NATIONAL RESEARCH UNIVERSITY
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PRICE-QUANTITY COMPETITION OF FARSIGHTED FIRMS: TOUGHNESS VS. COLLUSION

BASIC RESEARCH PROGRAM

WORKING PAPERS

SERIES: ECONOMICS
WP BRP 93/EC/2015

Price-Quantity Competition of Farsighted Firms: Toughness vs. Collusion*

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Abstract

The paper examines an interaction of boundedly rational firms that are able to calculate their gains after reaction of an opponent to their own deviations from the current strategy. We consider an equilibrium concept that we call a Nash-2 equilibrium. We discuss the problem of existence and possible multiplicity of such equilibria, relation to infinite rationality approach of folk theorem and security considerations of equilibrium in secure strategies. For a number of models (Bertrand with homogeneous and heterogeneous product, Cournot, Tullock competition) the Nash-2 equilibrium sets are obtained and considered as tacit collusion or strong competition in dependence of additional security considerations.

JEL Classification: C72, D03, D43, D70, L13.

Keywords: Nash-2 equilibrium, secure deviation, secure profile, Bertrand model, Cournot duopoly, differentiated product, Tullock contest, tacit collusion, tough competition.

*I acknowledge the financial support from the Russian Federation Government under Grant No. 11.G34.31.0059. I am deeply indebted to Jacques-Francois Thisse, Sergey Kokovin, Nikolay Bazenkov and Alexei Parakhonyak for fruitful discussions and comments.

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1 Introduction

A wide range of economic situations is characterized by long-run evolution. Strategies of each agent include attempts to predict how other players will behave in future. Success of such attempts depends largely on the analytical and computational abilities of agents. In accordance with classical (Nash) theory of rationality each player is a maximizer under his beliefs that her opponents also maximize ("rational-expectations").

For modeling long-run interactions, game-theorists widely use repeated games. Explicit examination of dynamic games often leads to totally different results in comparison with one-shot games. The most well-known example is Prisoner's Dilemma. Intuitively many economists agree that quite often the realistic outcome of such games is a tacit collusion [23, Tirole, 1988, Chapter 6] which is a sort of quasi-cooperative solution supported by *credible threats*. However, this result can be explained with different theoretical assumptions within the framework of canonical rationality theory.

In both one-stage and long-run settings there are experimental evidence that are not consistent with Nash equilibrium predictions. A number of intuitive contradictions is contained in [10, Goeree, Holt, 2001]. Seeking for a compromise between this "ideal" model and experiments has led to various bounded rationality concepts. Surveys on underlying ideas, classification, direction for further development are contained in [7, Crawford, 2013], [12, Hastard, Selten, 2013]. We are not going to immerse into the huge number of existing concepts, and will only consider an iterated strategic thinking process (see, for instance, [4, Binmore, 1988]).

A rational player can take in account how the opponents will respond when she makes a decision whether to deviate from the current strategy or not, and how exactly to deviate. A number of similar models of agent cognitive hierarchy, or adhering to other terminology k-level rationality, are developed in [8, Crawford at al., 2013] [5, Camerer, Ho, Chong, 2004], [22, Stahl, 1993]. Some empirical studies support k-level rationality approach ([5, Camerer, Ho, Chong, 2004], [19, Kawagoe, Takizawa, 2009]). The important point of hierarchical models is that each player assumes that other players have a lower level of rationality. It means that players of level-0 are strategically naive, while k-level players ($k > 0$) best respond on some beliefs on how their opponents are distributed by lower levels of rationality.

We also must mention some farsighted solution concepts based on the idea of k-level rationality: the largest consistent set [6, Chwe, 1994], noncooperative farsighted stable set [20, Nakanishi, 2007], farsighted pre-equilibrium [18, Jamroga, Melissen, 2011].

The reasonable degree of farsightness is an open question. The smallest one is *two*. In this case a player takes account of the opponent's best responses (see [11, Halpern, Rong, 2010] for cooperative equilibrium, and [3, Bazenkov, Korepanov, 2014] for equilibrium in double best responses).

The another possibility is to study players that have the same level of rationality, and it is a common belief. In such a situation a player can't predict with certainty how her opponents will respond on her deviations, and should include in the reasoning all profitable responses of the opponents. This idea underlies the concept of Nash-2 equilibria introduced in [21, Sandomirskaia, 2014][1].

One more approach is to introduce a *security* as an additional motivation for players' behavior. Two (the closest to our ideas) second-stage-foreseeing concepts that have been proposed are bargaining set based on the notion of threats and counter-threats (for cooperative games, see [2, Aumann, Maschler, 1964]) and equilibrium in secure strategies (EinSS, see [13, Iskakov M., Iskakov A., 2012a]). The idea of both concepts is that players worry not only about own first-stage payoffs and opponents' responses, but also about security against harmful actions ("threats") of the opponents.

The key point of this paper is that Nash-2 equilibrium concept includes secure and non-secure situations, and they are regarded as various degrees of competition toughness between agents. Secure situations correspond to strong low-profits competition, while non-secure situations are treated as tacit collusion.

