}{z_1 - \tilde{z}},\ r = \frac{R(v\tilde{z} - 1)}{z_1 - \tilde{z}},\ \textit{and } R = z_1 \frac{2 - 2\sqrt{1 - v\tilde{z}} - v\tilde{z}}{\tilde{z}^2}$$

Proof See Appendix □

The previous proposition although states necessary and sufficient condition for the optimality of free sampling does not provide much intuition for the result. The following proposition elaborates on the result.

Proposition 10 *The following holds:*

(a) If $z_1 > 1$ and $\tilde{z} > \frac{1 - z_1(1-v)}{v}$ then it is optimal to use the free sampling strategy.

(b) If $z_1 < 1$ and $\tilde{z} < \frac{1 - z_1(1-v)}{v}$ then the free sampling is not optimal.

Proof See Appendix □

In particular, Proposition 10 implies that if $z_1 > 1$ and $\tilde{z} > 1$ and thus at each time step as a result of purchase more consumers become aware of the product then the free sampling strategy is optimal for any quality v. Conversely, if $z_1 < 1$ and $\tilde{z} < 1$ the free sampling is never optimal, since by sacrificing gains from the first consumer the producer at each time step gets access to less and less consumers.

More interesting are cases where both z_1 and $\tilde{z}$ are not higher or lower than 1. In these cases the product quality comes into play and affects optimality condition in a non-monotonic way. Figure 1 depicts the area where free sampling is optimal for different values of product quality v. In the case when $z_1 < 1 < \tilde{z}$, product quality v higher than

$\frac{1-z_1}{\tilde{z}-z_1}$ is a necessary condition for the optimality of free sampling. Indeed, if v is sufficiently small then even by giving the product for free, the seller is not able to benefit from a high spreading efficiency of the network, since the diffusion, in expected terms, is mostly limited to neighbors of the first consumer. However, when v is sufficiently high, by sacrificing gains from the first consumer the producer is able to generate enough word of mouth thanks to a sufficiently high spreading efficiency $\tilde{z}$.

In the case when $\tilde{z} < 1 < z_1$, product quality v lower than $\frac{z_1-1}{z_1-\tilde{z}}$ is a sufficient condition for the optimality of free sampling. As before, when v is sufficiently small the diffusion is mostly limited to the first consumer. Thus by giving the product for free to the first consumer the producer ensures access to z_1 consumers, which is higher than one. In contrast when the product quality is sufficiently high there is no need of giving it for free, since with a high probability the first consumer buys it anyway.

Overall, the proposition shows that the effect of the product quality on the optimality of free sampling is different and depends on whether average connectivity z_1 and spreading efficiency $\tilde{z}$ are higher or lower than 1. The result is somehow similar to the one for the expected number of potential consumers as a function of product quality v, which was increasing or decreasing depending on the particular values of z_1 and $\tilde{z}$.

Proposition 11 *If $\tilde{z} < 1$ it is never optimal to give the product for free beyond the first consumer. Contrary, if $\tilde{z} > 1$ it is always optimal for the innovator to delay charging the price till the next step.*

Proof See Appendix □

If the firm gives the product for free to a neighbor of consumer then it gets access on average to $\tilde{z}$ consumers. To be optimal this should be higher than one in order to make up for foregone profits of giving the product for free. Thus when $\tilde{z} > 1$ the innovator will prefer to postpone charging the price as long as possible, since at each time step it gets an access to a higher number of potential buyers. Note that combination of Proposition and Proposition implies that there exist $z_1 > 1$, such that for any $\tilde{z} < 1$ it is optimal to give the product for free to the first consumer only.

In the model there is no cost of production and giving the product for free has only opportunity costs of foregone profits. However, one can find a sufficiently small cost c, such that $P - c$ multiplied by the number of new consumers that buy the product will be higher than c, the cost of giving the product for free to the first consumer. Thus the result should continue to hold even when there is a cost of production of each unit, assuming that the cost is sufficiently small.

If we want to find examples of such strategy in the real world we should look at businesses which use extensively word of mouth communication for marketing the product and for which the cost of providing the product to an additional consumer is sufficiently low. One of the examples may be Dropbox company that in two years after the launch grew to more than 4 million users. The company offers a storage space in the internet for user's files. At 2008 when the Dropbox company started to provide cloud space this was an innovative product and majority of users were not aware about it. That is why the company relied mostly on the word of mouth marketing in advertisement of the product. More traditional methods like advertising the product through search engines simply did not work due to a high innovative component of the product. The company uses a freemium business model. Under this model a user gets some free space just by signing and additional space can be bought at some price. To boost the adoption in the beginning the company used different promotions upon which a user were able to increase free space. To win the mobile phone market Dropbox offers promo with 50Gb of free space for two years on the top of 2Gb that you get with free account just by installing their mobile application on some models of mobile phones.

6 Bonuses

One of the popular methods to fuel word of mouth communication is use of bonuses. A consumer gets a bonus for each neighbor buying the product. In contrast to discounts, which influence decision of all buyers in the same way, bonuses allow seller to differentiate consumers with respect to individual spreading efficiency. In this section we study optimality of bonuses strategy and its impact on the optimal price for the Poisson random network.

We assume that when consumer receives information about the product she also becomes aware about bonuses program proposed by the innovator. The bonuses program works in the following way. When a consumer buys the product and recommends it to her neighbors, she receives bonus b for each neighbor buying the product. Thus now the consumer's decision to buy the product has an additional term that represents bonuses that consumer expects to get from purchases of neighbors. If a neighbor buys the product with probability $\bar{q}_1$ then the expected share of neighbors who buy the product is also $\bar{q}_1$. Hence, the expected probability to buy the product for a randomly selected consumer with k links is the following:

$$q(k) = v - P + b\bar{q}_1 k$$

To have always meaningful $q(k)$ we assume that product quality v and bonus b are sufficiently low and thus $v + bk < \gamma$ for all k that are in support of $p(k)$. When a consumer with k links gets the information about the product from a neighbor she can earn bonuses only by recommending it to other $k-1$ neighbors and that is why a neighbor with k links buys the product with the following expected probability:

$$q_1(k) = v - P + b\bar{q}_1(k-1)$$

Weighting this probability by the degree distribution of neighbor we can formulate the following self-consistency condition for $\bar{q}_1$:

$$\bar{q}_1 = v - P + b \sum_{k=1}^{\infty} \bar{q}_1 (k-1) \xi(k)$$

Solving for $\bar{q}_1$ we get:

$$\bar{q}_1 = \frac{(v-P)z_1}{z_1 - bz_2}$$

We skip the derivation of the demand function into appendix and present the result as the following lemma.

