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ESTIMATION AND FILTERING OF NONLINEAR MS-DSGE MODELS

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Estimation and filtering of nonlinear MS-DSGE models

Abstract

This article suggests and compares the properties of some nonlinear Markov-switching filters. Two of them are sigma point filters: the Markov switching central difference Kalman filter (MSCDKF) and MSCDKFA. Two of them are Gaussian assumed filters: Markov switching quadratic Kalman filter (MSQKF) and MSQKFA. A small scale financial MS-DSGE model is used for tests. MSQKF greatly outperforms other filters in terms of computational costs. It also is the first or the second best according to most tests of filtering quality (including the quality of quasi-maximum likelihood estimation with use of a filter, RMSE and LPS of unobserved variables).

Keywords: regime switching, second-order approximation, non-linear MS-DSGE estimation, MSQKF, MSCDKF

Jel-codes: C13; C32; E32

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models are a key instrument of macroeconomic analysis. They are widely used by central banks and other organizations across the world [Tovar (2009)]. The Markov switching dynamic stochastic general equilibrium (MS-DSGE) is a generalization of DSGE models which suggests a switching of regimes. The different regimes have different parameter values (such as for preferences, technologies, and the exogenous process).

The linearized DSGE models are the main focus of the empirical literature [Tovar (2009)]. However, linear approximations cannot capture important effects such as the influence of risk on economic decisions (which makes them inappropriate for some themes such as asset pricing). The second order approximation is much closer to the true solution [Collard and Juillard (2001)]. An additional advantage of nonlinear approximations usage is sharper likelihood [An and Schorfheide 2007, Pichler (2008)]. Recent papers developed some deterministic filters which greatly outperform previously used particle filters [Andreasen (2013), Ivashchenko (2014), Kollmann (2015)].

Linearized MS-DSGE models are used in the almost all papers [Liu et. all. (2011); Lhuissier and Zabelina (2015)]. Techniques for nonlinear approximations of MS-DSGE models were developed recently [Maih (2014), Foerster et. all. (2014)] in contrast to well known nonlinear approximation techniques for DSGE models [Collard and Juillard (2001), Schmitt-Grohe and Uribe (2004)].

The aim of this paper is to suggest a fast deterministic filter for the estimation of nonlinear MS-DSGE models. Maih and Bining suggest a filter which is based on idea of Sigma point Kalman filters [Binning and Maih (2015)]. However, it has strange collapsing rule (variance is equal to the weighted average of variance conditions on regimes). Thus, filters are constructed and compared in terms of speed and accuracy. The Markov switching quadratic Kalman filter (MSQKF) is a Gaussian assumed filter and uses collapsing before forecasting similar to [Binning and Maih (2015)]. The Markov switching quadratic Kalman filter accurate (MSQKFA) is a Gaussian assumed filter and uses collapsing after forecasting similar to [Kim (1994)]. These two filters are generalizations of the quadratic Kalman filter from [Ivashchenko (2014)]. The other two filters are Markov switching central difference Kalman filter (MSCDKF) and Markov switching central difference Kalman filter accurate (MSCDKFA) which are generalizations of the central difference Kalman filter [Andreasen (2013)]. We do not use particle filters because of the much higher computational costs for the same level of accuracy compared to deterministic filters [Andreasen (2013), Ivashchenko (2014), Kollmann (2015)].

The rest of the paper is organized as follow: Section 2 describes the general filtering scheme and each of the filters. The MS-DSGE model for testing is described in Section 3. Section 4 presents results and discussion. Some conclusions are drawn in Section 5.

2. Filtering scheme

2.1 The general filtering scheme

The purposes of filter usage in DSGE related themes are: to compute the model variable vector X_t density condition on vectors of observed variables $Y_1, \ldots, Y_t$; to compute the observed variables $Y_1,\ldots, Y_t$ density and likelihood. The phrase “compute density” means to compute the parameters of the density approximation (usually moments of density). In some specific cases this approximation is equal to the density (for example in a normal distribution). The procedure of the most filters can be presented as loop of the following steps:

1. Compute initial density X_t
2. Compute density of Y_t as function of density X_t
3. Compute likelihood of Y_t
4. Compute conditional density of $X_t|Y_t$
5. Compute density of X_{t+1} as functions of density $X_t|Y_t$
6. Move to step 2

The 4 filters differ only at step 5. The suggested model of the data generating process (DGP) is (1)–(3) and a discrete Markov-switching process for the regime variable r_t, where $X_{state,t}$ is the vector of state variables (a subset of model variable vector X_t) and ε_t and u_t are vectors of independent shocks (model innovations and measurement errors) that have a zero-mean normal distribution. δ is a constant equal to one and related to perturbation with respect to uncertainty. It is a second order approximation of the MS-DSGE model that can be computed with RISE toolbox [Maih (2015)].

$$Y_t = HX_t + u_t \tag{1}$$

$$Z_t = \begin{bmatrix} X_{state,t} & \delta & \varepsilon_t \end{bmatrix} \tag{2}$$

$$X_{t+1} = A_{0,r_{t+1}} + A_{1,r_{t+1}} Z_t + A_{2,r_{t+1}} \left(Z_t \otimes Z_t \right) \tag{3}$$

The difference from the usual DSGE model second order approximation is the existence of regime dependence. The each of filter steps is described below.

2.2 Compute the density of Y_t as function of the density of X_t

The initial information for this step is that the density of X_t is a normal mixture. The linear equation for the observed variables (1) means that the density of Y_t is a normal mixture with the same probabilities of regimes and following expectations and variances (conditional on the regime):

$$E_s(Y_t) = E_s(HX_t + u_t) = HE_s(X_t) \tag{4}$$

$$V_s(Y_t) = V_s(HX_t + u_t) = HV_s(X_t)H.' + V(u_t) \tag{5}$$

2.3 Compute the likelihood of Y_t

The initial information for this step is that the density of Y_t is a normal mixture. It means that the likelihood can be computed according to formula (6).

$$lik(Y_t) = \sum_{s=1}^{N_S} p(r_t = s) lik(Y_t \mid r_t = s) =$$
$$= \sum_{s=1}^{N_S} p(r_t = s) \frac{1}{(2\pi)^{N_Y/2} \left|V_s(Y_t)\right|^{1/2}} \exp\left(-0.5 (Y_t - E_s(Y_t))' (V_s(Y_t))^{-1} (Y_t - E_s(Y_t)) \right) \tag{6}$$

2.4 Compute the conditional density of $X_t|Y_t$

The initial information for this step is that the density of X_t is a normal mixture and the vector of observation Y_t. The linear equation (1) allows the computation conditional on the regime and observation density the same way as the Kalman filter (7)–(9).

$$K_s' = (V_s(Y_t))^{-1} H V_s(X_t) \tag{7}$$

$$E_s(X_t \mid Y_t) = E_s(X_t) + K_s(Y_t - E_s(Y_t)) \tag{8}$$

$$V_s(X_t \mid Y_t) = (I_{N_X} - K_s H) V_s(X_t) (I_{N_X} - K_s H)' \tag{9}$$

$$p(r_t = s \mid Y_t) = \frac{p(r_t = s; Y_t)}{p(Y_t)} = \frac{p(Y_t \mid r_t = s) p(r_t = s)}{p(Y_t)} \tag{10}$$

Equation (10) shows the probability of regime s on condition of the observed variables. $p(Y_t)$ is the likelihood (computed at step 2.3) and $p(Y_t|r_t=s)$ has a normal density.

