NATIONAL RESEARCH UNIVERSITY
HIGHER SCHOOL OF ECONOMICS

Kristian Behrens, Yasusada Murata

ON QUANTITATIVE SPATIAL ECONOMIC MODELS

BASIC RESEARCH PROGRAM
WORKING PAPERS

SERIES: ECONOMICS
WP BRP 184/EC/2018

On quantitative spatial economic models*

Kristian Behrens† Yasusada Murata‡

March 7, 2018

Abstract

Quantitative spatial economics (QSE) specifies various components such as preferences, production technology, and frictions for the movement of goods, people, and ideas. Despite the long literature on endogenous location decisions, the question of how these specifications affect resulting spatial equilibria has not been systematically explored. In this paper we start with workhorse models of QSE based on different specifications of preferences and show that spatial equilibria in those models can be generated using the conditional logit model by McFadden (1974). Our result suggests that existing models of QSE have a common origin in one of the oldest location choice models.

Keywords: quantitative spatial economics; location choice; logit; spatial equilibrium

JEL Classification: F12; F14; R12; R13

*We thank Esteban Rossi-Hansberg and other participants at the 7th European Meetings of the Urban Economics Association (Copenhagen), the 6th International Conference "Industrial Organization and Spatial Economics" (Saint Petersburg), the 12th North-American Meetings of the Urban Economics Association (Vancouver), and the 2017 SMU Conference on Urban and Regional Economics (Singapore) for valuable comments and suggestions. Behrens gratefully acknowledges financial support from the CRC Program of the Social Sciences and Humanities Research Council (SSHRC) of Canada for the funding of the Canada Research Chair in Regional Impacts of Globalization. Murata gratefully acknowledges financial support from JSPS KAKENHI Grant Numbers JP26285061 and JP16H02018. The study has been funded by the Russian Academic Excellence Project '5-100'. All remaining errors are ours.

†Department of Economics, Université du Québec à Montréal (ESG-UQAM), Canada; National Research University Higher School of Economics, Russian Federation; and CEPR, UK. E-mail: behrens.kristian@uqam.ca

‡Nihon University Population Research Institute (NUPRI); University Research Center, Nihon University, Japan; and National Research University Higher School of Economics, Russian Federation. E-mail: murata.yasusada@nihon-u.ac.jp

1 Introduction

Perhaps the defining characteristic of spatial economics is that agents choose their location endogenously. There are two different strands of literature on endogenous location decisions. The new economic geography such as Krugman (1991) and Helpman (1998) typically analyzes location choices of homogeneous workers, whereas the conditional logit model by McFadden (1974) consists of a common utility part and an idiosyncratic term, thus allowing consumers to have heterogeneous preferences over different locations.

These seminal works have paved the way for quantitative spatial economics (QSE). As stated by Redding and Rossi-Hansberg (2017), QSE requires a structural model that combines various components such as preferences, production technology, and frictions for the movement of goods, people, and ideas. Despite the long literature on endogenous location decisions, the question of how the specifications of these components affect resulting spatial equilibria has not been systematically explored.

In this paper we start with one component of first-order importance, namely, preferences, and show that the McFadden (1974) model using idiosyncratic preferences, together with a spatial equilibrium condition, can generate the Helpman (1998) model based on common preferences.[1] Our result sheds new light on QSE, as these two different models have been quantified independently by many authors. For example, the latter model (or its variation) has been taken to data by Hanson (2005), Redding and Sturm (2008), and Davis and Ortalo-Magné (2011), whereas the former has been quantified more recently by Brülhart et al. (2012), Diamond (2016), and Behrens et al. (2017).

Our result implies that the McFadden (1974) model and the Helpman (1998) model can deliver the same equilibrium distribution of population across regions. The other variables—such as regional wages and price indices—are determined conditional on the population distribution. Hence, these variables can also be the same between the two models if the underlying

[1]Unlike the Krugman (1991) model, where the spatial equilibrium involves a corner solution (or full agglomeration in a single region), there are generically no such corner solutions in the Helpman (1998) model. This may explain why the latter has been widely used for quantitative analysis.

assumptions (e.g., production technology, trade costs, etc.) are the same. Furthermore, other things being equal, a shock in fixed costs, trade costs, total population, or the supply of the nontraded good can lead to the same change in the equilibrium spatial distribution of population in both the McFadden (1974) model and the Helpman (1998) model. Thus, these two different models can predict the same quantitative change in equilibrium wages and price indices.