Lemma 2 *If seller uses bonuses the demand function is the following:*

$$D = (v - P + z_1 \bar{q}_1 b) + (z_1(v-P) + (z_2 + z_1)\bar{q}_1 b) \frac{z_1(v-P) + z_2 \bar{q}_1 b}{z_1 - (v-P)z_2 - (< k^3 > -2z_2 - z_1)\bar{q}_1 b},$$

where $\bar{q}_1 = \frac{(v-P)z_1}{z_1 - bz_2}$ *and* $< k^3 >$ *is the third uncentered moment of the degree distribution.*

Proof See Appendix □

Using the obtained demand function we can find the profit function of the innovator. The difference from the usual profit function is that for each consumer that buys the product the producer gets $P - b$ instead of the price P. This applies to all consumers apart of the first, since she gets a direct advertisement from the producer and thus no one receives bonuses for that. Thus the profit function is a sum of the following two terms. The first one is $P - b$ multiplied by the demand. And the second is b multiplied by the probability that the first consumer buys the product which is $\bar{q} = \sum_{k=0}^{\infty} q(k)p(k)$. In the case of Poisson network we know that $z_2 = z_1^2$ and $< k^3 >= z_1 \left(1 + 3z_1 + z_1^2\right)$. Substituting these values into the profit function we get the following expression:

$$(v-P)\left(\frac{(1+b+b^2)(P-b)}{1-(b+(b+1)(v-P))z_1} + \frac{b}{1-bz_1}\right)$$

We assume that there is no giant cascade of sales and thus b should be lower than $\frac{1-vz_1}{(1+v)z_1}$. It is interesting to note that under no giant cascade of sales condition the model does not allow for the appearance of financial pyramid. More, precisely however high is the bonus if $P > v$ the demand is zero. It is particularly interesting, since network in our model is a stochastic graph and thus backward induction argument does not work, since there is always a probability that your neighbor also has neighbors and thus will buy the product.

Proposition 12 *In the case of Poisson random network, using bonuses is always optimal strategy. Moreover, for sufficiently small z_1 the optimal price is increasing function in the value of bonus.*

Proof See Appendix □

The Proposition 12 implies that in the case of Poisson random network it is always optimal to use bonuses strategy to fuel propagation of word of mouth in the network. Moreover, the higher is the bonus that consumers receive the higher is the optimal price. The main idea of such strategy is to facilitate spreading of the information among consumers with highest connectivity by offering them bonuses, but at the same time to charge a higher price. In this way the innovator ensures that the diffusion takes place on the network core of highly connected consumers.

7 Freemium business model

In this section we study a freemium business model. The freemium business model assumes that a consumer may choose between two options - freemium and premium services. Under the freemium option a consumer gets limited services for free, while the premium option assumes access to all services, but consumer should pay some price. A consumer can select one of these two plans. In the further analysis we consider two cases. In the first one the valuation of premium option does not depend on the actual amount of services that consumer requires. In the second case, the valuation of the premium option linearly increases in the needs of consumer.

We study the freemium business model using a simplified version of the Dropbox company problem. Assume that under the freemium option a consumer gets storage space w for free, while with the premium option a consumer gets unlimited storage space for a fixed price P. A consumer can select one of these two plans or stay with an outside option. As before consumers differ in outside option that they have γ_i. In addition they also differ in the amount of space ω_i that they need to store their files. We assume that ω_i is distributed according to $U[0, 1]$.

7.1 Threshold utility function

Assume that the consumer's utility function is a threshold function. If consumer i gets a space that is greater or equal to ω_i her utility is 1 minus the price she needs to pay for it. Thus when a consumer becomes aware of the premium and freemium offers she first checks whether a free space covers her needs. This happens with the probability $P(\omega_i < w) = w$. In this case consumer gets utility 1 from consumption and since she pays nothing it is always better than the outside option. If the free space provided by the firm is not enough, she checks whether the utility she gets by buying the product is higher than the outside option $P(1 - P > \gamma_i | \omega_i > w)$.

Given that valuation of the premium service does not depend on the space that consumer needs the probability that a consumer buys the product is simply $q_b = 1 - P$. Thus a combined probability that a consumer buys the product or uses the free space is the following:

$$q = w + (1 - w)(1 - P) = 1 - P(1 - w)$$

Note that out of those who adopt the product only fraction $\frac{q_b}{q}$ are actually buying it. Substituting q to the demand function from Lemma 1 and multiplying it by $\frac{q_b}{q}$ we get the following demand function:

$$D = (1 - P)(1 - w)\left(1 + \frac{(1 - P(1 - w))z_1}{1 - (1 - P(1 - w))\tilde{z}}\right)$$

Proposition 13 *If the valuation of the premium option does not depend on the required space it is never optimal to provide consumers with free space. The results holds even though there is no cost of providing storage space.*

The firm would benefit most from the freemium model if consumers check first premium option and only then the freemium option. In this case consumers who are not ready to buy the product still with some probability use the freemium option and spread the information about the product. However, the model works other way round and consumers first check the freemium option. The share w of informed consumers chooses freemium option and the firm looses profits by not selling to $w(1-P)$ share of informed consumers. Proposition 13 implies that a higher spread of information does not compensate for the foregone profits when the valuation of the premium option does not depend on the required space.

7.2 Linear utility function

In this part we consider the case when valuation of the premium service is proportional to the amount of space that a consumer needs. Thus if the free space provided under the freemium option is not enough ($\omega_i > w$) a consumer buys the product if $\omega_i - P > \gamma_i$. Assume that price P is higher than free space w. A consumer buys the product with the following probability:

$$q_b = \frac{1}{1 - w}\int_P^1 \int_0^{\omega - P} 1 \, d\gamma \, d\omega = \frac{(1 - P)^2}{2(1 - w)}$$

or if $P < w$ then

$$q_b = \frac{1}{1 - w}\int_w^1 \int_0^{\omega - P} 1 \, d\gamma \, d\omega = \frac{1}{2}(1 - P + w - P)$$

Thus the probability that a consumer buys the product or uses the free space option is the following:

$$q = \begin{cases} w - P + \frac{1}{2}(1 + P^2), & P > w \\ w + \frac{1}{2}(1-w)(1 - P + w - P), & otherwise \end{cases}$$

Note that out of those who adopt the product only $\frac{q_b}{q}$ are buying it. Substituting q to the demand function from Lemma 1 and multiplying by $\frac{q_b}{q}$ we get the following demand:

$$D = \begin{cases} \frac{(1-P)^2}{2}\left(1 + \frac{z_1\left((1-P)^2+2w\right)}{2-\tilde{z}((1-P)^2+2w)}\right), & P > w \\ \frac{1}{2}(1-w)(1-P+w-P)\left(1 - \frac{z_1}{\tilde{z}}\left(1 - \frac{2}{2-\tilde{z}((1-w)(1-P+w-P)+2w)}\right)\right), & otherwise \end{cases}$$

It is easy to note that the first part of the demand function is increasing in w and thus the optimal value of w is always higher or equal to P. Taking the derivative of the second part with respect to w and substituting $w = P$ one can show that the derivative is positive and thus w^* is always higher then P^*. The following proposition summarizes the result:

Proposition 14 *If the valuation of the premium option linearly increases in the required space then it is always optimal to use the freemium business model. Moreover, the optimal space* $w^* > P^*$.

