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CONTINUOUS SPATIAL MONOPOLISTIC COMPETITION: MATCHING GOODS WITH CONSUMERS

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Continuous Spatial Monopolistic Competition: Matching Goods with Consumers

Abstract

Our new approach enriches the general additive monopolistic competition model (AMCM)—with a space of product characteristics: consumers' "ideal varieties". Unlike Hotelling, such *partially localized competition* involves intersecting zones of service among (continuously distributed) producers. Then, the uniform equilibrium firms' density increases with growing population, as with the usual AMCM. However, now increasing/decreasing prices are determined by the increasing/decreasing elasticity of elementary utility (instead of demand elasticity in AMCM). A new characteristic — the firm's range of service — decreases. Such *finer matching* between buyers and sellers becomes a new source of welfare gain from a thicker market, unlike the variety benefit in AMCM. The free-entry competition remains socially excessive under some natural preferences.

Keywords: monopolistic competition, spatial competition, optimal product diversity, gains from trade, finer matching.

JEL Codes: L11, L13.

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1 Introduction

Motivation. Some similar markets generate different, broad or narrow niches: e.g., half of the beer market in the US is covered by only three brands, whereas no brand has even ten percent of the beer market in the UK. Analogously, in a bigger city we typically observe more restaurants with narrow specialization: Italian, Japanese and other, which is not typical in small towns. Why?

In markets for differentiated products, one can observe that individuals typically vary in their "ideal goods", e.g. favourite type of beer or coffee — yet choose something different from time to time. Thus, *love for variety* is struggling with *love for ideal product type*. The reason can lie in multi-dimensional characteristics; for instance, a lady would like her ideal *size*, but also explores various *brands and fashions*, possibly supplied only with other sizes. Then there will be a trade-off, which results in a non-equal mixture of ideal and non-ideal sizes (varieties) in the consumption bundle. Somewhat similarly, consumers in a city quite often buy food from the nearest shop but also use other shops from time to time. Such behaviour generates an overlap of the range of service of the shops. On a country-wide scale, we also observe overlapping trade areas of various firms, though closer clients are served more frequently. Summarizing, many real markets show partially-localized consumer preferences and thereby *partially-localized competition.*

Our goal is to build a proper model of such competition and understand the market-size effects and their importance for welfare. Looking from another angle, we question the robustness of the mainstream theory of monopolistic competition to consumers' heterogeneity (a dimension orthogonal to firm heterogeneity). As we shall see, some but not all standard theoretical conclusions for price effects, welfare distortions and market-size gains remain valid.

In essence, our new model fills the gap between two traditional polar views on competition: spatial competition and monopolistic competition. Namely, we combine the (free-entry version of) Hotelling's (1929) consumer ideal points with Chamberlinian (1933) love for variety, in a simple but general way. Seeking for the *simplest*, most parsimonious combination, we maintain all elements of the monopolistic competition theory, but only replace the traditional representative consumer by spatially heterogenous consumers.[4] How does this single novelty modify the nature of competition and comparative statics of the free-entry equilibrium? For comparison, we use Zhelobodko et al. (2012), henceforth ZKPT, as a benchmark.

Setting. Consumers are continuously and uniformly distributed along a circumference similar to the Salop (1979) model. This space can represent geographical locations or space of consumers' tastes, i.e., ideal points (addresses) among varieties of the differentiated good. In contrast to inelastic demand in Salop, here each consumer combines various quantities of ideal and non-ideal varieties in her consumption bundle due to her love for variety. Consumers are identical in preferences with the same (unspecified additive) utility function and same income, but differ in their locations. Naturally, everyone prefers varieties (firms) located closer rather than farther away, so that the demand gradually fades with distance. Such behaviour is described by a linear "cost of distance" introduced in two versions: either as a disutility of distance from the ideal, or as a monetary cost for transporting (or adjusting) the good to the "ideal". The monetary version is better suited for the geographical interpretation of the model (or for production-components goods), whereas disutility of distance has more appeal for consumer goods and the space of their product characteristics.

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[4] Other space-and-variety models are discussed later on in the literature review.

Following Chamberlin (1933) and Dixit-Stiglitz (1977), our market exhibits free entry and increasing returns in producing a differentiated good so that the number (continuum) of firms is endogenously determined unlike Hotelling (1929)). Homogeneous firms simultaneously choose their prices and their locations, taking as given the density of consumers and the current local intensity of competition. Gross demand of a firm is the sum of all consumers' demand schedules within its range of service, where the distance cost allows for positive demand. In the most general version of our approach, the market equilibrium should consist of three *curves* in the address space: (i) density of firms, (ii) their prices, (iii) competition intensity (marginal utility of money). However, this paper deals only with the basic model having a uniform density of consumers and uniform distribution of firms. Such an equilibrium boils down to three *scalars*: the mass of firms, price, and the intensity of competition. These three variables of the model converge to the pattern of the standard spaceless model of monopolistic competition presented in ZKPT, when the circular space of consumers shrinks to a point, or the distance cost shrinks to zero.

Under positive distance costs, the new model enables richer predictions than ZKPT in terms of the range of service and allows for two regimes. Under utilities which have a finite derivative at zero (choke-price) like constant absolute risk aversion (CARA), some sufficiently high distance costs generate *partial coverage* of consumers by service of every firm. By contrast, decreasing distance costs eventually turn this world first into *full-coverage regime*, and then, at the limit, into the usual spaceless additive monopolistic competition.

Among results, the basic technical achievement is a convenient reformulation of a firm's aggregated demand in the form of a "consumer surplus" of the elementary utility function (taken at the consumption of the ideal variety). With this, our new uniform spatial model becomes almost as *simple and tractable* as the usual monopolistic competition. This technique enables us to expand some of theoretical results to the world of heterogenous consumers.

The market-size effects are adressed as in ZKPT. Should a *thicker market* make our firms more numerous and large, simultaneously pushing their prices down? Our answer is generally positive for both regimes: full or partial coverage of consumers by the firm's service. More specifically, under partial coverage, a growing population density generally leads to: (i) more firms entring the market; (ii) keener competition; (iii) less individual consumption of each variety; (iv) more localized competition (smaller range of service), however, (v) both the price behaviour and firm size depend on the elasticity of elementary utility. Namely, prices and mark-ups *go down*, firms increase in size under natural condition of decreasing elasticity of utility (DEU), whereas the opposite effects take place under the opposite condition.

This outcome is similar to ZKPT, where the necessary and sufficient condition for the "pro-competitive" effect of the market size on prices is an increasingly elastic demand (IED). The difference can be better understood by analysing market size impacts in two stages. First, we aggregate the demand of heterogeneous consumers, second, to the aggregate demand we apply the main equation of ZKPT model — the elasticity of revenue must equal the elasticity of cost. As we have explained, the aggregate demand takes the form of a consumer surplus. Therefore, now IED is replaced by DEU condition. Although none of the two conditions, IED and DEU, implies the other, their intuitive interpretation is quite similar: the demand is "not too convex," which is supposed realistic by those economists who support variable elasticity of substitution (Eckel and Neary, 2010).

A decrease in distance-costs affects equilibria in the same way as the increasing population density. The intensity of competition and the density of firms increase, and prices also react analogously to the market expansion. Another effect, specific to the decreasing distance-cost parameter, is an (eventually) increasing range of service. We show that the usual monopolistic competition of ZKPT is really a limiting case of the spatial competition when the distance cost fades away.

Intuitively, the absolute market size (length of the circumference) impacts the market quite differently from the relative market size (population density). Namely, under partial coverage of consumers by service, the length does not matter for equilibrium prices and firm sizes. Indeed, imagine a globe where each location (type of agent) trades only with its close neighbours. Then, the size of the globe is immaterial. However, this independence ceases to hold when the distance cost decreases to the level when each firm services the whole globe. Under full coverage, the world size works similar to the distance cost. Both regimes can have their implications in economic geography, where this model could be an appropriate continuous substitute of similar network models (see Ushchev and Zenou, 2015).

Welfare is analysed by decomposing consumer gains into two components. The first part represents welfare as if all the consumed varieties were ideal, while the second part accounts for losses from the consumption of non-ideal varieties. We show that in a thicker market these losses are smaller, i.e., a supply of varieties is *better matched* to heterogeneous consumers' tastes. This highlights a new source of gains from market expansion. This *better match* to heterogeneous consumers' tastes was always the heart of the verbal explanation of love for variety in monopolistic competition. Finally, our model makes this argument explicit, instead of leaving it obscure or hidden in the utility of the "representative consumer".

In a thicker market, a consumer spends her budget for varieties closer to her ideal, which become cheaper at the same time (under the natural DEU assumption). This double benefit yields a *positive effect* on welfare. At the same time, a variety per se (consumed by a person) expands less than the mass of firms due to the shrinking range of service. In other words, the mass (density) of firms in the market is incompletely translated here into the variety consumed, unlike spaceless models. Through these lenses, the "price index", usual for trade literature, can be just a *misleading* measure of welfare gains from a thicker market or trade, a concept non-obviously related to welfare. Indeed, consuming more food (varieties) which one does not like is not a welfare gain, compared to less but preferred variety.

Distortion. Further, the question since Dixit and Stiglitz (1977) is variety distortion. Does free entry let too many or too few firms into the market, from a social viewpoint? The usual trade-off here means that less variety could save entry costs and thereby bring cheaper goods from bigger firms. Dixit and Stiglitz establish a welfare criterion for a spaceless market: any DEU preference creates excessive competition, while IEU yields insufficient competition, CES being the borderline case with no distortion. In our model, under partial coverage, we find some necessary and sufficient condition on the elementary utility which guarantees excessive entry; CES is not borderline here because it does not allow for partial coverage.

Demand aggregation. Although our results in general corroborate the monopolistic competition theory, a theorist should be more interested in a *difference* of spatial monopolistic competition from the usual spaceless one. The first observation is that under partial coverage by service, the *range of service* becomes a new and important market characteristic. A more fundamental question is: how does heterogeneity modify the shape of the gross demand function? As Osharin et al. (2014) show, heterogeneity combined with aggregation can make the demand function more convex. For instance, a quadratic elementary utility $u(q) = q - q^2/2$ (dependent on individual consumption q) generates a *linear* individual demand function $q = 1 - (p + t)$, which depends upon price p and the cost-of-distance coefficient t. In our model the firm's gross demand Q appears *quadratic* in price: $Q = (1-p)^2/2t$, whihc is quite different from individual demand, more convex.[5] Similarly, a spatial aggregate of CES individual demands, generates a CES demand with a different, *higher* degree of convexity, which is situation-specific, dependent upon equilibrium variables. These simple examples show why our spatial model differs in some theoretical predictions from the usual one. They explain also a *spatial* version of demand aggregation: in spite of very similar consumers, a "permanent representative consumer" is a false concept. Instead, a situation-specific aggregate of consumers is relevant, dependent on distance costs and the intensity of competition. Through this example, we express a word of caution about the interpretation of empirical results. Demand characteristics, elasticity of substitution, and estimated *gains from trade*, inferred from individual level data can (and should, because our model) substantially *differ* from those estimated from market level data within studies like Arkolakis et al. (2015). Therefore, addressing consumer heterogeneity is necessary to reconcile such differences and reinterpret the market effects.

The rest of the paper is organized as follows. The next section reviews the related literature. Section 3 presents the core model, and Section 4 establishes the market effects and formalizes our welfare argument. The conclusion summarizes and Appendix contains proofs and the analysis of the version of the model with utility cost of distance.

2 Literature review

Literature. There are many papers on new trade theory with monopolistic competition, but with discrete locations (countries or cities), and often with too specific preferences. This discrete technique and focus on specific functions both shadow the links between general preference properties (concavity, elasticity) and market effects; instead, a continuous space enables algebraic tools of integration which reveal sufficient (or even necessary and sufficient) conditions on the demand structure for important market effects.

Several early attempts to combine continuous space and free entry were pioneered by Lancaster's (1966, 1975) approach to product "characteristics." The "spokes model" by Chen and Riordan (2007) also pursues the same goals as ours, but exploits an exotic space: exogenously expanding dimensions of product characteristics and "trade through the hub."This may help to model the fashion industry, but it is difficult to reconcile with our focus on a fixed geographical space or a space of characteristics like colour or design. Among other models serving us a benchmark, Picard and Tabuchi (2010) was the starting point for a continuous distribution of firms on the circumference. They found that stable equilibria are given by discrete distributions of firms and workers rather than continuous distributions.[6]

Closer to bringing together the "love for variety" and "address" models of product differentiation, are several papers on multi-product firms in trade. They highlight the same "better

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[5] By the way, our preliminary inquiry suggests that in some cases, heterogeneity and related excessive demand convexity may destroy the usual assumption of concave profit. In this case, multiple local maxima arise, but we leave aside this complex problem.

[6] Among other continuous models of economic geography, Allen and Arkolakis (2014) propose a continuous model of economic geography based on CES gravity equations in generating trade flows under given topography (pairwise location-specific trade frictions). Mobile consumers-workers generate local outputs and equalize welfare across locations populated. Unlike ours, the focus is on predicting a stable economic geography under given topography in continuous way, extending thereby the network-style discrete geography modeling. The latter is summarized in Behrens and Robert-Nicoud (2014), where heterogeneous agents choose cities conditional on their talent.

match" effect of competition — exploiting the notion of "core competence" and some specific details. Feenstra and Ma (2008) explore a model where firms choose their optimal product scope by balancing the net profits from a new variety against the costs of "cannibalizing" their own sales. This can yield same or wider scope of service under opening trade. In contrast, Eckel (2009), Eckel and Neary (2010) and Eckel and Irlacher (2014) develop a series of models of "flexible manufacturing" which highlight a new source of gains from trade – productivity increases as firms become "leaner and meaner", concentrating on their core competence – but also a new source of losses from trade: product variety may fall. Similarly, analysing firm level trade data, Mayer et al. (2011), Arkolakis and Muendler (2011) and Carballo et al. (2013) show that exporters tend to skew their export sales towards the best performing products – their core competencies. In terms of our model, these effects can be viewed as focusing on a smaller part of the market.