2.5 Compute density of X_{t+1} as functions of the density of $X_t|Y_t$ (version MSQKF)

The initial information for this step is the density for the vector of model variables X_t (normal mixture).

The first step is the computation of the expectation and variance of vector X_t on the condition of the future state (11)–(12).

$$\mathrm{E}_{s,1} = E(X_t \mid r_{t+1} = s) = \sum_{k=1}^{N_s} \frac{p(r_t = k) p(r_{t+1} = s \mid r_t = k)}{p(r_{t+1} = s)} E_k(X_t) \tag{11}$$

$$V_{s,1}=\left(\sum_{k=1}^{N_s}\frac{p(r_t=k)p(r_{t+1}=s\,|\,r_t=k)}{p(r_{t+1}=s)}\Big(E_k(X_t)E_k(X_t)'+V_k(X_t)\Big)\right)-E_{s,1}(E_{s,1})' \quad (12)$$

The next step is the approximation (collapsing rule): we believe that the density of vector X_t is a normal mixture with regime probabilities $p(r_{t+1}=s)$ and Gaussian densities with moments $E_{s,1}$ and $V_{s,1}$.

Knowledge of the conditional density of X_t gives us the density of Z_t. It allows us to compute the conditional moments of the future vector of variables X_{t+1}.

$$Z_{0,t,r_{t+1}}=Z_{t,r_{t+1}}-E\left(Z_{t,r_{t+1}}\right) \quad (13)$$

$$X_{t+1,r_{t+1}}=A_{0,r_{t+1}}+A_{1,r_{t+1}}Z_{t,r_{t+1}}+A_{2,r_{t+1}}\left(Z_{t,r_{t+1}}\otimes Z_{t,r_{t+1}}\right)=B_{0,r_{t+1}}+B_{1,r_{t+1}}Z_{0,t,r_{t+1}}+B_{2,r_{t+1}}\left(Z_{0,t,r_{t+1}}\otimes Z_{0,t,r_{t+1}}\right) \quad (14)$$

$$E\left(X_{t+1,r_{t+1}}\right)=B_{0,r_{t+1}}+B_{2,r_{t+1}}vec\left(V\left(Z_{t,r_{t+1}}\right)\right)=B_{0,r_{t+1}}+B_{2,r_{t+1}}vec\left(V_{r_{t+1}}\right) \quad (15)$$

$$vec\left(V\left(X_{t+1,r_{t+1}}\right)\right)=\left(B_{0,r_{t+1}}\otimes B_{0,r_{t+1}}\right)vec\left(V_{r_{t+1}}\right)+\left(B_{2,r_{t+1}}\otimes B_{2,r_{t+1}}\right)\begin{pmatrix}vec\left(V_{r_{t+1}}\right)\otimes vec\left(V_{r_{t+1}}\right)+\\+vec\left(vec\left(V_{r_{t+1}}\right)\otimes V_{r_{t+1}}\right)\end{pmatrix} \quad (16)$$

The formulas (13)–(16) are similar to formulas from [Ivashchenko (2014)]. The difference is that these formulas became formulas for moments, conditional on the regime. The last action of this step is an approximation. We suggest that the density of X_{t+1} is a normal mixture with moments according to (15)–(16).

2.6 Compute the density of X_{t+1} as functions the of density of $X_t|Y_t$ (Version MSQKFA)

Knowing the conditional density of X_t gives us the density of Z_t. However, this density is conditional on r_t, while equation (3) is conditional on r_{t+1}. Thus, we have to use all possible combinations of regimes, and compute the expectation and variance of the X_{t+1} condition on r_t and r_{t+1}.

$$Z_{0,t,r_t}=Z_{t,r_t}-E\left(Z_{t,r_t}\right) \quad (17)$$

$$X_{t+1,r_t,r_{t+1}}=A_{0,r_{t+1}}+A_{1,r_{t+1}}Z_{t,r_t}+A_{2,r_{t+1}}\left(Z_{t,r_t}\otimes Z_{t,r_t}\right)=B_{0,r_{t+1}}+B_{1,r_{t+1}}Z_{0,t,r_t}+B_{2,r_{t+1}}\left(Z_{0,t,r_t}\otimes Z_{0,t,r_t}\right) \quad (18)$$

$$E\left(X_{t+1,r_t,r_{t+1}}\right)=B_{0,r_{t+1}}+B_{2,r_{t+1}}vec\left(V\left(Z_{t,r_t}\right)\right)=B_{0,r_{t+1}}+B_{2,r_{t+1}}vec\left(V_{r_t}\right) \quad (19)$$

$$vec\left(V\left(X_{t+1,r_t,r_{t+1}}\right)\right)=\left(B_{0,r_{t+1}}\otimes B_{0,r_{t+1}}\right)vec\left(V_{r_t}\right)+\left(B_{2,r_{t+1}}\otimes B_{2,r_{t+1}}\right)\begin{pmatrix}vec\left(V_{r_t}\right)\otimes vec\left(V_{r_t}\right)+\\+vec\left(vec\left(V_{r_t}\right)\otimes V_{r_t}\right)\end{pmatrix} \tag{20}$$

Formulas (17)–(20) are almost the same as (13)–(16). The difference is the conditions for conditional moments. The expectation and variance of the future vector X_{t+1} conditional on the future regime (r_{t+1}) are the following:

$$E_s\left(X_{t+1}\right)=E\left(X_{t+1}\mid r_{t+1}=s\right)=\sum_{k=1}^{N_s}\frac{p(r_t=k)p(r_{t+1}=s\mid r_t=k)}{p(r_{t+1}=s)}E\left(X_{t+1}\mid r_t=k,r_{t+1}=s\right) \tag{21}$$

$$V_s=\left(\sum_{k=1}^{N_s}\frac{p(r_t=k)p(r_{t+1}=s\mid r_t=k)}{p(r_{t+1}=s)}\left(E_{k,s}\left(X_{t+1}\right)E_{k,s}\left(X_{t+1}\right)'+V_{k,s}\left(X_{t+1}\right)\right)\right)-E_s\left(X_{t+1}\right)\left(E_s\left(X_{t+1}\right)\right)' \tag{22}$$

The last action is an approximation (collapsing rule): we suggest that the density of X_{t+1} is a normal mixture with regime probabilities $p(r_{t+1})$ and conditional moments computed according to (21)–(22).

2.7 Compute the density of X_{t+1} as functions of the density of $X_t|Y_t$ (Version MSCDKF)

The beginning of this approach is similar to MSQKF. The first step is the computation of the expectation and variance of vector X_t on condition of the future state (11)–(12). The next step is the approximation (collapsing rule): we believe that the density of vector X_t is a normal mixture with regime probability $p(r_{t+1}=s)$ and Gaussian densities with moments $E_{s,1}$ and $V_{s,1}$. Knowing the conditional density of X_t gives us the density of Z_t.