In contrast to the case of the threshold utility function, when the valuation of the premium option is increasing in the required space, the probability that a consumer prefers the freemium option is negatively correlated with the probability that she finds the premium option appealing. Thus consumers who choose to use freemium option with sufficiently high probability are not willing to buy the product at first place. That is why the firm does not loose much by offering the free space. Hence, Proposition 13 states that gains of offering some free space always dominates the foregone profits from given the product for free.

8 Conclusions

In some situations word of mouth communication is the only effective mean of spreading information about a product among consumers. This is especially the case for innovative products for which traditional means of advertisement are costly and usually are not efficient. The main focus of this article is on the optimal pricing strategy for innovative goods when marketing campaign of a producer relies mostly on word of mouth communication.

In the paper we show that the optimal price is a non-monotonic function of the product quality. At first the price increases with the product quality, but after some threshold decreases. When the product quality is low enough the diffusion is mostly limited to the first consumer and the optimal price is close to the one set by the monopolist when all consumers are aware of the product. However, as the product becomes of sufficiently high quality the perimeter of the diffusion wave increases and the informational gains of cutting the price outweigh losses.

Some producers find it optimal to give a product for free to a set of consumers to boost word of mouth. By sacrificing profits from purchase of the first consumer, the producer ensures that all her neighbors become aware of the product. In the paper we show that the product quality has a different impact on the optimality of the free sampling, which depends on the network characteristics. When the average connectivity is lower than 1 and spreading efficiency is higher than 1 a sufficiently high product quality is a necessary condition for the optimality of free sampling. In contrast, when the opposite is true, a sufficiently low product quality is a sufficient condition for the optimality of free sampling.

Another popular methods to fuel word of mouth communication among consumers is to offer bonuses for recommendations that lead to a product purchase. We show that it is always optimal to offer bonuses to fuel the propagation of word of mouth. Moreover, the higher is the bonus that consumers receive, the higher is the optimal price. The main idea is to facilitate spreading of the information among the most connected consumers who constitute network, while making major part of profits on low connected individuals.

One of the models that recently gains popularity in the business community is a freemium model. Under this model the producer offers limited amount of services for free and full services at some price. We show that even when there is no costs of providing services, if the valuation of the premium option is a threshold function then it is not optimal to provide any services for free. In contrast, when the valuation of the premium option linearly increases in the amount of services that consumer uses then it is always optimal to propose consumers a freemium option.

References

[1] Campbell, A., 2010, "Word of Mouth and Percolation in Social Networks," mimeo, Yale University.

[2] Chuhay, R., 2012, "Strategic Diffusion of Information in Social Networks with Homophily," mimeo, Higher School of Economics.

[3] Candogan, O., Bimpikis, K., Ozdaglar, A., 2010, "Optimal pricing in the presence of local network effects," Internet and Network Economics pp. 118-132

[4] Galeotti, A. and Goyal, S., 2008, "A Theory of Strategic Diffusion," Rand Journal of Economics, 2009, pp. 509-532.

[5] Galeotti, A. and Mattozzi, A., 2011, "'Personal Influence': Social Context and Political Competition," American Economic Journal-Micro. 2011. 3(1), pp. 307-327.

[6] Iribarren, J.L. and Moro, E., 2011, "Affinity Paths and information diffusion in social networks," Social Networks, vol. 33, pp. 134-142

[7] Leskovec, J., Adamic, L. A. and Huberman, B. A., 2007, "The Dynamics of Viral Marketing," ACM Transactions on the Web (TWEB), 1(1), 5.

[8] Lopez-Pintado, D. and Watts, D. J., 2008, "Mass Media versus Word of Mouth," mimeo, Universidad Pablo de Olavide.

[9] Newman, M.E.J., Strogatz S.H., and Watts D.J., 2001 "Random graphs with arbitrary degree distributions," Physical Review E 64, 026118.

9 APPENDIX

Proof of Lemma 1

A random graph with an arbitrary degree distribution given by $p(k)$ can be described by means of probability generating functions. A pseudo-generating function $F_0(x)$ for a distribution $p(k)$ is given by:

$$F_0(x) = \sum_{k=0}^{\infty} qx^k p(k) \tag{1}$$

This is a polynomial expression with argument x, where the coefficient on the k-th power is a probability that a randomly chosen individual has exactly k neighbors and buys the product. A probability generating function encapsulates all information about the degree distribution, and thus completely characterizes a random network. The prefix pseudo indicates that for $x = 1$ it does not sum to 1. This happens since not all consumers buy the product. Actually, $F_0(1) = q$, which is the probability that a randomly chosen consumer buys the product given that she is aware of it.

A degree distribution of a neighbor of a randomly chosen consumer plays an important role in the further analysis. Note that it is not the same as the degree distribution of a randomly selected consumer, since the more links consumer has the greater is the probability that she will be encountered as a neighbor. A consumer with k links has k-times higher probability to be selected as a neighbor of a randomly chosen consumer than a consumer with one link. Therefore, the probability to have a neighbor with k links is proportional to $kp(k)$. After normalization we obtain a degree distribution of neighboring consumer $\xi(k)$, which is the following:

$$\xi(k) = \frac{kp(k)}{\sum_{j=1}^{\infty} jp(j)} = \frac{kp(k)}{z_1},$$

where a normalizing factor z_1 is the average number of links that a randomly chosen consumer possesses. Using the degree distribution of neighboring consumer, we can find the expected number of second neighbors z_2, which is number of consumers that are situated 2 links away from the current consumer. Note, each neighbor with degree k has $k-1$ additional links and consumer has in expected terms z_1 neighbors, thus $z_2 = z_1 \sum_{k=1}^{\infty}(k-1)\xi(k)$. A generating function that characterizes the degree distribution of consumer's neighbor is:

$$F_1(x) = q \sum_{k=0}^{\infty} \xi(k) x^k$$

Generating functions characterizing the probability that a neighboring consumer of type i has k links apart of the link, which led to this consumer is given by:

$$\hat{F}_1(x) = q \sum_{k=1}^{\infty} \xi(k) x^{k-1}$$

Let us denote by $H_1(x)$ a probability generating function over sizes of buyers components, induced by recommendation from one consumer to another consumer with k links. If the consumer does not buy the product, a component is empty. This happens with probability $1 - q \sum\limits_{k=1}^{\infty} \xi(k) = 1 - \hat{F}_1(1)$. With a complementary probability the consumer buys the product and relays information to neighbors. The further spreading of information is subject to analogous considerations for $k-1$ additional links and is described by $\hat{F}_1(H_1(x))$. We get the following self-consistency condition for $H_1(x)$:

