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A SIMPLE THEORETICAL SETUP FOR THE EVALUATION OF STERILIZED INTERVENTION EFFECTIVENESS IN A SMALL OPEN COMMODITY EXPORTING ECONOMY

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A SIMPLE THEORETICAL SETUP FOR THE EVALUATION OF STERILIZED INTERVENTION EFFECTIVENESS IN A SMALL OPEN COMMODITY EXPORTING ECONOMY

This paper constructs a theoretical general equilibrium model for exchange rate determination in a small open commodity exporting economy based on an imperfect capital market a la Gabaix-Maggiori and appropriate for estimation on high frequency data and could be used for the evaluation of sterilized intervention effectiveness. To find empirical confirmation of the theoretical setup validity I use Russian daily statistics to estimate the model and investigate the reaction of the Russian ruble-US dollar exchange rate to sterilized interventions in the form of foreign currency repo auctions conducted by the Bank of Russia in the period of 2014-2017. I also estimate a vector error correction model on the same dataset and use it as important empirical benchmark for the theoretical model. The empirical analysis revealed a temporary statistically significant effect of sterilized intervention on exchange rate level, which peaked eight working days after the auction day. The combination of theoretical and empirical approaches demonstrates the effectiveness of the portfolio and the ineffectiveness of signalling channels in the transmission mechanism of the sterilized intervention instrument in Russian case.

JEL Classification: E58; F32.

Keywords: sterilized interventions; intervention effectiveness; repo auctions; commodity export; imperfect capital market; Russia.

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1. Introduction

This paper constructs a theoretical model for exchange rate determination in a small open commodity exporting economy. The model is based on a general equilibrium two country theoretical setup proposed by [Gabaix and Maggiori, 2015] appropriate for modelling many current topics in international macroeconomics. I extend their setup for the case of small open commodity exporting economy to analyse the effectiveness of sterilized intervention in foreign exchange (forex) market. As in the reference model I keep the real part of the economy as simple as possible, introduce international capital market imperfection, and mainly ignore the traditional monetary policy instrument, concentrating on the alternative one. I assume that the home country exports a commodity good which is not included in the household CES utility function. Monetary authorities use an interest rate rule which stabilizes non-tradable goods production, and perform sterilized interventions to influence the exchange rate. The model is configured for analysing high frequency (daily, weekly) dynamics and the commodity price is the only high frequency exchange rate fundamental. The effect of other fundamental factors is not directly identified and is only included in the balance of payments shock.

For empirical confirmation of the model and testing its ability for evaluating sterilized intervention effectiveness, I collected daily Russian statistics on foreign currency repo auctions, Russian ruble-US dollar exchange rates and oil prices. The model is solved and estimated by maximum likelihood (ML). A comparison of the structural theoretical model estimation with the vector error correction model (VECM) estimation gives additional confirmation of the model validity and infers the effectiveness of the sterilized interventions made by the Bank of Russia (BoR) in the form of forex repo auctions. The analysis revealed a temporary statistically significant effect of sterilized intervention on the exchange rate which peaked eight working days after the auction day. The other empirical finding is the asymmetric reaction of the exchange rate on positive and negative intervention shocks. The response of the Russian ruble-US dollar exchange rate on a positive sterilized intervention shock (the increase of lending to commercial banks in US dollars) has the correct sign and is statistically significant while its response on a negative shock is statistically insignificant. The impulse response function of Russian ruble-US dollar exchange rate on the sterilized intervention shock shows that there is portfolio channel effectiveness and signalling channel ineffectiveness of the sterilized intervention instrument.

In this paper I elaborate a general equilibrium model of exchange rate determination and verify it using high frequency data, searching for robust relationships between the exchange rate and its fundamental. The main barrier for implementing this research agenda is the famous result of
[Meese and Rogoff, 1983] showing the inability of a fundamental-based model to beat random walk in out-of-sample forecasting. This result has survived many empirical verifications ([Engel et al., 2007], [Rossi, 2013]), and “had a pessimistic effect on the field of empirical exchange rate modelling in particular, and international finance in general” ([Frankel and Rose, 1995]). This paper is at the junction of two approaches to resolving the exchange rate disconnect from its traditional fundamentals problem. The first approach relies on financial factors of exchange rate dynamics grounded on financial market imperfections. [Gabaix and Maggiory, 2015] surveyed modern papers in favour of this new financial channel and believe financial factors may explain the exchange rate disconnect puzzle. In this paper I use the [Gabaix and Maggiory, 2015] model for financial market imperfection but mostly rely upon the second approach, which is based on the idea of using of commodity price dynamics as a fundamental factor. [Rogoff, 1996] assumed that “if one could find a sufficiently volatile real shock one could potentially go a long way toward resolving many empirical puzzles in international finance”. [Chen and Rogoff, 2003] demonstrate that a commodity price shock has volatility characteristics comparable with exchange rates, and prove that for three commodity exporting OECD countries New Zealand, Australia and Canada commodity prices influence the real exchange rate with elasticity estimates between 0.5 and 1. [Rossi, 2013] note that the main idea of using commodity price as fundamentals is that “typically, exchange rates are endogenously determined in equilibrium together with other macroeconomic variables, so it is difficult to predict exchange rate changes based on reduced-form models. However, if it were possible to identify an exogenous shock to exchange rates, that would cleanly predict exchange rate fluctuations.”

The choice of high frequency data for analysis is based on two main arguments. The first argument is the work of [Ferraro et al., 2013] in which the relationship between exchange rates and commodity prices is analysed using data with different frequencies (from daily to quarterly). They analysed the most difficult case of the three OECD countries mentioned above: Canadian-US dollar. They found surprisingly little systematic relationship between oil prices and the exchange rate in the monthly and quarterly data. In contrast such a relationship is rather robust in the daily data. They repeated [Meese and Rogoff, 1983] exercises for oil price dynamics and found that contemporaneous realized oil prices predict the Canadian-U.S. dollar exchange rate better than a random walk. They also repeated the test for Norwegian krone-US dollar exchange rate and oil prices, South African rand-US dollar exchange rate and gold prices, Chilean peso-US dollar exchange rate and copper prices, Australian-US dollar exchange rate and oil prices, and

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3 [Meese and Rogoff, 1983] used actual (not forecasted) fundamentals in making their pseudo out-of-sample forecasts so it undoubtedly indicates problems of explaining exchange rate by macroeconomic fundamentals dynamics.
demonstrated the ability of commodity price dynamics to beat a random walk in Meese-Rogoff experiments. [Ferraro et al., 2013] also repeated and confirmed negative Meese-Rogoff result for the only traditional fundamental factor available at daily frequency, the interest rate differential. The second argument is practical and concerns the data sample which is short for Russia and requires high frequency analysis (daily or weekly).

This paper is also related to the literature on the theoretical and empirical analysis of sterilized intervention effectiveness. [Cespedes et al., 2012], [Escude, 2013], [Castillo, 2014], [Benes et al., 2015], [Ghosh et al., 2015], [Gabaix and Maggiori, 2015], [Shulgin, 2015] demonstrate in a general equilibrium framework that an imperfect capital market makes this instrument effective and a welfare improving tool of monetary policy. [Cespedes et al., 2012] show that sterilized intervention allows relaxing financial collateral constraint, which is biding in a crises time. [Gabaix and Maggiori, 2015] use a model with an imperfect financial market based on capital flows to demonstrate that sterilized intervention may improve welfare in both countries involved. [Benes et al., 2015] model the trade-off between the insulation effect of sterilized intervention and the limited exchange rate adjustability effect. [Escude, 2013], [Castillo, 2014] and [Shulgin, 2015] build dynamic stochastic general equilibrium models with two monetary policy instruments for the Argentinian, Guatemalan and Russian economies respectively and reveal the role of interventions in welfare optimization exercises. [Ghosh et al., 2015] present both theoretical proof of welfare improvement and empirical evidence of the effectiveness of sterilized intervention and conclude that 9 out of 13 empirical papers on emerging market economies from 1999 to 2011 witness the effectiveness of sterilized intervention. There are several other comparative studies on intervention effectiveness. [Menkhoff, 2013] reviews many empirical papers evaluating intervention effectiveness in Latin America, Asia, Eastern Europe and finds that interventions in emerging economies were quite successful in the sense of influencing the level of exchange rate. [Daude et al., 2016] used quarterly panel data on 18 emerging economies in their error correction model to prove the effectiveness of foreign exchange market interventions. [Fratzscher et al., 2015] analyse 33 countries and find that about 80% of all interventions are effective according to different criteria. Intervention ineffectiveness might be revealed for some country for some period but most comparative studies are quite optimistic in evaluation of this instrument.

The empirical part of this paper discusses a stabilizing program of forex repo auctions implemented by the BoR. [Domanski et al., 2016] discuss different related issues of foreign market interventions in EMEs and present their proof of the effectiveness of interventions in Russia after adopting a free floating exchange rate at the end of 2014. The Hungarian case of forex tenders
described by [Balogh et al., 2013] helps us to understand why Russian repo auctions to resolve the problem with banking liquidity might influence the Russian ruble-US dollar exchange rate.

The rest of the paper is organized as follows. Section 2 presents the model of the exchange rate determination based on capital flows a la Gabaix-Maggiori. In Section 3 I perform an estimation of the model and compare the results with VECM estimated on the same data. Section 4 concludes.

2. The Model

The model is inspired by the modern reincarnation of old-fashioned hand-book models performed by [Gabaix and Maggiori, 2015]. They keep the model quite simple and at the same time propose a full-fledged micro-founded general equilibrium model which includes capital market imperfection. To make a reasonable parametrization of the model on high frequency data I introduce a commodity good and hence restrict the application of the model to commodity exporting countries with a floating exchange rate. Adhering to [Gabaix and Maggiori, 2015] overall, I change many particular elements of the model.