Unlike this important "flexible manufacturing" and core competency literature that have direct empirical justification, we put distance costs on the consumer side, avoid multi-product firms and "cannibalization" motives, and focus on a closed economy. Otherwise our "range of service" looks isomorphic to the "firm scope," and our distance is the same as their "distance from their core competency." More essentially, unlike all of these studies, we exploit *unspecified* additive preferences and continuous number of versions per firm, to find the *simplest general conditions* on the demand curvature that provide price-decreasing competition with tighter firm specialization.

3 Spatial model with uniform equilibria

In this section we set up a simplified version of our model. We assume that consumers are distributed uniformly over the circumference of arbitrary length. A point on the circumference can be viewed as a geographic location or a specific product in the product characteristic space. Firms are free to choose any point on the circumference to enter. For now we constrain our attention to the case where firms are also distributed uniformly over the circumference. In what follows, we label it a uniform equilibrium. The concept of uniform equilibria may be criticized because they need not be stable and because non-uniform consumer distribution is unlikely to give rise to a uniform distribution of firms. However, without this basic model more complicated equilibria are difficult to comprehend. To support an approximately-uniform, or at least continuous distribution of firms, we introduce, in a reduced form, an external *dispersion force* that pushes one firm away from another. It represents the price of land and other congestion forces common to economic geography but not modelled here explicitly. In this case the tendency towards dispersion of firms looks more plausible.

In what follows we consider a version of a spatial model with a monetary transport cost. The adjustment cost for consuming products produced further away from a consumer's location enters the budget constraint. This formulation is common in economic geography, and it is also fits well to the case when our "consumer" is actually a firm that consumes some intermediate good, incurring cost for adjusting the good to fit its exact needs. We also consider a version of the model where the transport cost directly enters the utility function. This variation has more appeal for the consumption goods, because here "distance" from one's favourite variety has some disutility value. Since the results appear to be very similar, we relegate the discussion of the "disutility of distance" version of the model to the appendix.

3.1 General model setup

Consumers and varieties. The consumers are identical except for their "addresses." As in Hotelling (1929), any consumer type is characterized by her bliss point x in some commodity space Ω, i.e., her favourite variety of the differentiated good. The types are uniformly distributed with density L along the circular space of product characteristics, the circumference $\Omega = [-S,\, S]$ of length $2S$, where 0 is any given point (such Salop's "race-track economy" is a proxy for a "long" linear interval). Each consumer supplies one unit of a numeraire good (for instance, labour) to the market, in exchange for all varieties she consumes. Following the Chamberlinian tradition, each variety is produced by a single firm and each firm produces a single product. There is a continuum of firms and a firm's type denoted $y \in [-S,\, S]$ refers to its location on the circumference. The firm's "address" means its targeted type of consumers, whereas (endogenous) density μ_y is the measure of such firms in the same location. In this section the density $\mu_y \equiv \mu > 0$ is a scalar, assumed to be the same at each location $y \in \Omega$. In addition, we assume mill pricing by the firms, i.e., a firm at y charges a f.o.b. price p_y for its product. Because of symmetry, after firms optimize their prices, the price distribution will become also uniform with $p_y \equiv p > 0$.

Importantly, ranges of the service of various firms do intersect with each other, because consumers love variety. However, they love different varieties unequally. The bliss-point variety is slightly preferred to other varieties. For instance, one can imagine a consumer occasionally using many restaurants in her city but preferring the closest ones. In the "monetary" version of distance, we suppose that either adjusting the non-ideal variety to consumer's tastes is costly, or carrying a purchase home from a remote shop is costly. Specifically, we assume adjustment costs $q \cdot \tau(\theta)$ for buying q and carrying it home from distance θ, where $\tau(\cdot)$ is an increasing function of distance, assumed to be linear for exposition simplicity.

Hence, in both versions the remote varieties will be consumed in smaller amounts than close varieties. In particular, extremely remote varieties may not be consumed. In equilibrium each consumer x has an (endogenous) range $\hat{\theta}$ of varieties (firm types) that she wishes to buy, where $\hat{\theta} \in (0, S]$ denotes the distance to go shopping, or the *range of service*, uniform among consumers. An equilibrium may result in a small range $\hat{\theta} < S$ which means "partial coverage" of the circumference Ω by each firm's service. Another possibility is "full coverage by service" $\hat{\theta} = S$, occurring when the cost of distance is small enough to buy products (in different quantities) from all firms.

Now we can formulate the consumer's optimization problem. Given the (uniform) price distribution p and the firm distribution μ, the consumer seeks to maximize her utility subject to the budget constraint:

$$\max_{q_{xy} > 0} \; \mu \int_{\Omega} u(q_{xy})dy$$

$$\text{s.t. } \mu \int_{\Omega} (p + \tau(x, y)) q_{xy} dy = 1$$

Here a consumer located at x, who buys quantity q_{xy} from a firm located at y, enjoys the direct utility $u(q_{xy})$ and bears some losses $q_{xy}\tau(x, y)$. The elementary utility function $u(\cdot)$ is assumed to be increasing, thrice differentiable and concave, thus generating love for variety. If a consumer does not consume a variety, her utility from it is $u(0) = 0$, i.e., the existence of a variety per se does not generate any benefits. For some results we shall need also a choke-price assumption $u'(0) < \infty$.

One can see that our total utility is additive in its elementary utilities over the whole range of varieties. Such unspecified elementary utility function $u(\cdot)$ will allow us to relate arising market effects to the features of preferences as in ZKPT, and to contrast the results with non-spatial monopolistic competition.

The transport cost function $\tau(x, y)$ depends on the distance between x and y and represents the monetary cost per unit of consumption. The distant varieties are worse than the ideal variety, by assumption $\tau(x, x) = 0$, $\tau(x, y) > 0\,(x \neq y)$. Though our model allows for more general forms, for simplicity here we assume the transport cost to be linear in distance. Given that our space is a unit circumference, linearity implies the shortest (right or left) distance in the form:

$$\tau(x, y) = t \cdot \min\left\{|x - y|, 2S - |x - y|\right\}.$$

The transport cost can describe situations where space is geographical and a consumer spends her money to bring varieties home, or when some costly adjustment is needed, like adjusting the size of clothing.

Using the Lagrange multiplier λ, the utility maximization yields the demand of consumer x for variety-type y expressed as:

$$q_{xy} = D(\lambda_x p_y + \lambda_x \tau(x, y)),$$

where

$$D(P) \equiv {u'}^{-1}(P) \vee 0 \;\; \forall P > 0$$

is the demand that equals the inverse derivative of the elementary utility when positive, otherwise zero. Naturally, here the Lagrange multiplier $\lambda = \lambda(\mu, \mathbf{p})$ is not the argument but the result of the consumer's optimization under a given vector $(\mu, \mathbf{p})$ of all prices and densities on Ω, which is the true argument of her demand function.

Producers. As we have seen, the solution to the consumer problem gives rise to the location-specific individual demand functions $\mathbf{d}_{xy}$:

$$\mathbf{d}_{xy}(\mu, \mathbf{p}, \lambda) = D\left(\lambda_x(\mu, \mathbf{p}) \cdot p_y + \lambda_x(\mu, \mathbf{p})\tau(x, y)\right).$$

It shows how much a consumer at x buys from a firm located at y, under given prices $\mathbf{p}$ and market situation $\mu, \lambda(\mu, \mathbf{p})$. Each producer takes the demand functions and the level of competition λ as given when choosing her price to maximize its profit. Following the monopolistic competition literature, we assume a constant marginal cost m of production and a fixed cost F to operate in the market. In addition, for some special results we introduce a reduced form of the dispersion force into the model through the location-specific fixed cost $F_y = F(\mu_y)$, which is a non-decreasing function of the density of firms at location y. However, in the main part of our paper we treat the cost F as constant, assuming that its dependence on the density is weak enough to be neglected in our comparative statics analysis.[7]

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[7] Nevertheless, we introduce a dispersion force for two reasons. First, the dispersion forces are especially important conceptually in the interpretation relevant to economic geography. Indeed, concentration of firms in a particular location raises the price of land and increases the congestion costs. Second, theoretically, the presence of a dispersion force can eliminate potential instability of continuous uniform equilibria that we study. The latter consideration can be relevant, since preliminary computer simulations reveal that continuous equilibria need not be always stable (stability means that any small deviation of an equilibrium curve will return to the original curve).

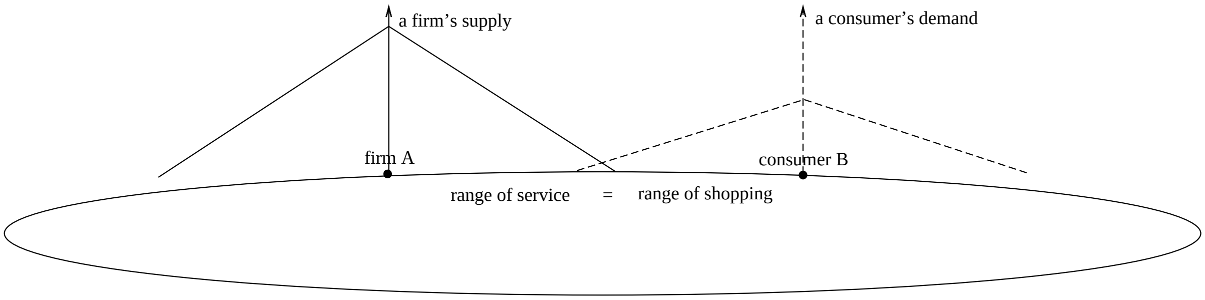

Figure 1: A circular space where each agent's demand/supply fades with distance.

Formally, a producer y chooses her price p_y to maximize her profit $\Pi_y(\cdot)$ as:

$$\max_{p_y \geq 0} \Pi_y(\mu, \mathbf{p}, \lambda) = \max_{p_y \geq 0} \ (p_y - m) \int_\Omega L\mathbf{d}_{xy}(\mu, \mathbf{p}, \lambda) dx - F(\mu). \tag{1}$$

Equilibrium. Entry into any location is free, so that profits must vanish at each location:

$$\Pi_y(\mu, \mathbf{p}, \lambda) = 0 \ \ \forall y. \tag{2}$$

Symmetric equilibrium is a bundle $\left\{p, \lambda, \mu, \{q_{xy}\}_{(x,y)\in\Omega^2}\right\}$ of price, competition level, density of firms and location-specific consumption quantities which solve all consumer and producer optimization programs, and satisfy the free-entry condition (2).[8] The labour balance in the economy follows from the budget constraint.

This general definition of equilibrium is valid for both versions of the model and both cases: full or partial coverage of the market by a firm. In the following analysis of each version, we shall specify the equilibrium definition in more detail, to simplify exposition in each case.

[Figure 1 is about here.]

Figure 1 illustrates the idea of continuous spatial competition: a (circular) space and each agent's demand/supply fading in distance. A firm's gross output equals the integral of its pointwise supply. The density of firms at the consumer's location determines the size of her consumption here. Similarly, the density of consumers provides the size of a firm's pointwise sales. That is why two triangles may have unequal height. However, they have equal base, because the range of any firm's service equals the distance, that consumers are willing to travel for shopping. On this picture, consumer B does not buy from firm A, located outside her range of shopping.

4 Equilibrium analysis

Full or partial coverage by service. From the complementary slackness condition of optimization, it can be seen that if the derivative $u'(0)$ is small enough relative to the distance cost t, then $q_{xy} = 0$ for all firms y located sufficiently far from x (zero demand). In the opposite case, it might be that $q_{xy} > 0$ for every pair x and y on space Ω (for instance, it must be the case when derivative $u'(0) = \infty$). We shall call the former case *partial coverage* because a firm does

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[8] In our accompanying paper, we also study asymmetric equilibria, where consumers need not locate uniformly and firms may behave differently even under symmetric consumer distribution.

not serve every consumer, and distinguish it *full coverage*; these are the two possible regimes of the model. As we show later, this distinction is quite important because the comparative statics of these two kinds of equilibria differs. To comprise both cases, the length $\hat{\theta}$ of coverage (radius of service) can be found as:

$$\hat{\theta}(p) = \min\left\{ \frac{1}{t}\left(\frac{u'(0)}{\lambda} - p \right), S \right\}.$$

4.1 Equilibria with partial coverage

We start our analysis with the case of partial coverage of the market by a firm. Because we consider only uniform equilibria (firms are identical up to rotation), it is sufficient to focus on a firm located at $y = 0$ and on its price $p \equiv p|_{y=0}$. Recall that the elementary demand function is $D(P) = u'^{-1}(P) \vee 0$ whenever the positive inverse of the marginal utility exists, otherwise demand is zero (here $P \equiv p + \tau(x, y)$). With this notation, the gross consumption per consumer Q and the firm's profit can be written as:

$$\Pi_{y=0}(p, \lambda) = (p - m) \cdot L \cdot Q(p, \lambda) - F(\mu)$$

where

$$Q(p, \lambda) \equiv 2 \int_0^{\hat{\theta}(p)} D(\lambda p + \lambda \tau(\theta, 0)) d\theta.$$

We should emphasize that when maximizing profit, producers take the intensity of competition λ as given. Here variable $\theta \equiv |x - y|$ denotes the consumer-producer distance, i.e., the shortest way from any consumer-type $\theta \in [0, \hat{\theta}]$ to a firm located at 0. Aggregate output LQ sold by the firm is the sum of quantities sold to all consumers between the limiting points $-\hat{\theta}(p)$ and $\hat{\theta}(p)$. Density L of consumers at each location factorizes the total output of the firm sold to all consumers served (LS is the total population).