The next step is computing the points around the mean, conditional on the regime and corresponding to their future values of model variable vector (23)–(25). The matrix U can be computed with Cholesky factorizations, eigenvalue decomposition, singular value decomposition (implemented in the filters code) or other techniques. The parameter h is a tuning parameter. The recommended value for normal density is $h^2=3$.

$$Z_{t,r_{t+1},j}=\begin{cases}-hU_{r_{t+1},-j} & j=-n_Z:-1\\ 0 & j=0\\ hU_{r_{t+1},j} & j=n_Z:1\end{cases} \tag{23}$$

$$V_{r_{t+1},1} = U_{r_{t+1},1:n_Z} U_{r_{t+1},1:n_Z}{}' \tag{24}$$

$$X_{t+1,r_{t+1},j} = B_{0,r_{t+1}} + B_{1,r_{t+1}} Z_{t,r_{t+1},j} + B_{2,r_{t+1}} \left(Z_{t,r_{t+1},j} \otimes Z_{t,r_{t+1},j} \right) \tag{25}$$

The last step is the approximation: we believe that the density of X_{t+1} is a normal mixture with regime probabilities $p(r_{t+1})$ and conditional moments computed according to (26)–(29).

$$E_s(X_{t+1}) = E(X_{t+1} \mid r_{t+1} = s) = X_{t+1,s,0} + \sum_{k=1}^{N_Z} \frac{\left(X_{t+1,s,k} + X_{t+1,s,-k} - 2X_{t+1,s,0} \right)}{2h^2} = \\ = \sum_{\substack{k=-N_Z \\ k \neq 0}}^{N_Z} \frac{X_{t+1,s,k}}{2h^2} + \frac{h^2 - N_Z}{h^2} X_{t+1,s,0} \tag{26}$$

$$V_s = \frac{1}{4h^2} \hat{B}_{1,s} \hat{B}_{1,s}{}' + \frac{h^2 - 1}{4h^2} \hat{B}_{2,s} \hat{B}_{2,s}{}' \tag{27}$$

$$\hat{B}_{1,s} = [X_{t+1,s,1} - X_{t+1,s,-1}, X_{t+1,s,2} - X_{t+1,s,-2}, \ldots, X_{t+1,s,N_Z} - X_{t+1,s,-N_Z}] \tag{28}$$

$$\hat{B}_{2,s} = [X_{t+1,s,1} + X_{t+1,s,-1} - 2X_{t+1,s,0}, \ldots, X_{t+1,s,N_Z} + X_{t+1,s,-N_Z} - 2X_{t+1,s,0}] \tag{29}$$

2.8 Compute the density of X_{t+1} as functions of the density of $X_t|Y_t$ (Version MSCDKFA)

The beginning of this approach is similar to MSQKFA. Knowing the conditional density of X_t gives us the density of Z_t. However, this density is a condition of r_t, while equation (3) is a condition of r_{t+1}. Thus, we have to use all possible combinations of regimes, and compute the expectation and variance of the X_{t+1} conditional on r_t and r_{t+1}.

The next step is computing the points around the mean conditional on the regime, and corresponding to their future values of the model variable vector for each possible future regime (30)–(32). The formulas (30)–(32) differ from (23)–(25) only in the regime conditions.

$$Z_{t,r_t,j} = \begin{cases} -hU_{r_t,-j} & j = -n_Z : -1 \\ 0 & j = 0 \\ hU_{r_t,j} & j = n_Z : 1 \end{cases} \tag{30}$$

$$V_{r_t} = U_{r_t,1:n_Z} U_{r_t,1:n_Z}{}' \tag{31}$$

$$X_{t+1,r_t,r_{t+1},j} = B_{0,r_{t+1}} + B_{1,r_{t+1}} Z_{t,r_t,j} + B_{2,r_{t+1}} \left(Z_{t,r_t,j} \otimes Z_{t,r_t,j} \right) \tag{32}$$

The next step is the approximation: we approximate the moments of the X_{t+1} condition for the regimes r_t and r_{t+1} according to (33)–(36).

$$E_{r_t,r_{t+1}}(X_{t+1}) = E(X_{t+1} \mid r_t, r_{t+1}) = X_{t+1,r_t,r_{t+1},0} + \sum_{k=1}^{N_Z} \frac{\left(X_{t+1,r_t,r_{t+1},k} + X_{t+1,r_t,r_{t+1},-k} - 2X_{t+1,r_t,r_{t+1},0}\right)}{2h^2} = \\ = \sum_{\substack{k=-N_Z \\ k\neq 0}}^{N_Z} \frac{X_{t+1,r_t,r_{t+1},k}}{2h^2} + \frac{h^2 - N_Z}{h^2} X_{t+1,r_t,r_{t+1},0} \tag{33}$$

$$V_{r_t,r_{t+1}} = \frac{1}{4h^2} \hat{B}_{1,r_t,r_{t+1}} \hat{B}_{1,r_t,r_{t+1}}{}' + \frac{h^2 - 1}{4h^2} \hat{B}_{2,r_t,r_{t+1}} \hat{B}_{2,r_t,r_{t+1}}{}' \tag{34}$$

$$\hat{B}_{1,r_t,r_{t+1}} = [X_{t+1,r_t,r_{t+1},1} - X_{t+1,r_t,r_{t+1},-1}, X_{t+1,r_t,r_{t+1},2} - X_{t+1,r_t,r_{t+1},-2}, \ldots, X_{t+1,r_t,r_{t+1},N_Z} - X_{t+1,r_t,r_{t+1},-N_Z}] \tag{35}$$

$$\hat{B}_{2,r_t,r_{t+1}} = [X_{t+1,r_t,r_{t+1},1} + X_{t+1,r_t,r_{t+1},-1} - 2X_{t+1,r_t,r_{t+1},0}, \ldots, X_{t+1,r_t,r_{t+1},N_Z} + X_{t+1,r_t,r_{t+1},-N_Z} - 2X_{t+1,r_t,r_{t+1},0}] \tag{36}$$

The expectation and variance of the future vector X_{t+1} condition on the future regime (r_{t+1}) is computed the same way as MSQKFA (formulas (21)–(22)). The last approximation (collapsing) assumes that the density of X_{t+1} is a normal mixture with regime probabilities $p(r_{t+1})$ and conditional moments computed according to (21)–(22) and (33)–(36).