The structure of the paper is the following. Section 2 defines Nash-2 equilibrium and equilibrium in secure strategies, provides an interpretation for secure and non-secure part of Nash-2 set, and explains the connection with folk theorem approach. In Section 3 we present some ideas how to select among multiple Nash-2 equilibrium profiles, and introduce a measure of feasibility on the set of Nash-2 equilibria. In Section 4 we turn to application of the concept to a number of classical IO models: Bertrand competition, Cournot competition, and Tullock rent-seeking contest.

2 Definition, related concepts and interpretation

Consider a 2-person non-cooperative game in the normal form

$$G = (i \in \{1,2\}; s_i \in S_i; u_i : S_1 \times S_2 \to R),$$

where s_i, S_i and u_i are the strategy, the set of all available strategies and the payoff function, respectively, of player i, $i = 1, 2$. Henceforth, we will deal only with pure strategies.

Let us remind the formal definition of Nash-2 equilibrium formulated in [21, Sandomirskaia, 2014].

Definition 1 (profitable secure deviation)**.** A deviation s'_i of player i at strategy profile $s = (s_i, s_{-i})$ is *profitable and secure* if $u_i(s'_i, s_{-i}) > u_i(s_i, s_{-i})$ and for any strategy s'_{-i} of

[1] In fact, the paper [14, Iskakov M., Iskakov A., 2012b] introduces the idea of Nash-2 equilibrium as "threatening-proof profile", but it hasn't been developed at that time. Since 2014 they have simultaneously started to studying the similar to Nash-2 equilibrium concept named equilibrium contained by counter-threats [16, Iskakov M., Iskakov A., 2014].

player $-i$ such that $u_{-i}(s'_i, s'_{-i}) > u_{-i}(s'_i, s_{-i})$

$$u_i(s'_i, s'_{-i}) \geq u_i(s_i, s_{-i}).$$

Definition 2 (NE-2)**.** A strategy profile is *a Nash-2 equilibrium* if no player has a profitable secure deviation.

When we add a requirement of security we obtain an equilibrium in secure strategies [13, Iskakov M., Iskakov A., 2012a]. Strictly speaking, the definition of equilibrium in secure strategies is the following.

Definition 3 (secure profile)**.** A *threat* of player i to player $-i$ at strategy profile s is a strategy s'_i such that

$$u_i(s'_i, s_{-i}) > u_i(s_i, s_{-i}) \quad \text{and} \quad u_{-i}(s'_i, s_{-i}) < u_{-i}(s_i, s_{-i}).$$

The strategy profile s is said to *pose a threat* from player i to player $-i$. A strategy profile s is *secure* for player i if s poses no treats from other players to i.

Definition 4 (EinSS)**.** A strategy profile is an *equilibrium in secure strategies* (EinSS) if

- it is secure,
- no player has a profitable secure deviation.

Obviously, the following relation takes place.

Proposition 1 (Iskakov & Iskakov, 2012)**.** *Any NE is an EinSS. Any EinSS is a NE-2.*

So, the set of NE-2 can be naturally divided into two sets: secure profiles (EinSS) and non-secure (”risky”) outcomes (NE-2 \ EinSS). Secure part can be regarded as a tough competition where agents should protect themselves against any possible threats, even non-credible. It often leads to the situations with low profits as players in such situations have nothing to lose.

On the other hand, risky situations are characterised with the following: agents have opportunities to harm one to another, but they do not actualize these threats as they are not credible. In a number of situations such a cautious behavior enables agents to hold on higher profits than in case when players care about the security. For example, in Hotelling linear city model [21, Sandomirskaia, 2014] players do not undercut one another according to Nash-2 deviation principle. These outcomes can be considered as situations of tacit collusion. And really, if explicit collusion is a NE-2 then it is in NE-2 \ EinSS.

Theorem 1. *If a collusion outcome is not Nash equilibrium then it is not a secure profile.*

Proof. Let $(s_1^c, s_2^c) = \arg\max_{s_1,s_2}(u_1(s_1,s_2) + u_2(s_1,s_2))$. Assume that it is secure. It means that it poses no threats from one to another. The two cases are possible: there no profitable deviations and for any profitable deviation $s_1 \to s_i'$ of player i the another player is not worse off $u_{-i}(s_i', s_{-i})$.

In the first case we deal with NE. In the second case $u_i(s_i', s_{-i}) + u_{-i}(s_i', s_{-i}) > u_1(s_1^c, s_2^c) + u_2(s_1^c, s_2^c)$ and this contrary to the fact that (s_1^c, s_2^c) is collusive outcome. □

So, NE-2 is a rather wide concept, and it includes NE, EinSS, and even sometimes cooperative behavior. Is the set of possible gains at NE-2 profiles as wide as the set of equilibrium payoffs that is established in Folk theorem?

Folk theorems are a class of theorems about possible Nash equilibrium payoffs in an infinitely repeated games. Following [9, Fudenberg, Tirole, 1991] recall the basic notions.