Integral Q of the (inverse) derivative of u can be easily simplified for the case of linear cost function $\tau(\theta, 0) = t\theta$. Namely, we consider D (whose argument runs from the minimal "price" λp to the maximal "price" $\lambda p + \lambda t\hat{\theta}$), and argue that integrating D is the same as integrating its inverse u' whose argument runs from 0 to maximum value $q_0 = D(\lambda p)$, which is the maximal purchase occurring near the consumer's bliss-point. Essentially, instead of integrating consumer demand over the locations, we integrate it now over the quantity range. Technically, it amounts to substitution of variables: $q = D(\lambda p + \lambda t\theta)$, or changing the axis of integration in the price-quantity space (the demand "triangle"). Then, any firm's gross output LQ can be represented as

$$LQ(p, \lambda) = 2L \int_0^{\hat{\theta}(p)} D(\lambda p + \lambda t\theta) d\theta = \frac{2L}{\lambda t} \int_{D(\lambda p)}^{0} q d(D^{-1}(q) - \lambda p) = -\frac{2L}{\lambda t} \int_0^{D(\lambda p)} q du'(q) \quad (3)$$

$$= \frac{2L}{\lambda t} \left[-D(\lambda p) u'(D(\lambda p)) + \int_0^{D(\lambda p)} u'(q) dq \right] = \frac{2L}{\lambda t} [u(D(\lambda p)) - \lambda p D(\lambda p)].$$

This magnitude LQ is similar to the "consumer surplus" in spaceless IO models and decreases in p and λ. In fact, it is the surplus of the consumer located exactly at the firm's location $x = y$. Of course, this simplified aggregate demand structure relies on the assumption of linear transport cost $\tau(\theta) = t\theta$. Consequently, under uniform equilibrium with partial coverage of consumers, any producer's profit can be rewritten without an integral, and the free-entry condition becomes as simple as

$$\Pi(p,\lambda) = (p-m)\frac{2L}{\lambda t}[u(D(\lambda p)) - \lambda p D(\lambda p)] - F(\mu) = 0. \tag{4}$$

Differentiating such profit (4) w.r.t. price p, we arrive at the firm's first-order condition (FOC):

$$\Pi_p = \frac{2L}{\lambda t}\left[u(D(\lambda p)) - \lambda p D(\lambda p) - (p-m)\lambda D(\lambda p)\right] = 0. \tag{5}$$

Furthermore, differentiating the FOC expression (5) we get the producer's second-order condition for profit maximization:

$$\Pi_{pp} = \frac{2L}{t}\left[-(p-m)\lambda D'(\lambda p) - 2D(\lambda p)\right] < 0.$$

This strict inequality is assumed to hold in the neighborhood of equilibrium. It is guaranteed, whenever the elasticity of marginal utility is larger than 1/2.

Thus, the producer's optimality condition $\Pi_p(p,\lambda) = 0$ together with the free-entry condition $\Pi(p,\lambda) = 0$ determine the equilibrium pair of price and competition intensity (p,λ). Using them, other equilibrium magnitudes of interest can be obtained via the consumer's first-order condition and the budget constraint: consumption, density of firms and range of service.

For further analysis, now we rewrite the firm's first-order and free-entry conditions in (p, q_0) variables, where $q_0 = D(\lambda p)$ is the consumption of the ideal variety, instead of (p,λ), using the fact that $\lambda = \frac{u'(q_0)}{p}$. Then the firm's first-order and free-entry conditions can be conveniently reformulated as the link between markup $\frac{p}{m}$ and the elasticity of utility $\varepsilon_u(q_0) \equiv \frac{u'(q_0)q_0}{u(q_0)}$, involving also the fixed cost F:

$$\frac{1}{\varepsilon_u(q_0)} - 1 = 1 - \frac{m}{p}, \qquad (\frac{p}{m} - 1)^2 \frac{2Lm^2}{t} q_0 = F(\mu). \tag{6}$$

Expressing price $p = m + \sqrt{\frac{tF(\mu)}{2Lq_0}}$, these two can be reduced to a single equation w.r.t. quantity q_0 consumed "near the firm:"

$$\frac{1}{1 + \sqrt{\frac{tF(\mu)}{2m^2Lq_0}}} = 2 - \frac{1}{\varepsilon_u(q_0)}, \tag{7}$$

or an equation w.r.t. price:

$$1 = \left(2 - \frac{m}{p}\right) \cdot \varepsilon_u\left(\frac{F(\mu)t}{2(p-m)^2 L}\right). \tag{8}$$

Observe that positivity in equation (7) imposes condition $\varepsilon_u(q_0) > 1/2$ on the elasticity (otherwise no equilibrium exists), and that when cost $F(\mu) = F =$const., either of the two equations (7) and (8) is sufficient to find an equilibrium, otherwise both must be solved jointly with the consumer's budget constraint. Under a reasonable assumption that elasticity $\varepsilon_u \in (1/2, 1)$ decreases from $\varepsilon_u(0) = 1$, the right-hand side of (8) increases in p from a negative value to larger than 1, and therefore *a unique equilibrium exists* (under more general, increasing ε_u, we did not find any counterexample).

Also we note that under fixed cost F, a single exogenous "relative market size" parameter:

$$\hat{L} \equiv \frac{2m^2 L}{tF} \tag{9}$$

characterizes some market effects, notably, it governs all changes in the equilibrium consumption and mark-up. Now we summarize this equilibrium characterization.

Lemma 1. *Under linear distance cost $\tau(\theta) = t\theta$ and uniform equilibrium with partial coverage of consumers, any producer's profit takes the simple form (4), whereas the equilibrium price, quantity and intensity of competition are determined by (6), (7) and (8). Relative market size $\hat{L}$ (9), also reflecting three kinds of costs, is the main exogenous parameter, whereas absolute size S of space does not affect equilibrium.*

In other words, for equilibrium analysis, any impact of changes in cost composition F/m^2, transport cost t, and/or population L, altogether can be studied in the same fashion through varying parameter $\hat{L}$. The independence of equilibria from the *absolute size* S of the market is in sharp contrast with a spaceless economy, or the full coverage economy (a hybrid between the partial coverage and the spaceless economy). Ina geographical interpretation, such independence looks reasonable: whenever a firm trades only within some limited area, the competition on the other side of the globe becomes immaterial to it, as does the globe size, only the *population density* in the neighbourhood matters.

Comparative statics. We have characterized the equilibrium in the case of partial coverage by service. Now we study it regarding our question of interest: how does the equilibrium react to changes in the relative market size $\hat{L}$? In particular, should increasing market size or decreasing transport cost lead to *lower* prices through the intensified competition? The answer depends on the properties of utility function, as we find now.

In what follows, we extensively use the consumer's demand $q_0 = D(\lambda p)$ (denoting the utility-maximizing consumption of her ideal variety) and the elasticity $\varepsilon_u(q) = \frac{qu'(q)}{u(q)}$ of the elementary utility.[9] The next Proposition shows, that it is this elasticity characteristic ε_u that governs the comparative statics of prices; namely, that DEU leads to pro-competitive effects, whereas increasing $\varepsilon_u(q)$ leads to anti-competitive effects under increasing market size.

Proposition 2. *Consider a model with partial market coverage, and $F(\mu) = F =$const. Then, an increase in the local market size (population density) L or any increase in L/tF leads to: (i) an increase in the competition intensity λ; (ii) a decrease in the purchase of the ideal variety q_0; (iii) a price decrease (increase) – whenever elasticity $\varepsilon_u(\cdot)$ is a decreasing (increasing) function.*

Growing ratio "density over distance cost" L/t makes a firm's output LQ change opposite to the price.

Growing ratio "density over fixed cost" L/F leads to increasing entry of firms μ and a decreasing range of service $\hat{\theta}$ (i.e., the competition becomes more targeted to specific consumers), which, under DEU, implies increasing consumer's welfare.

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[9] Before proceeding to the proposition, we should address a caution that $\varepsilon_u(q)$ is not immune to affine transformations of the elementary utility function. At the first glance, under our assumption of separable additive aggregate utility, an affine transformation of $u(\cdot)$ must not change the equilibrium outcome. However, in the derivation of the aggregate demand for the firm's product, we have used a normalization assumption $u(0) = 0$. Without it, the result of our comparative statics analysis would depend on another, more cumbersome elasticity $\tilde{\varepsilon_u}(q) = \frac{qu'(q)}{u(q)-u(0)}$.

Proof. Since L/tF enters the equilibrium conditions jointly, any impact of the transport cost or fixed cost on price/quantity follows immediately from the results below regarding the population density L, which we focus on here.

From the first equilibrium condition in (6), we have $1/2 < \varepsilon_u(q_0) < 1$. Totally differentiating both conditions w.r.t. L, we obtain:

$$-\frac{\varepsilon_u'(q_0)}{\varepsilon_u^2(q_0)}\frac{dq_0}{dL} = \frac{m}{p^2}\frac{dp}{dL}$$

and

$$2Lq_0\frac{dp}{dL} + (p-m)L\frac{dq_0}{dL} + (p-m)q_0 = 0.$$

It follows from the first equation that price p and ideal quantity q_0 comove for DEU and move in the opposite direction for IEU. Combining the equations we get:

$$\left[p - m - 2q_0\frac{\varepsilon_u'(q_0)p^2}{\varepsilon_u^2(q_0)m}\right]\frac{dq_0}{dL} = -\frac{(p-m)q_0}{L} \tag{10}$$

Using the fact that $\varepsilon_u'(q_0) = \left(\frac{q_0u'(q_0)}{u(q_0)}\right)' = \frac{u'(q_0)u(q_0)+q_0u''(q_0)u(q_0)-q_0u'^2(q_0)}{u^2(q_0)}$ we can rewrite the expression in the square brackets as

$$\begin{aligned}
p - m - 2q_0\frac{\varepsilon_u'(q_0)p^2}{\varepsilon_u^2(q_0)m} &= p - m - 2q_0\frac{u'(q_0)u(q_0)+q_0u''(q_0)u(q_0)-q_0u'^2(q_0)}{q_0^2u'^2(q_0)}\frac{p^2}{m} \\
&= p - m - 2q_0\left[\frac{1}{q_0\varepsilon(q_0)} + \frac{u''(q_0)}{u'(q_0)}\cdot\frac{1}{\varepsilon(q_0)} - \frac{1}{q_0}\right]\frac{p^2}{m} \\
&= p - m - 2\left[\frac{1}{\varepsilon(q_0)} - 1\right]\frac{p^2}{m} - 2q_0\frac{u''(q_0)}{u'(q_0)}\cdot\frac{1}{\varepsilon(q_0)}\frac{p^2}{m} \\
&= p - m - 2\left(1 - \frac{m}{p}\right)\frac{p^2}{m} - 2q_0\frac{u''(q_0)}{u'(q_0)}\cdot\frac{1}{\varepsilon(q_0)}\frac{p^2}{m} \\
= (p-m)(1 - 2\frac{p}{m}) - 2q_0\frac{u''(q_0)}{u'(q_0)}\cdot\frac{1}{\varepsilon(q_0)}\frac{p^2}{m} &= (p-m)\frac{-1}{\varepsilon_u(q_0)}\cdot\frac{p}{m} - 2q_0\frac{u''(q_0)}{u'(q_0)}\cdot\frac{1}{\varepsilon(q_0)}\frac{p^2}{m} \\
&= -\frac{1}{\varepsilon_u(q_0)}\cdot\frac{p^2}{m}\cdot\frac{u''(q_0)}{u'(q_0)}\left[\frac{p-m}{p}\cdot\frac{u'(q_0)}{u''(q_0)} + 2q_0\right].
\end{aligned}$$

The firm's second-order condition can be expressed as $\frac{p-m}{p}\cdot\frac{u'(q_0)}{u''(q_0)} + 2q_0 > 0$, which is exactly the bracketed term. Thus, the bracketed term in (10) is positive, and hence $\frac{dq_0}{dL} < 0$, implying that consumption of the ideal variety always decreases with the population density. The result for the increasing/decreasing price behaviour under IEU/DEU follows from decreasing q_0 and equations (6). A firm's output LQ always changes opposite to price because of the free-entry condition $(p-m)LQ = F$.

Using $\Pi(p,\lambda) = 0$ together with $\Pi_p = 0$ and manipulating, we have

$$\frac{d\lambda}{dL} = -\frac{\Pi_L}{\Pi_\lambda} = -\frac{F(\mu)/L}{-\frac{1}{\lambda}F(\mu) + (p-m)\frac{2L}{\lambda t}[-pD(\lambda p)]} > 0.$$

This means that the intensity of competition λ increases with the population density regardless of the nature of preferences. In addition, we get

$$\frac{d\lambda}{dt} = -\frac{\Pi_t}{\Pi_\lambda} = -\frac{-F(\mu)/t}{-\frac{1}{\lambda}F(\mu) + (p-m)\frac{2L}{\lambda t}[-pD(\lambda p)]} < 0$$

and

$$\varepsilon_t^\lambda = -\frac{t}{\lambda}\frac{d\lambda}{dt} = \frac{F(\mu)}{F(\mu) + (p-m)\frac{2L}{t}[pD(\lambda p)]} = 1 - \varepsilon_u(q_0) < \frac{1}{2}$$

Thus, the intensity of competition increases when distance costs decrease. However, it does not increase too fast because $\varepsilon_t^\lambda < 1/2$ implies that both λt and $\lambda^2 t$ decrease when the distance cost decreases.