3 The MS-DSGE model and tests descriptions

The model that is used for the test is a financial one that is similar to the one used in [Ivashchenko (2014)]. The system of rational expectation equations (restrictions and first-order conditions) in terms of stable variables contains the following: a budget restriction (37), an exogenous rule for dividend growth (38), an exogenous number of bonds bought by the government (39), and an amount of stocks equal to one (40), and the optimal conditions of (41)–(43) with an additional exogenous process ($z_{A,S,t}$, $z_{A,B,t}$, $z_{A,C,t}$). The description of the variables is presented in Table 1.

$$e^{c_t} + b_t + q_t = e^{r_{t-1} - s_t} b_{t-1} + q_{t-1}\left(1 + e^{d_t}\right) + z_{I,t} \tag{37}$$

$$d_t - d_{t-1} + s_t = z_{D,t} \tag{38}$$

$$b_t = z_{B,t} \tag{39}$$

$$q_t = 1 \quad (40)$$

$$e^{\lambda_t + z_{A,S,t}} = E_t e^{\lambda_{t+1} + \ln(\beta) + \gamma(s_{t+1} - z_{P,t+1})}\left(1 + e^{d_{t+1}}\right) \quad (41)$$

$$e^{\lambda_t + z_{A,B,t}} = E_t e^{\lambda_{t+1} + r_t - s_{t+1} + \ln(\beta) + \gamma(s_{t+1} - z_{P,t+1})} \quad (42)$$

$$\gamma c_t = \lambda_t + c_t + z_{A,C,t} \quad (43)$$

Tab. 1. DSGE model variables

Variable	Description	Stationary variable
B_t	Value of bonds bought by households at period t	$b_t = B_t / S_t$
C_t	Consumption at time t	$c_t = \ln(Z_{P,t} C_t / S_t)$
D_t	Dividends at time t	$d_t = \ln(D_t / S_t)$
R_t	Interest rate at time t	$r_t = \ln(R_t)$
S_t	Price of stocks at time t	$s_t = \ln(S_t / S_{t-1})$
Q_t	Amount of stocks bought by households at period t	$q_t = Q_t$
Λ_t	Lagrange multiplier corresponding to budget restriction of households at period t	$\lambda_t = \Lambda_t$
$Z_{A,B,t}$	Exogenous process corresponding to near-rationality of households with its bond position	$z_{A,B,t} = Z_{A,B,t}$
$Z_{A,C,t}$	Exogenous process corresponding to near-rationality of households with its consumption	$z_{A,C,t} = Z_{A,C,t}$
$Z_{A,S,t}$	Exogenous process corresponding to near-rationality of households with its stocks position	$z_{A,C,t} = Z_{A,C,t}$
$Z_{B,t}$	Exogenous process corresponding to bond amount sold by the government	$z_{B,t} = Z_{B,t}$
$Z_{D,t}$	Exogenous process corresponding to dividends growth	$z_{D,t} = Z_{D,t}$
$Z_{I,t}$	Exogenous process corresponding to households income	$z_{T,t} = Z_{T,t}$
$Z_{P,t}$	Exogenous process corresponding to price level	$z_{P,t} = \ln(Z_{P,t} / Z_{P,t-1})$

All exogenous processes are AR(1) with following parameterization (44):

$$z_{*,t} = \eta_{0,*,t}(1 - \eta_{1,*,t}) + \eta_{1,*,t} z_{*,t-1} + \varepsilon_{*,t} \quad (44)$$

The difference of this MS-DSGE from that used in [Ivashchenko (2014)] is the following: $\eta_{1,A,B}=0$, $\eta_{1,A,C}=0$, $\eta_{1,A,S}=0$; there is switching (with 2 regimes) for parameters $\eta_{1,*}$ and standard deviation of all exogenous shocks. The MS-DSGE model is estimated with quarterly data from 1985 Q4 to 2015 Q3. The following data are used: MSCI USA price return, MSCI USA gross return ($obs_{pg,t}$), and the 3-month euro-dollar deposit rate. The first 4 quarters are used as a pre-sample (for better initialization of the filter). The maximum likelihood estimation is used.

A few tests of filtering quality are done. The first is a test of the estimation quality. The second order approximation of the MS-DSGE model with parameter values from the maximum likelihood estimation is used as DGP. 120 observations were generated 100 times. The model is estimated for each of the generated data with the different filters (MSQKF, MSQKFA, MSCDKF, MSCDKFA and a filter that uses a linear approximation of the model [Kim (1994)]). RMSE of estimation (relative to the linear filter) is presented in Table 2. The designation for the linear approximation based filter is MSKFA.

The estimation results give a small probability of regime switching (3.5% and 8.53%). Persistent regimes are common results for MS-DSGE models. However, this can influence filter performance. Thus, the previous test is repeated with a change in DGP parameters (the transition probability is set to 0.45). The corresponding results are presented at Table 3.

The next test is the estimation of computational costs and their dependence on the number of variables. The simple modifications of the model are done for this test. The lags are changed in equation (44). Instead of the first lag we use the second, the fifth and the tenth. This forces the creation of auxiliary variables and increases the size of the model. The time of filtering is measured for these modifications of the model.

The last tests are related to filtering quality. RMSE of filters are calculated. Formula (45) describes RMSE for the updated values of variables. Table 5 presents RMSE for model variables in the last period. The same 100 calculations (as for estimation test) are used.

$$RMSE_{updated\,,} = \sqrt{\sum_{i_{draw}=1}^{N_{draw}} \left(X_{t,i_{draw}} - E\left(X_{t,i_{draw}} \mid Y_{1,i_{draw}},\ldots,Y_{t,i_{draw}} \right) \right)^2 / N_{draw}} \quad (45)$$

The density filtering quality is measured by log-predictive-score (LPS): the likelihood of unobserved variables according to the updated variable density ($p(regime_t,\ X_t|Y_1,\ldots,Y_t)$). The numeric conditional variance matrix of the updated model variables has 4 eigenvalues that are close to zero (3 observed variables and 1 static variable). These values are counted as exactly zero for computing LPS.

4 Results and discussion

RMSE of the parameter estimation is presented in Table 2. All filters that use the second order estimation are much more accurate than the linear one. RMSE became about 10 times smaller. This improvement is much larger than for the DSGE model [Ivashchenko (2014)]. It is related to using a shorter sample (120 observations vs 400 observations) and quarterly data (which makes the estimated standard deviation of shocks larger). Markov switching gives additional advantages to nonlinear filters because it estimates regime probabilities more accurately. The errors in regime identification make the estimation of regime specific parameters doubtful.