We find that the specification of the common utility part in the McFadden (1974) model is crucial for these results. Indeed, other existing models of QSE can also be generated from the McFadden (1974) model using different specifications of the common utility part. Thus, without loss of generality, we can focus on one of the oldest location choice models, thereby reducing the number of spatial economic models that have to be quantified. Our result is useful since, as illustrated by Redding and Rossi-Hansberg (2017), there are too many combinations of different components to quantify each of all possible models of QSE. It also allows us to sharpen the quantitative analysis by indicating when different models of QSE yield the same quantitative results.

Our work is closely related to the literature on product differentiation showing that discrete choice models with heterogeneous agents can generate the same aggregate demand as representative agent models (see Anderson et al., 1992; Thisse and Ushchev, 2016). Yet, our paper on location choice is different from the existing work on product choice. In their aggregate demand analysis, representative agents consume all available varieties, whereas heterogeneous agents choose only one variety. In our paper, both representative and heterogeneous consumers choose only one location.

The rest of the paper is organized as follows. Section 2 introduces the model of common preferences and that of heterogeneous preferences. In Section 3 we compare these two models and derive our main result. Section 4 turns to the related literature, where we deal with other existing models of QSE and discuss the similarities and differences between QSE and aggregate demand analysis. Section 5 concludes.

2 Models

Consider an economy with R regions, indexed by $r = 1, ..., R$. Assume that there is a mass L of consumers, each of whom chooses one of the regions and supplies one unit of labor inelastically. We assume that each region is endowed with an exogenous stock of a nontraded good, $H_r > 0$, in perfectly inelastic supply.[2] We start with the Helpman (1998) model with common preferences and then turn to the McFadden (1974) model with idiosyncratic preferences.

2.1 Common preferences

Consider the Helpman (1998) model based on common preferences. Let $V_r = D_r^{1-\mu} h_r^{\mu}$, denote the utility of choosing region r, where $D_r = [\sum_{s=1}^{R} \int_{\Omega_{sr}} d_{sr}(\omega)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega]^{\frac{\sigma}{\sigma-1}}$ is the consumption index of a differentiated traded good with Ω_{sr} and $d_{sr}(\omega)$ being the set of varieties and the quantity of variety ω produced in region s and consumed in region r, $\sigma > 1$ is the elasticity of substitution between any pair of varieties, $h_r = H_r / L_r$ is the consumption of the nontraded good (e.g., amenity, housing, or local public services), L_r is the population in region r, and μ is the expenditure share of the nontraded good.

Each variety is produced by a monopolistically competitive firm that incurs fixed and marginal costs, as well as trade costs, in terms of labor, which is the sole factor of production. We assume that trade costs have an iceberg form, τ_{sr}, where $\tau_{sr} > 1$ for $s \neq r$ and $\tau_{sr} = 1$ for $s = r$. A firm in region s thus maximizes the profit $\pi_s(\omega) = \sum_{r=1}^{R} [p_{sr}(\omega) - \tau_{sr} m w_s] d_{sr}(\omega) - w_s F$, where w_s, m, and F are the wage rate in region s and the marginal and fixed labor requirements, respectively. As a result, the profit-maximizing price is given by $p_{sr}(\omega) = [\sigma/(\sigma-1)]\tau_{sr} m w_s$.

In this setting, a larger population in region r lowers the price index of the traded good $P_r = [\sum_{s=1}^{R} \int_{\Omega_{sr}} p_{sr}^{1-\sigma}(\omega) \mathrm{d}\omega]^{1/(1-\sigma)}$ and increases the consumption index D_r because consumers purchase a greater range of local varieties without incurring trade costs. Thus, the presence

[2] The nontraded good is interpreted as housing in Helpman (1998) or amenity in Redding and Strum (2008).

of the traded good induces an agglomeration force. In contrast, a larger L_r reduces the consumption of the nontraded good h_r, which generates a dispersion force. The relative strength of the agglomeration and dispersion forces is governed by the expenditure share μ.