Reservation utility or minmax value of player i is

$$r_i = \min_{s_{-i}} \max_{s_i} u_i(s_i, s_{-i}).$$

A payoff profile is said to be feasible if it lies in the convex hull (Conv) of the set of possible payoff profiles of the stage game.

The folk theorem states that *any feasible payoff profile that strictly dominates the reservation utility can be realized as a Nash equilibrium payoff profile, with sufficiently large discount factor.*

On the opposite, the following property of Nash-2 concept is obvious.

Proposition 2. *Any NE-2 payoff weakly dominates the reservation utility.*

Indeed, if a player gets less than minmax value then she has a profitable secure deviation to the strategy which ensures it.

Denote by $\Pi_{NE-2} = \{(u_1(s_1,s_2), u_2(s_1,s_2)) |\, (s_1,s_2) \in \text{NE-2}\}$ the set of payoffs in NE-2 situations in a game, by Π_∞ the set of NE payoffs approved by the folk theorem. Let $\text{Int}(A)$ be the interior of the set A.

The following theorem argues that NE-2 payoff set is at least as small as folk theorem set but never greater than it.

Theorem 2.

$$\text{Int}(\text{Conv}(\Pi_{NE-2})) \subseteq \text{Int}(\Pi_\infty)$$

In fact, the inclusion is strict for a large number of models like Bertrand, Cournot, Hotelling competition and others. Nevertheless, equality is held for a special class of games, namely for a strictly competitive games with some extra condition on the minmax profiles described in [21, Theorems 3, 4, Sandomirskaia, 2014].

3 Multiplicity of equilibria

In [21, Sandomirskaia, 2014] it was proven that NE-2 exists in almost any game with compact strategy sets, and shown that in most cases it isn't unique. In other words, whenever a game does not have NE-2, any small "perturbation" of payoffs yields NE-2 existence. Moreover, the question how to select the unique equilibrium from the set of NE-2 still remains open. The answer is supposed to vary for different models.

In situation with strict competition between firms one can choose, for instance, an equilibrium in secure strategies as the most attractive. In Hotelling linear city model EinSS concept provides the unique equilibrium corresponding to dumping pricing [15, Iskakov M., Iskakov A., 2013].

The totally different approach is to choose the collusion outcome like in Bertrand or Cournot model or, at least, Pareto efficient profiles in the set of NE-2.

Alternative way of solving the problem is to introduce the measure on the set of NE-2 that reflects with which probability a concrete equilibrium can be realized. This can be done in different ways, and we present here one of them.

Firstly remind the definition of secure path.

Definition 5. A path of profiles $\{(s_i^t, s_{-i}^t)\}_{t=1,\dots,T}$ is called a *secure path* if each its arc $(s_i^t, s_{-i}^t) \to (s_i^{t+1}, s_{-i}^{t+1}) = (s_i^{t+1}, s_{-i}^t)$ contains a secure profitable deviation s_i^{t+1} for some player i.

The idea is the following. We suppose that originally players randomly get to any game profile s with equal probabilities $\nu_0(s) = \frac{\mu(s)}{\mu(S_1 \times S_2)}$, where $\mu(A)$ is a measure of the set A. However, if the profile s is not NE-2, then there exists a secure path from this profile to some NE-2. Denote the set of all NE-2 that can be reached from the profile s by means of some secure path by NE-2$_s$. For simplicity we assume that all profiles in NE-2$_s$ are reached with equal probabilities. So, the final probability of each NE-2 profile to be realised is

$$\nu(s) = \frac{\mu(s)}{\mu(S_1 \times S_2)} + \sum_{\tilde{s}: s \in \text{NE-2}_{\tilde{s}}} \frac{\mu(\tilde{s})}{\mu(\text{NE-2}_{\tilde{s}})\mu(S_1 \times S_2)}, \quad \forall s \in \text{NE-2}.$$

These probabilities form the *measure of feasibility* on the set of NE-2.

If a NE-2 profile s is not reachable from any point of $S_1 \times S_2$ (we will call it isolate), then $\nu(s) = \nu_0(s)$.

For the sake of visualization in the case of discrete action sets let us construct a directed graph Γ. The nodes of Γ are the game profiles. The directed link from node s to node s' exists if there is a secure path from s to s', and there are no secure paths starting at s'.

In this graph the nodes with zero outdegree $deg^+(s)$ are NE-2 equilibria. The links demonstrate how not NE-2 profiles transmit their initial probabilities to NE-2 profiles by secure paths. Here for all $s \in$ NE-2 the number of all profiles from which a secure path to s

exists equals to the indegree $deg^-(s)$ of s in Γ. If $\forall s \in \Gamma$ $deg^+(s) \leq 1$, then

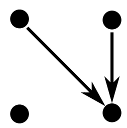
$$\nu(s) = \frac{1}{|S_1| \cdot |S_2|} \left(1 + deg^-(s)\right), \quad \forall s \in \text{NE-2},$$

$|A|$ is the number of elements in the set A.

Let us give several examples.

Example 1.