Tab. 2. RMSE of parameters estimation

parameter	parameter value	RMSE				
		MSKFA	MSQKF / MSKFA	MSQKFA / MSKFA	MSCDKF / MSKFA	MSCDKFA / MSKFA
$p(reg_{t+1}=2 \mid reg_t=1)$	3.50E-02	4.02E-02	**1.55E-01**	1.59E-01	1.70E-01	1.66E-01
$p(reg_{t+1}=1 \mid reg_t=2)$	8.53E-02	5.60E-02	1.89E-01	**1.78E-01**	1.86E-01	2.26E-01
$ln(\beta)$	-6.21E-02	3.82E-02	**4.01E-02**	5.82E-02	4.88E-02	4.11E-02
γ	2.55E-01	6.02E-01	1.71E-02	1.31E-02	1.31E-02	**1.18E-02**
$\eta_{0,B}$	-3.26E-01	2.51E+00	1.33E-02	1.09E-02	1.21E-02	**1.06E-02**
$\eta_{0,D}$	-1.09E-02	3.71E-02	1.48E-01	2.18E-01	**1.34E-01**	1.73E-01
$\eta_{0,I}$	1.29E+01	8.35E+00	1.29E-01	**8.62E-02**	9.40E-02	1.22E-01
$\eta_{0,P}$	3.10E-02	3.24E-02	3.09E-01	2.79E-01	**2.42E-01**	2.59E-01
$\eta_{1,B}$ *(reg. №1)*	-7.81E-01	7.85E-01	1.60E-01	**1.37E-01**	1.43E-01	1.51E-01
$\eta_{1,D}$*(reg. №1)*	7.56E-01	1.15E+00	5.62E-02	6.15E-02	7.03E-02	**5.34E-02**
$\eta_{1,I}$*(reg. №1)*	8.74E-01	8.37E-01	1.25E-01	1.92E-01	**7.31E-02**	1.64E-01
$\eta_{1,P}$*(reg. №1)*	9.75E-01	2.23E-01	3.65E-02	3.61E-02	3.11E-02	**2.70E-02**
std of ε_{AB} *(reg. №1)*	3.14E-01	1.22E+00	2.08E-01	2.03E-01	2.54E-01	**1.90E-01**
std of ε_{AC} *(reg. №1)*	4.59E-02	2.66E+00	1.71E-02	**1.53E-02**	1.90E-02	1.75E-02
std of ε_{AS} *(reg. №1)*	3.60E-03	6.20E-01	**6.85E-02**	7.48E-02	9.33E-02	8.80E-02
std of ε_{B} *(reg. №1)*	6.34E-01	3.33E+00	3.66E-02	3.69E-02	**3.06E-02**	3.74E-02
std of ε_{D} *(reg. №1)*	4.86E-01	4.09E+00	**1.60E-02**	1.89E-02	2.20E-02	1.66E-02
std of ε_{I} *(reg. №1)*	2.74E+00	5.16E+00	2.06E-01	2.13E-01	**1.86E-01**	2.04E-01
std of ε_{P} *(reg. №1)*	3.01E-01	2.25E+00	9.30E-03	6.74E-03	**6.52E-03**	7.76E-03
$\eta_{1,B}$ *(reg. №2)*	9.90E-01	6.25E-01	2.88E-01	2.66E-01	3.14E-01	**2.31E-02**
$\eta_{1,D}$*(reg. №2)*	-8.28E-02	3.21E-01	2.74E-01	**1.76E-01**	3.86E-01	**1.76E-01**
$\eta_{1,I}$*(reg. №2)*	9.90E-01	5.70E-01	1.73E-01	1.13E-01	1.66E-02	**3.80E-02**
$\eta_{1,P}$*(reg. №2)*	-1.91E-01	5.28E-01	**1.64E-01**	2.95E-01	2.51E-01	2.81E-01
std of ε_{AB} *(reg. №2)*	1.00E-06	2.58E-01	4.74E-03	1.76E-01	**4.07E-03**	2.02E-01
std of ε_{AC} *(reg. №2)*	1.00E-06	1.08E+00	1.81E-04	2.35E-04	**1.64E-04**	4.89E-03
std of ε_{AS} *(reg. №2)*	1.00E-06	1.56E-01	**3.59E-03**	5.20E-02	2.04E-01	2.79E-02
std of ε_{B} *(reg. №2)*	8.33E-03	4.14E-01	**1.44E-02**	7.43E-02	1.50E-02	8.18E-02

std of ε_D *(reg. №2)*	7.79E-02	8.51E-01	1.93E-02	7.90E-02	**1.69E-02**	7.80E-02
std of ε_I *(reg. №2)*	5.69E-01	1.69E+00	**1.42E-01**	4.43E-01	4.11E-01	3.01E-01
std of ε_P *(reg. №2)*	2.88E-03	9.68E-01	1.40E-01	9.33E-03	**7.06E-03**	7.03E-02
mean	-	1.38E+00	**1.05E-01**	1.23E-01	1.15E-01	1.08E-01
root-mean-square	-	2.27E+00	**1.40E-01**	1.62E-01	1.65E-01	1.41E-01
median	-	7.05E-01	9.65E-02	8.26E-02	**7.17E-02**	7.99E-02

The comparison of nonlinear filters demonstrates the small advantage of MSQKF according to mean and root-mean-squared aggregate measures. The use of the median gives some advantage to MSCDKFA. However, it is not a dominance of one filter over another. Each of the filters produces the best quality of estimation for some of the parameters. There is a notable spread in the relative performance from 1.64e-4 (std of ε_{AC} *(reg. №2)* MSCDKF) to 4.43e-1 (std of ε_I *(reg. №2)* MSQKFA). The usage of absolute RMSE instead of relative influences the choice of the best filters. MSQKF and MSCDKF are the best according to mean RMSE, while MSCDKF is the best according to root-mean-square RMSE and median RMSE. Thus, the estimation quality produced by all filters is very close, and advantage of MSQKF is insignificant.

However, there was a result for persistent regimes; if regimes are not persistent then the picture is different (see Table 3). RMSE for most variables and filters became smaller. The improvement of linear filter quality is greater than for others. Thus, the relative performance of nonlinear filters worsens. This can be explained by the better identification of regime probabilities.

It was natural to expect that the later implementation of collapsing rule (MSQKFA and MSCDKFA) should produce a better quality of estimation (especially in situations of low regime persistence). However, this does not happen. The best relative performance was achieved by MSQKF, and MSCDKF was better than MSCDKFA.