The location choice problem of the representative agent is given by $\max_{r=1,\ldots,R}\{V_r\}$. The consumption index D_r is positive and finite for all regions as long as trade costs are finite. Since H_r is also positive and finite, we have $\lim_{L_r\to 0} V_r = \infty$, which implies that each region attracts at least some population. Thus, with finite trade costs and with the expenditure share being $\mu \in (0,1)$, it follows that $V_r = V_s$ for any r and $s \neq r$. Hence, in such a case, the spatial equilibrium is defined as the population distribution $\{L_r\}_{r=1}^R$ satisfying

$$D_r^{1-\mu}\left(\frac{H_r}{L_r}\right)^{\mu} = D_s^{1-\mu}\left(\frac{H_s}{L_s}\right)^{\mu} \quad \Rightarrow \quad \left(\frac{D_r}{D_s}\right)^{\frac{1-\mu}{\mu}}\frac{H_r}{H_s} = \frac{L_r}{L_s}. \tag{1}$$

In the extreme case where μ goes to zero, consumers choose a location based solely on the consumption of the differentiated good. Since the price index is lower in the larger region, all consumers end up in a single region. However, in the other extreme case where μ goes to one, consumers care only about the consumption of the nontraded good, which leads to the equalization of H_r/L_r across regions.

In what follows we relate (1) to the spatial equilibrium condition based on the McFadden (1974) model of location choice.

2.2 Idiosyncratic preferences

Consider the McFadden (1974) model of idiosyncratic preferences. Let $V_r^\ell = U_r + \varepsilon_r^\ell$ denote the utility of agent ℓ from choosing region r, where U_r and ε_r^ℓ are the common utility part and the idiosyncratic term that follows a Gumbel distribution with zero mean and variance equal to $\pi^2\beta^2/6$. The choice probability of region r is then given by

$$\mathbb{P}_r = \Pr\left(V_r^\ell > \max_{s\neq r} V_s^\ell\right) = \frac{\exp(U_r/\beta)}{\sum_{s=1}^R \exp(U_s/\beta)}. \tag{2}$$

A larger U_r increases the choice probability of region r, which induces agglomeration. In the extreme case where β goes to zero, consumers choose the region with the highest U_r with probability one. However, in the other extreme case where β goes to infinity, the choice probability becomes $\mathbb{P}_r = 1/R$ for all r, regardless of the distribution $\{U_r\}_{r=1}^R$. Thus, β governs the dispersion force, like the expenditure share μ for the nontraded good in the case of common preferences.

In the case of idiosyncratic consumers, the spatial equilibrium $\{L_r\}_{r=1}^R$ is given by the condition that the choice probability of region r equals the population share of region r as follows (see Murata, 2003; Kline and Moretti, 2014; Behrens et al., 2017):[3]

$$\mathbb{P}_r = \frac{\exp(U_r/\beta)}{\sum_{s=1}^R \exp(U_s/\beta)} = \frac{L_r}{\sum_{s=1}^R L_s}. \tag{3}$$

Taking the ratio between regions r and $s \neq r$, the spatial equilibrium condition implies that

$$\frac{\exp(U_r/\beta)}{\exp(U_s/\beta)} = \frac{L_r}{L_s}. \tag{4}$$

In what follows, we compare the two spatial equilibrium conditions: expression (4) based on idiosyncratic preferences; and expression (1) based on common preferences.

3 Comparison

3.1 Spatial equilibrium

We have so far derived the two spatial equilibrium conditions (1) and (4). Since the right-hand side is identical in both expressions, the spatial equilibrium condition (4) using the McFadden

[3]Tabuchi and Thisse (2002) consider two types of agents: workers follow the choice probability (2); and farmers are immobile and equally distributed between regions. Thus, in their case farmers are excluded from L_r in (3). Alternatively, farmers may be regarded as following (2) with $\beta = \infty$. Diamond (2016) assumes that the variance of the idiosyncratic term differs across demographic groups. Brülhart et al. (2012) analyze a "two-country" case, where (3) holds for all regions within each country.