Let us assume that our small open economy is inhabited by identical households with a utility function:

$$U_t = \sum_{j=0}^{\infty} \beta^j \ln C_{t+j},$$

where $\beta$ is the intertemporal discount factor; $C_t$ is the aggregate consumption determined by the CES function:

$$C_t = \left(\left(C_{N,t}\right)^{\delta-1} + \left(C_{F,t}\right)^{\delta-1}\right)^{\frac{\delta}{\delta-1}},$$

where $\delta > 0$ is the elasticity of the substitution of home non-tradable goods and foreign tradable goods; $C_{N,t}$ and $C_{F,t}$ are the consumption of non-tradable and imported tradable goods, respectively.

As in [Gabaix and Maggiori, 2015] I fix the price of non-tradable goods at $P_{N,t+j} = 1$ for $\forall j$ and use them as the numéraire in the economy. Let us also assume that the level of non-tradables production is constant for all periods: $Y_{N,t+j} = 1$ for $\forall j$. The equilibrium condition for the non-tradable goods market is trivial: $Y_{N,t} = C_{N,t} = 1$.

FOC for the intra-temporal consumption optimization problem gives:

\[^4\text{To achieve nontradables production stability monetary authorities should use appropriate interest rate rule (see equation (6))}\]
where \( P_{F,t} \) is the home currency price of imported tradable goods.

The law of one price in tradable goods market gives:

\[
P_{F,t} = S_t P_{F,t}',
\]

where \( S_t \) is the exchange rate determined as the price of foreign currency in terms of home currency; \( P_{F,t}' = 1 \) is the normalized at unity foreign currency price of an imported good.

Expressed in units of foreign currency, the nominal import is:

\[
P_{F,t}^{*} C_{F,t} = S_t^{*} \left( \frac{P_{F,t}}{P_{N,t}} \right)^{1-\delta} P_{N,t} C_{N,t} = S_t^{\delta}
\]

Households can use domestic bonds with nominal return \( R_t \) to perform intertemporal consumption optimization.

FOC for the intertemporal utility optimization problem requires:

\[
E_t \left( \beta R_t \frac{P_{N,t}}{P_{N,t+1}} \frac{\partial U_t}{\partial C_{N,t+1}} / \frac{\partial U_t}{\partial C_{N,t}} \right) = 1
\]

Using the fact that \( C_{N,t} = \left( \frac{P_{N,t}}{P_t} \right)^{\delta} C_t \) and the definition of the consumer price level \( P_t^{1-\delta} = P_{N,t}^{1-\delta} + P_{F,t}^{1-\delta} \) we can rearrange equation (5) to get a Euler equation for optimal consumption:

\[
E_t \left( \beta R_t \frac{1 + S_t^{1-\delta}}{1 + S_{t+1}^{1-\delta}} \right) = 1
\]

Higher interest rate in the financial market is associated with more consumption in the future, which for the case of \( \delta > 1 \) can be achieved by cheaper imports and hence a more appreciated home currency in the future.

Euler equation (6) is equivalent to a monetary policy rule in which monetary authorities set interest rates to stabilize nominal income in the non-tradable sector \( P_{N,t} Y_{N,t} \). Expected home currency depreciation creates an expenditure switching effect: demand switches from imported to non-tradables in the future. Monetary authorities should cut interest rates to compensate for this effect and stabilize the home production of non-tradables. Such a monetary policy rule fully insulates the national economy from external shocks.
In traditional monetary models with rational expectations, interest rate dynamics absorb information about exchange rate fundamentals\(^5\). In the model I simplify the real part of the economy and as a result simplify the monetary policy rule and abstract from most traditional exchange rate fundamentals.\(^6\) The effectiveness of the conventional monetary policy instrument is beyond doubt but it is usually not used for forex market regulation hence shocks in the nominal interest rate are not introduced into the model directly and their effect on the exchange rate is included only in the balance of payments shock. I further concentrate on an alternative monetary instrument in the form of a sterilized intervention in the foreign exchange market.

Let us assume that every period the home economy exports units of a commodity at price \(P_{Oil,t}^*\), which follow a random walk process:

\[
P_{Oil,t}^* = P_{Oil,t-1}^* e^\left(\varepsilon_{Oil,t}\right),
\]

(7)

where \(\varepsilon_{Oil,t}\) is the commodity price shock with zero expectation and non-zero variance.

The trade balance in the economy:

\[
TB_t = P_{Oil,t}^* - S_t^{-\delta}
\]

(8)

Other participants of the foreign exchange market are monetary authorities performing sterilized interventions, and financiers clearing the foreign currency market.

Following [Gabaix and Maggiori, 2015] I assume that the financier absorbs medium-term imbalances which may appear in the forex market after fluctuations in trade balance, capital flows, and interventions. The financier bears the risks of open uncovered positions in foreign currency and requires a premium for risk. To substantiate such a risk premium let us assume that financier invests \(F_t\) units of foreign currency in foreign bonds with nominal return \(R^*_t\) and borrows funds in the home currency \(F_tS_t\) at nominal return \(R_t\). The value of her firm in foreign currency is:

\[
V_t = E_t \left[ R^*_t \left( \frac{S_t}{S_{t+1}} - R_t \right) F_t \right]
\]

(9)

For simplicity I assume that the financier consumes all of her profit out of small open home economy and hence use foreign discount factor \(\beta^*\).

[Gabaix and Maggiori, 2015] assumed that the financier’s ability to get funding is limited by possible losses which lenders could bear if financier diverts funds. Losses are expected to be

\(^5\) [Mark, 2005], [Engel and West, 2006] and [Clarida and Waldman, 2008] clarified the role of the monetary policy rule concept in explaining exchange rate dynamics.

\(^6\) As it was mentioned above the model is oriented on high frequency data while daily information on macroeconomic fundamentals is unavailable. Some empirical studies (see for example [Andersen et al., 2003], [Faust et al., 2007], [Chaboud et al., 2008], [Fratzscher, 2009]) discuss the role of new information in exchange rate dynamics. Main finding of the literature on news is the existence of very short living effect of news on exchange rate.
quadratic in her liabilities $S_i F_i$ because the more funds the financier manages, the greater the complexity of her firm, and the larger the share of divertible funds. If a diverted portion is linear in the absolute value of financier’s liabilities: $\Gamma |S_i F_i|$ then the total divertible funds in home currency are quadratic: $[S_i F_i] \Gamma |S_i F_i| = \Gamma (S_i F_i)^2$, where $\Gamma > 0$.\footnote{To get additional information about decomposition of $\Gamma$ see [Gabaix and Maggiori, 2015]}

The financier solves the problem of constrained optimization:

$$ \max_{F_i} V_i = E_i \left[ \beta^\prime (R_i^\prime - \frac{S_i}{S_{i+1}} - R_i) F_i \right] \quad \text{subject to} \quad V_i \geq \Gamma S_i F_i^2 $$ \hspace{1cm} (10)

Making a simplifying assumption about foreign interest rate dynamics: $\beta^\prime R_i^\prime = 1$, the solution to that problem is:

$$ F_i = \frac{1}{\Gamma} E_i \left( \frac{1}{S_i} - \frac{1}{S_{i+1}} R_i \right). $$ \hspace{1cm} (11)

The higher the expected devaluation of the home currency, the higher the financier’s demand for foreign assets. Other factors for that demand are the foreign and home interest rate differential and the term called in [Gabaix and Maggiori, 2015] ‘risk bearing capacity’, which is the reciprocal of $\Gamma$.

Equation (11) links the interest rate differential with the demand for foreign assets and allows the influence of interest rates and the devaluation expectation on the balance of payments to be found. An alternative technique of introducing such an influence in the model is the portfolio balance approach ([Kouri, 1976], [Dornbusch and Fischer, 1980]). More recent related research on the risk premium for foreign assets/debts acquired in small open economy models are discussed in [Schmitt-Grohé and Uribe, 2003], [Lubik, 2007], [Benigno, 2009].

Now I can formulate the balance of payments equation which determines the exchange rate:

$$ F_i = P_{oil,t}^* - S_i^{\delta} + Z_i + X_i + F_{i-1} R_{i-1}^*, $$ \hspace{1cm} (12)

where $Z_i$ is the impact created by sterilized intervention in period $t$; $X_i$ is AR(1) process driven by the balance of payments shocks, which will be defined later.

### 2.1 The model without unit root series

Let us get rid of the unit root series of equations and linearize some of them. Oil price is an integrated series and makes some other series integrated. Let us divide both parts of the balance of payments equation (12) by $P_{oil,t}^*$.
\[ f_t = 1 - (1 + \tilde{S}_t)^{-\delta} + z_t + x_t + \frac{1}{\beta^*} f_{t-1} \exp(\epsilon_{Oilt}) , \]  

(13)

where \( f_t = \frac{F_t}{P_{Oilt}} \); \( z_t = \frac{Z_t}{P_{Oilt}} \); \( x_t = \frac{X_t}{P_{Oilt}} \); \( \tilde{S}_t = \frac{S_t - \bar{S}}{\bar{S}} \); \( \bar{S}_t = (P_{Oilt}^*)^{\frac{1}{\beta^*}} \); \( R_{t-1}^* = \frac{1}{\beta^*} \).

The exchange rate has no deterministic steady state value, so it is worth introducing a deviation from following random walk process the steady state value \( \bar{S}_t \).