Tab. 3. RMSE of parameters estimation in case of high probability of switching

parameter	RMSE				
	MSKFA	MSQKF / MSKFA	MSQKFA / MSKFA	MSCDKF / MSKFA	MSCDKFA / MSKFA
$p(reg_{t+1}=2\| reg_t=1)$	9.36E-02	**1.85E-01**	2.23E-01	2.48E-01	2.79E-01
$p(reg_{t+1}=1\| reg_t=2)$	1.09E-01	**1.72E-01**	1.99E-01	2.44E-01	2.42E-01
$ln(\beta)$	1.75E-02	**7.81E-02**	8.43E-02	1.41E-01	1.18E-01
γ	8.92E-02	2.00E-01	**1.98E-01**	2.19E-01	2.46E-01
$\eta_{0,B}$	2.64E-01	**1.95E-01**	2.00E-01	2.70E-01	2.64E-01
$\eta_{0,D}$	3.02E-02	**2.13E-01**	2.20E-01	2.54E-01	2.64E-01
$\eta_{0,I}$	5.36E+00	**1.87E-01**	1.99E-01	2.59E-01	2.96E-01
$\eta_{0,P}$	2.71E-02	2.87E-01	**2.86E-01**	3.54E-01	3.43E-01
$\eta_{1,B}$ *(reg. №1)*	5.35E-01	**1.39E-01**	1.53E-01	1.61E-01	1.73E-01
$\eta_{1,D}$*(reg. №1)*	5.01E-01	**1.11E-01**	1.14E-01	1.31E-01	1.36E-01
$\eta_{1,I}$*(reg. №1)*	2.72E-01	**7.28E-02**	8.48E-02	9.00E-02	9.23E-02
$\eta_{1,P}$*(reg. №1)*	6.55E-01	**1.20E-01**	1.26E-01	1.29E-01	1.44E-01
std of ε_{AB} *(reg. №1)*	2.66E-01	**2.22E-01**	2.36E-01	2.70E-01	2.69E-01
std of ε_{AC} *(reg. №1)*	4.80E-02	7.01E-01	7.03E-01	**6.85E-01**	6.69E-01
std of ε_{AS} *(reg. №1)*	7.85E-02	**3.10E-01**	3.26E-01	3.14E-01	3.82E-01
std of ε_{B} *(reg. №1)*	2.62E-01	4.38E-01	**4.35E-01**	4.95E-01	5.06E-01
std of ε_{D} *(reg. №1)*	3.80E-01	**1.17E-01**	1.22E-01	1.34E-01	1.44E-01
std of ε_{I} *(reg. №1)*	1.42E+00	2.79E-01	**2.78E-01**	3.68E-01	4.04E-01
std of ε_{P} *(reg. №1)*	2.35E+00	**1.34E-02**	1.61E-02	1.99E-02	2.41E-02
$\eta_{1,B}$ *(reg. №2)*	5.00E-02	**1.92E-01**	2.50E-01	2.44E-01	3.21E-01
$\eta_{1,D}$*(reg. №2)*	9.07E-02	**4.18E-01**	4.44E-01	4.49E-01	4.93E-01
$\eta_{1,I}$*(reg. №2)*	1.21E-02	1.61E+00	1.69E+00	**1.59E+00**	1.79E+00
$\eta_{1,P}$*(reg. №2)*	2.35E-01	5.16E-01	**4.74E-01**	5.88E-01	5.82E-01
std of ε_{AB} *(reg. №2)*	7.10E-03	1.66E-01	1.38E-01	**1.27E-01**	1.43E-01
std of ε_{AC} *(reg. №2)*	7.18E-04	2.03E-01	**1.60E-01**	1.70E-01	1.80E-01
std of ε_{AS} *(reg. №2)*	4.60E-03	1.23E-01	1.18E-01	1.24E-01	**1.16E-01**
std of ε_{B} *(reg. №2)*	1.11E-02	**6.33E-01**	6.49E-01	6.61E-01	6.60E-01
std of ε_{D} *(reg. №2)*	4.05E-02	**2.64E-01**	2.69E-01	2.75E-01	2.71E-01
std of ε_{I} *(reg. №2)*	4.08E-01	**5.23E-01**	5.48E-01	5.59E-01	6.38E-01
std of ε_{P} *(reg. №2)*	1.42E+00	**6.86E-03**	1.02E-02	1.33E-02	1.30E-02
mean	5.01E-01	**2.90E-01**	2.99E-01	3.19E-01	3.40E-01
root mean of MSE	1.15E+00	**4.15E-01**	4.30E-01	4.33E-01	4.70E-01
median	1.01E-01	**1.97E-01**	2.10E-01	2.51E-01	2.66E-01

Table 4 demonstrates the dependence of computation time on the number of variables. It shows the time of the likelihood calculation excluding the time for solution approximations. The growth of the model size leads to a much higher share of the time required for solution approximation. It makes the use of all filters almost equivalent for large models. The MSKF sometimes is faster than the time required for a second order solutions because it requires only a linear solution. The MSQKF is the fastest nonlinear filter.

Tab. 4. Time for likelihood calculation (sec) for model with 2 states

n_X	10	14	26	46
n_Z	14	18	30	50
Solution	5.77E-02	9.21E-02	6.80E-01	2.35E+01
MSKF (including solution)	9.51E-02	1.20E-01	6.49E-01	2.23E+01
MSQKF	1.58E-01	2.90E-01	4.98E-01	1.10E+00
MSQKFA	2.65E-01	5.09E-01	9.17E-01	2.93E+00
MSCDKF	2.98E-01	3.85E-01	7.52E-01	1.89E+00
MSCDKFA	5.35E-01	7.04E-01	1.42E+00	4.36E+00

* PC used: Intel core i5 3.4 GHz; 8 Gb RAM; Windows 7.

The time for filtering (without the time for the solution) for version of filters with late collapsing is about 1.8 times longer. MSQKF filtering is 1.3–1.9 times faster than MSCDKF . However, there is not a monotone dependence of the advantage on the model size. This differs from the situation with computational costs of QKF and CDKF [Ivashchenko (2014)] because of the improved quality of the filter code.

Tab. 5. RMSE of updated unobserved variables

Variable	Parameters	MSKFA	MSQKF	MSQKFA	MSCDKF	MSCDKFA
c_t	estim	2.37E+01	5.23E-01	5.34E-01	**4.81E-01**	5.46E-01
c_t	true	2.66E+00	4.56E-01	4.56E-01	**4.55E-01**	4.55E-01
d_t	estim	1.53E+01	**9.54E-01**	1.03E+00	1.05E+00	9.98E-01
d_t	true	1.99E+01	1.39E+00	**1.39E+00**	3.14E+00	3.26E+00
λ_t	estim	1.69E+01	3.91E-01	3.97E-01	**3.57E-01**	4.02E-01
λ_t	true	1.72E+00	3.43E-01	3.43E-01	**3.40E-01**	3.40E-01
$z_{B,t}$	estim	7.28E+00	**4.53E-01**	4.69E-01	4.74E-01	4.89E-01
$z_{B,t}$	true	1.51E+01	**4.70E-01**	4.71E-01	9.32E-01	9.55E-01
$z_{D,t}$	estim	4.72E+00	2.97E-01	**2.85E-01**	3.11E-01	3.01E-01
$z_{D,t}$	true	4.88E+00	3.29E-01	**3.29E-01**	4.65E-01	5.11E-01
$z_{I,t}$	estim	1.84E+01	5.96E+00	5.70E+00	**5.67E+00**	6.41E+00
$z_{I,t}$	true	7.54E+01	**7.08E+00**	7.09E+00	9.85E+00	9.81E+00
$z_{P,t}$	estim	6.00E+00	2.83E-01	**2.71E-01**	3.03E-01	2.98E-01
$z_{P,t}$	true	8.67E+00	2.89E-01	**2.88E-01**	6.91E-01	6.63E-01
mean (RMSE / RMSE of MSKFA)	estim	100.00%	8.64%	**8.49%**	8.59%	9.17%
mean(RMSE / RMSE of MSKFA)	true	100.00%	**9.52%**	9.52%	12.76%	12.96%
median(RMSE / RMSE of MSKFA)	estim	100.00%	6.22%	**6.04%**	6.51%	6.38%
median(RMSE / RMSE of MSKFA)	true	100.00%	7.00%	**7.00%**	13.06%	13.01%

The next test checks quality of filtering. Table 5 presents RMSE of the updated unobserved variables ($E(X_t|Y_1,\ldots,Y_t)$) for true and estimated values of the parameters. MSQKF and MSQKFA produce almost the same quality of filtering for the true values of the parameters. MSCDKF and MSCDKFA produce much worse performances. However, for estimated parameter values all 4 nonlinear filters are very similar. MSQKFA is slightly better, but choice between the MSQKF and the MSCDKF depends on the aggregate measure. Thus, the predictable advantage of late collapsing is observed for filtering quality (in contrast to estimation quality).

An interesting detail is related to the comparison of filtering quality in the true and estimated points of the parameter space. The quality of the filters improves for 5 variables and worsens for 2 (c and limdaa). It is unclear why all the filters improve the same variable filtering quality. This could be related to some properties of the variance matrix. MSKFA has large changes in filtering quality while for MSQKF and MSQKFA the change in filtering quality is small. MSCDKF and MSCDKFA have a notable improvement and a minor decrease of quality respectively. The result is a significant improvement of the relative quality of filtering for these two filters for estimated parameter values.