	L	R
T	1	-1
B	0	0

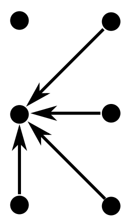


Strategy profile (B,R) is NE and NE-2, and profile (B,L) is NE-2, but not NE. (B,L) is an isolated NE-2, thus $\nu(B, L) = 1/4$.

$deg^-(B, R) = 2$. Thereby, $\nu(B, R) = \frac{1}{4}(1 + 2) = 3/4$.

Example 2.

	L	R
T	(2/3, 1/3)	(-1, 2)
C	(1/2, 1/2)	(1, 0)
B	(1, 0)	(0, 1)



Here NE-2 set consists of two strategy profiles (C,L) and (T,L) with profits (1/2, 1/2) and (2/3, 1/3), respectively. NE doesn't exist.

(T,L) is an isolated NE-2, thus $\nu(T, L) = 1/6$.

$deg^-(C, L) = 4$. Thereby, $\nu(C, L) = \frac{1}{6}(1 + 4) = 5/6$.

Example 3 (Bertrand model with homogeneous product).

Consider the classical Bertrand model with two firms producing a homogeneous product with equal marginal costs c. Let p_1 and p_2 be the prices proposed by firms 1 and 2, respectively. Consumers buy the product with lowest price, the demand being a linear function of the price $Q(p) = 1 - p$. If the prices are equal then consumers choose each firm with equal probabilities. NE-2 concept establishes that an equilibrium might be with any price level $p = p_1 = p_2 \in [c, 1]$. In particular, the set of NE-2 includes the collusive (monopoly) price level $p = \frac{1+c}{2}$.

In this case there is a secure path from each profile (p_1, p_2), $p_1 \neq p_2$, $p_1, p_2 \in [c, 1]$, to NE-2 profile (p, p) with $p \in [c, \min(p_1, p_2)]$.

Explicit calculations yield:

$$\nu(p, p) = \frac{2}{1 - c} \left(\ln \frac{1 - c}{p - c} - \frac{1 - p}{1 - c} \right), \quad \forall p \in [c, 1].$$

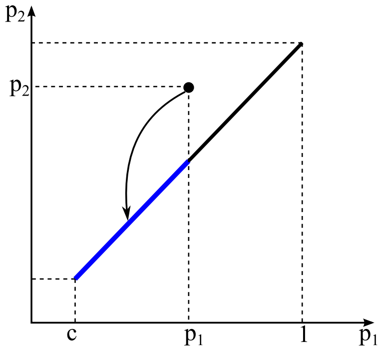


Figure 1: The structure of secure paths in Bertrand model

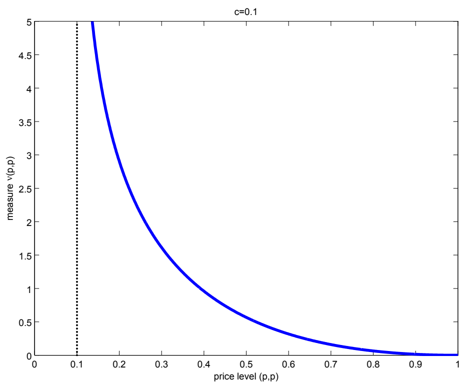


Figure 2: The measure of feasibility on the set of NE-2 in Bertrand model with $c = 0.1$

One can think about this measure function in the sense that the probability to come into the ϵ-neighbourhood of the prices (p, p) is $\int_{p-\varepsilon}^{p+\varepsilon} \nu(x)dx$. Note that collusion price level $(\frac{1+c}{2}, \frac{1+c}{2})$ has a positive measure of feasibility.

4 Application to IO models

Let us turn out now to some applications of Nash-2 equilibrium concept to well known models of price-quantity competition. We will start with Cournot duopoly with homogeneous product, linear demand, and equal marginal costs, and demonstrate in terms of NE-2 whether the possibilities for collusion or more strong competition actually exist. Then we examine the general model of price competition of firms producing imperfect substitutes. Finally we will discuss the computer solution of rent-seeking game (Tullock contest) and outline the difference between secure-but-strong-competitive and risky-but-collusive outcomes.

4.1 Cournot duopoly

Let two firms produce q_1 and q_2 units of homogeneous product, respectively, with equal constant marginal costs c per unit. We assume the equilibrium price $p(Q)$ to be a linear decreasing function $p(Q) = 1 - Q$ of total output $Q = q_1 + q_2$. The benefit of i-th firm is

$$\pi_i(q_1, q_2) = q_i \cdot (p(Q) - c) = q_i(1 - q_1 - q_2 - c).$$

At NE firms produce by one third of maximal total output which ensures non-negative prices on the market

$$q_1^* = q_2^* = \frac{1-c}{3}, \qquad \pi_1^* = \pi_2^* = \left(\frac{1-c}{3}\right)^2.$$

Theorem 3. *NE-2 are* (q_1, q_2) *s.t.*
a) they are in the set

$$\left\{ \left(b; \frac{1-c-b}{2}\right) \cup \left(\frac{1-c-b}{2}; b\right) \mid \quad b \in \left[\frac{1-c}{3}; 1-c\right) \right\}.$$

b)

$$q_1 = q_2 \in (0, (1-c)/3)$$

including collusive outcome $(1-c)/4$, $(1-c)/4$.