Here we can see how the model explains the long run elasticity of the exchange rate with respect to oil prices. This elasticity equals \( -\frac{1}{\delta} \) and depends on the elasticity of the substitution of home non-tradable and foreign tradable goods \( \delta \). The bigger \( \delta \), the less the exchange rate change needs to restore the current account balance in the case of a change in oil prices\(^8\).

Let us also divide both parts of optimal financier demand for foreign assets (11) by \( P_{Oilt}^* \) and take the Taylor expansion around steady state of its right hand side:

\[ f_t = \frac{1}{\Gamma P_{Oilt}^*} E_t \left( \frac{1}{\bar{S}_t} \tilde{S}_t + \frac{1}{\bar{S}_{t+1}} \tilde{S}_{t+1} + \tilde{S}_t + \tilde{R}_t^* - \tilde{R}_t \right) , \]

(14)

where \( \tilde{R}_t = \frac{R_t - \bar{R}}{\bar{R}} \) is the deviation of the nominal return from its steady state \( \bar{R} = \frac{1}{\beta^*} \). We assumed \( \tilde{R}_t^* = \frac{1}{\beta^*} \) and therefore \( \tilde{R}_t^* = 0 \).

Rearranging (14) gives:

\[ f_t = \gamma (E_t \tilde{S}_{t+1} - \tilde{S}_t) , \]

(15)

where \( \gamma = \frac{1}{\Gamma P_{Oilt}^*} = \frac{1}{\Gamma (P_{Oilt}^*)^{\frac{1}{\beta^*}}} \); assuming \( \gamma \) is constant means that risk bearing capacity is expected to be an increasing function of \( P_{Oilt}^* \). I also ignore the fact that \( E_t (\exp(\epsilon_{Oilt+1})) \neq 1 \). These two assumptions simplify the model without missing significant economic effects.

Let us take the Taylor expansion of the right hand side of the Euler equation (6):

\[ E_t(\tilde{R}_t) - (1 - \delta) \frac{S_{t+1}^{-\delta}}{1 + \tilde{S}_{t+1}^{-\delta}} (\tilde{S}_{t+1} - \tilde{S}_t) = 0 \]  

(16)

\(^8\) [Chen and Rogoff, 2003] used simple flexible prices Balasa-Samuelson model to give alternative explanation of exchange rate elasticity w.r.t. commodity price. In their model with logarithmic utility function it equals one minus import share in consumption (0.75 in their example). For the case of more labor intensive production of non-tradable w.r.t. production of exported goods this elasticity should be more than 0.75. In a simplest sticky prices model exchange rate should be more than proportionate to exchange rate (elasticity more than 1) to accommodate changes in relative prices.
Ignoring the fact that steady state value of import share in consumption \( \frac{S_{t+1}^{1-\delta}}{1+S_{t+1}^{1-\delta}} \) follows random walk I denote \( k_{lm} = \frac{S_{t+1}^{1-\delta}}{1+S_{t+1}^{1-\delta}} \) and assume it is constant. Rearranging (16) gives a linearized version of the Euler equation (monetary policy rule):

\[
\tilde{R}_t = (1-\delta)k_{lm} (E_t\tilde{S}_{t+1} - \tilde{S}_t)
\]

(17)

Substituting (17) into (15) gets rid of the nominal return \( R_t \) of the equations:

\[
f_t = \lambda (E_t\tilde{S}_{t+1} - \tilde{S}_t),
\]

where \( \lambda \equiv \gamma (1+(\delta-1)k_{lm}) \).

(18)

Linearizing (13) gives:

\[
f_t = \delta \tilde{S}_t + z_t + x_t + \frac{1}{\beta^t} f_{t-1}
\]

(19)

It is time to define the dynamics of latent \( x_t \):

\[
x_t = \rho_x x_{t-1} + \varepsilon_{x,t}, \quad \rho_x \in (0, 1)
\]

(20)

where \( \varepsilon_{x,t} \) is the balance of payments shock with a zero mean and non-zero variance.

2.2 Transmission mechanism for the sterilized intervention setup

To introduce sterilized interventions into the model we need enough information about the country specific institutional setup for performing foreign exchange market interventions. Getting detailed information on bank balance sheets is usually impossible hence it is reasonable to use ad hoc equations which describe foreign currency supply/demand in the forex market. I propose a setup which is appropriate for the outright buying/selling of international reserves and for operations with forex derivatives (repos, swaps etc.) to regulate banking liquidity. The main idea of the setup is that commercial banks play the role of intermediaries in the supply/demand of foreign currency. This setup takes into account the ability of an asymmetric reaction of commercial banks of the same size to increase and decrease sterilized intervention instruments. The simple setup consists of two equations:

\[
z_t = \xi_t, \quad \xi \in (0, 1)
\]

(21)

\[
l_t = (1-\xi)l_{t-1} + \psi \varepsilon_{f_t-N} + \chi \varepsilon_{f_t-M}, \quad \psi, \chi \in (0, 1)
\]

(22)

9 In this composite shock both shock in capital flows and trade balance shock are mixed up because I have no ability to identify them on daily data.
where \( l_t \) is the disposable foreign currency volume which commercial banks can supply to or demand from the forex market in the period \( t \); \( z_t \) is banks’ actual foreign currency supply to or demand from the forex market created by sterilized interventions; \( \varepsilon_{I^+,t} \geq 0 \) and \( \varepsilon_{I^-,t} \leq 0 \) are the positive and negative parts of sterilized intervention shock \( \varepsilon_{I,t} = \varepsilon_{I^+,t} + \varepsilon_{I^-,t} \) respectively, which are equal to the volume of foreign currency involved in sterilized intervention in period \( t \)\(^{10}\).

Equation (21) says that only part \( \xi \) of whole disposable volume \( l_t \) is actually supplied in period \( t \). This may happen if banks use the foreign currency they got after the sterilized intervention for their own purposes (for example for covering short or medium term shortages of foreign currency). Equation (22) defines the dynamics of \( l_t \) assuming \( N \) days delay in supplying foreign currency after the positive intervention day and \( M \) days delay in demanding foreign currency after the negative intervention day. This might reflect institutional features of interventions (for example a delay in the actual currency delivery after the auction date) or delays created by the average duration of payments banks perform. As assumed, both delays (\( N \) and \( M \)) and the reaction coefficients (\( \psi \) and \( \chi \)) could be different for positive and negative interventions. For simplicity I ignore all interest payments in equations (21) and (22).

The outright buying/selling of international reserves can be described by the next parameter set: \( \xi = 1, \chi = 1, \psi = 1, M = 0 \) and \( N = 0 \). Other principal types of interventions are repo auctions to regulate banking liquidity with the parameters: \( \xi \in (0, 1), \chi \in (0, 1), \psi \in (0, 1), M > 0, N > 0 \).

Determined by equations (21) and (22) the setup demonstrates that interventions conducted in the period \( t \) influence foreign currency supply in periods \( t + N, t + N + 1 \) and so on. The exchange rate in contrast will react to interventions in the same period \( t \) because rational agents get information about future foreign currency supply changes in the period of the intervention \( t \). This means that portfolio and information channels are expected to be active in the model and may transmit a sterilized intervention shock to the economy.

Resolving the linearized model can be found in Appendix A.

### 3. Estimation of the model

The model is estimated using Russian daily statistics. I use maximum likelihood (ML) estimation because of the lack of reliable prior information about coefficients. The likelihood

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\(^{10}\) Both \( l_t \), \( \varepsilon_{I^+,t} \), and \( \varepsilon_{I^-,t} \) are in capital letters because they enter equation for \( f_t = \frac{F_t}{P_{Oil,t}} \) and hence expressed in terms of the exporting commodity goods.
function is calculated by standard Kalman filter. Modes of the distribution are found by the Ratto algorithm. Confidence intervals for impulse-response functions (IRF) are calculated by the Markov Chain Monte-Carlo method with Metropolis-Hastings (MCMCMH) algorithm. I use Dynare software [Adjemian et al., 2011] to make all the numerical calculations.

3.1 Foreign currency repo auctions in Russia

At the end of 2014 the BoR rejected the exchange rate adjustment rule introduced in practice in February 2009 after the World financial crises\textsuperscript{11}. The initial plan was turning to floating regime for the Russian ruble in 2015 but severe negative shocks at the end of 2014 provoked speculative activity based on the predictability of the rule and compelled the BoR to hasten the transition. Problems of adaptation to the new forex market conditions coincided with capital outflow caused by the sanctions and the fall in oil prices which together were called a ‘perfect storm’. The BoR decided to maintain the currency market by supplying US dollars through repo auctions in foreign currency to support banking liquidity and prevent ruble devaluation. Interestingly the BoR did not call such support ‘intervention’ so as not to mix it up with the outright selling/buying of foreign currency from international reserves it performed before 2015. But such repos in foreign currency were indeed sterilized interventions in the foreign exchange market. At the maximum in April 2015 the liabilities of Russian banks to the BoR on repos in foreign currency were about 35 billion US dollars (about 8 per cent of international reserves stock).