(1974) model boils down to (1) in the Helpman (1998) model when

$$\frac{\exp(U_r/\beta)}{\exp(U_s/\beta)} = \left(\frac{D_r}{D_s}\right)^{\frac{1-\mu}{\mu}} \frac{H_r}{H_s}.$$

The foregoing expression is satisfied if

$$\frac{U_r}{\beta} = \ln\left[\kappa\left(D_r^{\frac{1-\mu}{\mu}} H_r\right)\right] \quad \Rightarrow \quad U_r = \frac{\beta}{\mu}\ln\left[\kappa^{\mu}\left(D_r^{1-\mu} H_r^{\mu}\right)\right], \tag{5}$$

where $\kappa > 0$ is a constant. Hence, we can establish the following proposition.

Proposition 1 *Assume that $U_r = \beta \ln \kappa + (\beta/\mu)\ln(D_r^{1-\mu} H_r^{\mu})$. Then, the spatial equilibrium condition* (4) *using the McFadden (1974) model generates the spatial equilibrium condition* (1) *in the Helpman (1998) model. Hence, other things being equal, these two models yield the same spatial equilibria $\{L_r\}_{r=1}^{R}$ for any given set of parameter values.*

Several comments are in order. First, the spatial equilibrium condition (1) in the Helpman (1998) model may be viewed as a special case of (4) based on the McFadden (1974) model with the common utility part being $U_r = \beta \ln \kappa + (\beta/\mu)\ln(D_r^{1-\mu} H_r^{\mu})$. In that sense, the spatial equilibrium condition (4) is more flexible than (1) since the former does not impose such a restriction.

Second, when the spatial equilibrium condition using the McFadden (1974) model boils down to the spatial equilibrium condition in the Helpman (1998) model, these two conditions yield the same spatial equilibria $\{L_r\}_{r=1}^{R}$. The other variables such as price indices, wages, and rents are determined conditional on $\{L_r\}_{r=1}^{R}$, so that those variables are also the same if the underlying assumptions (e.g., production technology, trade costs, land ownership, etc.) are the same. In such a case, all equilibrium conditions (including the spatial equilibrium condition) are identical between the two models.

Third, if these underlying assumptions are the same, Proposition 1 further implies that for any given shock in fixed costs F, trade costs τ_{rs}, population L, or the supply of the nontraded good H_r, both the McFadden (1974) model and the Helpman (1998) model predict the same quantitative change in the equilibrium allocation. This quantitative property is in sharp contrast to the qualitative result in Robert-Nicoud (2005) who finds common features such as comparative statics and the number and stability of equilibria in different spatial economic models with homogeneous agents.

Fourth, the common utility part of the McFadden (1974) model in Proposition 1 can be rewritten as $U_r = \beta \ln \kappa + (\beta/\mu) \ln(D_r^{1-\mu} H_r^{\mu}) = \beta \ln \kappa + (\beta/\mu) \ln[(D_r^{1-\mu} h_r^{\mu}) L_r^{\mu}] = \beta \ln \kappa + (\beta/\mu) \ln V_r + \beta \ln L_r$. Thus, when $\kappa = 1$ and $\beta = \mu$, we have $U_r = \ln(D_r^{1-\mu} H_r^{\mu}) = \ln V_r + \beta \ln L_r$. Hence, the common utility part U_r in the McFadden (1974) and the utility V_r in the Helpman (1998) model differ by $\beta \ln L_r$ (except that V_r is log-transformed). The reason is that when $\mu > 0$, the nontraded good is congestible in the Helpman (1998) model, whereas it is not in the McFadden (1974) model. We show in Section 3.2 that a similar result holds when comparing the change in the (expected) equilibrium utilities in both models.