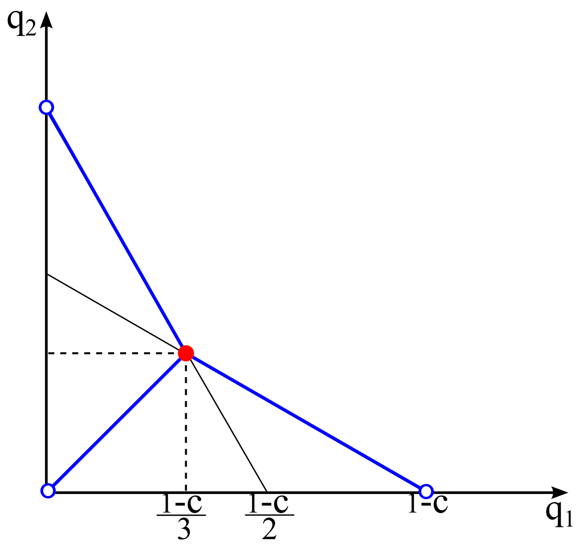


Figure 3: Red point is NE, NE-2. Blue lines are NE-2.

Proof is in Appendix.

It is to be stressed that the set a) also consists of secure situations. For $b \in \left(\frac{1-c}{3}; \frac{2(1-c)}{3}\right)$ such situations are fruitful for the firm that overproduces, and for any b they are bad for the another firm (in comparison with NE profits).

The set NE-2 \ EinSS includes collusive outcome, so collusion is strategically (tacitly) supported. The profiles with $q_1 = q_2 \in \left(\frac{1-c}{4}, \frac{1-c}{3}\right)$ cover all intermediate situations between Nash competition and cooperative behavior.

NE-2 set provides a number of regimes with various degree of toughness from competitive till collusive. Related discussion can be found in [1, d'Aspremont, 2007].

4.2 Bertrand competition with differentiated product

Consider the general model of price competition between two firms producing imperfect substitutes with marginal costs equal c_1 and c_2, respectively. Coefficient of substitution is

$\gamma \in [0, \infty)$. Firms' demand curves are:

$$q_1 = 1 - p_1 - \gamma(p_1 - p_2), \quad q_2 = 1 - p_2 - \gamma(p_2 - p_1).$$

The firms' profits are

$$\pi_1(p_1, p_2) = (p_1 - c_1)(1 - p_1 - \gamma(p_1 - p_2)).$$

$$\pi_2(p_1, p_2) = (p_2 - c_2)(1 - p_2 - \gamma(p_2 - p_1)).$$

When $\gamma = 0$ it corresponds to the case of monopoly. When $\gamma \to \infty$ the product becomes more and more homogeneous.

NE prices are

$$p_1^* = \frac{2 + 3\gamma + 2(1+\gamma)^2 c_1 + \gamma(1+\gamma)c_2}{(2+3\gamma)(2+\gamma)}, \quad p_2^* = \frac{2 + 3\gamma + 2(1+\gamma)^2 c_2 + \gamma(1+\gamma)c_1}{(2+3\gamma)(2+\gamma)},$$

when $p_1^* \geq c_1$, $p_2^* \geq c_2$.

If marginal costs are equal, then $p_1^* = p_2^* = \frac{1+(1+\gamma)c}{2+\gamma} > c$. As $\gamma \to \infty$ we face to classical Bertrand model.

Let us describe the set of NE-2 profiles p_1, p_2.

Note firstly that two following conditions mean that markup and demand at equilibrium should be non-negative.

$$p_1 \geq c_1, \quad p_2 \geq c_2, \qquad a)$$

$$q_1(p_1, p_2) \geq 0, \quad q_2(p_1, p_2) \geq 0. \qquad b)$$

The next condition states that only excessive prices (in comparison with best response level) can be NE-2:

$$p_1 \geq \frac{1 + \gamma p_2 + c_1(1+\gamma)}{2(1+\gamma)}, \quad p_2 \geq \frac{1 + \gamma p_1 + c_2(1+\gamma)}{2(1+\gamma)}. \qquad c)$$

One more claim is that at NE-2 firms get not less then their guaranteed benefits (reservation utility):

$$\pi_1(p_1, p_2) \geq \frac{(1 - c_1(1+\gamma))^2}{4(1+\gamma)}, \quad \pi_2(p_1, p_2) \geq \frac{(1 - c_(1+\gamma))^2}{4(1+\gamma)}. \qquad d)$$

The last condition directly concerns the absence of secure profitable deviations:

$$\begin{aligned} &\left(\frac{1-c_1}{2} - \frac{\gamma(1+\gamma)(p_2-c_2)}{2(1+2\gamma)}\right)\left(\frac{1+2\gamma+\gamma^2 c_2-(1+\gamma)^2 c_1}{2(1+\gamma)} + \frac{3}{2}(p_2 - c_2)\right) \leq \pi_1(p_1, p_2), \\ &\left(\frac{1-c_2}{2} - \frac{\gamma(1+\gamma)(p_1-c_1)}{2(1+2\gamma)}\right)\left(\frac{1+2\gamma+\gamma^2 c_1-(1+\gamma)^2 c_2}{2(1+\gamma)} + \frac{3}{2}(p_1 - c_1)\right) \leq \pi_2(p_1, p_2). \end{aligned} \qquad e)$$

Proof is in Appendix.