[Domanski et al., 2016] claim that “authorities have increasingly used forex derivatives or related instruments. This has allowed them to provide hedges against forex risk and influence forex market liquidity and the exchange rate while economizing on the use of international reserves and retaining foreign reserve buffers”. They also demonstrate that for four EMEs (Russia, Brazil, Peru and Turkey) interventions to provide liquidity for the banking sector in difficult times effectively influenced [Karnaukh et al., 2015] market illiquidity measure. To understand why foreign currency repo auctions may at the same time influence exchange rates we compare Russian foreign currency repos with Hungarian foreign currency tenders described by [Balogh et al., 2013]. They claim that “Magyar Nemzeti Bank (MNB) introduced this instrument of banking liquidity support after the September 2011 when the Hungarian Parliament ratified legislation that allowed households to repay their foreign currency denominated mortgages at a preferential predetermined exchange rate. This legal act created an open foreign currency position of significant but uncertain size on the balance sheet of Hungarian banking system. […] MNB decided to commit to delivering foreign currency necessary for the program in an organized structured manner to minimize the motivation

\textsuperscript{11}Russian exchange rate rule before and after the World financial crises is estimated in [Shulgin, 2017] on the base of DSGE methodology.
for speculative sales of forex”. Having the ability to control the amount of Euro banks demanded as a result of program participation MNB prevented speculative pressure on forex. We might describe this case in terms of the sterilized intervention setup above as $\chi = 0$, $\psi = 0$, because the whole volume distributed by MNB through foreign currency tenders was not supplied in the forex market but used by Hungarian banks for the mortgage repayments of their clients.\(^{12}\)

The Russian case of foreign currency repo auctions is also a structured way to resolve the problem of currency outflow in the period of sanctions of unknown size and at an unknown time. The first difference between the Russian case and the Hungarian one is that the Russian banks’ problem with foreign currency financing is expected to be temporary while the Hungarian mortgage repayments is an example of permanent capital outflow. That is why the BoR used derivative forex facility (repos) while MNB was forced to lose international reserves. The second difference is that MNB had information about the volume of foreign currency Hungarian banks demanded, while the BoR had no ability to get such information. That is why Russian banks might borrow excessive foreign currency through the repo auctions in a highly volatile market. It looks like speculative behaviour, but it had no serious negative consequences for the Russian monetary system because such behaviour reduced depreciation pressure on the Russian ruble while the temporary nature of repos avoided the consequences of international reserve losses. In terms of the sterilized intervention setup it means that $\psi \in (0, 1)$. Another feature of the Russian foreign currency repo auctions was that banks did not have full confidence in their ability to borrow foreign currency at a specific repo auction. It created a stimulus for banks to borrow foreign currency through repo auctions in advance that, in terms of the sterilized interventions setup, means that $N > 0$. Taking into account two days delay in foreign currency delivery after the auction date we may refine this claim to $N > 2$.

The data sample used also covers the period of turning Russian repo auctions program off hence we should think about the transmission mechanism for negative interventions $e_{t-2} \leq 0$. We similarly think that banks may buy in the forex market some part of the volume needed to pay their debts and in terms of the sterilized intervention setup it means that $\chi \in (0, 1)$. The period of negative interventions happened in calmer economic times and banks might reserve the volume needed to pay their debts. In the sterilized intervention setup it could mean that $M \leq 2$.

\(^{12}\) Balogh et al., 2013] notice that Hungarian tenders covered about 60 per cent of EUR 4353 millions of actual repayment. The remaining 40 per cent was purchased by banks from other sources. Related forint sales might contribute to 12 per cent depreciation of forint in the end of 2011.
3.2 Data

The time series used in the estimation includes 618 daily observations from 6 November 2014 to 20 April 2017 and are shown in the Fig. 1. The oil price is the price of the OPEC basket in US dollars per barrel. The forex rate is the average spot rate (with delivery TODAY) on the Moscow Interbank Currency Exchange (MICEX) measured in Russian rubles per US dollar. The calculation of the sterilized intervention shock is based on the results of repo auctions which were transformed to correspond to introduced in the model shock $\varepsilon_{I,t}$:

$$
\varepsilon_{I,t} = \frac{V_{repo,t}}{\bar{Y}_{Oil,t} \bar{P}_{Oil,t}} \frac{\bar{P}_{Oil,t}}{P_{Oil,t}},
$$

(23)

where $V_{repo,t}$ is the volume of US dollars distributed through the repo auction on day $t$ with currency delivery at $t+2$; $\bar{Y}_{Oil,t} \bar{P}_{Oil,t}$ is the average daily export of oil, gas and oil products taken from Russian balance of payments of 2015-2016; $\bar{P}_{Oil,t}$ is the average oil price (OPEC basket) in period of 2015-2016.
Fig. 1. Daily statistics on Russian ruble-US dollar exchange rate $S_t$, oil price $P_{Oil,t}$ and positive $\varepsilon_{I^+,t}$ and negative $\varepsilon_{I^-,t}$ sterilized interventions used for the model estimation. Source: Bank of Russia

Fig. 1 demonstrates the cointegration of the $S_t$ and $P_{Oil,t}$ series confirmed by all the available statistical tests at 1% significance level. It also shows that repo auctions were significant in volume and were conducted mainly before May 2015. Negative sterilized interventions (turning off the repo auctions program) dominated from May 2015 and were more time-smoothed than the positive ones. The BoR lent about 35 billion USD to commercial banks at the peak in April 2015, which was about 8% of its international reserves.
3.3 Results of the model estimation

First an appropriate delay $N$ in the monetary transmission equation (22) was chosen. The model was estimated with different $N$ and it was found that $N = 8$ corresponds to the estimate with largest likelihood function.

![Graph showing maximum likelihood $L$ in different versions of the model estimation with different delay $N$](image)

Fig. 2. Maximum likelihood $L$ in different versions of the model estimation with different delay $N$.

The results of the ML estimation with most stable delays $N = 5, 9$ are represented in Tab. A1 of the Appendix B.

The foreign discount factor was fixed in all estimations at $\beta^* = 0.9998$ which corresponds to about a 5% annual interest rate. As assumed in the sterilized intervention setup not all the volume distributed through repo auctions is supplied in the forex market but only the share $\psi$. Similarly not all the volume needed to pay off repo auctions debts is demanded from the forex market, but only the share $\chi$. It is assumed that the shares $\psi$ and $\chi$ are not necessarily the same. The estimations demonstrate that this assumption is justified by the data. The coefficient $\chi$ is estimated at close to zero with an extremely low level to standard deviation ratio. This is the reason why in all tests in which I estimated the effect of a negative sterilized intervention shock on the endogenous variables is statistically insignificant. The coefficient $\psi$ is estimated at about 0.1 with a moderate standard deviation in most versions of the model. The estimates of the coefficient $\xi$ differ significantly for models with different $N$ and this is normal for such a procedure because improper model specification may lead to a biased estimate. The coefficient $\xi$ is due to modelling the delay at the moment when banks can supply foreign currency distributed through the repo auctions and the moment when banks actually supply foreign currency to the forex market. This delay is summed up with $N$ days delay in equation (22) and if, for instance, $N$ in the model is smaller than the actual delay in the data, the estimate of $\xi$ will also be smaller. The last effect arises to postpone the reaction of exchange rates to the repo auction and hence to partially compensate for bias in $N$. A similar situation occurs with the estimates of the coefficient $\lambda$ which determine one of the two

\[ 13 \text{ In all model estimations presented here } M = 2 \]
stable roots of foreign exchange rate dynamics (see Appendix A). The estimate of $\lambda$ is closely related to the estimate of another stable root $(1 - \xi)$. If one of the roots estimates is biased ($\xi$ in our case) the second one will be biased too.

The results in Tab. A1 demonstrate the high sensitivity of parameter estimates to the lag specification in the transmission mechanism (22) and confirm the choice of $N$. I perform the estimation of VECM based on the same dataset. The results of the VECM estimation are represented in the Tab. A3 and A4.

An additional check of the model is a comparison of the impulse-response functions (IRFs) of the model with the IRFs of the VECM. Sterilized interventions are assumed to be endogenous in the VECM and to make a comparison possible I retrieve the response of the positive and negative sterilized interventions to the structural shock in the VECM and perform a model simulation with the retrieved series. I concentrate on the structural shock of the positive sterilized intervention because the response of the exchange rate to that shock is statistically significant (Fig. A3) unlike the response of the exchange rate to the negative sterilized intervention shock (Fig. A4). I call the simulated series of exchange rate as simulated IRF.

Fig. 3 demonstrates simulated IRF for different $N$ in comparison with the IRF in the VECM. We can see that simulated IRF for the model with $N = 8$ is close to the IRF of the VECM.

Fig. 3. IRFs of exchange rate $S_t$ on positive sterilized intervention shock $\varepsilon_{I+,t}$ in VECM (with 95% confidence interval) and in the model with $N = 6, 8, 9$. 
We also see that simulated IRFs for the model with different $N$ are also close to each other and to compare them I calculate the sum of the squared differences between the simulated IRF $S_{tIRF_N}^{-}$ and the IRF of the exchange rate on the positive sterilized intervention shock in the VECM $S_{tIRF_{VECM}}^{-}$ for first 50 periods. It is possible measure $M$ of models quality valuation:

$$M = \sum_{j=0}^{50} (S_{tIRF_{VECM}^{-}} - S_{tIRF_N^{-}})^2$$ (24)

The results of that calculation for the model with different $N \in \{4, 10\}$ are shown in Fig. 4 which confirms the choice of eight working days delay in equation (22) $N = 8$ made earlier on the base of likelihood function (Fig. 2).

Fig. 4. The measure $M$ characterizes similarity of IRF of exchange rate on positive sterilized intervention shock of the VECM and the model with different $N$.

Fig. 3 and A3 demonstrate the statistically significant response of the exchange rate to a positive sterilized intervention shock within the period 7-10 working days after the repo auction date. The response has the correct sign because the more foreign currency distributed through repo auctions, the more the exchange rate of home currency appreciates. This is statistical proof of the effectiveness of sterilized intervention made by VAR methodology on Russian daily data.