$$H_1(x) = 1 - \hat{F}_1(1) + x\hat{F}_1(H_1(x))$$

A leading factor x accounts for the fact that the consumer buys the product. On the basis of $H_1(x)$ we can define $H_0^(x)$ - generating function describing the size of buyers components resulting from an advertisement to a randomly chosen consumer. Since a randomly chosen consumer does not buy the product with the probability $1 - F_0(1)$ we have:

$$H_0(x) = 1 - F_0(1) + xF_0(H_1(x))$$

The derivative of the generating function evaluated at $x = 1$ gives us the first moment of a distribution. That is why the number of consumers who eventually buy the product if we advertise it to a randomly chosen consumer is $\frac{\partial H_0}{\partial x}$ evaluated at $x = 1$. With the abuse of notation we assume that all function are being evaluated at point $x = 1$:

$$H_{0x} = F_0 + F_{0x} H_{1x}$$

We can find H_{1x} by solving $H_{1x} = \hat{F}_1 + \hat{F}_{1x} H_{1x}$. The solution is $H_{1x} = \frac{\hat{F}_1}{1-\hat{F}_{1x}}$. Thus we can find the number of consumers who buy the product if one consumer receives direct advertisement:

$$H_{0x} = F_0 + F_{0x} \frac{\hat{F}_1}{1 - \hat{F}_{1x}} \tag{2}$$

Note that $F_0 = q$, $F_0 x = z_1$, $\hat{F}_1 = q$, $\hat{F}_{1x} = q\frac{z_2}{z_1}$ and $q = v - P$. The demand for the product therefore is given by the following expression:

$$D(v, P) = (v - P)\left(1 + \frac{(v - P)z_1}{1 - (v - P)\tilde{z}}\right)$$

Proof of Proposition 1

Taking the derivative we get

$$D'_v(v, P) = \frac{(2 - (v - P)\tilde{z})(v - P)(z_1 - \tilde{z}) + 1}{(1 - (v - P)\tilde{z})^2}$$

The numerator of the expression represents upward sloping parabola in $\tilde{z}$. Thus taking the derivative and equating to 0 we can find its minimum, which occurs at the point $\frac{1}{v-P} + \frac{z_1}{2}$. Note that by NGC condition $\tilde{z} < 1$. Thus the minimum of the numerator when $0 < \tilde{z} < 1$ is at the point where $\tilde{z} = 1$. Substituting it into the numerator we get $((v - P)(2 - (v - P))(z_1 - 1) + 1)$ which attains the minimum when $v - P = 1$ in which case it is zero. The same holds for $\tilde{z}$:

$$D'_{\tilde{z}}(v, P) = \frac{(v - P)^3 z_1}{(1 - (v - P)\tilde{z})^2} > 0$$

The price elasticity of demand is:

$$\varepsilon_P = P\left(\frac{1}{v - P} + \frac{z_1}{(1 + (v - P)(z_1 - \tilde{z}))(1 - (v - P)\tilde{z})}\right)$$

Note that $\tilde{z}$ enters only the denominator of the second term in the brackets, which is decreasing in $\tilde{z}$. Thus the price elasticity of demand increases in $\tilde{z}$.

Proof of Proposition 2

Taking the derivative of the number of potential buyers with respect to q we get:

$$\frac{z_1(1-q(2-q\tilde{z}))-(1-q\tilde{z})^2}{(1-q\tilde{z})^2} \tag{3}$$

The denominator is always positive and thus the derivative is positive if $\tilde{z}(z_1-\tilde{z})q^2 + 2(\tilde{z}-z_1)q + z_1 - 1 > 0$. Solving we get the following two roots

$$q_1 = \frac{1}{\tilde{z}} - \frac{\sqrt{z_1(\tilde{z}-z_1)(\tilde{z}-1)}}{\tilde{z}(\tilde{z}-z_1)};\ q_2 = \frac{1}{\tilde{z}} + \frac{\sqrt{z_1(\tilde{z}-z_1)(\tilde{z}-1)}}{\tilde{z}(\tilde{z}-z_1)};$$

It is easy to show the following:

- If $1 < \tilde{z} < z_1$ then both q_1 and q_2 are complex and expression (3) is always positive.
- If $1 < z_1 < \tilde{z}$ then $q_1 < 0$ and $q_2 > 0$ and expression (3) is always positive.
- If $\tilde{z} < z_1 < 1$ then $q_1 < 0$ and $q_2 > 0$ and expression (3) is always negative.
- If $z_1 < \tilde{z} < 1$ then both q_1 and q_2 are complex and expression (3) is always negative.
- If $z_1 < 1 < \tilde{z}$ then $0 < q_1 < \frac{1}{\tilde{z}}$ and $q_2 > \frac{1}{\tilde{z}}$ thus the derivative first increases and then decreases.
- If $\tilde{z} < 1 < z_1$ then $0 < q_1 < \frac{1}{\tilde{z}}$ and $q_2 > \frac{1}{\tilde{z}}$ thus the derivative first decreases and then increases.

Lemma 3 *Function $\frac{\partial\pi(v,P)}{\partial P}$ is convex in P and crosses horizontal axes from above.*

Proof

Taking the derivative of profit with respect to price we get the following FOC:

$$\frac{\partial\pi}{\partial P} = \frac{(v-2P)-(v-P)^2(v-2P)(z_1-\tilde{z})\tilde{z}-(v-P)(Pz_1+(v-2P)(2\tilde{z}-z_1))}{(1-(v-P)\tilde{z})^2} = 0 \tag{4}$$

Lets denote this derivative by $F(v,P)$. Evaluating it at the end points we get

$$F(v,0) = \frac{v(v(z_1-\tilde{z})+1)}{(1-v\tilde{z})} > 0, \quad F(v,\frac{v}{2}) = -\frac{v^2 z_1}{(2-v\tilde{z})^2} < 0$$

The second derivative of F with respect to P is

$$F''_{PP}(v,P) = \frac{6z_1(1-v\tilde{z})}{(1-(v-P)\tilde{z})^4} \geq 0$$

Thus we can conclude that $F(v,P)$ is convex function that crosses axes x ones from above.

Proof of Proposition 3

The optimal price is lower than $P^*_{FI} = \frac{v}{2}$

Substituting $P = \frac{v}{2}$ into FOC for price we get $-\frac{v^2 z_1}{(v\tilde{z}-2)^2}$, which is negative. Thus by Lemma 3 the optimal price is lower than $P^*_{FI} = \frac{v}{2}$.