As we can observe NE-2 set becomes more asymmetric as the difference between marginal costs increases (see Fig. 4 – 6). On the other hand, it becomes narrower and elongate as $\gamma \to \infty$ (see Fig. 7 – 9) and this asymmetry ceases to play an important role.

Note that in case of $c_1 = c_2 = c$ the collusion profile $p_1 = p_2 = (1+c)/2$ is inside NE-2 set. Nevertheless, there exist another outcomes on the Pareto boundary of NE-2 profit set.

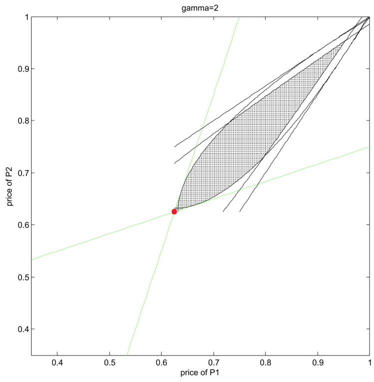


Figure 4: $c_1 = c_2 = 0.5$, $\gamma = 2$. Red point is NE, EinSS, NE-2. Shaded area is NE-2.

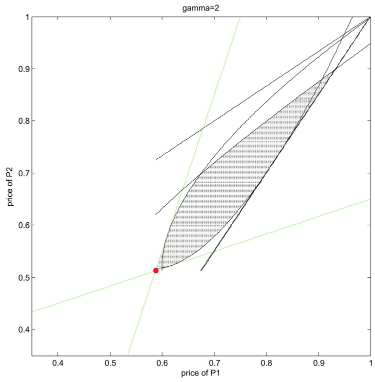


Figure 5: $c_1 = 0.5$, $c_2 = 0.3$, $\gamma = 2$. Red point is NE, EinSS, NE-2. Shaded area is NE-2.

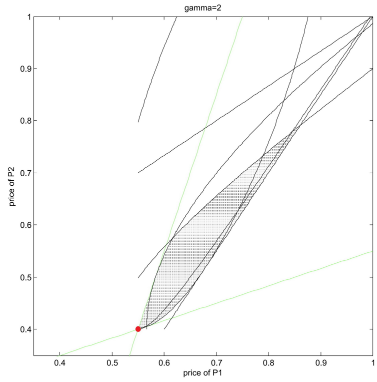


Figure 6: $c_1 = 0.5$, $c_2 = 0.1$, $\gamma = 2$. Red point is NE, EinSS, NE-2. Shaded area is NE-2.

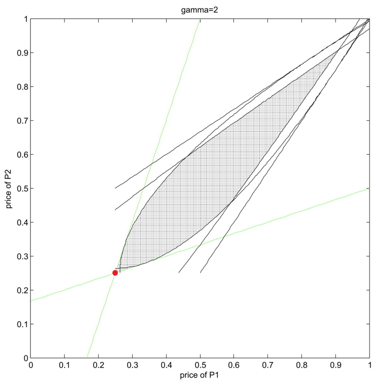


Figure 7: $c_1 = c_2 = 0$, $\gamma = 2$. Red point is NE, EinSS, NE-2. Shaded area is NE-2.

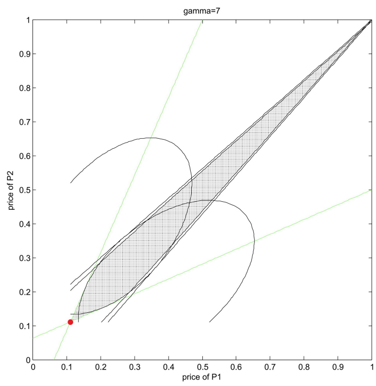


Figure 8: $c_1 = c_2 = 0$, $\gamma = 7$. Red point is NE, EinSS, NE-2. Shaded area is NE-2.

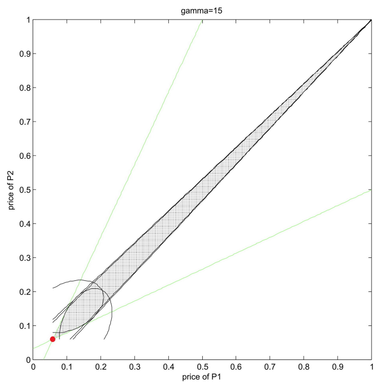


Figure 9: $c_1 = c_2 = 0$, $\gamma = 15$. Red point is NE, EinSS, NE-2. Shaded area is NE-2.