RMSE of filtered variables ($E(X_t|Y_1,\ldots,Y_{t-1})$) can be used as a measure of filter quality. The results are presented in Table A1 (see appendix). It produces almost the same picture: MSQKF and MSQKFA are better for true parameter values; the close performance of filters for the estimated values of parameters (MSCDKF is better according to the mean while the MSQKFA is better according to the median). The difference is a larger improvement of MSKFA compared to other filters for the move from the true to the estimated values of the parameters.

The quality of regime identification is another important property of the filters. Figures 1–2 show the mean RMSE of regime 1 probabilities ($p(regime=1|\ Y_1,\ldots,Y_t)$) for the true and the estimated parameters values. MSKF has a significantly worse performance, which motivates us to drop it from the plots. The first 4 observations (used for pre-filtering and having a significantly worse performance) are also dropped.

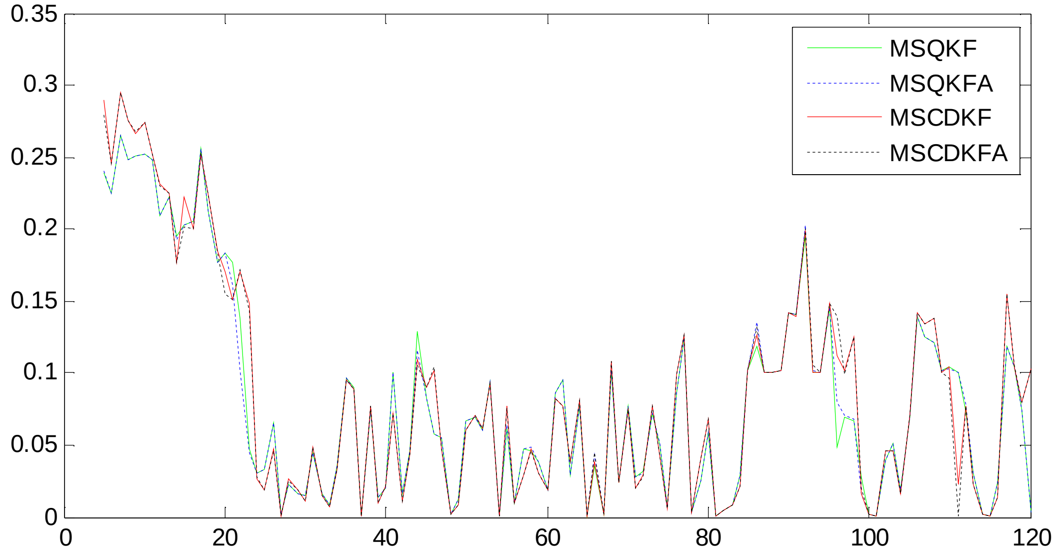

Fig. 1. RMSE of updated regime 1 probability (the true values of parameters).

The performance of the 4 filters are very similar for true parameter usage. There are some time points where the MSQKF and the MSQKFA have a slightly better performance but the difference is small. The average error is about 5–10%. The average performance does not change for the estimated parameters. However, the performance of filters became less correlated and a smaller number of observations were required for achieving an "average" result. It should be noted that the small probabilities of switching makes the Monte-Carlo estimation with 100 draws not a very accurate measure of regime identification quality.

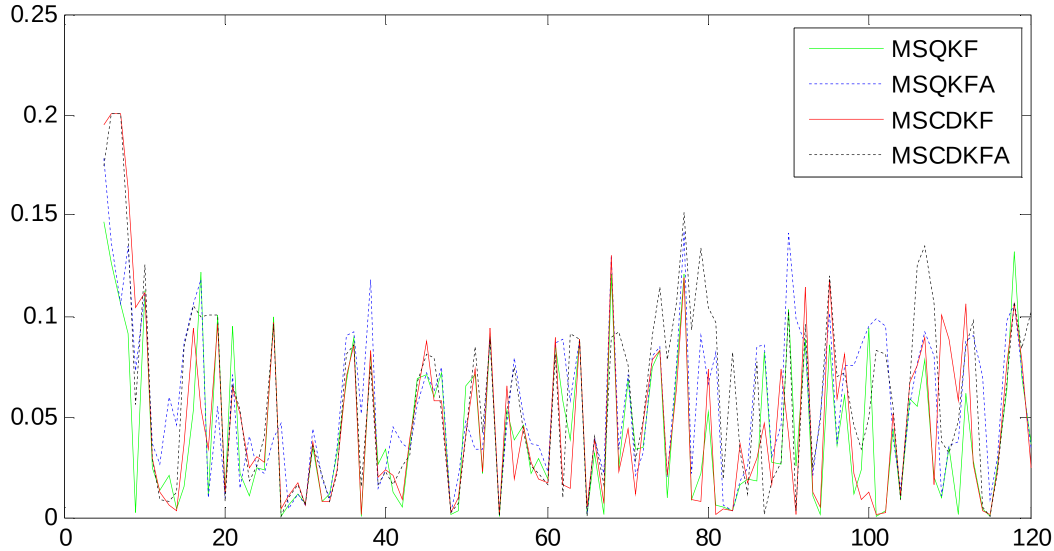

Fig. 2. RMSE of updated regime 1 probability (the estimated values of parameters).

The filters produce not a point estimation of unobserved variables, but a density estimation. LPS is a measure of density fit quality. It is the log-likelihood (according to the filtered density) of the unobserved variables. Figures 3–4 present plots for the median LPS of the four nonlinear filters. MSKFA is dropped because of its significantly worse performance. The first 4 observations are also dropped. The median is used because of outliers near the moments of switching.

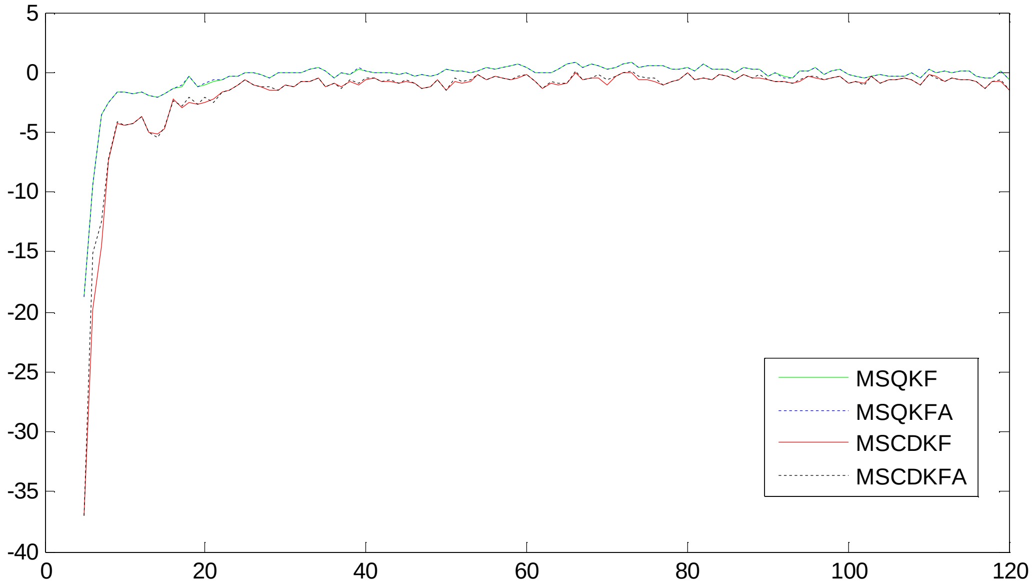

Fig. 3. The median LPS of updated unobserved variables, including regime (the true values of parameters).