Last, the common utility part U_r in the McFadden (1974) model does not have to be of the Cobb-Douglas upper-tier and CES-lower tier form. For example, Behrens et al. (2017) assume that $U_r = \ln[\exp(\mu_0 + \mu_1 D_r^{\text{VES}} + \mu_2 H_r^o + \mu_3 H_r^u)]$, where D_r^{VES} is the utility from consuming the differentiated good based on a variable elasticity of substitution (VES) specification, H_r^o and H_r^u capture the observed and unobserved amenities, and μ_0, μ_1, μ_2, and μ_3 are parameters. Hence, as already mentioned, the McFadden (1974) model is more flexible than the Helpman (1998) model.

3.2 Welfare

Even when the McFadden (1974) model and the Helpman (1998) model generate the same spatial equilibrium, the welfare change driven by some shock can be different for the following reasons. First, since $V_r = V_s$ holds for any r and $s \neq r$ in the Helpman (1998) model, the (expected) equilibrium utility is given by $V^* = \mathbb{E}[\max_s V_s] = \max_s V_s$. Note that there are no random variables in the Helpman (1998) model.

Second, Small and Rosen (1981) show that the expected utility $\mathbb{E}\left[\max_s V_s^{\ell}\right]$ in the Mc-

Fadden (1974) model is given by:[4]

$$\mathbb{E}\left[\max_s \left(U_s + \varepsilon_s^\ell\right)\right] = \beta \ln \left[\sum_{s=1}^{R} \exp\left(\frac{U_s}{\beta}\right)\right] = \beta \ln \left[\kappa \sum_{s=1}^{R} \left(D_s^{\frac{1-\mu}{\mu}} H_s\right)\right].$$

Note that the last equality is obtained from (5), which holds when the McFadden (1974) model and the Helpman (1998) model yield the same spatial equilibrium condition. The spatial equilibrium condition in the Helpman model implies that $D_s^{1-\mu}(H_s/L_s)^\mu = D_r^{1-\mu}(H_r/L_r)^\mu$ holds for all $s = 1, ..., R$, so that

$$\mathbb{E}\left[\max_s \left(U_s + \varepsilon_s^\ell\right)\right] = \beta \ln \left\{\kappa \sum_{s=1}^{R} \left[D_s^{1-\mu} \left(\frac{H_s}{L_s}\right)^\mu L_s^\mu\right]^{\frac{1}{\mu}}\right\} = \beta \ln \left\{\kappa \left[D_r^{1-\mu} \left(\frac{H_r}{L_r}\right)^\mu\right]^{\frac{1}{\mu}} L\right\},$$

where $L = \sum_{s=1}^{R} L_s$ is the total population. Recalling that V^* denotes the equilibrium utility in the Helpman (1998) model, the expected equilibrium utility in the McFadden (1974) model can be rewritten as

$$\mathbb{E}\left[\max_s \left(U_s + \varepsilon_s^\ell\right)\right] = \beta \ln \left[\kappa (V^*)^{\frac{1}{\mu}} L\right] = \frac{\beta}{\mu} \ln V^* + \beta \ln(\kappa L). \tag{6}$$

Note that when $\kappa = 1$ and $\beta = \mu$, the foregoing expression can be simplified as follows:

$$\mathbb{E}\left[\max_s \left(U_s + \varepsilon_s^\ell\right)\right] = \ln V^* + \beta \ln L, \tag{7}$$

which is different from the equilibrium utility V^* in the Helpman (1998) model.

As seen from (7), except that V^* is log-transformed, the (expected) equilibrium utility in the Helpman (1998) model and the expected equilibrium utility in the McFadden (1974) model differ by $\beta \ln L$, conditional on the same spatial equilibrium. This difference depends on the size of the economy L and the 'degree of taste heterogeneity' β.

Thus, for the same positive shock to population L in both models, utility increases less

[4] This definition of the expected utility is different from the one in Desmet, Nagy, and Rossi-Hansberg (2018) who consider the expected utility conditional on choosing a particular region. Our expected utility does not impose this condition, i.e., it is computed before location choices are made.

in the Helpman (1998) model. The reason is that the consumption of the nontraded good is congestible in the Helpman (1998) model, i.e., V_r and thus V^* depend on $h_r = H_r/L_r$. However, it is not congestible in the McFadden (1974) model, i.e., U_r depends on H_r as can be seen from Proposition 1. Hence, even though both models generate the same change in the equilibrium allocation, their welfare implications are different.