Usually intervention effectiveness is measured on the basis of the exchange rate reaction on the day of the intervention or 1-2 days after it but [Disyatat and Galati, 2007] found that interventions in the Czech koruna market in 2001-2002 had no contemporaneous effect on the exchange rate level but cumulated interventions had a statistically significant although economically limited shorter-term effect. In the case of the Russian ruble in 2014-2017 we can see a statistically insignificant exchange rate reaction in first six days after the repo auction. This demonstrates the ineffectiveness of the signalling channel in a sterilized intervention transmission mechanism. This inference is in line with the fact that the BoR never mentioned foreign currency repo auctions as its foreign currency intervention tool.

The IRF analysis made in Appendix C reveals other important effects in the dataset. Fig. A4 shows that the exchange rate reaction to a negative sterilized intervention shock is weak and statistically insignificant. It means that turning the repo auctions program off had no statistically
significant effect on Russian ruble-US dollar exchange rate. This inference is close to the result of [Guimarães and Karacadag, 2004] who report an asymmetric reaction of Mexican peso and Turkish lira to US dollar sales and purchases. In both countries in a period of difficult circumstances international reserves purchases were inefficient while stabilizing international reserves sales had statistically significant impact on the exchange rate.

Fig. A5 demonstrates the reaction of policy instruments on the exchange rate shock. An unexpected home currency depreciation induces a positive statistically significant response in the cumulated positive sterilized interventions which may witness a “leaning against wind” behaviour of the BoR. Fig. A3 and A5 are typical for monetary policy instrument vs. target relationships. A home currency depreciation shock provokes authorities to increase the foreign currency volume distributed through repo auction which has a reverse effect on the exchange rate. As a result of that leaning against wind policy exchange rate dynamics become more stable and smoothed.

Fig. A6 demonstrates the statistically significant response of Russian ruble-US dollar exchange rate on the oil price shock. Oil price shock has a permanent effect on both the exchange rate and the oil price because these series are cointegrated. The instantaneous effect is also statistically significant and a little bit smaller than the long run effect therefore the shock in fundamentals does not create overshooting.

3.4 Bayesian IRFs in the model

After equation (22) was appropriately estimated and justified by VECM analysis I can check the effectiveness of sterilized intervention on the basis of the model itself. To do that I use the MCMCMH algorithm which give a Gaussian approximation of the likelihood function around its maximum i.e. around the parameter modes [An and Shorfheide, 2007]. This approximation can be used in the Monte-Carlo method of means and confidence intervals of IRFs calculation implemented in Dynare.

The results of the MCMCMH algorithm realization with 2 chains of 100000 iterations each are represented in the Tab. A2. The IRF of the exchange rate on a positive sterilized intervention shock and its 90% confidence intervals are calculated on the basis of the MCMCMH algorithm parameter distribution and further called Bayesian IRFs\textsuperscript{14}. They are shown in Fig. 5 in comparison with the IRF in the VECM with 90% confidence interval bootstrapped by the Hall algorithm. The IRF in the VECM is more persistent than in the model because of the estimated autoregression of the positive intervention shock (see the right part of Fig. A3). We may conclude that the exchange rate response in the model is statistically significant at the 10% significance level for all periods and

\textsuperscript{14} The term ‘Bayesian IRF’ refers the Bayesian approach to the model interpretation.
hence may infer a signalling channel of sterilized intervention effectiveness. This inference contrasts with the results from the VECM analysis. One possible explanation for this is that the model simplifies the transformation of information about repo auctions into actual forex agent behaviour. In reality banks do not have complete information about how repo auctions influence the exchange rate. The resulting Russian ruble-US dollar exchange rate dynamics revealed only after actual foreign currency supply increase happens.

Both the VECM and the structural theoretical model estimations witness the effectiveness of the portfolio channel of sterilized intervention effectiveness. An analysis of 90% confidence intervals demonstrates that repo auctions have a statistically significant effect on the exchange rate 7–15 working days after the auction date.

![Graph](image)

Fig. 4. IRF of Russian ruble-US dollar exchange rate $S_t$ on positive sterilized intervention shock $\varepsilon_{t^+,I}$ (with corresponding 90% confidence intervals) in VECM and Bayesian IRF in the model

I do not use the Bayesian technique to demonstrate the ineffectiveness of a negative sterilized intervention shock because the MCMCMH algorithm does not give an appropriate approximation of the parameter distribution with an extremely low mean to standard deviation ratio. The results of the ML and VECM estimations (Tab. A1, A3, A4, Fig. A4) evidence the ineffectiveness of a negative sterilized intervention shock in the form of repo auctions. This result is coherent with many other studies emphasizing the importance of the historical context of intervention effectiveness analysis. The BoR introduced foreign currency repo auctions in 2014 to withstand in the period of significant negative balance of payments shocks. The forward-looking...
behaviour of Russian banks in response to the foreign currency liquidity support program provided a stabilizing effect on the Russian ruble-US dollar exchange rate. This side effect did not appear in a period of turning program off.

3. Conclusion

This paper contributes to open economy macroeconomics in two main ways. The first is the construction of a general equilibrium model based on a Gabaix-Maggiory imperfect capital market setup appropriate for modelling high frequency dynamics in a small open commodity exporting economy. The model is appropriate for estimation using daily statistics and includes two ingredients which help weakening the exchange rate disconnect problem: an imperfect capital market and commodity price dynamics which is the only exchange rate fundamental in the model. I provide an additional ad hoc setup for analysing sterilized interventions of a different nature.

The second contribution of the paper is the investigation of sterilized intervention effectiveness using Russian daily statistics of repo auctions the BoR conducted during the period of extreme negative balance of payments shocks 2014-2015. It demonstrates the ability to use a theoretical setup for the evaluation of sterilized interventions programs in different countries. The theoretical model is estimated by the ML method supplemented by the MCMCMH algorithm. It allows statistical inferences about the impulse response functions significance to be made. I also estimate the VECM on the same dataset to confirm the results, and the combination of theoretical and empirical approaches allows me to address the question of different channels of the of sterilized intervention mechanism effectiveness. The main finding concerns proof of portfolio channel effectiveness and signalling channel ineffectiveness in the sterilized intervention transmission mechanism of the instrument. I also found an asymmetric reaction of the Russian ruble-US dollar exchange rate to the positive and negative sterilized intervention shocks. The response of the exchange rate level on a positive shock (an increase of lending to commercial banks in US dollars) has the correct sign and is statistically significant while its response to a negative shock is statistically insignificant.

Finally I conclude that this theoretical setup is appropriate for estimations using high frequency data. It creates reasonable restrictions on a series and complements empirical approach to sterilized interventions effectiveness evaluation.
4. Literature


Guimarães, RF & Karacadag, C 2004, „The empirics of foreign exchange intervention in emerging market countries: The cases of Mexico and Turkey”, Working Paper no. 04/123.


Appendix A. Linearized model resolving

Let us bring all linearized equations of the model together and complete the system by equations which allow relating model with observable series:
\[ f_t = \lambda (E_t \tilde{S}_{t+1} - \tilde{S}_t), \quad \text{(A1)} \]
\[ f_t = \delta \tilde{S}_t + z_t + x_t + \frac{1}{\beta^*} f_{t-1} \quad \text{(A2)} \]
\[ x_t = \rho_x x_{t-1} + \varepsilon_{x,t}, \quad \rho_x \in (0,1) \quad \text{(A3)} \]
\[ z_t = \xi_t \quad \xi \in (0,1) \quad \text{(A4)} \]
\[ l_t = (1-\xi)l_{t-1} + \psi \varepsilon_{1-t-N} + \chi \varepsilon_{1-t-2}, \quad \psi, \chi \in (0,1) \quad \text{(A5)} \]
\[ \Delta p_{Oil,t} = \varepsilon_{Oil,t} \quad \text{(A6)} \]
\[ \Delta s_t = -\frac{1}{\delta} \Delta p_{Oil,t} + \tilde{S}_t - \tilde{S}_{t-1} \quad \text{(A7)} \]
\[ S_t = S_{t-1} \exp(\Delta s_t), \quad \text{(A8)} \]

where \( \Delta s_t \) and \( \Delta p_{Oil,t} \) are logarithmic differences in exchange rate and oil price, respectively; equation (A6) follows from the random walk process for oil prices (7); equation (A7) follows from the definitions of \( \tilde{S}_t \) and \( \tilde{S}_t \) made above. Equations (A6) and (A7) allow introducing observable variables in the models while the equation (A8) lets accounting cumulated changes in foreign exchange rate. Variable \( \tilde{S}_t \) is forward-looking; \( f_t, x_t, l_t \) are state variables; \( z_t \) is static variable; \( \varepsilon_{x,t}, \varepsilon_{Oil,t}, \varepsilon_{1-t-N}, \varepsilon_{1-t-2} \) are exogenous shocks.