Now, we focus on $\hat{\theta}$. As q_0 decreases, $\lambda p = u'(q_0)$ increases and λ increases too. Therefore, the radius of service $\hat{\theta} = \frac{u'(0)-\lambda p}{\lambda t}$ decreases with the population density L.

Finally, consider the density of firms μ. Exploiting the consumer's budget constraint (through changing the integration variable), we have

$$\frac{1}{2\mu} = \int_0^{\hat{\theta}} (p + t\theta) D(\lambda p + \lambda t\theta) d\theta$$

$$= \frac{1}{\lambda t}\int_{D(\lambda p)}^{0} q\frac{u'(q)}{\lambda} du'(q) = \frac{1}{2t\lambda^2}\int_0^{D(\lambda p)} q d(-u'^2)$$

The latter integrand does not depend on any equilibrium variable, whereas the upper limit of integration $q_0 = D(\lambda p)$ decreases with market size, as we have proven. Therefore, the entire integral decreases. In addition, the intensity of competition λ increases, and thus, the increasing population density leads to additional entry, which means an increase in the density of firms μ. Consequently, under DEU, each consumer's welfare $U = \mu\int_\Omega u(q_{xy})dy$ increases because of more varieties and cheaper goods. **Q.E.D.**

These comparative statics results generally look intuitive. Higher consumer density should attract more firms to each location. This shift intensifies local competition and pushes the consumption of each individual variety down because more varieties are readily available to consumers. As a consequence, one would expect decreasing prices. Indeed, this is really the case under the natural DEU assumption. Essentially, we have classified all markets according to the elasticity ε_u into two categories: DEU-markets react pro-competitively to the relative market size (a drop in prices under higher competition), while those with IEU behave anti-competitively. It looks plausible, that the DEU case is more realistic, at least, it is widespread in theory. For instance, the widely used linear demand, CARA and HARA (hyperbolic-absolute risk aversion) utility functions, all generate DEU.[10] Notably, these three kinds of preferences generate similar pro-competitive effects in the usual spaceless monopolistic competition models as well, but for *a different reason:* not because of DEU, but because of IED, see ZKPT. In principle, a combination of the properties IED+DEU is common among preferences and considered natural but not guaranteed.

Why in spatial competition does increasing or decreasing elasticity of *utility* govern prices unlike increasing or decreasing *demand* elasticity in ZKPT? The difference stems from the

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[10] CES utility is neutral in this respect, but irrelevant for this subsection, being incompatible with partial coverage of consumers.

fact that now gross demand is the aggregate of the local demands u'^{-1} of various consumers (varying in distance from firms). Integrating u'^{-1} can be looked upon as integrating u', that is why maximizing profit looks like maximizing utility u. Put differently, what is crucial for price change is the elasticity of a firm's aggregate demand Q. This aggregate of heterogeneous demand does not directly inherit the properties of individual demand. In other respects, explanations for price change are the same: any sufficiently flat *gross* demand curve generates rather natural effects, whereas all too convex gross demands enable paradoxical price changes in response to increasing competition.

The impact of decreasing costs t on the radius $\hat{\theta}$ is more involved. Cheaper transport leads to an expansion of the radius directly, whereas equilibrium forces, as in the case of increasing population density, push the radius of service down indirectly. The sign of the net effect of these two effects is unclear. Analogously, the effect on the density of firms μ when the distance cost t decreases, is ambiguous.

4.2 Equilibria with complete coverage

When transport costs decrease, each firm eventually covers the entire market. However, unlike the ZKPT spaceless model, quantities are decreasing in distance to consumers. This case is difficult analytically because there are two different market operating modes. Intuitively, as the transport cost gets sufficiently small, the model converges to the spaceless one, and the comparative statics is governed by the elasticity of individual demand (marginal utility), as in the ZKPT model. On the other hand, when the space is "weakly" covered, i.e. consumption of the most remote varieties is very small, it behaves similar to the model with partial coverage, where comparative statics is governed mostly by the elasticity of elementary utility. Therefore, in this subsection the comparative statics should be between partial coverage and spaceless regimes, so that it depends on both the elasticity of utility and the elasticity of marginal utility. Because of this difficulty, here we provide incomplete characterization of the full-coverage regime, focusing only on the popular and arguably natural case of not too-convex demands: IED+DEU preferences (including CARA, HARA and quadratic utility, explained in the section "Pro-competitive preferences" after Proposition 2). We assume also naturally bounded demand elasticity $-\frac{qu''(q)}{u'(q)} < 1$ in equilibrium.

To study full coverage, in addition to the consumption quantity of an ideal variety $q_0 = D(\lambda p)$, we introduce a notation for the quantity of the least preferred variety $q_1 \equiv D(\lambda p + \lambda t S)$ on the circumference Ω. This (after changing the variables of integration in a similar fashion as before) allows us to express a firm's gross demand LQ as the difference between two consumer surpluses at q_0 and at q_1:

$$LQ(p,\lambda) = 2L\int_0^S D(\lambda p + \lambda t\theta)d\theta = \frac{2L}{\lambda t}\int_{q_0}^{q_1} q\,du'(q) \tag{11}$$

$$= \frac{2L}{\lambda t}\left[q_1 u'(q_1) - q_0 u'(q_0) - \int_{q_0}^{q_1} u'(q)dq\right] = \frac{2L}{\lambda t}[u(q_0) - \lambda p q_0 - u(q_1) + (\lambda p + \lambda t S)q_1].$$

In other words, the firm's total demand is proportional to the difference in consumer surpluses between its closest and its farthest consumers, where Q is the individual gross consumption per unit of firm density μ. Again, this relatively straightforward representation relies on the linear distance cost (this restrictive assumption is not uncommon in the literature). As in the case of partial coverage, the free-entry condition and the firms' profit maximizing behaviour (FOC) become the equilibrium conditions in (p, λ) variables:

$$\Pi(p, \lambda) = (p - m)LQ(p, \lambda) - F(\mu) = 0, \tag{12}$$

$$\Pi_p = LQ(p, \lambda) + (p - m)\frac{2L}{\lambda t}[-\lambda D(\lambda p) + \lambda D(\lambda p + \lambda tS)] = 0. \tag{13}$$

Under constant cost $F(\mu) = F$ (which we assume further), (p, λ) variables can be determined by these two conditions. Similar to the case studied before, consumption of varieties q_0 and q_1 and density of firms μ can be derived from (p, λ) using a consumer's optimality condition and the budget constraint.

Using $q_0 \equiv D(\lambda p)$ and $q_1 \equiv D(\lambda p + \lambda tS)$, all our equilibrium conditions can be transformed into four equations w.r.t. (λ, p, q_0, q_1):

$$\frac{p - m}{\lambda}[u(q_0) - u'(q_0)q_0 - u(q_1) + u'(q_1)q_1] = \frac{Ft}{2L}, \tag{14}$$

$$(q_0 - q_1)(p - m)^2 = \frac{Ft}{2L}, \qquad u'(q_0) = \lambda p, \qquad u'(q_1) = \lambda p + \lambda tS \tag{15}$$

that can be reduced to two explicit equations in q_0 and q_1, constructed from an arbitrary utility function $u(\cdot)$. It can be seen that (unlike the previous case) the size of the world S matters now, though the relative market size parameter L/F again plays an important role in comparative statics. Another comparative statics with distance cost t enables us to compare the partial coverage model with the spaceless model, which will be discussed after Proposition 2.

Now we conduct comparative statics with a certain restriction on the demand convexity or flatness at the equilibrium point:

$$p\frac{u(q_0) - u(q_1)}{u'(q_0)} > (p - m)\left[\frac{u'(q_1)}{u''(q_1)} - \frac{u'(q_0)}{u''(q_0)}\right]. \tag{16}$$

The analytical complexity of this kind of equilibria precludes a complete characterization of the comparative statics. Nevertheless, we show now that the market behaves pro-competitively when individual demand is relatively flat.

Proposition 3. *Consider a model with constant F and complete market coverage. An increase in the relative local market size L/F leads to an increase in the intensity of competition λ. It also leads to a decreasing price p (a pro-competitive reaction) if and only if condition (16) holds, which is guaranteed, when the ratio $-u''(q)/u'(q)$ is an increasing function. Moreover, each firm's output LQ changes oppositely to price under growing L/t.*

Proof. We may focus on the market reaction to L, since reaction to L/F is analogous (as one can see from equations (14), dependent on L/F fraction, rather than L and F separately). First, we study λ by totally differentiating the free-entry condition (13), exploiting itself and FOC $\Pi_p = 0$ in equilibrium. After some algebra, we can sign as follows:[11]

$$\frac{d\lambda}{dL} = -\frac{\Pi_L}{\Pi_\lambda} = \frac{F/L}{F(\mu)/\lambda + (p - m)\frac{2L}{\lambda^2 t}[\lambda p D(\lambda p) - (\lambda p + \lambda tS)D(\lambda p + \lambda tS)]} > 0,$$

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[11] Another way to know the sign of the total derivative $\lambda'_L > 0$ is to express $d\Pi/dL = (p-m)\cdot[Q'_\lambda \cdot \lambda'_L + Q/L] = 0$ and further exploit the negative partial derivative $Q'_\lambda < 0$.

where the inequality follows from the free-entry condition in the form (14):

$$\frac{F(\mu)}{(p-m)\lambda}+\frac{2L}{\lambda^2 t}[\lambda p D(\lambda p)-(\lambda p+\lambda tS)D(\lambda p+\lambda tS)]=\frac{2L}{\lambda^2 t}[u(D(\lambda p))-u(D(\lambda p+\lambda tS))]>0.$$

Also, plugging $\frac{(p-m)}{\lambda}[u(q_0)-u'(q_0)q_0-u(q_1)+u'(q_1)q_1]\frac{2L}{Ft}=1$ into $\mathcal{E}_\lambda$ we can express the total elasticity as

$$\mathcal{E}_\lambda\equiv\frac{d\lambda}{dL}\cdot\frac{L}{\lambda}=\frac{u(q_0)-u'(q_0)q_0-u(q_1)+u'(q_1)q_1}{u(q_0)-u(q_1)}.$$

Then, substituting the same expression, we get

$$\frac{q_0-q_1}{[u(q_0)-u'(q_0)q_0-u(q_1)+u'(q_1)q_1]^2}=\frac{2L}{\lambda^2 Ft}.$$

Second, totally differentiating FOC $\Pi_p=0$, we express the cross-derivatives as: $\Pi_{pp}\frac{dp}{dL}+\Pi_{p\lambda}\frac{d\lambda}{dL}+\Pi_{pL}=0$. Since $\Pi_{pL}=L\Pi_p=0$, we have

$$\frac{dp}{dL}=-\frac{\Pi_{p\lambda}}{\Pi_{pp}}\cdot\frac{d\lambda}{dL}.$$

Since $\Pi_{pp}<0$ in equilibrium (because of the SOC) and $\frac{d\lambda}{dL}>0$ as established, the sign of the comparative statics of the price with respect to the market size coincides with the sign of the cross derivative $\Pi_{p\lambda}$ of the profit function. The last step is to characterize this sign:

$$\Pi_{p\lambda}=\frac{2L}{\lambda t}[-pD(\lambda p)+(p+t/2)D(\lambda p+\lambda t/2)]+$$

$$+(p-m)\frac{2L}{\lambda t}[-D(\lambda p)+D(\lambda p+\lambda t/2)-\lambda pD'(\lambda p)+(\lambda p+\lambda t/2)D'(\lambda p+\lambda t/2)].$$

We now rewrite this in terms of variables $q_0=D(\lambda p)=u'^{-1}(\lambda p)$ and $q_1=D(\lambda p+\lambda t/2)$ using the fact that $D'(\tilde{p})=\frac{1}{u''(D(\tilde{p}))}$:

$$\Pi_{p\lambda}\propto\frac{-q_0u'(q_0)+q_1u'(q_1)}{\lambda}+(p-m)\left[-q_0-\frac{u'(q_0)}{u''(q_0)}+q_1+\frac{u'(q_1)}{u''(q_1)}\right]$$

$$=\frac{u(q_1)-u(q_0)}{\lambda}+(p-m)\left[-\frac{u'(q_0)}{u''(q_0)}+\frac{u'(q_1)}{u''(q_1)}\right],$$

where we have used FOC in terms of q_1 and q_0. Now replacing $\lambda=u'(q_0)/p$ and simplifying, we come to (16) as a necessary and sufficient condition for $\Pi_{p\lambda}<0$, and hence $\frac{dp}{dL}<0$. Furthermore, our assumption of increasing function $-u''(q)/u'(q)$ yields also a weaker inequality (16), as we need.

A firm's output LQ always changes inversly to the margin $(p-m)$ under growing L because of the free-entry condition $(p-m)LQ=F$. **Q.E.D.**

Pro-competitive preferences Under condition (16), the comparative statics is pro-competitive, i.e. prices decrease with the increasing local market size, because of increasing competition. What does this condition mean? As we have said, it corresponds to a low (in some sense) convexity of demand. In particular, linear demand $D = a - bq$ (which is very flat) satisfies (16) because the right-hand side becomes negative, whereas the left-hand side is always positive. Similarly, the CARA utility given by $u(q) = 1 - e^{-\alpha q}$, also satisfies (16) because it has constant characteristic $-u''(q)/u'(q)$ and the right-hand side of (16) is zero. As for the more convex demand, generated by HARA utility $u(q) = (a + q)^\rho$ $(\rho < 1, a \geq 0)$, it has a decreasing characteristic $-u''(q)/u'(q)$ (as well as its particular case CES), nevertheless, after some tedious algebra one can show that condition (16) hold whenever $a > 0$.