Last, when $\kappa = 1$ and $\beta = \mu$, we can rewrite (6) as follows

$$\mathbb{E}\left[\max_s \left(U_s + \varepsilon_s^\ell\right)\right] = \ln\left[D_r^{1-\mu}(H_r/L_r)^\mu\right] + \beta \ln L = U_r^* + \beta \ln\left(\frac{R\overline{L}}{L_r}\right) \quad \forall r,$$

where U_r^* is the common utility part in the McFadden (1974) model evaluated at equilibrium, and where $\overline{L}$ denotes the average population size of the R regions, so that $L = R\overline{L}$. As one can see, if $L_r = \overline{L}$ for all r, i.e., all regions have the same size, then the utility increases in the number of regions R. This is a standard feature of discrete choice models, where the utility is increasing in the range of the choice set.

4 Related literature

In this section, we deal with other existing models of QSE and discuss the similarities and differences between QSE and aggregate demand analysis.

4.1 Armington model

Allen and Arkolakis (2014) develop a model of QSE based on Armington (1969). In their model, the utility function is specified as $V_r = D_r H_r L_r^{-\widetilde{\mu}}$. The spatial equilibrium condition is then given by

$$D_r H_r L_r^{-\widetilde{\mu}} = D_s H_s L_s^{-\widetilde{\mu}} \quad \Rightarrow \quad \frac{(D_r H_r)^{1/\widetilde{\mu}}}{(D_s H_s)^{1/\widetilde{\mu}}} = \frac{L_r}{L_s}, \tag{8}$$

where the right-hand side is the same as that of (4). Thus, the spatial equilibrium condition (4) using the McFadden (1974) model boils down to the spatial equilibrium condition (8) in the Allen and Arkolakis (2014) model when

$$\frac{\exp(U_r/\beta)}{\exp(U_s/\beta)} = \frac{(D_r H_r)^{1/\widetilde{\mu}}}{(D_s H_s)^{1/\widetilde{\mu}}},$$

which implies

$$\frac{U_r}{\beta} = \ln\left[\widetilde{\kappa}(D_r H_r)^{1/\widetilde{\mu}}\right] \quad \Rightarrow \quad U_r = \frac{\beta}{\widetilde{\mu}} \ln\left[\widetilde{\kappa}^{\widetilde{\mu}}(D_r H_r)\right],$$

where $\widetilde{\kappa} > 0$. Hence, we can establish the following proposition.

Proposition 2 *Assume that* $U_r = \beta \ln \widetilde{\kappa} + (\beta/\widetilde{\mu}) \ln(D_r H_r)$. *Then, the spatial equilibrium condition* (4) *using the McFadden model (1974) generates the spatial equilibrium condition* (8) *in the Allen and Arkolakis (2014) model. Hence, other things being equal, these two models yield the same spatial equilibria* $\{L_r\}_{r=1}^R$ *for any given set of parameter values.*

Note that Proposition 2 may be viewed as a variation of Proposition 1. Indeed, when $\widetilde{\kappa} = 1$ and $\beta = \widetilde{\mu}$, the common utility part U_r in the McFadden (1974) model reduces to $\ln(D_r H_r)$. This is consistent with the Helpman (1998) model as seen from the discussion after Proposition 1.

4.2 Fréchet model

We have so far considered the McFadden (1974) model, i.e., the additively separable model, $V_r^\ell = U_r + \varepsilon_r^\ell$, with respect to the common utility part U_r and the idiosyncratic term ε_r^ℓ, where the latter follows a Gumbel distribution.