Substituting \( \tilde{S}_t \) from (A2) into the (A1) formulates forward looking expression for \( f_t \):
\[ f_t = \mu \left[ E_t f_{t+1} + \frac{1}{\beta^*} f_{t-1} + (1-\rho_x) x_t + \xi^2 l_t - \xi (\psi \varepsilon_{1-t-N} + \chi \varepsilon_{1-t-2}) \right], \quad \text{(A9)} \]
where \[ \mu = \frac{1}{2 + \frac{\delta}{\lambda} + \frac{1-\beta^*}{\beta^*}} < \frac{1}{2}. \]

Characteristic polynomial for (A9) is:
\[ \mu y^2 - y + \frac{\mu}{\beta^*} = 0, \quad \text{(A10)} \]
with polynomial roots:
\[ y_{1,2} = 1 + 0.5 \left( \frac{\delta}{\lambda} + \frac{1-\beta^*}{\beta^*} \right) \pm \sqrt{0.25 \left( \frac{\delta}{\lambda} + \frac{1-\beta^*}{\beta^*} \right)^2 + \frac{\delta}{\lambda}} \quad \text{(A11)} \]

The roots in (A11) satisfy Blanchard-Kahn condition because the root \( y_1 \in (0,1) \) corresponds to the stable solution for \( f_t \) while the root \( y_2 > 1 \) means that the solution for \( \tilde{S}_t \) is unique.
Solution could be found by the undefined coefficients method:

\[ f_t = k_f f_{t-1} + k_x x_t + k_l l_t + k_0 e_{f_{t-1}} + k_i e_{f_{t-2}} + \ldots + k_{N-1} e_{f_{t-N+1}} + k_0 e_{f_{t-N}} + k_i e_{f_{t+1}}, \quad (A12) \]

where \( k_f, k_x, k_l, k_{j}, k_0, k_i \) are coefficients which could be found after substitution of the (A12) into the (A9):

\[
\begin{align*}
(1-\mu k_f) k_f &= \frac{\mu}{\beta^*} \\
(1-\mu k_f) k_x &= \mu k_x \rho_x + \mu (1-\rho_x) \\
(1-\mu k_f) k_l &= \mu k_l (1-\xi) + \mu \xi^2 \\
(1-\mu k_f) k_{j} &= \mu k_{j+1}, \quad i = 0, N-2 \\
(1-\mu k_f) k_{N-1} &= \mu \psi (k_l - \xi) \\
(1-\mu k_f) k_0 &= \mu k_i \\
(1-\mu k_f) k_i &= \mu \chi (k_l - \xi) \\
\end{align*}
\]

The quadratic equation for \( k_f \) (A13) corresponds to the characteristic polynomial (A9) with stable root:

\[
k_f = y_1 = 1 + 0.5 \left( \frac{\delta + 1-\beta^*}{\lambda} \right) - \sqrt{0.5 \left( \frac{\delta + 1-\beta^*}{\lambda} \right)^2 + \frac{\delta}{\lambda}} \in (0, 1) \quad (A20)
\]

Other coefficients are:

\[
k_x = \frac{\mu (1-\rho_x)}{1-\mu (k_f + \rho_x)} \in (0, 1) \quad (A21)
\]

\[
k_l = \frac{\mu \xi^2}{1-\mu (k_f + 1-\xi)} \in (0, 1) \quad k_l \in (0, \xi) \quad (A22)
\]

\[
k_{N-1} = \psi k_f \beta^* (k_l - \xi) \quad k_{N-1} < 0 \quad (A23)
\]

\[
\frac{k_{j}}{k_{j+1}} = \frac{1}{k_f \beta^*} > 1 \quad i = 1,...,N-1 \quad (A24)
\]

\[
k_i = \chi k_f \beta^* (k_l - \xi) \quad k_i < 0 \quad (A25)
\]

\[
\frac{k_i}{k_0} = \frac{1}{k_f \beta^*} > 1 \quad (A26)
\]

Expressing \( \tilde{S}_t \) from (A2) gives the solution for exchange rate dynamics:

\[
\tilde{S}_t = \frac{1}{\delta} \left( f_t - \frac{1}{\beta^*} f_{t-1} - \xi l_t - x_t \right) \quad (A27)
\]
Impulse response functions on balance of payments shock

Let us assume one st.d. balance of payments shock happens at period $t=0$: $\varepsilon_{x,0} = \sigma_x$ while other shocks are zeros.

The solution for $f_t$ could be simplified as:

$$
\begin{align*}
  f_t &= k_f f_{t-1} + k_x x_t, \\
  x_t &= \rho_x x_{t-1} + \varepsilon_{x,t}. 
\end{align*}
$$

(A28)

Getting rid $x_t$ of (A28) gives the AR(2) process for $f_t$ with stable roots $k_f$ and $\rho_x$:

$$
  f_t (1-k_f L)(1-\rho_x L) = k_x \varepsilon_{x,t},
$$

(A29)

where $L$ is the lag operator.

If $k_f + \rho_x > 1$ IRF of financier’s long position in foreign currency $f_t$ on balance of payments shock $\varepsilon_{x,t}$ will be a humped function.

Substituting (A29) into (A27) lets representing impulse-response function of $\tilde{S}_t$ as minimal and causal ARMA(2,1) process:

$$
\tilde{S}_t (1-k_f L)(1-\rho_x L) = -\frac{1}{\delta} \left(1-k_x + \left(\frac{1}{\beta} - k_f\right) L\right) \varepsilon_{x,t},
$$

(A30)

where AR roots are the same as in the (A29); MA root is $k_f \rho_x$.

Graphical representation of IRFs is shown on Fig. A1.

---

Fig. A1. Impulse-response functions on the balance of payments shock $\varepsilon_{x,0} = \sigma_x$ calculated for parameters: $\delta = 2.632$, $\lambda = 25.65$, $\rho_x = 0.9508$, $\beta = 0.9998$, $\mu = 0.475$, $k_f = 0.727$, $k_x = 0.1157$, $\sigma_x = 0.0353$.

Impulse response functions on positive sterilized intervention shock
Assume one st.d. positive sterilized intervention shock happens at period \( t=0 \): \( \varepsilon_{t-0} = \sigma_i \), while other shocks are zeros. The solution for \( f_i \) could be rewritten as:

\[
\begin{align*}
  f_i &= k_f f_{i-1} + k_l l_i + \sum_{j=0}^{N-1} k_j \varepsilon_{i-1-j} \\
  l_i &= (1 - \xi) l_{i-1} + \psi \varepsilon_{i-1-N}
\end{align*}
\]  
(A31)

The system (A31) can be contingently divided into two subperiods: before and after the moment of actual foreign currency supply increase.

The first one characterizes forward-looking solution in presence of future anticipated foreign currency supply change. It covers periods \( t = 0, N-1 \) when there is no actual foreign currency supply increase but households anticipate it and forecast exchange rate appreciation at period \( t+N \). As a result of their forward-looking behaviour both \( f_i \) and \( \tilde{S}_i \) for \( t = 0, N-1 \) react on the shock \( \varepsilon_{i-0} \). Taking into consideration (A23) and (A24) the system (A31) for the first \( N-1 \) periods can be rewritten as:

\[
\begin{align*}
  f_i &= k_f f_{i-1} + q_i \\
  q_i &= \frac{1}{k_f \beta^*} q_{i-1} + \psi (k_i - \xi) (k_f \beta^*)^N \varepsilon_{i-1}, \quad \text{for } t = 0, N-1
\end{align*}
\]  
(A32)

where \( q_i \) is the contribution of forward-looking behaviour of households into the dynamics of \( f_i \). The system (A32) diverges because it can be rewritten as an AR(2) process for \( f_i \) with one unstable root \( \frac{1}{k_f \beta^*} > 1 \):

\[
f_i (1 - k_f L) (1 - \frac{1}{k_f \beta^*} L) = \psi (\xi - k_i) (k_f \beta^*)^N \varepsilon_{i-1}
\]  
(A33)

ARMA(2,1) process for exchange rate also diverges:

\[
\tilde{S}_i (1 - k_f L) (1 - \frac{1}{k_f \beta^*} L) = -\frac{1}{\sigma} \psi (\xi - k_i) (k_f \beta^*)^N (1 - \frac{L}{\beta^*}) \varepsilon_{i-1},
\]  
(A34)

where AR roots are \( k_f \) and \( \frac{1}{k_f \beta^*} > 1 \); MA root is \( \frac{1}{\beta^*} > 1 \).

There is no expected foreign currency supply increase in the second subperiod \( t = N, \infty \) hence the system becomes stable and its behaviour is close to the case of balance of payments shock. The system (A31) for periods \( t = N, \infty \) reduces to:

\[
\begin{align*}
  f_i &= k_f f_{i-1} + k_l l_i \\
  l_i &= (1 - \xi) l_{i-1} + \psi \varepsilon_{i-1-N}
\end{align*}
\]  
(A35)
The system (A35) can be rewritten as AR(2) process for \( f \) with stable roots \( k_f \) and \( 1 - \xi \):

\[
f_i (1 - k_f L)(1 - (1 - \xi)L) = k_i \psi e_{i,t-N}^{*}
\]

(A36)

Substituting (A36) into the (A27) allows finding solution for \( \tilde{S}_i \) as minimal and causal ARMA(2,1) process:

\[
\tilde{S}_i (1 - k_f L)(1 - (1 - \xi)L) = -\frac{1}{\delta} \left( \xi - k_i + \frac{k_i}{k_f} \right) e_{i,t-N}^{*},
\]

(A37)

where AR roots are \( k_f \) and \( 1 - \xi \); MA root is \( k_f (1 - \xi) \).

Graphical representation of IRFs derived above is shown on the Fig. A2.

Fig. A2. Impulse-response functions on the positive sterilized intervention shock \( e_{i,t,0} = \sigma_i \), calculated for parameters: \( N = 8 \), \( \delta = 2.632 \), \( \lambda = 25.65 \), \( \xi = 0.984 \), \( \beta^* = 0.9998 \), \( \mu = 0.475 \), \( k_f = 0.727 \), \( k_i = 0.712 \), \( \sigma_i = 0.919 \).

In the first subperiod \( t < N \) exchange rate \( \tilde{S}_i \) and financier’s position in foreign currency \( f \), correlate positively because expected appreciation of home currency stimulates financer to invest in home assets. In the second subperiod \( t \geq N \) \( \tilde{S}_i \) and \( f \) correlate negatively as in the case of balance of payment shock because returning back to the equilibrium after the positive foreign currency supply shock results in home currency depreciation stimulating foreign assets acquisition.