The positive right-hand side need not dominate over the positive left-hand side in (16). In particular, calculations show that under CES the condition (16) reduces to $[1 - (x_1/x_0)^\rho]\,(1-\rho)/\rho > [1 - x_1/x_0]\,(p-m)/p$, which is not obviously satisfied. Unlike spaceless models, we could expect from CES a price-changing effect of a bigger market in a spatial model (CES must generate full coverage). However, plugging the CES utility into the full-coverage equilibrium conditions (14)-(15), we can reduce them to

$$\frac{p-m}{p} = \frac{1-\rho}{\rho}\left[\frac{1-\left(1+\frac{tS}{p}\right)^{\frac{\rho}{\rho-1}}}{1-\left(1+\frac{tS}{p}\right)^{\frac{1}{\rho-1}}}\right],$$

which under $t \to 0$ degenerates into the well-known condition on markup $\frac{p-m}{p} = 1 - \rho$ used in spaceless monopolistic competition. Importantly, this equation *does not* depend on population density L, but on distance cost t and size S of the world. This means that the CES utility is again, as in the spaceless model, a very special preference (avoiding some important market effects) not because of constant elasticity of demand, but because of constant elasticity of utility.

Convergence to a spaceless model. We would argue that full coverage is an *intermediate regime* between partial coverage and a spaceless economy. Indeed, one can see from (13) that when the distance cost t vanishes, the difference $q_0 - q_1$ vanishes too. This explains the convergence of the full-coverage equilibrium – to the usual spaceless equilibrium explored in ZKPT. This convergence is more evident from equation (11) which becomes $Q(p, \lambda) = 2LS \cdot D(\lambda p)$ under $t = 0$, where $2LS$ denotes the *total* population in the economy, whereas L denotes the population density. Under such an expression of output Q, both profit maximization and free-entry conditions take their usual spaceless form, as in ZKPT. Looking at a similar convergence in the opposite direction ($t \to \infty$), one can see that the full-coverage equilibrium equations in the form (11)-(12)-(13) degenerate into partial-coverage equations (3)-(4)-(5) at some stage of growing t, namely, when cost t is big enough to annihilate (the farthest from home) consumption $q_1 \to 0$.

[Figure 2 is about here.]

Similarly important for robust modelling is the below lemma. Under full coverage, it states that our necessary and sufficient condition (16) for a price decrease converges in $t \to 0$ to similar condition under spaceless competition, known from ZKPT: it requires increasing elasticity of the inverse demand: $\left[-\frac{qu''(q)}{u'(q)}\right]' > 0$ (see Appendix).

Lemma 4. *The pro-competitive condition (16) converges to the similar ZKPT condition* $\left[-\frac{qu''(q)}{u'(q)}\right]' > 0$*, when distance cost* $t \to 0$.

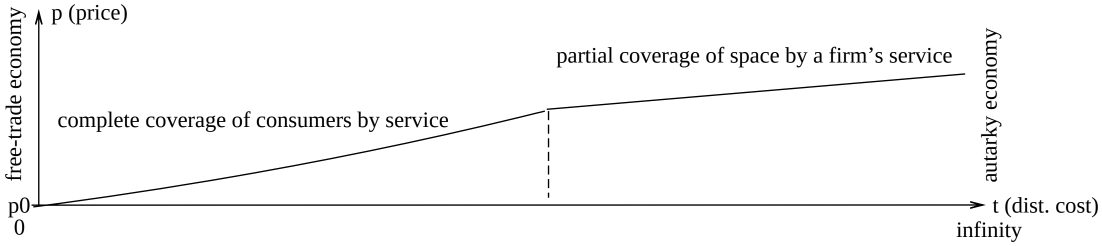

Figure 2: Evolution of price under growing distance cost t through 3 regimes: from usual spaceless competition ($t = 0$) through complete coverage, then through partial coverage to almost autarky.

4.3 Effects on welfare

As we argued above, our model highlights the new gains from market expansion. In addition to more available varieties and (potentially) cheaper products, consumers benefit from the greater availability of the varieties they prefer more, i.e., from a better match between produced and consumed varieties. This argument has important implications for the welfare of trade liberalization since this channel of gains from trade is largely unexplored in the literature. In this subsection, we use the model to formalize this argument.

To clarify intuition, we start with a particular example. Suppose individual preferences are linear-quadratic, thus, the elementary utility function has the form $u(q) = q - \frac{\gamma}{2}q^2$. Denote the number of varieties consumed by a particular consumer by M and the total consumption volume by $Q = \int q_i di$. Then, simple algebra shows that consumers utility is equal to:

$$U = \int u(q_i)di = Q - \frac{\gamma}{2}\frac{Q^2}{M} - \frac{\gamma}{2}M{\cdot}\mathrm{var}(q)$$

This aggregate utility decomposition is the first step towards establishing our argument. Here, var(q) denotes variance in consumption volumes of different varieties. Such utility decomposition does not depend on the spatial nature of our model. However, in the spaceless world of ZKPT all varieties are consumed in the same quantity so that the last term is equal to zero in equilibrium. The first two terms have the same interpretation in our model as in ZKPT: utility is increasing in the total consumption and in the number of varieties. Hence, comparative statics effects show that with the expansion of the market each consumer gains from increasing entry of firms and from (potentially) lower prices, manifested as an increase in the total consumption. Our model introduces an additional potential source of gains from a larger market: the increasing availability of the most preferred varieties can induce a consumer to concentrate her consumption on a smaller set of varieties leading to a decrease in the variance of her consumption bundle, and hence larger welfare. The next proposition confirms this intuition.

Proposition 5. *Consider a model with a monetary cost of distance with partial market coverage, $F(\mu) = F =$const and $u(q) = q - \frac{\gamma}{2}q^2$. Then, an increase in the local market size (population density) L leads to welfare gains from better fit of consumed varieties to consumer preferences, i.e., variance $2\mu\hat{\theta}var(q)$ decreases.*

Proof. First, we express the equilibrium conditions for a particular case of linear quadratic preferences. Since $u(q) = q - \frac{\gamma}{2}q^2$, we have $u'(q) = 1 - \gamma q$, $u''(q) = -\gamma$, $\varepsilon_u(q) = \frac{1-\gamma q}{1-\frac{\gamma}{2}q}$ and

$\varepsilon_u'(q) = \frac{\gamma/2}{(1-\frac{\gamma}{2}q)^2}$. With this at hand, equilibrium conditions become

$$\frac{1-\frac{\gamma}{2}q_0}{1-\gamma q_0} = 2 - \frac{m}{p} \qquad \text{and} \qquad (p-m)^2\frac{2L}{t}q_0 = F$$

In addition, consumer optimization implies that $q_x = q_0 - \lambda\frac{t}{\gamma}x$, and therefore, $\hat{\theta} = \frac{\gamma q_0}{\lambda t}$. The next step is to understand the behaviour of number of firms μ and variance of consumption volumes $\text{var}(q)$. We start with the number of firms. Budget constraint implies:

$$1 = 2\mu\int_0^{\hat{\theta}}(p+tx)q_x dx = \frac{2\mu}{\lambda}\int_0^{\hat{\theta}}u'(q_x)q_x dx = \frac{2\mu}{\lambda}\int_0^{\hat{\theta}}(q_x-\gamma q_x^2)dx = \frac{2\mu}{\lambda}\int_{q_0}^{0}(q-\gamma q^2)d\frac{-\gamma q}{\lambda t} = \frac{2\gamma\mu}{\lambda^2 t}\bigg(\frac{q_0^2}{2} - \gamma$$

Therefore,

$$\mu = \frac{\lambda^2 t}{\gamma q_0^2\left(1-\frac{2}{3}\gamma q_0\right)}$$

Now, we express the variance in consumption volumes as a function of equilibrium variables.

$$\text{var}(q) = \frac{1}{2\mu\hat{\theta}}2\mu\int_0^{\hat{\theta}}q_x^2 dx - \left(\frac{1}{2\mu\hat{\theta}}2\mu\int_0^{\hat{\theta}}q_x dx\right)^2 = \frac{1}{\hat{\theta}}\int_{q_0}^{0}q^2 d\frac{-\gamma q}{\lambda t} - \left(\frac{1}{\hat{\theta}}\int_{q_0}^{0}q d\frac{-\gamma q}{\lambda t}\right)^2$$

$$\text{var}(q) = \frac{\gamma}{\hat{\theta}\lambda t}\frac{q_0^3}{3} - \left(\frac{\gamma}{\hat{\theta}\lambda t}\frac{q_0^2}{2}\right)^2 = \frac{q_0^2}{3} - \frac{q_0^2}{4} = \frac{q_0^2}{12}$$

It immediately follows from this expression that variance in consumption volumes decreases with the expansion of the market because consumption of a single variety $q_0$decreases as we have shown before. However, to establish gains from better matches we need to show that variance adjusted for the number of varieties, $\mu\hat{\theta}\text{var}(q)$, decreases. From above it follows that $\mu\hat{\theta}\text{var}(q) = \frac{\lambda q_0/12}{1-\frac{2}{3}\gamma q_0}$.

To show that this expression is decreasing with the market size we need to show that

$$\frac{L}{\mu\hat{\theta}\text{var}(q)}\frac{d}{dL}\mu\hat{\theta}\text{var}(q) = \frac{L}{\lambda}\frac{d\lambda}{dL} + \frac{L}{q_0(1-\frac{2}{3}\gamma q_0)}\frac{dq_0}{dL} < 0 \tag{17}$$

We use the results, established in the Proposition 1:

$$\frac{L}{q_0}\frac{dq_0}{dL} = -\frac{(p-m)}{\left[p-m-2q_0\frac{\varepsilon_u'(q_0)p^2}{\varepsilon_u^2(q_0)m}\right]}$$

$$\frac{L}{\lambda}\frac{d\lambda}{dL} = \frac{p-m}{2p-m}$$

Thus,

$$\frac{L}{\mu\hat{\theta}\text{var}(q)}\frac{d}{dL}\mu\hat{\theta}\text{var}(q) = \frac{p-m}{2p-m} - \frac{(p-m)}{(1-\frac{2}{3}\gamma q_0)\left[p-m-2q_0\frac{\varepsilon_u'(q_0)p^2}{\varepsilon_u^2(q_0)m}\right]}$$

$$= \frac{p-m}{2p-m} - \frac{(p-m)}{(1-\frac{2}{3}\gamma q_0)\left[p-m-q_0\frac{\gamma p^2}{(1-\gamma q)^2 m}\right]} = \frac{p-m}{2p-m}\left(1 - \frac{2-m/p}{(1-\frac{2}{3}\gamma q_0)\left[1-m/p-q_0\frac{\gamma p}{(1-\gamma q)^2 m}\right]}\right)$$

$$= \frac{p-m}{2p-m} \left(1 - \frac{1-\gamma q_0/2}{(1-\gamma q_0)(1-\frac{2}{3}\gamma q_0)\left[\frac{\gamma q_0/2}{1-\gamma q_0} - q_0 \frac{\gamma}{(1-\gamma q)^2}\frac{1-\gamma q_0}{1-\frac{3}{2}\gamma q_0}\right]}\right) = \frac{p-m}{2p-m} \Bigg(1 - \frac{1-\gamma q_0/2}{(1-\frac{2}{3}\gamma q_0)\Big[\gamma q_0/2 -}$$

Therefore, $\mu\hat{\theta}\text{var}(q)$ is decreasing, whenever

$$\frac{1-\gamma q_0/2}{(1-\frac{2}{3}\gamma q_0)\left[\gamma q_0/2 - \frac{\gamma q_0}{1-\frac{3}{2}\gamma q_0}\right]} > 1$$

After some tedious algebra we arrive at the following equivalent condition:

$$\frac{7}{6}(\gamma q_0)^3 - \frac{17}{6}(\gamma q_0)^2 + \frac{5}{2}\gamma q_0 - 1 < 0$$

It is straightforward to verify that the left-hand side of this expression is increasing in γq_0 and the condition holds true for $\gamma q_0 = 1$. Since in equilibrium the marginal utility must be positive, γq_0 cannot exceed 1, and hence, the condition holds. Therefore consumers gain from better matches with the varieties consumed, not only from total consumption Q. **Q.E.D.**

5 Conclusions

This theoretical paper makes an attempt to bridge two traditions in modelling markets with horizontal product differentiation. Combining the Hotelling's (1929) "address economy" with Chamberlinian Dixit-Stiglitz (1977) monopolistic competition, we develop a model that features both spatial and price competition under variable elasticity of substitution among varieties of a differentiated product. The preference structure employed allows our consumers to have an ideal product and love for variety at the same time, consuming a range of varieties but in different quantities. This novelty intends to reflect real life, where consumers stick to their favourite types of product most of the time, but occasionally deviate from them. The model attempts to better formalize the idea, that love for variety observed in the aggregate demand stems not only from personal preference for variety, but also from heterogeneity of preferences, and therefore might appear stronger (or weaker) in the aggregate than on the individual level.

Despite its complexity, this approach turns out *tractable* in a number of important respects and cases. In particular, a uniform equilibrium displays clear analytical results when the product space is a circumference with symmetric (uniformly distributed) consumers, bearing linear distance costs, either in monetary or in utility terms. We show that in both these versions of our model, the market behaves pro-competitively under reasonable assumptions: *prices (and markups) decrease* in response to increasing relative local market size (population density). This implies the entry of additional firms — whenever demand is not too convex, which includes many natural additive preference specifications: CARA, HARA, etc. Specifically, under partial coverage of the market by service (when not every consumer buys from each firm), the necessary and sufficient condition for such a pro-competitive effect is the decreasing elasticity of the elementary utility— instead of the decreasing elasticity of its derivative, known in spaceless monopolistic competition. Another effect (unknown in spaceless competition) is the *shrinking range of service*: a thicker market and more intense competition makes each firm more targeted to their "core competence," i.e., to a specific consumer taste.