Optimal price decreases in $\tilde{z}$

Taking the derivative of FOC for price with respect to $\tilde{z}$ we get:

$$\frac{(v-P)^2(v-2P)z_1^2}{(z_1-(v-P)z_2)^2} - \frac{2P(v-P)^2 z_1^3}{(z_1-(v-P)z_2)^3},$$

which is positive if optimal price P^* is lower than $\bar{P}_{\tilde{z}} = \frac{3v\tilde{z}-4+\sqrt{(v\tilde{z}-4)^2-8v\tilde{z}}}{4\tilde{z}}$. Substituting $\bar{P}_{\tilde{z}}$ into FOC for price we get $F(v, \bar{P}_{\tilde{z}}) > 0$, which by Lemma 3 implies that $P^* > \bar{P}_{\tilde{z}}$ and thus the optimal price decreases in $\tilde{z}$.

Proof of Proposition 4

Optimal price first increases in v ***and then decreases***

Taking the derivative of price elasticity with respect to v we get

$$(\varepsilon_P)'_v = P\left(\frac{\tilde{z}^2}{(1-(v-P)\tilde{z})^2} - \frac{1}{(v-P)^2} - \frac{(z_1-\tilde{z})^2}{((v-P)(z_1-\tilde{z})+1)^2}\right) \tag{5}$$

Lets denote previous derivative by $G(v, P)$ and by $G^*(v) = G(v, P^*(v))$. Substituting $v = 0$ to the $G(\cdot)$ function we get:

$$\begin{aligned} G(0,P) &= P\left(\frac{\tilde{z}^2}{(P\tilde{z}+1)^2} - \frac{1}{P^2} - \frac{(\tilde{z}-z_1)^2}{(P(\tilde{z}-z_1)+1)^2}\right) \\ &= -P\left(\frac{(1+2P\tilde{z})}{P^2(1+P\tilde{z})^2} + \frac{(\tilde{z}-z_1)^2}{(P(\tilde{z}-z_1)+1)^2}\right) < 0 \end{aligned}$$

Hence $G(0, P)$ is less than zero for all price values and thus $G^*(0) < 0$. Lets now consider the other extreme value of v. Assume that $\tilde{z} > 1$ then the maximal value v is $\frac{1}{\tilde{z}}$. Substituting it to the (4) we get the following expression for the optimal price:

$$P^*\left(\frac{1}{\tilde{z}}\right) = \frac{2z_1-\tilde{z}}{2(z_1-\tilde{z})\tilde{z}} \tag{6}$$

The solution is non-negative if $\tilde{z} \geq 2z_1$ or $\tilde{z} < z_1$. The SOC at $P^*\left(\frac{1}{\tilde{z}}\right)$ is $\frac{2(z_1-\tilde{z})}{\tilde{z}}$ and $\frac{(2z_1-\tilde{z})}{z_1-\tilde{z}} < 1$. Thus (6) is a solution if $\tilde{z} \geq 2z_1$. If $\tilde{z} < 2z_1$ then the optimal price is zero. Substituting $v = \frac{1}{\tilde{z}}$ into $G^*(\cdot)$ we get that if $\tilde{z} > 1$ then function is the following:

$$G^*\left(\frac{1}{\tilde{z}}\right) = \begin{cases} -\frac{2(z_1-\tilde{z})\left(8z_1^2-8z_1\tilde{z}+\tilde{z}^2\right)}{(2z_1-\tilde{z})\tilde{z}}, & \max\{1, 2z_1\} \leq \tilde{z} \\ \infty, & 1 < \tilde{z} < \max\{1, 2z_1\} \end{cases}$$

Thus $G^*\left(\frac{1}{\tilde{z}}\right) > 0$ if $1 < \tilde{z} < 2z_1(2+\sqrt{2})$. It is easy to check that (4) is continuous in v and thus by Lemma 3 the optimal price P^* is also continuous in v. One also can note that given no giant cascade condition $G(v, P)$ is continuous in both v and P. Thus there exists $\tilde{z}_c < 1$ such that $G^*\left(\frac{1}{\tilde{z}}\right) > 0$ if the following holds:

$$\tilde{z}_c < \tilde{z} < 2z_1(2+\sqrt{2}) \tag{7}$$

Taking derivative of $G(v, P)$ with respect to v we get:

$$G'_v(v, P) = 2P\left(\frac{1}{(v-P)^3} + \frac{(z_1-\tilde{z})^3}{(2+(v-P)(z_1-\tilde{z}))^3} + \frac{\tilde{z}^3}{(1-(v-P)\tilde{z})^3}\right)$$

The last term in the brackets is positive. The sum of first two terms is also positive:

$$\frac{2(1+(v-P)(z_1-\tilde{z}))((v-P)^2(z_1-\tilde{z})^2+2(2+(v-P)(z_1-\tilde{z})))}{(v-P)^3(2+(v-P)(z_1-\tilde{z}))^3} > 0$$

Thus $G(v, P)$ is increasing function of v for all $P \in \left[0, \frac{v}{2}\right]$. Lets consider the derivative

$$\frac{\partial}{\partial v}G^*(v) = G'_v(v, P^*(v)) + G'_P(v, P^*(v))\frac{\partial P^*(v)}{\partial v}$$

We want to prove that there is at most one point where $G^*(v)$ changes sign. Indeed, assume that $G^*(v_0) = 0$ and there exists v_1, s.t. $G^*(v_1) = 0$. We know that $G^*(0) < 0$ thus for the second point to exists the derivative $\frac{\partial}{\partial v}G^*(v_1)$ should be less than zero. By assumption $G^*(v_1) = \frac{\partial \varepsilon_P(v_1)}{\partial v} = 0$ and thus $\frac{\partial P^*(v)}{\partial v}$ should be equal to zero. We got a contradiction since $\frac{\partial}{\partial v}G^*(v_1) = G'_v(v_1, P) > 0$. Thus we can conclude that if there is v_0 s.t. $G^*(v_0) = 0$ then $\forall v > v_0$ function $G^*(v) \geq 0$.

The last statement in particular implies that there is v_0 s.t. $G^*(v_0) = 0$ only if $G^*\left(\frac{1}{\tilde{z}}\right) \geq 0$. Thus we can conclude: if condition (7) holds then the optimal price first increases in v and then decreases.

Proof of Proposition 5

The first order condition in the case of Poisson degree distribution is the following:

$$F = \frac{v - 2P - (v-P)^2 z_1}{(1-(v-P)z_1)^2} = 0$$

By second order condition the derivative of F with respect to P is less than zero. The derivative of F with respect to z_1 is the following:

$$\frac{\partial F}{\partial z_1} = \frac{(v-2P)^2 - (v-P)^3 z_1 - P}{(1-(v-P)z_1)^3}$$

The denominator of the derivative is positive and the nominator is negative since by the first order condition $v - 2P = (v-P)^2 z_1$. Thus by implicit function theorem the derivative $\frac{\partial P^*}{\partial z_1}$ is negative and the optimal price decreases in z_1.