Appendix B. Results of the model estimation

Tab. A1. Results of ML estimation of the model with different delays \( N \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( N = 5 )</th>
<th>( N = 6 )</th>
<th>( N = 7 )</th>
<th>( N = 8 )</th>
<th>( N = 9 )</th>
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</thead>
<tbody>
<tr>
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<td>Mode</td>
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<td>( \sigma_i )</td>
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<td>0.919</td>
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<td>95%</td>
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<td>-----------------------------------------------</td>
<td>-------</td>
<td>-----------</td>
<td>-------</td>
<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>$\sigma(\varepsilon_{Oil,t})$</td>
<td>0.0229</td>
<td>0.0029</td>
<td>0.0229</td>
<td>0.0229</td>
<td>0.0229</td>
</tr>
<tr>
<td>$\sigma(\varepsilon_{x,t})$</td>
<td>0.0345</td>
<td>0.0026</td>
<td>0.0347</td>
<td>0.0353</td>
<td>0.0381</td>
</tr>
<tr>
<td>$\sigma(\varepsilon_{I^+,t})$</td>
<td>0.9191</td>
<td>0.0263</td>
<td>0.9191</td>
<td>0.9191</td>
<td>0.9191</td>
</tr>
<tr>
<td>$\sigma(\varepsilon_{I^-,t})$</td>
<td>0.4908</td>
<td>0.0140</td>
<td>0.4908</td>
<td>0.4908</td>
<td>0.4908</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>0.9534</td>
<td>0.0152</td>
<td>0.9539</td>
<td>0.9508</td>
<td>0.9433</td>
</tr>
<tr>
<td>$\delta$</td>
<td>2.597</td>
<td>1.41</td>
<td>2.635</td>
<td>2.6322</td>
<td>2.6059</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>16.72</td>
<td>6.27</td>
<td>19.05</td>
<td>25.65</td>
<td>47.45</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.1234</td>
<td>0.0619</td>
<td>0.521</td>
<td>0.984</td>
<td>1.507</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.1185</td>
<td>0.0523</td>
<td>0.0994</td>
<td>0.0993</td>
<td>0.0561</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.0220</td>
<td>0.0576</td>
<td>-0.0048</td>
<td>-0.0065</td>
<td>-0.0096</td>
</tr>
<tr>
<td>$\beta^*$</td>
<td>0.9998</td>
<td>0.9998</td>
<td>0.9998</td>
<td>0.9998</td>
<td>0.9998</td>
</tr>
</tbody>
</table>

Maximized likelihood 2023.50 2028.85 2030.23 2034.18 2025.69

Tab. A2. Results of Markov Chain Monte-Carlo method with Metropolis-Hastings algorithm
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(e_{\text{Oil}}) )</td>
<td>Standard deviation of the oil price shock</td>
<td>0.0229</td>
<td>0.0007</td>
<td>0.0230</td>
<td>0.0219</td>
<td>0.0241</td>
</tr>
<tr>
<td>( \sigma(e_{\text{x},t}) )</td>
<td>Standard deviation of the balance of payments shock</td>
<td>0.0354</td>
<td>0.0025</td>
<td>0.0366</td>
<td>0.0321</td>
<td>0.0413</td>
</tr>
<tr>
<td>( \sigma(e_{I,+}) )</td>
<td>Standard deviation of positive sterilized intervention shock</td>
<td>0.9191</td>
<td>0.0262</td>
<td>0.9213</td>
<td>0.8772</td>
<td>0.9656</td>
</tr>
<tr>
<td>( \sigma(e_{I,-}) )</td>
<td>Standard deviation of negative sterilized intervention shock</td>
<td>0.4908</td>
<td>0.0140</td>
<td>0.4917</td>
<td>0.4685</td>
<td>0.5156</td>
</tr>
<tr>
<td>( \rho_x )</td>
<td>AR(1) coefficient of balance of payments shock</td>
<td>0.9508</td>
<td>0.0145</td>
<td>0.9494</td>
<td>0.9254</td>
<td>0.9745</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Elasticity of the substitution of home non-tradable goods and foreign tradable goods</td>
<td>2.6395</td>
<td>0.1431</td>
<td>2.6815</td>
<td>2.4190</td>
<td>2.9186</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Coefficient which characterize risk bearing capacity of financier</td>
<td>25.91</td>
<td>9.11</td>
<td>29.49</td>
<td>12.99</td>
<td>45.44</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Share of disposable currency volume spent on the forex in current period</td>
<td>0.9786</td>
<td>0.1585</td>
<td>0.8653</td>
<td>0.7227</td>
<td>1.0000</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Supplied in the forex share of foreign currency volume distributed through the repo auctions</td>
<td>0.0994</td>
<td>0.0275</td>
<td>0.1065</td>
<td>0.0598</td>
<td>0.1504</td>
</tr>
<tr>
<td>( \chi )</td>
<td>Demanded from the forex share of foreign currency volume needed to pay off repo auction debts</td>
<td>0*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \beta^* )</td>
<td>Foreign discount factor</td>
<td>0.9998</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Notes:** The parameter is not estimated because for stable work of MCMC-MH algorithm the likelihood function should be concave enough w.r.t. the parameter.

**Appendix C. Results of vector errors correction model (VECM) estimation**

The VECM model is estimated on the same daily dataset from 6 November 2014 to 20 April 2017. All calculations are made in JMulti software (Lutkepohl and Kratzig, 2004) where I use next setup of the VECM:
\[ \Delta y_t = \alpha \kappa \eta \begin{bmatrix} y_{t-1} \\ 1 \end{bmatrix} + \sum_{j=1}^{20} \Gamma_j \Delta y_{t-j} + u_t \]  

(A38)

\[ u_t = B e_t, \]  

(A39)

where \( y_t = [s, c e_{I_t}, c e_{I_t}, p_{Oil,t}] \) is the vector of four observable endogenous variables; \( c e_{I_t} \) and \( c e_{I_t} \) are cumulated positive and negative sterilized intervention shocks, respectively; \( \alpha \) is the loading coefficients vector; \([\kappa \ \eta]\) are coefficients in cointegration vector \( \begin{bmatrix} y_{t-1} \\ 1 \end{bmatrix} \); \( \Gamma_j \) are 4x4 parameter matrices; \( u_t \) is four-dimensional unobservable zero mean white noise processes; \( B \) is the matrix of contemporaneous effects of structural shocks \( e_t \).

To include log of oil price \( p_{Oil,t} \) in cointegration vector I treat it as endogenous variable, but also I set corresponding restrictions on coefficients in \( \Gamma_j \), \( \alpha \) and \( \kappa \) to estimate it as exogenous random walk process. I don’t include sterilized interventions variables \( c e_{I_t} \) and \( c e_{I_t} \) in the cointegration equation and set corresponding restrictions on \( \alpha \) and \( \kappa \).

VECM is estimated with 20 lags by two stage procedure: at the first stage simple two stage (S2S) estimation allows imposing restrictions on cointegration vector; at the second stage generalized least of squares (GLS) method is used to take into account restrictions on \( \alpha \) and \( \Gamma_j \) (Lutkepohl and Kratzig, 2004).

Estimated on the first stage cointegration vector is:

\[ \begin{bmatrix} \kappa \ \eta \end{bmatrix} \begin{bmatrix} y_{t-1} \\ 1 \end{bmatrix} = s_{t-1} + 0.429 p_{Oil,t-1} - 5.773, \]  

(A40)

where numbers in parenthesis are standard deviations; numbers in squared brackets are t-values.

Result of estimation on the second stage is represented in the Tab. A3.