These two effects belong to the topics of interest in theory and empirical testing (Allen and Arkolakis 2014, Carballo et al. 2016). They closely relate also to discussion about the gains from trade or a thicker market (Arkolakis et al. 2015). Instead of variety being the only reason for gains under CES Dixit-Stiglitz model, or variety and cheaper goods in its VES version, our model sets a *better match* of goods to tastes as a new source of gains. Actually, economists always had this benefit in mind, when speaking about the representative consumer's love for variety, but now it is modelled explicitly, without proxies. This approach undermines the price index concept as an adequate measure of welfare gains.

What follows from these findings for theory is, first, a more rigorous notion of *partially localized market competition.* Yes, each firm directly competes mainly with few neighbours of her "core competency," but indirectly each firm competes with all other firms. Moreover, its high (low) markup has nothing to do with the large (small) mass of its direct competitors, or strategic behavior, unlike the common view in IO. The mass of direct (local) competitors may decrease through the decreasing range of service simultaneously with decreasing mark-up, and for reasonable demand shapes it is indeed the case. Second, when developed towards economic geography, this model should generate a continuous version of the "market potential" concept (at each point of space), like network models arising now. This may entail new continuous models of agglomeration. Third, some estimates of preferences and gains from a bigger market may be revised through heterogeneity, as we explained in the Introduction.

We believe that our framework will pave the way for the future research. If various extensions of our model turn out to be tractable, then the implications of this new modelling strategy could modify many topics of IO, trade, and economic geography. The reason for this hope is realism: this strategy enables us to treat competition as *partially localized*, and a firm's demand as an aggregate demand of *heterogeneous consumers.* These two features make a lot of difference for many economic questions, that may be revised now through continuous-spatial lenses. Maybe, after almost a century of numerous parallel developments in Hotelling's and Chamberlinian frameworks, and several fresh attempts to combine them, these two competing concepts of competition can be bridged in a more simple fashion.

Among the extensions left outside the scope of this paper, there is a need for multi-dimensional space, possibly with edges, for continuous models of economic geography, and for comparisons with data.

However, the most urgent extension is a possibility of non-uniform distribution of firms, even in the same homogenous circular economy. Can clusters or other spatial distributions of firms arise? It could be the case that free entry of firms leads to their grouping, or the standardization of products in the characteristic space—"minimal differentiation principle," as was believed by Hotelling. In economic geography, such an outcome would mean spatial agglomeration of firms, like shopping malls or cities, stemming from competition *per se*, without any additional agglomeration force! More formally, this is the question of the multiplicity of equilibria and the stability of the uniform equilibrium. Our preliminary inquiry shows that under very flat demands and monetary cost of distance, the uniform equilibrium can be unstable. Then, standardization (clusters of firms on the circumference) may occur as a typical stable equilibrium outcome. However, cumbersome clarification of this important issue is left for future work.

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Appendix

6 Analysis of disutility of distance

Consumers and varieties. Now we turn to an alternative formulation of our model. Instead of costly adjustment, or bearing a monetary cost for transporting home the varieties produced elsewhere, a consumer now obtains a reduced utility from consuming all varieties different from her "ideal variety" (in size or location). In other words, distance costs now enter the utility function rather then the budget constraint. In all other respects the setup remains the same as before. Now the consumer problem becomes

$$\max_{q_\theta > 0} 2\int_0^{\hat{\theta}} \mu(u(q_\theta) - q_\theta t\theta)d\theta \tag{18}$$

$$\text{s.t.} \quad 2\int_0^{\hat{\theta}} \mu p q_\theta d\theta = 1.$$

Here, as before, $\hat{\theta} \in (0, S]$ is the range of consumption, with $\hat{\theta} = S$ representing the case of full coverage, when a person consumes all varieties present. For a symmetric model, it makes no difference, to study location $x \equiv 0$ or any other, so, we focus on the distance between consumer 0 and producer θ. We denote, as before, the demand function $D(\cdot) \equiv {u'}^{-1}(\cdot) \vee 0$ (that equals the derivative whenever positive). Solving FOC, we obtain the demand for a variety (firm) θ given by

$$q_\theta = D(\lambda p + t\theta).$$

Here λ is again the Lagrange multiplier of the budget constraint, i.e. the marginal utility of income and, at the same time, the intensity of competition. One can observe the basic difference between the two setups: since costs of the mismatch between consumers and producers are now non-monetary, they are not multiplied by the marginal utility of money λ in the demand function. In other words, there is no need for the auxiliary "translation" of monetary costs into utility units.

Producers. As before, there is a continuum of producers, and each producer takes the intensity of competition λ and the demand schedule as given when maximizing her profit in price:

$$\max_{p \geq 0} \Pi(p, \lambda) = \max_{p \geq 0} 2(p - m)L \int_0^{\hat{\theta}(p)} D(\lambda p + t\theta)d\theta - F(\mu).$$

This producer's problem is similar to the previous, monetary-distance cost problem, only $\lambda t\theta$ has turned into $t\theta$. We again simplify the objective function using the change of the integration axes: instead of integrating over locations, we integrate consumption over quantities. This reformulation enables relatively simple aggregate demand representation. Thus, in the case of partial coverage $\hat{\theta} < S$, any firm's free-entry condition can be rewritten as

$$\Pi(p, \lambda) = 2(p - m)\frac{L}{t}[u(D(\lambda p)) - \lambda p D(\lambda p)] - F(\mu) = 0. \tag{19}$$

which in terms of q_0 becomes:

$$\Pi(q_0, \lambda) = 2\left[\frac{u'(q_0)}{\lambda} - m\right]\frac{L}{t}[u(q_0) - q_0 u'(q_0)] - F(\mu) = 0. \tag{20}$$

Analogously, for the case of full coverage ($\hat{\theta} = S$), profit takes the form:

$$\Pi(p,\lambda) = 2(p-m)\frac{L}{t}\left\{u(D(\lambda p)) - \lambda p D(\lambda p) - [u(D(\lambda p + tS)) - (\lambda p + t/2)D(\lambda p + tS)]\right\} \tag{21}$$
$$- F(\mu)$$

Equilibrium. As previously, we allow the firms to relocate in space and enter/exit the market. Thereby, in equilibrium profits must vanish at each location: $\Pi(p,\lambda) = 0$, see (19). This free-entry condition alongside with the firm's FOC optimality conditions define an equilibrium in (p,λ) variables. All other equilibrium variables $(q,\mu,\hat{\theta})$ can be derived from (p,λ) through the consumer's optimality condition and the budget constraint.

Symmetric equilibrium is a bundle $(p,\mu,\lambda,\mathbf{q},\hat{\theta})$ including price, mass of firms, marginal utility of income, consumption quantities, and the radius of service, that satisfies consumers' and producers' optimization conditions (including the budget constraint), and the free-entry condition.

6.1 Partial-coverage equilibria under disutility of distance

We start with the case of only partial coverage: $\hat{\theta} < S$. As in the monetary cost version of the model, we require the firm's second-order condition to hold in equilibrium:

$$\Pi_{pp}(p,\lambda) = -2 - \frac{p-m}{p}\cdot\frac{u'(q_0)}{q_0 u''(q_0)} < 0 \tag{22}$$

First, observe that the first- and second-order conditions (22) for profit maximization essentially do not differ from ones in the previous version of the model and the output is $Q = \frac{2L}{t}\left[u(D(\lambda p)) - \lambda p D(\lambda p)\right]$. Indeed, the only novelty in a firm's objective function is absent multiplier λ in expression $\lambda t\theta$. However, λ is treated parametrically by the producer, so, the logic remains the same. This observation (using previous analysis) allows for a straightforward characterization of equilibrium in variables (p,λ):

$$\frac{u(D(\lambda p))}{\lambda p D(\lambda p)} = 2 - \frac{m}{p}, \qquad (p-m)\left[u(D(\lambda p)) - \lambda p D(\lambda p)\right] = \frac{tF(\mu)}{2L}$$

or (using $q_0 = D(\lambda p)$) characterization in variables (p,q_0):

$$\frac{u(q_0)}{u'(q_0)q_0} = 2 - \frac{m}{p}, \qquad (p-m)\left[u(q_0) - u'(q_0)q_0\right] = \frac{tF(\mu)}{2L}. \tag{23}$$

Using $\varepsilon_u(q_0) \equiv \frac{u'(q_0)q_0}{u(q_0)}$, these equations can be reduced to a single equation

$$\left[\frac{1}{2-\frac{1}{\varepsilon_u(q_0)}} - 1\right]\left[u(q_0) - u'(q_0)q_0\right] = \frac{tF(\mu)}{2Lm}.$$

The latter, in the case of DEU , demonstrates an increasing left-hand side expression (conveniently for comparative statics).

The difference of new equilibrium equations from previous $2 - \frac{1}{\varepsilon_u(q_0)} = \frac{1}{1+\sqrt{\frac{tF(\mu)}{2m^2 L q_0}}}$ (or $\frac{1}{\varepsilon_u(q_0)} = 2 - m/p$ and $(p-m)^2 q_0 = \frac{tF(\mu)}{2L}$) — stems from the the absence of multiplier λ in one term of the free-entry condition. This relatively simple characterization of the equilibrium allows us to study the comparative statics with respect to the market size and the disutility cost.

Proposition 6. *Consider the model with disutility of distance, partial market coverage and fixed $F(\mu) = F$. Then an increase in the relative market size L/tF leads to: (i) an increase in the intensity of competition λ; (ii) a decrease in the purchase of the ideal variety q_0; (iii) a price decrease (increase) whenever $\varepsilon_u(\cdot)$ is a decreasing (increasing) function; (iv) each firm's output Q always changes opposite to price. (v) Expanding ratio L/F leads to a decrease in the service range $\hat{\theta}$ (to more localized competition), and to increasing density of firms μ, in this case decreasing price guarantees increasing welfare. (vi) A decrease in the distance cost t leads to more (less) entry, i.e. increasing (decreasing) μ—whenever $\varepsilon_u(\cdot)$ is a decreasing (increasing) function.*

Proof. We start again by noticing that elements of relative market size L/tF enters the free-entry condition only together (as this ratio) and we can focus on derivatives in L.

To study the intensity of competition λ, we totally differentiate in L the free-entry condition and express the result in partial derivatives:

$$\Pi_p \frac{dp}{dL} + \Pi_\lambda \frac{d\lambda}{dL} + \Pi_L = 0,$$

where $\Pi_p = 0$ because of the profit maximization. Therefore,

$$\frac{d\lambda}{dL} = -\frac{\Pi_L}{\Pi_\lambda} = \frac{F(\mu)/L}{2(p-m)LpD(\lambda p)/t} > 0,$$

so, the intensity of competition increases. Moreover,

$$\varepsilon_\lambda = \frac{p-m}{p} < 1.$$

To study q_0, we again study the equilibrium through quantities, and make use of the fact that consumption of a variety produced by the closest firm is $q_0 = D(\lambda p)$, so that $\lambda p = u'(q_0)$. The zero profit and free-entry conditions become

$$\frac{u(q_0)}{q_0 u'(q_0)} = 2 - m/p, \qquad (p-m)\left[u(q_0) - q_0 u'(q_0)\right] L = tF(\mu)/2$$

Totally differentiating them we obtain

$$-\frac{\varepsilon_u'(q_0)}{\varepsilon_u^2(q_0)} \cdot \frac{dq_0}{dL} = \frac{m}{p^2} \cdot \frac{dp}{dL}$$

and

$$\left[u(q_0) - q_0 u'(q_0)\right] L \cdot \frac{dp}{dL} - q_0 u''(q_0)(p-m)L \cdot \frac{dq_0}{dL} + \left[u(q_0) - q_0 u'(q_0)\right](p-m) = 0.$$

Again, from the first equation we see that price and quantity comove whenever the elasticity of utility is decreasing, but move opposite when the elasticity of utility is increasing. Combining the two equations we obtain:

$$\left[-\frac{\varepsilon_u'(q_0)}{\varepsilon_u^2(q_0)} \cdot \frac{p^2}{m} \cdot \left(u(q_0) - q_0 u'(q_0)\right) - q_0 u''(q_0)(p-m)\right] \cdot \frac{dq_0}{dL} = -\frac{\left[u(q_0) - q_0 u'(q_0)\right](p-m)}{L}$$

The right-hand side here is clearly negative. We now call the bracketed term on the left-hand side $[B]$, and study it using the fact that the firm's first-order condition entails $u(q_0)-q_0u'(q_0) = \frac{p-m}{p}q_0u'(q_0)$:

$$\begin{aligned}
[B] &= -\frac{\varepsilon_u'(q_0)}{\varepsilon_u^2(q_0)}\frac{p^2}{m}\left[u(q_0)-q_0u'(q_0)\right] - q_0u''(q_0)(p-m) \\
&= -q_0u''(q_0)\left[p-m+\frac{u'(q_0)u(q_0)+q_0u''(q_0)u(q_0)-q_0u'^2(q_0)}{q_0^2u'^2(q_0)}\cdot\frac{p^2}{m}\cdot\frac{u(q_0)-q_0u'(q_0)}{q_0u''(q_0)}\right] \\
&= -q_0u''(q_0)\left[p-m+\left(\frac{1}{q_0\varepsilon_u(q_0)}+\frac{u''(q_0)}{u'(q_0)\varepsilon_u(q_0)}-\frac{1}{q_0}\right)\frac{p^2}{m}\cdot\frac{p-m}{p}\cdot\frac{q_0u'(q_0)}{q_0u''(q_0)}\right] \\
&= -q_0u''(q_0)(p-m)\left[1+\left(\frac{1}{q_0\varepsilon_u(q_0)}+\frac{u''(q_0)}{u'(q_0)\varepsilon_u(q_0)}-\frac{1}{q_0}\right)\frac{p}{m}\cdot\frac{u'(q_0)}{u''(q_0)}\right] \\
&= -q_0u''(q_0)(p-m)\left[1+\frac{p}{m}\cdot\frac{1}{\varepsilon_u(q_0)}+\frac{1}{q_0}\left(\frac{1}{\varepsilon_u(q_0)}-1\right)\frac{p}{m}\cdot\frac{u'(q_0)}{u''(q_0)}\right] \\
&= -q_0u''(q_0)\cdot(p-m)\left[1+\frac{p}{m}(2-\frac{m}{p})+(1-\frac{m}{p})\frac{p}{m}\cdot\frac{u'(q_0)}{q_0u''(q_0)}\right] \\
&= -q_0u''(q_0)\cdot(p-m)\frac{p}{m}\cdot\left[2+\frac{p-m}{p}\cdot\frac{u'(q_0)}{q_0u''(q_0)}\right] > 0.
\end{aligned}$$

The latter expression has its first term clearly positive since $u''(\cdot) < 0$, while the bracketed term is positive because of the firm's second-order condition (the same as in the case of the monetary cost of distance), expressed as

$$2+\frac{p-m}{p}\cdot\frac{u'(q_0)}{q_0u''(q_0)} > 0. \tag{24}$$

Altogether it implies a negative impact $\frac{dq_0}{dL} < 0$, i.e. consumption of an ideal variety decreases with the market size. Then the decrease in price $\frac{dp}{dL} < 0$ follows from the discussion above and the second-order equilibrium condition (22) (the same as under disutility of distance). Output Q always changes opposite to price because of free-entry equation $(p-m)Q = F$.