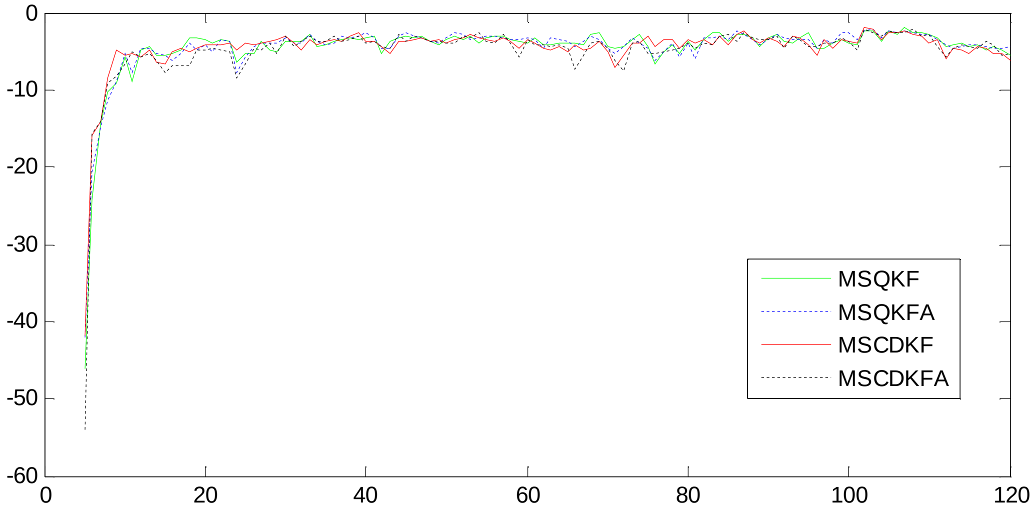

Fig. 4. The median LPS of updated unobserved variables, including regime (the estimated values of parameters).

It is clear that MSQKF and MSQKFA slightly outperform MSCDKF and MSCDKFA for true parameter values. However, the advantage of these filters is unclear for estimated values. The median value of the median LPS (for samples: 5–120 and 21–120) are: -1.64e+12 and -1.67e+12 for MSKF; -3.77 and -3.68 for MSQKF; -3.70 and -3.50 for MSQKFA; -3.95 and -3.77 for MSCDKF and; -4.08 and -3.95 for MSCDKFA. Thus, MSQKFA is the best while MSQKF is the second best.

5 Conclusions

his paper develops filters for the estimation of nonlinear MS-DSGE models. 4 nonlinear filters are constructed and tested with a small-scale MS-DSGE model. They are Gaussian assumed filters (MSQKF and MSQKFA) and sigma point filters (MSCDKF and MSCDKFA).

The test of estimation quality shows a small advantage for MSQKF. However, this is sensitive to aggregate performance measures, and all filters give very similar results. The advantage of MSQKF increases for DGP with higher switching probabilities.

The test of computational costs shows a significant advantage for MSQKF. However, for models with a large number of parameters almost all the time is spent finding the solution approximation. MSQKF is about 60% faster (without taking in account the time for solution approximation) than the second fastest filters (MSQKFA and MSCDKF) depending on the size of the model.

The RMSE-test of filtering quality shows MSQKFA is better. MSQKF is the first or the second best according to the most aggregate measures. The advantage of MSQKF and MSQKFA is significant for true parameter values. The advantage decreases for estimated parameter values. The density-test of filtering shows a small advantage for MSQKFA and MSQKF. This advantage is clear and stable for true parameters. However, it became less stable for estimated parameters.

Thus, MSQKF is overall the best investigated filters according to filtering quality, and it also has much lower computational costs.

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Appendix

Tab. A1. RMSE of filtered unobserved variables

Variable	Parameters	MSKF	MSQKF	MSQKFA	MSCDKF	MSCDKFA
c_t	estim	1.87E+01	1.09E+00	**1.05E+00**	1.05E+00	1.13E+00
c_t	true	6.61E+00	1.09E+00	**1.09E+00**	1.26E+00	1.25E+00
d_t	estim	8.58E+00	5.87E+00	5.84E+00	5.81E+00	**5.63E+00**
d_t	true	8.03E+00	6.00E+00	6.00E+00	**5.94E+00**	5.94E+00
λ_t	estim	1.32E+01	8.13E-01	**7.87E-01**	7.89E-01	8.38E-01
λ_t	true	4.94E+00	8.21E-01	**8.21E-01**	9.45E-01	9.38E-01
s_t	estim	2.07E+01	6.30E+00	6.28E+00	**6.11E+00**	6.18E+00
s_t	true	1.73E+01	**6.29E+00**	6.29E+00	6.74E+00	6.73E+00
r_t	estim	1.39E+01	3.12E+00	3.12E+00	**3.03E+00**	3.09E+00
r_t	true	3.02E+01	**3.25E+00**	3.25E+00	3.84E+00	3.84E+00
$z_{B,t}$	estim	7.19E+00	7.32E-01	**7.18E-01**	7.26E-01	7.20E-01
$z_{B,t}$	true	1.14E+01	7.41E-01	**7.36E-01**	7.40E-01	7.41E-01
$z_{D,t}$	estim	1.79E+00	4.43E-01	**4.36E-01**	4.30E-01	4.30E-01
$z_{D,t}$	true	3.36E+00	**4.41E-01**	4.41E-01	5.41E-01	5.38E-01
$z_{I,t}$	estim	1.64E+01	5.86E+00	**5.66E+00**	5.77E+00	6.68E+00
$z_{I,t}$	true	5.66E+01	**7.44E+00**	7.44E+00	8.70E+00	8.77E+00
$z_{P,t}$	estim	5.00E+00	4.74E-01	**4.59E-01**	4.79E-01	4.64E-01
$z_{P,t}$	true	8.59E+00	4.88E-01	**4.88E-01**	7.43E-01	7.47E-01
$obs_{pg,t}$	estim	1.91E+01	6.10E+00	6.08E+00	**5.92E+00**	5.99E+00
$obs_{pg,t}$	true	1.63E+01	**6.10E+00**	6.10E+00	6.53E+00	6.53E+00
mean (RMSE / RMSE of MSKFA)	estim	100.00%	24.54%	24.23%	**24.06%**	24.55%
mean (RMSE / RMSE of MSKFA)	true	100.00%	7.47%	**7.34%**	10.46%	9.91%
median (RMSE / RMSE of MSKFA)	estim	100.00%	23.59%	23.36%	**22.88%**	23.10%
median (RMSE / RMSE of MSKFA)	true	100.00%	5.79%	**5.71%**	8.75%	9.06%

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