Proof of Proposition 6

The first order conditions with respect to price and quality, which we denote by F and G are the following:

$$\begin{array}{l} F = \frac{\partial \pi}{\partial P} = D + P\frac{\partial D}{\partial P} = 0 \\ G = \frac{\partial \pi}{\partial v} = P\frac{\partial D}{\partial v} - \frac{\partial c}{\partial v} = 0 \end{array} \tag{8}$$

Lets denote by $P^*(v, z_1, \tilde{z})$ the solution for the optimal price from the first equation in (8) and substitute it the profit function. In this way we reduce our problem to just one variable v. Taking partial derivative of the result with respect to v we get:

$$\frac{\partial P^*}{\partial v}\left(D + P^*\frac{\partial D}{\partial P}\right) + P^*\frac{\partial D}{\partial v} - \frac{\partial c}{\partial v} = 0$$

Note that the demand function effectively depends on one parameter $q = v - P$ and thus $\frac{\partial D}{\partial P} = -\frac{\partial D}{\partial v}$. Substituting it and deriving the FOC with respect to $\tilde{z}$ we get:

$$\frac{\partial^2 P^*}{\partial v \partial \tilde{z}}\left(D + P\frac{\partial D}{\partial P}\right) + \frac{\partial P^*}{\partial v}\left(\frac{\partial D}{\partial \tilde{z}} + \frac{\partial D}{\partial P}\frac{\partial P^*}{\partial \tilde{z}} + \frac{\partial P^*}{\partial \tilde{z}}\frac{\partial D}{\partial P} + P^*\frac{\partial^2 D}{\partial P \partial \tilde{z}}\right) - \frac{\partial P^*}{\partial \tilde{z}}\frac{\partial D}{\partial P} - P^*\frac{\partial^2 D}{\partial P \partial \tilde{z}}$$

The first two terms are zero because first order condition for the price should hold for any v and $\tilde{z}$. From the second term we can express:

$$\frac{\partial D}{\partial P}\frac{\partial P^*}{\partial \tilde{z}} = -\frac{1}{2}\frac{\partial D}{\partial \tilde{z}} - \frac{1}{2}P^*\frac{\partial^2 D}{\partial P \partial \tilde{z}}$$

Substituting it to the third term we get:

$$\frac{1}{2}\left(\frac{\partial D}{\partial \tilde{z}} - P^*\frac{\partial^2 D}{\partial P \partial \tilde{z}}\right)$$

We know that $\frac{\partial D}{\partial \tilde{z}} > 0$ and $-\frac{\partial^2 D}{\partial P \partial \tilde{z}} = \frac{\partial^2 D}{\partial v \partial \tilde{z}}$, which is positive. Thus using the fact that by second order condition the second derivative of $\pi(v, P^*(v, z_1, \tilde{z}), z_1, \tilde{z})$ with respect to v is negative by implicit function theorem we can conclude that $\frac{\partial v^*}{\partial \tilde{z}} > 0$.

Noting that $\frac{\partial^2 D}{\partial v \partial z_1} > 0$ we can show that an increase in the average connectivity which leads to a higher $\tilde{z}$ leads to a higher optimal quality.

Proof of Proposition 7

Substituting $z_2 = z_1^2$ into the first order conditions we get:

$$F = (v - P)(1 - z_1(v - P)) - P = 0$$
$$G = (v - P) - (1 - z_1(v - P))c'(v) = 0$$

Taking the full derivative with respect to z_1 of both conditions and solving for $\frac{dv}{dz_1}$ and $\frac{dP}{dz_1}$ we get:

$$\frac{dv}{dz_1} = \frac{(v - P)\,(v - P + (2 - z_1(v - P))c'(v))}{2c''(v)(1 - z_1(v - P))^2 - z_1 c'(v) - 1}$$

$$\frac{dP}{dz_1} = \frac{(v - P)\,(v - P + (1 - (v - P)z_1)\,(c'(v) - (v - P)c''(v)))}{2c''(v)(1 - z_1(v - P))^2 - z_1 c'(v) - 1}$$

From **previous** analysis we know that $\frac{dv^*}{dz_1} > 0$, which implies that the denominator in both expressions is positive. The derivative $\frac{dP}{dz_1}$ is positive when the nominator is positive. Solving for $c'(v)$ FOC and substituting it to the nominator we get the following condition $2 - (1 - (v - P)z_1)c''(v) > 0$. Substituting the optimal price and taking into account that $\frac{\partial D}{\partial v} < \frac{\partial^2 c}{\partial v^2}$ we get the following condition:

$$\frac{1}{1 - vz_1} < c''(v) < \frac{2}{\sqrt{1 - vz_1}} \tag{9}$$

Lets denote by $v^*(z_1)$ the optimal price. Thus if $c''(v^*(0)) < 2$ for sufficiently small z_1 the optimal price increases in z_1. If $c''(v^*(1)) > \frac{2}{\sqrt{1-v^*(1)}}$ then for sufficiently high z_1 the optimal price decreases in z_1. Substituting the optimal price into the second FOC we get:

$$c'(v) = \frac{1}{z_1}\left(\frac{1}{\sqrt{1 - vz_1}} - 1\right) \tag{10}$$

Combining (9) and (10) we get that if $c''(v) - 2(c'(v) + 1)$ is quasi-concave function on the interval $[0, 1]$ and $c''(v^*(0)) < 2$ and $c''(v^*(1)) > \frac{2}{\sqrt{1-v^*(1)}}$ then the optimal price first increases in z_1 and then decreases.

Assume that the cost function is the following $c(v) = \frac{v}{1-v}$, The first derivative is $c'(v) = \frac{1}{(1-v)^2}$

Proof of Proposition 8

If $\tilde{z} > \frac{3}{4}$ then the optimal price in the case of free sampling $\hat{P}^* = \frac{\sqrt{1-v\tilde{z}}-(1-v\tilde{z})}{\tilde{z}}$ is non-monotonic function in v, which first increases and then decreases.

Taking the first derivative of profit function in the case of free sampling with respect to price we get $\frac{z_1(v-2P)-(v-P)^2 z_1 \tilde{z}}{(1-(v-P)\tilde{z})^2}$. The second derivative is $-\frac{2z_1(1-v\tilde{z})}{(1-(v-P)\tilde{z})^3}$, which is negative and thus the function is concave and the following critical point is maximum:

$$\hat{P}^* = \frac{\sqrt{1-v\tilde{z}} - (1-v\tilde{z})}{\tilde{z}} \tag{11}$$

Taking the first derivative of $\hat{P}^*$ with respect to v we get $1 - \frac{1}{2\sqrt{1-v\tilde{z}}}$. It is positive when $v = 0$ and is negative when $v > \frac{3}{4\tilde{z}}$. Thus if $\tilde{z} > \frac{3}{4}$ there are v such that the derivative is negative. Taking into account that the second derivative is always negative $-\frac{\tilde{z}}{4(1-v\tilde{z})^{3/2}}$ we can conclude that if $\tilde{z} > \frac{3}{4}$, $\hat{P}^*$ first increases in v and then decreases.