The next parameter of interest is the range of service $\hat{\theta}$. To understand its behavior, consider the demand there: $D(\lambda p + t\hat{\theta}) = 0$, or alternatively $\lambda p + t\hat{\theta} = u'(0)$. The last step is to note that $\lambda p = u'(q_0)$, thus,

$$\hat{\theta} = \frac{1}{t}\left[u'(0)-u'(q_0)\right]$$

It immediately follows that the impact of the market size L on the range of service – replicates the impact on the consumption of ideal variety q_0. Hence, the range of service decreases with market size ($\frac{d\hat{\theta}}{dL} < 0$). Put differently, an increase in the relative market size leads to more localized competition.

The last question is the response of the mass of firms to a change in the relative market size. Recall the aggregate labor balance: $\mu S\left(mLQ+F\right) = SL$. It can be expressed alternatively as:

$$\mu = \frac{1}{mQ+F/L} = \frac{1}{\frac{m}{t}\left[u(q_0)-q_0u'(q_0)\right]+F/L}.$$

If density of population L increases, both summands in the denominator decrease, therefore, the density of firms reacts positively to the expansion of the labor market. To understand the reaction of the density of firms to the change in the disutility parameter, consider the consumer budget constraint in the following form:

$$1 = 2\mu p \int_0^{\hat{\theta}} D(\lambda p + t\theta)d\theta = 2\mu p \frac{u(q_0) - q_0 u'(q_0)}{t} = \mu p \frac{F}{(p-m)L}$$

From this it is evident that the density of firms comoves with the prices in response to the change in transport cost. Thus, whenever the elementary utility function has DEU property, a decrease in transport cost leads to an increase in the density of firms, and vice versa. Observe, that the analysis in the version of the model with the disutility of distance is unambiguous because there is now mechanical effect of saving labor on the cheaper transport.

Q.E.D.

Thus, we have shown that under partial coverage by service, both versions of the model exhibit similar comparative statics: the market is pro-competitive whenever the elasticity of utility is a decreasing function. Other variables also behave naturally: an increase in the market size (population density) intensifies competition, leads to smaller consumption of each varieties and more localized competition. The intuition behind such result remains the same independently of the model version, i.e., the explanations given for the case of transport cost can be repeated for this section too.

6.2 Full-coverage equilibria under disutility of distance

Now we consider properties and comparative statics of equilibrium when service coverage is full. As in the version of the model with monetary distance cost, full coverage is substantially less tractable analytically. Again, the difference from monetary cost, is λ disappearing from expression λtS. We again denote by $q_0 = D(\lambda p)$ the consumption of the ideal variety and by $q_1 = D(\lambda p + tS)$ the consumption of the least liked variety, produced at the opposite extreme of the circumference. The firm's gross output becomes $Q = \frac{2L}{t}[u(q_0) - \lambda p q_0 - u(q_1) + (\lambda p + tS)q_1]$.

Our reference point in the analysis will be the situation when a consumer buys from each and every firm because of sufficiently low transport cost $t \approx 0$. Constructing any point as a departure from $t \approx 0$, and using the Taylor series expansion we can express the consumer surplus from the least preferred variety as:

$$u(D(\lambda p + tS)) - (\lambda p + tS)D(\lambda p + t/2) = u(D(\lambda p)) - \lambda p D(\lambda p) - D(\lambda p)tS + o(t).$$

Substituting this expression back into the profit definition for the case of full coverage and using only the first order approximation, we effectively obtain an approximation of the profit function:

$$\Pi(p, \lambda) = 2(p-m)LSD(\lambda p) - F(\mu).$$

In other words, under $t \approx 0$ the model collapses to a case with no distance, studied in ZKPT. As ZKPT shows, in this case the behavior of the elasticity of marginal utility (rather than utility itself) defines the direction of comparative statics effects with respect to market size. This observation sheds light on the model behavior in between the two extreme cases, i.e. when firms serve all consumers but the disutility from shopping far away is not sufficiently small.

Now we turn to formal analysis of the comparative statics under full coverage. Differentiating the free-entry condition

$$\Pi(p,\lambda) = 2(p-m)\frac{L}{t}\left[u(D(\lambda p)) - \lambda p D(\lambda p) - u(D(\lambda p + tS)) + (\lambda p + tS)D(\lambda p + tS)\right] - F(\mu) = 0 \tag{25}$$

with respect to price, we obtain the firm's first-order condition (using $q_0 = D(\lambda p)$, $q_1 = D(\lambda p + tS)$):

$$\Pi_p(p,\lambda) = 2\frac{L}{t}\left[u(q_0) - \lambda p q_0 - u(q_1) + (\lambda p + tS)q_1\right] - 2(p-m)\frac{L}{t}\lambda \cdot (q_0 - q_1) = 0, \tag{26}$$

used to derive the SOC

$$0 > \frac{d}{dp}\left\{\left[u(D(\lambda p)) - \lambda p D(\lambda p) - u(D(\lambda p + tS)) + (\lambda p + tS)D(\lambda p + tS)\right] - (p-m)\lambda \cdot (D(\lambda p) - D(\lambda p +\right.$$

$$\begin{aligned}
&= -2\lambda \cdot (q_0 - q_1) - (p-m)\lambda \cdot (D(\lambda p) - D(\lambda p + tS))' \\
&= -2\lambda(q_0 - q_1) - (p-m)\lambda^2\left[\frac{1}{u''(q_0)} - \frac{1}{u''(q_1)}\right] \Rightarrow \\
&2(q_0 - q_1) > -\frac{(p-m)}{p} \cdot \frac{u'(q_0)}{u''(q_0)} \cdot \left[1 - \frac{u''(q_0)}{u''(q_1)}\right].
\end{aligned}$$

We assume it to hold in equilibrium, which is guaranteed under $u'''(q) \geq 0$.

Under constant cost F, we can express equilibrium conditions for (q_0, q_1, p, λ) as equations

$$u'(q_0) = u'(q_1) - tS = \lambda p,$$

$$\zeta(q_0, q_1) \equiv \frac{\left[u(q_0) - u'(q_0)q_0 - u(q_1) + u'(q_1)q_1\right]}{(q_0 - q_1)u'(q_0)} = 1 - \frac{m}{p}, \tag{27}$$

$$\frac{(p-m)^2}{p}u'(q_0) \cdot (q_0 - q_1) = \frac{Ft}{2L}. \tag{28}$$

The complexity of this model in the case of full coverage hampers the comparative statics, but at least we can formulate one more "flatness" condition that may hold (or not) at the equilibrium. Namely, function $\zeta(q_0, D(u'(q_0) + tS))$ introduced in (27) should be increasing in q_0, this condition being reformulated as

$$\begin{aligned}
&-q_0\frac{u''(q_0)}{u'(q_0)} \cdot \left[\frac{(q_0 - q_1)u'(q_0)}{u(q_0) - u'(q_0)q_0 - u(q_1) + u'(q_1)q_1} + 1\right] \\
&> \frac{1 - u''(q_0)/u''(q_1)}{1 - q_1/q_0} \quad \forall q_0, q_1 = D(u'(q_0) + tS).
\end{aligned} \tag{29}$$

This restriction is obviously satisfied for a linear demand (the left-hand side being positive, comparable with 0.5, whereas the right hand side is equal to zero) and, hopefully, for some other demands. Then, it is possible to guarantee the pro-competitive behavior of the market when demand is not too convex in this sense.

Proposition 7. *Consider the version of the model with disutility of distance, full market coverage and fixed* $F(\mu) = F$. *Then, (i) an increase in relative market size* L/tF *leads to an increase in the intensity of competition* λ. *(ii) Increasing ratio* L/F *implies decreasing consumption* q_0 *of the ideal variety, and whenever (29) holds, price* p *decreases (pro-competitive effect), while per-consumer output* Q *always changes opposite to price* p.

Proof. As before, we analyze our comparative statics in market size through variable L, and results for parameter L/tF follow. One can write down the free-entry condition as:

$$\Pi(p,\lambda,L) = \frac{2L}{t}(p-m)\left\{u(D(\lambda p)) - \lambda p D(\lambda p) - \left[u\left(D\left(\lambda p + \frac{t}{2}\right)\right) - (\lambda p + \frac{t}{2})D\left(\lambda p + \frac{t}{2}\right)\right]\right\}$$
$$-F = 0.$$

Totally differentiating it with respect to L we express the result in partial derivatives:

$$\Pi_p \frac{dp}{dL} + \Pi_\lambda \frac{d\lambda}{dL} + \Pi_L = 0.$$

The first term here is zero because of the profit maximization. Hence

$$\frac{d\lambda}{dL} = -\frac{\Pi_L}{\Pi_\lambda} = \frac{tF}{2(p-m)L^2 p(q_0 - q_1)} > 0,$$

whereas related total elasticity is

$$**\mathcal{E}_{\lambda/L}** = \frac{d\lambda}{dL}\cdot\frac{L}{\lambda} = \frac{tF}{2(p-m)L\lambda p(q_0-q_1)} = \frac{p-m}{p} < 1,$$

which we have obtained by plugging $\lambda p \cdot (q_0 - q_1) = \frac{Ft}{2L}\cdot\frac{p}{(p-m)^2}$ from (28). In other words, as intuitively expected, an increase in the market size leads to (not too fast) increase in the intensity of competition measured by the marginal utility of income.

Further, to study consumption q_0, the two equilibrium equations can become $\zeta(q_0, D(u'(q_0) + tS))p = (p-m)$ and

$$\zeta^2(q_0, D(u'(q_0)+tS)) \cdot \frac{u'(q_0)}{\lambda} u'(q_0)\cdot(q_0 - q_1) = \frac{Ft}{2L}$$

which, as a function of q_0, becomes

$$\frac{\left[u(q_0) - u'(q_0)q_0 - u(D(u'(q_0)+tS)) + u'(D(u'(q_0)+tS))D(u'(q_0)+tS)\right]^2}{(q_0 - D(u'(q_0)+tS))} = \frac{Ft\lambda}{2L}.$$

We know that total elasticity $\mathcal{E}_{\lambda/L} < 1$, so, the right-hand side decreases in L, and an increasing (in q_0) left-hand side here should be a *necessary and sufficient* condition for decreasing equilibrium value of q_0. Using tedious but straightforward algebra, one can show that the SOC guarantees the left-hand side is decreasing in q_0. Thus, q_0 decreases in L.

Further, increasing function $\zeta(q_0, D(u'(q_0)+tS))$ is a necessary and sufficient condition for comovement of price p and quantity q_0, because equation (27) has its right-hand side increasing in p. Taking the derivative

$$\frac{d\zeta}{dq_0} = \frac{-u''(q_0)(q_0-q_1)(q_0-q_1)u'(q_0) - \left[u''(q_0)(q_0-q_1) + u'(q_0)\left(1 - \frac{u''(q_0)}{u''(q_1)}\right)\right]}{\left[(q_0-q_1)u'(q_0)\right]^2},$$

one, after some algebra, finds that positivity $\zeta' > 0$ is exactly the condition (27) assumed. So, our comovement is established, and decreasing q_0 entails decreasing p.

Finally, output $Q = F/(p - m)$ always changes opposite to price. **Q.E.D.**

The sufficient condition (27) for pro-competitive effect, exploited in Proposition 4—includes linear demand and many other reasonable functional forms. We emphasize that it confirms general intuitive conclusion from the literature that flat demands should generate pro-competitive market effects.

6.3 Welfare: excessive or insufficient entry

One of the most prevalent questions in theory of product differentiation, is whether the market equilibrium leads to insufficient or excessive firms' entry. In this section, we provide an answer specific to our framework, through comparing market outcome with social optimum.