**Tab. A3. Estimation of loading coefficients \( \alpha \) and parameter matrices \( \Gamma_j \)**

<table>
<thead>
<tr>
<th></th>
<th>( \Delta s_t )</th>
<th>( \varepsilon_{I_t} )</th>
<th>( \varepsilon_{I_t} )</th>
<th>( \Delta p_{Oil,t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha' )</td>
<td>-0.049 (0.014)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
\[ \Delta s_{t-1} \quad -0.0740 \quad -0.6514 \quad 1.2885 \quad - \\
(0.0408) \quad (2.7483) \quad (1.4946) \quad - \\
\varepsilon_{t' - 1} \quad -0.0016 \quad -0.0871 \quad 0.0030 \quad - \\
(0.0006) \quad (0.0411) \quad (0.0224) \quad - \\
\varepsilon_{t' - 1} \quad 0.0000 \quad 0.0553 \quad -0.0439 \quad - \\
(0.0011) \quad (0.0752) \quad (0.0409) \quad - \\
\Delta p_{t' - 1} \quad -0.0837 \quad -0.5071 \quad 1.2111 \quad - \\
(0.0268) \quad (1.8146) \quad (0.9868) \quad - \\
\Delta s_{t-2} \quad 0.0456 \quad 0.3891 \quad -0.8902 \quad - \\
(0.0410) \quad (2.7703) \quad (1.5066) \quad - \\
\varepsilon_{t' - 2} \quad 0.0021 \quad 0.0266 \quad 0.0248 \quad - \\
(0.0006) \quad (0.0415) \quad (0.0226) \quad - \\
\varepsilon_{t' - 2} \quad 0.0004 \quad 0.0069 \quad -0.0124 \quad - \\
(0.0011) \quad (0.0744) \quad (0.0405) \quad - \\
\Delta p_{t' - 2} \quad 0.0577 \quad 0.2518 \quad -1.1605 \quad - \\
(0.0279) \quad (1.8986) \quad (1.0325) \quad - \\
\Delta s_{t-3} \quad -0.0017 \quad -1.5178 \quad 2.4419 \quad - \\
(0.0405) \quad (2.7268) \quad (1.4829) \quad - \\
\varepsilon_{t' - 3} \quad 0.0016 \quad 0.0648 \quad 0.0249 \quad - \\
(0.0006) \quad (0.0418) \quad (0.0227) \quad - \\
\varepsilon_{t' - 3} \quad 0.0001 \quad -0.0508 \quad -0.0242 \quad - \\
(0.0011) \quad (0.0730) \quad (0.0397) \quad - \\
\Delta p_{t' - 3} \quad 0.0042 \quad -0.3095 \quad 1.0551 \quad - \\
(0.0280) \quad (1.8971) \quad (1.0317) \quad - \\
\Delta s_{t-4} \quad 0.0259 \quad 1.4307 \quad -0.3986 \quad - \\
(0.0324) \quad (2.7217) \quad (1.4799) \quad - \\
\varepsilon_{t' - 4} \quad 0.0008 \quad 0.1346 \quad 0.0332 \quad - \\
(0.0006) \quad (0.0419) \quad (0.0228) \quad - \\
\varepsilon_{t' - 4} \quad 0.0002 \quad -0.0116 \quad 0.0512 \quad - \\
(0.0011) \quad (0.0728) \quad (0.0396) \quad - \\
\Delta p_{t' - 4} \quad 0.0366 \quad 0.7891 \quad 0.4519 \quad - \\
(0.0279) \quad (1.8938) \quad (1.0299) \quad - \\
\Delta s_{t-5} \quad 3.0178 \quad -6.2125 \quad -3.5625 \quad - \\
(0.0340) \quad (2.7229) \quad (1.4803) \quad - \\
\varepsilon_{t' - 5} \quad 0.0002 \quad 0.2642 \quad -0.0103 \quad - \\
(0.0006) \quad (0.0435) \quad (0.0242) \quad - \\
\varepsilon_{t' - 5} \quad 0.0001 \quad -0.0606 \quad 0.3165 \quad - \\
(0.0011) \quad (0.0728) \quad (0.0396) \quad - \\
\Delta p_{t' - 5} \quad 0.0023 \quad -3.0725 \quad -1.0594 \quad - \\
(0.0276) \quad (1.8712) \quad (1.0176) \quad - \\
\Delta s_{t-6} \quad -0.0220 \quad 9.7974 \quad 0.7320 \quad - \\
(0.0404) \quad (2.7312) \quad (1.4853) \quad - \\
\varepsilon_{t' - 6} \quad -0.0007 \quad 0.0653 \quad -0.0234 \quad - \\
(0.0007) \quad (0.0455) \quad (0.0248) \quad - \\
\varepsilon_{t' - 6} \quad -0.0014 \quad -0.0564 \quad 0.0563 \quad - \\
(0.0011) \quad (0.0765) \quad (0.0416) \quad - \\
\Delta p_{t' - 6} \quad -0.0011 \quad 1.1871 \quad -0.2601 \quad - \\
(0.0275) \quad (1.8652) \quad (1.0144) \quad - \\
\Delta s_{t-7} \quad -0.0847 \quad 9.0883 \quad 1.4772 \quad - \\
(0.0405) \quad (2.7477) \quad (1.4942) \quad - \\
\varepsilon_{t' - 7} \quad -0.0027 \quad -0.0237 \quad 0.0160 \quad - \\
(0.0007) \quad (0.0456) \quad (0.0248) \quad - \\
\varepsilon_{t' - 7} \quad -0.0014 \quad -0.0103 \quad 0.0174 \quad - \\
(0.0011) \quad (0.0759) \quad (0.0413) \quad - \\
\Delta p_{t' - 7} \quad -0.0022 \quad 2.9817 \quad -0.4305 \quad - \\
(0.0273) \quad (1.8590) \quad (1.0110) \quad - \\
\Delta s_{t-8} \quad 0.0654 \quad 1.1637 \quad 0.5208 \quad - \\
(0.0408) \quad (2.7766) \quad (1.5100) \quad - \\
\varepsilon_{t' - 8} \quad -0.0014 \quad -0.0107 \quad -0.0170 \quad - \\
(0.0007) \quad (0.0461) \quad (0.0251) \quad - \\


| \( \varepsilon_{I^*} \) | \( I \rightarrow J \rightarrow 8 \) | \( \Delta p_{Oil,J \rightarrow 8} \) | \( \Delta s_{J \rightarrow 9} \) | \( \varepsilon_{I^*} \) | \( I \rightarrow J \rightarrow 9 \) | \( \Delta p_{Oil,J \rightarrow 9} \) | \( \Delta s_{J \rightarrow 10} \) | \( \varepsilon_{I^*} \) | \( I \rightarrow J \rightarrow 10 \) | \( \Delta p_{Oil,J \rightarrow 10} \) | \( \Delta s_{J \rightarrow 11} \) | \( \varepsilon_{I^*} \) | \( I \rightarrow J \rightarrow 11 \) | \( \Delta p_{Oil,J \rightarrow 11} \) | \( \Delta s_{J \rightarrow 12} \) | \( \varepsilon_{I^*} \) | \( I \rightarrow J \rightarrow 12 \) | \( \Delta p_{Oil,J \rightarrow 12} \) | \( \Delta s_{J \rightarrow 13} \) | \( \varepsilon_{I^*} \) | \( I \rightarrow J \rightarrow 13 \) | \( \Delta p_{Oil,J \rightarrow 13} \) | \( \Delta s_{J \rightarrow 14} \) | \( \varepsilon_{I^*} \) | \( I \rightarrow J \rightarrow 14 \) | \( \Delta p_{Oil,J \rightarrow 14} \) | \( \Delta s_{J \rightarrow 15} \) | \( \varepsilon_{I^*} \) | \( I \rightarrow J \rightarrow 15 \) | \( \Delta p_{Oil,J \rightarrow 15} \) | \( \Delta s_{J \rightarrow 16} \) |
|------------------|-------------------|-------------------|------------------|------------------|-------------------|-------------------|-------------------|------------------|-------------------|-------------------|-------------------|------------------|-------------------|-------------------|-------------------|------------------|-------------------|-------------------|------------------|------------------|------------------|-------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                  | 0.0018            | 0.0438            | 0.0362           | 0.0008           | -0.0118           | -0.0364           | 0.1635            | 0.0005           | -0.0099           | 0.0198            | 0.0630            | 0.0020            | 0.0006           | 0.0060           | 0.1045            | 0.0004           | -0.0001           | 0.0340            | -0.0117           | 0.0006           | 0.0005           | 0.0322           | 0.0004           | -0.0044           | -0.0656           | 0.0252           | 0.0014           | 0.0469           | -0.0302           |
The structural equation (A39) is estimated by maximum likelihood method (Breitung, Brüggemann, and Lütkepohl, 2004). Standard errors (put in parenthesis) are bootstrapped. Setting restrictions on coefficients of B matrix I assume that innovations in all endogenous can influence foreign exchange rate within the same working day, while other three endogenous can’t influence each other in the same period. Assumptions made above define over-identifying system and their validity has to be checked. The results are represented in Tab. A3.

**Tab. A4. Results of B matrix estimation**

<table>
<thead>
<tr>
<th></th>
<th>Δs₁</th>
<th>ε₁&lt;sup&gt;-&lt;/sup&gt;,&lt;sub&gt;j&lt;/sub&gt;</th>
<th>ε₁&lt;sup&gt;+&lt;/sup&gt;,&lt;sub&gt;j&lt;/sub&gt;</th>
<th>Δp&lt;sub&gt;Oil,j&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δs₁</td>
<td>0.0113</td>
<td>0.0010</td>
<td>0.0004</td>
<td>-0.0076</td>
</tr>
<tr>
<td></td>
<td>(0.0100)</td>
<td>(0.0007)</td>
<td>(0.0005)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td></td>
<td>[11.09]</td>
<td>[1.43]</td>
<td>[0.88]</td>
<td>[-14.56]</td>
</tr>
<tr>
<td>ε₁&lt;sup&gt;-&lt;/sup&gt;,&lt;sub&gt;j&lt;/sub&gt;</td>
<td>-</td>
<td>0.7804</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.1300)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

35
Here the VECM is over-identified with 3 degrees of freedom. LR test of over-identifying restrictions can’t reject the validity of the constraints imposed ($\chi^2(3) = 2.718$, prob. = 0.437).

Impulse-response functions

Impulse-response functions are shown on Fig. A3.

95% confidence intervals in VECM are bootstrapped by two methods: Hall algorithm (Hall, 1992) and Efron algorithm (Efron and Tibshirani, 1993).

Fig. A3. IRFs of exchange rate $S_t$ (left part) and cumulated positive $c\varepsilon_{I+}$ and negative $c\varepsilon_{I-}$ (right part) sterilized interventions on positive sterilized intervention shock $\varepsilon_{I+}$ in VECM with 95% confidence intervals bootstrapped by Efron algorithm ($Efrinf$ and $Efrsup$) and Hall algorithm ($Hallinf$ and $Hallsup$)

Fig. A4. IRFs of exchange rate $S_t$ (left part) and cumulated positive $c\varepsilon_{I+}$ and negative $c\varepsilon_{I-}$ (right part) sterilized interventions on negative sterilized intervention shock $\varepsilon_{I-}$ in VECM.
Fig. A5. IRFs of exchange rate $S_t$ (left part) and cumulated positive $c_{\varepsilon t'}$ (right part) sterilized interventions on exchange rate structural shock in VECM.

Fig. A6. IRFs of exchange rate $S_t$ sterilized interventions on oil price shock in VECM.

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