4.3 Tullock contest

In rent-seeking modeling one focuses on the manipulation of firms to gain monopolistic advantages in the market. It is a widespread way to examine the processes of political lobbying for government benefits or subsidies, or to impose regulations on competitors, in order to increase market share.

The contest success function translates the effort $x = (x_1, x_2)$ of the players into the probabilities that each player will obtain the resource R.

$$p_i(x_i, x_{-i}) = \frac{x_i^\alpha}{x_i^\alpha + x_{-i}^\alpha}, \quad x \neq 0, i = 1, 2.$$

If $x = (0, 0)$ then $p_i = p_{-i} = 1/2$.

The payoff function for each player

$$u_i(x_i, x_{-i}) = Rp_i(x_i, x_{-i}) - x_i.$$

Without loss of generality assume $R = 1$, $x_i \in [0,\ 1]$.

The players' behavior roughly depends on the value α. It can be treated as a responsiveness of the utility function on increasing the effort. When $\alpha \leq 2$ NE exists and equals to $\alpha/4$. In [17, Iskakov at al, 2013] the EinSS in Tullock model was obtained for all α. Here we present preliminary results on finding NE-2: the computer solution (see Fig. 10 – 12).

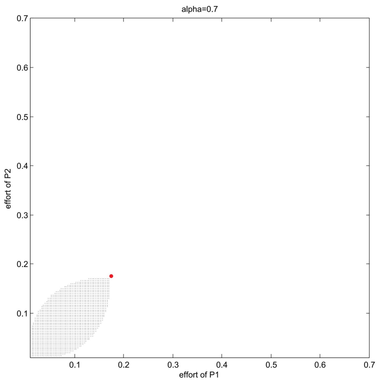


Figure 10: $\alpha = 0.7$. Red point is NE, EinSS, NE-2. Shaded area is NE-2.

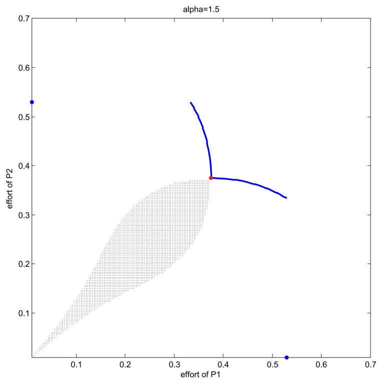


Figure 11: $\alpha = 1.5$. Red point is NE, EinSS, NE-2. Blue curve and points are EinSS, NE-2. Shaded area is NE-2.

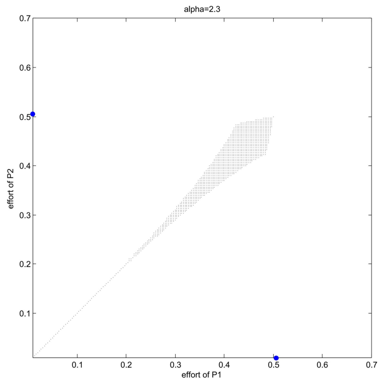


Figure 12: $\alpha = 2.3$. Blue points are EinSS, NE-2. Shaded area is NE-2.

All EinSS are Pareto dominated by some Nash-2 profile (see Fig. 13)

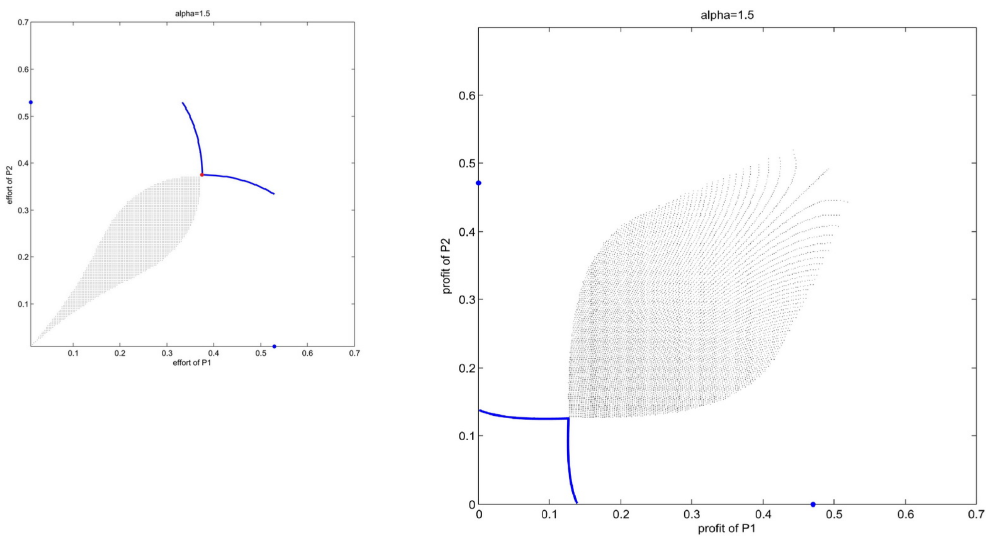


Figure 13: $\alpha = 1.5$. Efforts and profits. Curves and blue points are EinSS and NE-2 payoffs, shaded area is set of profits at NE-2.