We focus on the second-best optimum: social planner can choose prices and the number of entrants, but firms should have non-negative profit. In other words, we don't allow for cross-subsidization and lump-sum transfers from consumers to firms. We focus on the second best because we find it more relevant to real life regulation: we believe that direct transfers from consumers to firms are politically implausible. We proceed through the following (traditional) steps: first, we set up social planner problem, and show that it can be reduced to an unconstrained choice of one variable — the consumption of an ideal variety. Second, we show that the market equilibrium can be represented analogously to social planner problem but with the objective function being aggregate revenue in the economy, rather than utility. Afterwards, we show how comparison of these two objective functions allows us to make conclusions about the relation between social optimum and equilibrium outcome, and finally, we relate this comparison back to the primitives of our model.

We focus on the partial coverage case for its analytical tractability and intuitive appeal. We first analyze the disutility of distance version, but show later on that results carry over to the case of monetary cost of transport. We start with the social planner problem.

Social planner chooses price p^o and firms' density μ^o at each point of the circumference. Consumers are free to allocate their budget across varieties. Since consumers are identical, we consider the behavior of consumer located at point 0. Denote $q^o(p^o, \mu^o, x)$ the demand (consumption) of a variety produced at x by a consumer located at 0. Social planner maximizes utility of this consumer (and by symmetry, of every other consumer) subject to zero-profit constraint.[12] Thus, social optimum problem can be formulated as follows:

$$\max_{p,\mu} \mu \int_{\Omega} u(q^o(p,\mu,x)) - txq^o(p,\mu,x)dx$$

$$\text{s.t.} \quad \mu(mL \int_{\Omega} q^o(p,\mu,x)dx + F) = L$$

Solution to this problem is the second-best optimum of price and mass of firms (p^o, μ^o). Now we show that the social planner problem can be characterized as a choice of only one variable — the consumption of ideal variety q_0^o, i.e. a variety produced at the consumer's location. First,

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[12] Although our original assumption is that profits must be non-negative, it is easy to see that at social optimum profits must be zero. Otherwise, social planner could keep the number of varieties fixed and decrease their prices, thereby generating additional utility without violating non-negative profit.

recall that from consumer's optimality it follows that $u'(q^o(x)) = u'(q_0^o) + tx$. Consequently, the consumption range is $\hat{\theta} = \frac{u'(0)-u'(q_0)}{t}$. As we have shown before

$$\int_0^{\hat{\theta}} q^o(p, \mu, x)dx = \frac{u(q_0^o) - q_0^o u'(q_0^o)}{t}$$

Analogously, total utility is

$$U(q_0^o) = \int_0^{\hat{\theta}} u(q^o(p, \mu, x)) - txq^o(p, \mu, x)dx = \frac{1}{t}\int_{q_0^o}^0 u(q) - (u'(q) - u'(q_0^o))qdu'(q)$$

$$= -\frac{1}{t}u(q_0^o)u'(q_0^o) + \frac{1}{t}\int_0^{q_0^o} u'(q)u'(q_0^o)dq - \frac{1}{t}\int_0^{q_0^o} u'(q)u''(q)qdq$$

$$= -\frac{1}{2t}\int_0^{q_0^o} qdu'(q)^2 = -\frac{1}{2t}q_0^o u'(q_0^o)^2 + \frac{1}{2t}\int_0^{q_0^o} u'(q)^2 dq$$

Thus, social planner problem can be expressed as an unconstrained choice of the consumption of the ideal variety:

$$\max_{q_0^o} \frac{2U(q_0^o)}{\frac{F}{L} + 2m\frac{u(q_0^o)-q_0^o u'(q_0^o)}{t}}.$$

Now, to compare the social optimum with the market outcome, we show that the market equilibrium can be viewed as the solution to the problem of revenue maximization. This property of equilibrium holds in a large class of spaceless monopolistic competition models. We now show that introduction of space does not eliminate this useful property. Define the **revenue maximization problem** as a choice of the price and the mass of firms that maximizes aggregate revenue in the economy subject to the labor balance:

$$\max_{p,\mu} \mu p \int_\Omega q^o(p, \mu, x)dx \tag{30}$$

$$\text{s.t.} \quad \mu(mL\int_\Omega q^o(p, \mu, x)dx + F) = L$$

The next proposition formally establishes our claim. Denote by $R(q_0^e) = u'(q_0^e)\left[u(q_0^e) - q_0^e u'(q_0^e)\right]/t$ the "normalized revenue" of a firm (up to a multiplier perceived as a constant), as a function of the consumption of ideal variety by consumer.

Proposition 8. *Consider a version of the model with disutility of distance and partial coverage by service. Then, the solution to the revenue maximization problem (30) generates the market outcome. Moreover, the revenue maximization problem can be expressed as an unconstrained choice of the consumption of ideal variety in the following way:*

$$\max_{q_0^e} \frac{2R(q_0^e)}{\frac{F}{L} + 2m\frac{u(q_0^o)-q_0^o u'(q_0^o)}{t}}.$$

Proof. The second part of the proposition is straightforward. Plugging the expression for $\int_0^{\hat{\theta}} q(p, \mu, x)dx$ from above into the constraint problem, plugging μ from the labor balance constraint and using the consumer's choice optimality $u'(q_0^e) = \lambda p$, we get the result.

To prove the first part of the proposition, we characterize the solution of modified problem, and show that its first-order condition coincides with market equilibrium conditions. For simpler exposition, we introduce the following notation: firm's cost as a function of consumption of ideal variety$C(q_0^e) = 2mL\frac{u(q_0^o)-q_0^o u'(q_0^o)}{t} + F$. Then the revenue maximization problem is $\max_{q_0^e} \frac{2LR(q_0^e)}{C(q_0^e)}$, with the first-order condition:

$$R'(q_0^e) - \frac{R(q_0^e)}{C(q_0^e)}C'(q_0^e) = 0$$

Now, from the firm's point of view, choices of price or an ideal variety are interchangeable. Thus, in equilibrium firms maximize profit $\Pi(q_0^e) = 2LR(q_0^e)/\lambda - C(q_0^e)$. The first-order condition for the firm is $2LR'(q_0^e) - \lambda C'(q_0^e) = 0$. This, together with the free-entry condition, yields the equivalence result desired. **Q.E.D.**

Formulating the market equilibrium outcome as a solution to the revenue maximization problem, analogous to the social planner problem, allows us to use the well-developed comparative statics machinery (see Dhinga-Morrow??). In particular, if the derivative of one objective function is everywhere greater than that of another, the former's argmaximum point is higher than the latter's. The next proposition formalizes this intuition, and links it to the primitives of the model.

Proposition 9. *Consider the version of the model with partial coverage and disutility of distance. Assume also that the objective functions of welfare maximization and revenue maximization are both quasi-concave. (a) If the elasticity of total utility is larger (smaller) than the elasticity of total revenue* $\varepsilon_U(q) < (>)\varepsilon_R(q)$*, then firms are too small (too big) in equilibrium* $q_0^e < (>) q_0^o$*, and there is excessive (insufficient) entry of firms. (b) Condition* $\varepsilon_U(q) < (>)\varepsilon_R(q)$ *holds whenever*

$$\int_0^q u'(z)^2 dz > (<) \frac{qu'(q)^2}{2\varepsilon_u(q) - 1}. \tag{31}$$

Proof. We start with the first part of proposition. Consider the case $\varepsilon_U(q) < \varepsilon_R(q)$. Observe, that in the first-order condition of the social planner problem can be represented as $\varepsilon_U(q_0^o) = \varepsilon_C(q_0^o)$, i.e. the elasticity of total utility is equal to the elasticity of the total cost. It implies that, $\varepsilon_R(q_0^o) - \varepsilon_C(q_0^o) > 0$, i.e. the derivative of the objective function of the revenue maximization problem is positive at q_0^o, and therefore it's maximizer $q_0^e < q_0^o$. Since from consumer optimization problem $q(x) = D(u'(q_0) + tx)$, consumption at every distance is smaller than optimal, and therefore total output is smaller than optimal, i.e. firms are too small. Finally if firms are too small, labor balance immediately implies that there is too many of them. That proves the first part of the proposition.

Now we prove the second part of the proposition, which links well-known comparative statics intuition with the primitives of our model. First,

$$\varepsilon_U(q) = \frac{qU'(q)}{U(q)} = \frac{-2q^2u'u''}{-qu'^2 + \int_0^q u'^2 dz}$$

Second,

$$\varepsilon_R(q) = \frac{qR'(q)}{R(q)} = **\frac{qu''(u-2qu')}{u'(u-qu')}$$

Thus,

$$\varepsilon_U(q) < \varepsilon_R(q) \iff \frac{-2q^2u'u''}{-qu'^2 + \int_0^q u'^2 dz} < \frac{qu''(u-2qu')}{u'(u-qu')}$$

$$\iff \frac{-2qu'}{-qu'^2 + \int_0^q u'^2 dz} < \frac{u - 2qu'}{u'\,(u - qu')}$$

$$\iff -quu'^2 < (u - 2qu') \int_0^q u'^2 dz$$

$$\iff \frac{qu'^2}{2\varepsilon_u(q) - 1} < \int_0^q u'^2 dz. **$$

This completes the proof. **Q.E.D.**

Thus, in the Dixit-Stiglitz topic of "optimal product diversity" (as well as in comparative statics), we also observe some similarity with the spaceless competition and some difference. The role of elasticities of aggregate revenue and aggregate utility in the economy is the same. However, now these two are non-trivially linked to properties of individual demand, as in (31). Indeed, integrating a function we rarely keep all its properties (if it is not an exponent).

Turning to examples, one may ask: Which "natural" utilities bring excessive or insufficient firms' entry??

7 Equivalence of the models in the limit

Here we show that with $t \to 0$, not only the model converges to the spaceless monopolistic competition version of ZKPT, but also the comparative statics result converges as well.

Preliminaries. In ZKPT, the direction of the effect of the market expansion depends on the sign of the derivative of the relative love for variety $r_u(q) = -\frac{qu''(q)}{u'(q)}$, with $r'_u(q) > 0$ implying pro-competitive behavior. We are going to use the fact that

$$r'_u(q) = -\frac{u''(q)u'(q) + qu'(q)u'''(q) - q(u''^2}{(u'^2}.$$

As we show in the proof of Proposition 2, the pro- or anti-competitive effect of market expansion depends on the sign of

$$\Pi_{p\lambda} = \frac{u(q_1) - u(q_0)}{\lambda t} + \frac{p - m}{t}\left[\frac{u'(q_1)}{u''(q_1)} - \frac{u'(q_0)}{u''(q_0)}\right] \tag{32}$$

, with $\Pi_{\lambda p} < 0$ implying pro-competitive behavior. First, observe that $u'(q_1) = u'(q_0) + \lambda t S$. Thus for $t \to 0$, $q_1 - q_0 \to 0$. Moreover, using $q_1 = D(u'(q_0) + \lambda t S)$ and $D' = 1/u''$, thus, using Taylor expansion

$$q_1 = q_0 + \frac{\lambda S}{u''(q_0)} t + o(t^2) \tag{33}$$

Analogously,

$$u(q_1) = u(D(u'(q_0) + \lambda t S)) = u(q_0) + u'(q_0)\frac{\lambda S}{u''(q_0)} t + o(t)$$

and, using (33),

$$\frac{u'(q_1)}{u''(q_1)} = \frac{u'(q_0)}{u''(q_0)} + \frac{(u''(q_0))^2 - u'(q_0)u'''(q_0)}{(u''(q_0))^2} \cdot \frac{\lambda S}{u''(q_0)} t + o(t).$$

Plugging all these Taylor representations of q_1 and its functions together into (32) we arrive at:

$$\Pi_{p\lambda} \propto \frac{u'(q_0)}{u''(q_0)} S + (p-m)\frac{(u''(q_0))^2 - u'(q_0)u'''(q_0)}{(u''(q_0))^2} \cdot \frac{\lambda S}{u''(q_0)} + O(t).$$

Now, we need to establish limits of p for $t \to 0$, using the fact that $q_0 = D(\lambda p)$. As we have shown, in the limit $Q(\lambda, p) = 2LSD(\lambda p) = 2LSq_0$, and thus the first-order condition $\Pi_p = Q(p,\lambda) + (p-m)\frac{2L}{\lambda t}[-\lambda D(\lambda p) + \lambda D(\lambda p + \lambda t S)] = 0$ takes the form

$$2LSq_0 + 2L(p-m)\frac{\lambda S}{u''(q_0)} = 0.$$

$$1 = -\frac{(p-m)}{p} \cdot \frac{u'(q_0)}{q_0 u''(q_0)}.$$

And thus, at the limit $\frac{p-m}{p} = r_u(q_0)$ as in ZKPT. Plugging this formula into recent expression for the cross-derivative $\Pi_{p\lambda}$, in the limit $t \to 0$, we get:

$$\Pi_{p\lambda} \propto \frac{u'(q_0)}{u''(q_0)} + \frac{-q_0 u''(q_0)}{u'(q_0)} \frac{(u''(q_0))^2 - u'(q_0)u'''(q_0)}{(u''(q_0))^2} \frac{u'(q_0)}{u''(q_0)}$$

$$= \left(\frac{u'(q_0)}{u''(q_0)}\right)^2 \left[\frac{u''(q_0)}{u'(q_0)} - \frac{q_0(u''(q_0))^2 - q_0 u'(q_0)u'''(q_0)}{(u'(q_0))^2}\right]$$

$$= \left(\frac{u'(q_0)}{u''(q_0)}\right)^2 \frac{u''(q_0)u'(q_0) - q_0(u''(q_0))^2 + q_0 u'(q_0)u'''(q_0)}{(u'(q_0))^2} = -r'_u(q_0)\left(\frac{u'(q_0)}{u''(q_0)}\right)^2.$$

The sign of this expression governs price decrease, in the direction that we needed to prove. **Q.E.D.**

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