The optimal price in the case of free sampling $\hat{P}^*$ is higher than in the standard case if $z_1 > \tilde{z}$ and is lower otherwise.

Taking into account that $\pi(P)$ is quasi concave, price $\hat{P}^*$ will be higher than P^* if $\frac{\partial\pi(\hat{P}^*)}{\partial P} < 0$. Substituting (11) into (12) the condition reduces to the following:

$$\frac{(z_1 - \tilde{z})\left(v\tilde{z} - 2(1-\sqrt{1-v\tilde{z}})\right)}{\tilde{z}^2} < 0$$

The last condition holds whenever $z_1 < \tilde{z}$. Thus optimal price in the case of free sampling $\hat{P}^*$ is higher than in the standard case if $z_1 > \tilde{z}$ and is lower otherwise.

Proof of Proposition 9

Condition for the optimality of free sampling.

Substituting the optimal price in the case of free sampling to $\hat{\pi}$ we get:

$$\hat{\pi}^* = z_1 \frac{2\left(1-\sqrt{1-v\tilde{z}}\right) - v\tilde{z}}{\tilde{z}^2}$$

Profit function for the case when we sell to the first consumer is the following:

$$\pi(P) = P(v-P)\left(1 + \frac{(v-P)z_1}{1-(v-P)\tilde{z}}\right)$$

The first derivative with respect to price is:

$$\frac{(v-P)^2(v-2P)(\tilde{z}-z_1)\tilde{z} - (v-P)((3P-v)z_1 + 2(v-2P)\tilde{z}) + (v-2P)}{(1-(v-P)\tilde{z})^2} \tag{12}$$

The denominator is always positive by no giant cascade of sales condition. Substituting $P = 0$ we get $\frac{1+v(v(z_1-\tilde{z}))}{1+\tilde{z}-v} > 0$, and for $P = v$ we get $-v$. The derivative of the numerator with respect to P is $-2((2v-3P)(z_1-\tilde{z})+1)(1-(v-P)\tilde{z})$. Thus the derivative first decreases and then after $P = \frac{1}{3}\left(2v + \frac{1}{z_1 - \tilde{z}}\right)$ increases. However, we know that at $P = v$ the derivative is still negative and thus on the interval $P \in [0, v]$ function $\pi(P)$ is quasiconcave.

To find whether $\hat{\pi}^* \geq \pi(P)$ for any $P \in [0, v]$ we should identify, whether equation $\pi(P) = \hat{\pi}^*$ has roots on interval $[0, v]$. Rewriting, we get the following cubic equation:

$$(z_1 - \tilde{z})P^3 - (2v(z_1 - \tilde{z}) + 1)P^2 + (v^2(z_1 - \tilde{z}) + v - R\tilde{z})P - R(1 - v\tilde{z}) = 0, \tag{13}$$

where $R = z_1 \frac{2 - 2\sqrt{1 - v\tilde{z}} - v\tilde{z}}{\tilde{z}^2}$. Taking into account that $\pi(P)$ is quasiconcave function on $[0, v]$ there should exist two or none roots in this region. However, the cubic equation may have one or three real roots. Thus if (13) has just one real root then we can conclude $\hat{\pi}^* \geq \pi^*$. The condition is the following:

$$(2p^3 - 9pq + 27r)^2 + 4(3q - p^2)^3 \geq 0, \tag{14}$$

where $p = -2v - \frac{1}{z_1 - \tilde{z}}$, $q = v^2 + \frac{v - R\tilde{z}}{z_1 - \tilde{z}}$ and $r = \frac{R(v\tilde{z} - 1)}{z_1 - \tilde{z}}$.

Proof of Proposition 10

Condition for the optimality of free sampling. Intuition.

Taking the derivative of the difference $\hat{\pi}(P) - \pi(P)$ with respect to P and substituting $P = 0$ we get the following expression:

$$\eta = \frac{v(z_1 - 1 + v(\tilde{z} - z_1))}{1 - v\tilde{z}}$$

When η is higher than zero the profit function in the case of free sampling has a higher slope than the standard one at $P = 0$. We also can find an interior point of intersection of the two functions, which is is not zero or one. The point is $P_c = v + \frac{z_1 - 1}{\tilde{z} - z_1}$.

Lets consider first the case when $z_1 < 1$ and $\tilde{z} < 1$. It is easy to show that η is negative and P_c does not belongs to the interval $(0, 1)$. Thus we can conclude that for any $P \in (0, 1)$ function $\hat{\pi}(P)$ lies below $\pi(P)$, which implies that it is never optimal to use free sampling.

When $z_1 > 1$ and $\tilde{z} > 1$ we can show that η is positive for any v and P_c does not belongs to the interval $(0, 1)$. Thus we can conclude that for any $P \in (0, 1)$ function $\hat{\pi}(P)$ lies above $\pi(P)$, which implies that it always optimal to use the free sampling strategy.

If $z_1 < 1$ and $1 < \tilde{z} < \frac{1 - z_1(1 - v)}{v}$ then η is negative and P_c does not belong to $(0, 1)$. Thus we can conclude that in this case a necessary condition is $\tilde{z} > \frac{1 - z_1(1 - v)}{v}$.

When $z_1 > 1$ and $\frac{1 - z_1(1 - v)}{v} < \tilde{z} < 1$ one can show that η is positive and P_c does not belong to $(0, 1)$. Thus condition $\tilde{z} > \frac{1 - z_1(1 - v)}{v}$ is a sufficient condition for the optimality of free sampling.

Proof of Proposition 11

It is not optimal to give the product for free to any consumer beyond the first one if $\tilde{z} < 1$.

Profit function for the case of free sampling can be rewritten in the following way: $\hat{\pi}(P) = P(v - P)z_1 \frac{1}{1-(v-P)\tilde{z}}$. Term $(v - P)z_1$ is the probability that the first **neighbor** buys the product multiplied by the number of first neighbors z_1. If the innovator gives the product for free to the first neighbor the profit it gets is $\tilde{\pi}(P) = Pz_1 \left(\frac{1}{1-(v-P)\tilde{z}} - 1\right)$. One is subtracted to take into account that we do not sell the product to the first neighbor. Solving for the optimal price we get $\tilde{P}^* = \frac{v\tilde{z}-1+\sqrt{1-v\tilde{z}}}{\tilde{z}}$. Substituting the optimal prices $\hat{P}^*$ and $\tilde{P}^*$ into profit functions correspondingly and taking the difference we get:

$$\hat{\pi}(\hat{P}^*) - \tilde{\pi}(\tilde{P}^*) = \frac{z_1(1-\tilde{z})\left(2 - v\tilde{z} - 2\sqrt{1-v\tilde{z}}\right)}{\tilde{z}^2} = \frac{z_1(1-\tilde{z})\left(1 - \sqrt{1-v\tilde{z}}\right)^2}{\tilde{z}^2}$$

The difference is positive for $\tilde{z} < 1$ and is negative otherwise.