The set NE-2 \ EinSS seems to be intuitively clear: the "farsighted" players in some sense are engaged in tacit collaboration and make smaller efforts to reach the same probability of obtaining the resource. This is what we mean by tacit collusion.

5 Conclusion

The paper continues examination of rather novel concept of Nash-2 equilibrium by which a player takes into account possible reaction of an opponent to her actions. It contains the suggestion that secure part of NE-2 set can be regarded as strong competition, while risky outcomes are treated as tacit collusion. This is demonstrated with several examples of firms' competition: Cournot duopoly, Bertrand ones, and Tullock contests.

One more advantage of the Nash-2 concept is its existence for almost all games. On the other hand, this approach often provides multiple solutions, and the selection problem still remains open. In the paper we introduce a measure on the set of NE-2 reflecting the possibility for each equilibrium to be realized under the assumption that all game profiles are initially equiprobable.

We assume that multiplicity of theoretical predictions is a natural expression of great variety of real-life agents' behavior. However, experimental justification of such rationality type is desirable.

Appendix

Proof of theorem 3.

Reaction functions of both firms are

$$r_1(q_2) = (1 - c - q_2)/2, \qquad r_2(q_1) = (1 - c - q_1)/2.$$

Note that any decreasing of production of any player is profitable for her opponent. It immediately yields that if for one player it is profitable to decrease her price then another player (if she doesn't play her best response) has a profitable secure deviation. Therefore, such situations don't claim to be a NE-2.

If one player (for definiteness, let firm 1) plays her best response on the firm's 2 output, while firm 2 produces more than her best response level, then such a situation is NE-2. Really, firm 1 hasn't a profitable deviation, and any profitable deviation of firm 2 is decreasing the output: $q_2 \to q_2 - \varepsilon$, for some $\varepsilon > 0$. However, if firm 2 deviates then firm 1 acquires a profitable deviation $q_1 \to q_1 + \varepsilon - \delta$ with enough small $0 < \delta < \varepsilon$ such that the deviation $q_2 - \varepsilon$ is not secure for firm 2.

Now turn out to the case when both players produce less then best response level: $q_1 \leq (1 - c - q_2)/2$ and $q_2 \leq (1 - c - q_1)/2$. Assume first that $q_1 > q_2$ (the symmetric case is analogous). Then firm 2 has profitable secure deviation $q_2 \to 1 - c - q_1 - q_2 - \varepsilon$ with

$0 < \varepsilon < q_1 - q_2$. After this q_1 becomes greater than new best response level and any profitable deviation of firm 1 is decreasing q_1 which is good for firm 2.

The last possible situation is $q_1 \leq (1-c-q_2)/2$, $q_2 \leq (1-c-q_1)/2$, and $q_1 = q_2 = q$. Let us show that $(q,\, q)$, $q \leq (1-c)/3$, is NE-2. We will prove from the firm 1 point of view. Any profitable deviation of firm 1 has a form $q_1 \to q + \varepsilon$ with $0 < \varepsilon < 1 - c - 3q$. After this firm 2 has a profitable deviation $q_2 \to 1 - c - 2q - \varepsilon - \delta$ which provides non-security for firm 1 if $0 < \delta < \frac{q}{q+\varepsilon}(1 - c - 3q - \varepsilon)$.

This completes the proof.

Comments on the proof of conditions from Sec.4.2.

Let us start with the observation that, in contrast to Cournot duopoly, any increasing price of any player is profitable for her opponent. From this fact it follows that if one firm assign prices that is less than best response level then she has a profitable and secure deviation. It provides condition c).

Proposition 2 provides us condition d).

Now look at the residual area and establish which situations are NE-2. In this area firms propose prices more than at best response level (condition c)). Let us look on the situation from the perspective of firm 1. Any profitable deviation of firm 1 is decreasing the price: $p_1 \to \tilde{p}_1^\varepsilon = p_1 - \varepsilon$ with $\varepsilon \in \left(0;\, 2\left(p_1 - \frac{1+\gamma p_2 + c_1(1+\gamma)}{2(1+\gamma)}\right)\right)$. The most harmful response of firm 2 is maximal decreasing the price: $p_2 \to \tilde{p}_2^\varepsilon = 2 \cdot \frac{1+\gamma(p_1-\varepsilon)+c_2(1+\gamma)}{2(1+\gamma)} - p_2 + \delta$ with $\delta = +0$.

If (p_1, p_2) is NE-2 than for any ε firm 1 should get worse:

$$\pi_1(\tilde{p}_1^\varepsilon, \tilde{p}_2^\varepsilon) < \pi_1(p_1, p_2),$$

or, equivalently, $\max_\varepsilon \pi_1(\tilde{p}_1^\varepsilon, \tilde{p}_2^\varepsilon) < \pi_1(p_1, p_2)$.

Explicit calculation of this maximum gives condition e). The proof is finished.

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