Proof of Lemma 2

Evaluating generating functions we get:

$$F_0(1) = \sum_{k=0}^{\infty} p(k)q(k) = v - P + z_1\bar{q}_1 b$$

$$F'_{0x}(1) = \mu \sum_{k=0}^{\infty} kp(k)q(k) = z_1(v-P) + (z_2 + z_1)\bar{q}_1 b$$

$$\hat{F}_1(1) = \sum_{k=0}^{\infty} \xi(k)q_1(k) = (v - P + b) + \bar{q}_1 b \frac{z_2}{z_1}$$

$$\hat{F}'_{1x}(1) = \mu \sum_{k=1}^{\infty} (k-1)\xi(k)q_1(k) = \frac{z_2(v - P + b) + (< k^3 > -2z_2 - z_1)\bar{q}_1 b}{z_1}$$

Substituting into expression for the demand (2) we get:

$$D = (v - P + z_1\bar{q}_1 b) + (z_1(v-P) + (z_2 + z_1)\bar{q}_1 b)\frac{z_1(v-P) + z_2\bar{q}_1 b}{z_1 - (v-P)z_2 - (< k^3 > -2z_2 - z_1)\bar{q}_1 b}$$

Proof of Proposition 12

The FOC with respect to b is the following:

$$(P-v)\left(\frac{b^2+b+1}{z_1(b(P-v-1)+P-v)+1}-\frac{\left(b^2+b+1\right)z_1(b-P)(P-v-1)}{(z_1(b(P-v-1)+P-v)+1)^2}\right.$$
$$\left.+\frac{(2b+1)(b-P)}{z_1(b(P-v-1)+P-v)+1}+\frac{1}{bz_1-1}-\frac{bz_1}{(bz_1-1)^2}\right)=0$$

Substituting $b=0$ we get the following expression:

$$\frac{(v-P)(P-(v-2P)z_1+(v-P)^2z_1^2)}{(1-(v-P)z_1)^2}$$

An expression in the second brackets in the numerator is an upward sloping parabola in P with a positive root given by the following expression:

$$\hat{P}=v-\frac{1+2z_1-\sqrt{1+4z_1(z_1+1)(1-vz_1)}}{2z_1^2}$$

Thus if the optimal price is higher than $\hat{P}$ then the derivative $\frac{\partial\pi}{\partial b}\big|_{b=0}$ is positive. Substituting $P=\hat{P}$ and $b=0$ into the FOC for the price we get:

$$\frac{2v}{1+2z_1(1-v(z_1+1))+\sqrt{1+4z_1(z_1+1)(1-vz_1)}}$$

If $1+2z_1(1-v(z_1+1))>0$ then the expression is trivially positive. Assume that $1+2z_1(1-v(z_1+1))<0$ then the expression is positive if

$$\sqrt{1+4z_1(z_1+1)(1-vz_1)}>|1+2z_1(1-v(z_1+1))|$$

Squaring both sides and rearranging we get:

$$4vz_1(1+z_1)^2(1-vz_1)>0$$

Thus we can conclude that when $b=0$ the optimal price is always higher than $\hat{P}$, which in turn implies that $\frac{\partial\pi}{\partial b}\big|_{b=0}$ and thus using bonuses is always an optimal strategy.

The FOC with respect to price is:

$$\frac{(b^2+b+1)(v-P)\left(z_1\left(b^2-(b+1)v\right)+1\right)}{(z_1(b(P-v-1)+P-v)+1)^2}-\frac{b}{1-bz_1}-\frac{(b^2+b+1)(P-b)}{z_1(b(P-v-1)+P-v)+1}=0$$

Taking the derivative with respect to b and evaluating limit as z_1 approaches 0 we get plus infinity, which implies that for sufficiently small z_1 the optimal price is increasing in b.

Proof of Proposition 13

The derivative of the profit function with respect to w is the following:

$$P(1-P)\left(\frac{z_1}{\tilde{z}}\left(1-\frac{1-\tilde{z}}{(1-(1-P(1-w))\tilde{z})^2}\right)-1\right) \tag{15}$$

The second derivative is always negative:

$$-\frac{2(1-P)P^2 z_1(1-\tilde{z})}{(1-\tilde{z}(1-P(1-w)))^3}$$

Thus the profit function is concave in w. Note that the expression in the brackets in (15) increases in P. Evaluating (15) at $w=0$ and assuming that P equals to 1 we get in the brackets z_1-1, which is negative if $z_1<1$. Hence, for any P (15) at $w=0$ is always negative. Taking into account that profit is concave and when $z_1<1$ the first derivative is negative at 0 we can conclude that the optimal free space is zero.

Assume that $z_1>1$. The expression in the brackets in (15) can be rewritten as:

$$\frac{(1-w)^2(z_1-\tilde{z})\tilde{z}P^2+2(1-w)(z_1-\tilde{z})(1-\tilde{z})P-(1+z_1-\tilde{z})(1-\tilde{z})}{(1-(1-P(1-w))\tilde{z})^2}$$

The nominator represents upward sloping parabola in P with roots:

$$P_{1,2}=\frac{-(z_1-\tilde{z})(1-\tilde{z})\mp\sqrt{z_1(z_1-\tilde{z})(1-\tilde{z})}}{(1-w)(z_1-\tilde{z})\tilde{z}}$$

The smallest root is negative and thus the expression is positive only if $P>P_2$. Taking the derivative of the profit function with respect to P and substituting P_2 we get:

$$\frac{-z_1-(z_1-\tilde{z})(1-\tilde{z})-2\sqrt{z_1(z_1-\tilde{z})(1-\tilde{z})}}{\tilde{z}^2}$$

The expression does not depend on w and is always negative. Thus the optimal price is always lower than P_2, which in turn implies that the first order condition for w is always negative. Hence, it is not optimal to give any space for free.

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В данной статье рассматривается оптимальная ценовая стратегия инноватора, который учитывает присутствие сарафанного радио. В модели инноватор разрабатывает и продает новый продукт потребителям, которые изначально не знают о его существовании. Потребители общаются между собой и могут узнать о продукте и его качестве непосредственно из рекламы или от соседей, которые уже приобрели товар. Инноватор знает статистические свойства сети и выбирает ценовую стратегию, чтобы максимизировать прибыль. Мы показываем, что оптимальная цена сначала увеличивается, а затем уменьшается при росте качества продукта. Мы также показываем, что таргетирование потребителей с большим количеством связей с помощью бонусов за покупки, сделанные соседями, при одновременном повышении цены является оптимальной стратегией.

Ключевые слова: сарафанное радио, вирусный маркетинг, распространение информации, социальные сети, стратегия ценообразования

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Исследования по экономике и финансам

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Ценовая политика для инновационных продуктов с учетом эффекта сарафанного радио