Alternatively, we can think of a multiplicatively separable case, $\widetilde{V}_r^\ell = \widetilde{U}_r \times \widetilde{\varepsilon}_r^\ell$. Assuming that $\widetilde{\varepsilon}_r^\ell$ follows a Fréchet distribution, $\Pr(\widetilde{\varepsilon}_r^\ell \leq \varepsilon) = \exp(-\varepsilon^{-1/\widetilde{\beta}})$, the choice probability of region r is given by

$$\widetilde{\mathbb{P}}_r = \Pr\left(\widetilde{V}_r^\ell > \max_{s \neq r} \widetilde{V}_s^\ell\right) = \frac{\widetilde{U}_r^{1/\widetilde{\beta}}}{\sum_{s=1}^R \widetilde{U}_s^{1/\widetilde{\beta}}},$$

so that the spatial equilibrium is given by

$$\widetilde{\mathbb{P}}_r = \frac{\widetilde{U}_r^{1/\widetilde{\beta}}}{\sum_{s=1}^R \widetilde{U}_s^{1/\widetilde{\beta}}} = \frac{L_r}{\sum_{s=1}^R L_s}.$$

Taking the ratio between regions r and s, the spatial equilibrium condition implies that

$$\frac{\widetilde{U}_r^{1/\widetilde{\beta}}}{\widetilde{U}_s^{1/\widetilde{\beta}}} = \frac{L_r}{L_s}, \tag{9}$$

where the right-hand side is the same as that of (4). Thus, the spatial equilibrium condition (4) based on the Gumbel distribution boils down to the spatial equilibrium condition (9) based on the Fréchet distribution when

$$\frac{\exp(U_r/\beta)}{\exp(U_s/\beta)} = \frac{\widetilde{U}_r^{1/\widetilde{\beta}}}{\widetilde{U}_s^{1/\widetilde{\beta}}},$$

which implies

$$\frac{U_r}{\beta} = \ln\left(\widetilde{\kappa}\widetilde{U}_r^{1/\widetilde{\beta}}\right) \quad \Rightarrow \quad U_r = \frac{\beta}{\widetilde{\beta}}\ln\left(\widetilde{\kappa}^{\widetilde{\beta}}\widetilde{U}_r\right),$$

where $\widetilde{\kappa} > 0$ is a constant. Hence, we can establish the following proposition.

Proposition 3 *Assume that* $U_r = \beta \ln \widetilde{\kappa} + (\beta/\widetilde{\beta}) \ln \widetilde{U}_r$. *Then, the spatial equilibrium condition* (4) *based on the Gumbel distribution boils down to the spatial equilibrium condition* (9) *based on the Fréchet distribution. Hence, other things being equal, these two models yield the same spatial equilibria* $\{L_r\}_{r=1}^R$ *for any given set of parameter values.*

Note that when $\widetilde{\kappa} = 1$ and $\beta = \widetilde{\beta}$, the common utility part U_r in the McFadden (1974) model reduces to $\ln \widetilde{U}_r$. There are several models of location choice based on the Fréchet distribution, among others by Ahlfeldt et al. (2015) and Redding (2016) who focus on the Cobb-Douglas (or Cobb-Douglas upper-tier and CES lower-tier) specification for $\widetilde{U}_r$.

4.3 Aggregate demand analysis

We have so far considered the location choice problem. Turning to the product choice, there is a long literature showing that the McFadden (1974) model with heterogeneous consumers can generate the same aggregate CES demand derived from representative consumers (see

Anderson et al., 1992; Thisse and Ushchev, 2016). In this literature on product differentiation, representative consumers are assumed to purchase all varieties, whereas heterogeneous consumers choose only one variety. This differs from our analysis on location choice because both representative and heterogeneous consumers choose only one location.

Furthermore, in the Helpman (1998) model, consumers are ex ante identical, but they end up choosing different locations. In the aggregate demand analysis, representative agents consume an identical set of varieties, whereas heterogeneous agents consume a different variety. For these reasons, our result is different from the relationship between the CES and the logit in the existing literature on industrial organization.

5 Concluding remarks

In this paper we have shown that the conditional logit model by McFadden (1974), together with a spatial equilibrium condition, can generate workhorse models of QSE based on various specifications of preferences. Our result suggests that existing models of QSE have a common origin rooted in one of the oldest location choice models. This result is useful since there are too many models of QSE to quantify all of them. It also allows us to sharpen the quantitative analysis by indicating when different models of QSE yield the same quantitative results.

References

[1] Ahlfeldt, G., S. Redding, D. Sturm, N. Wolf. 2015. The economics of density: Evidence from the Berlin wall. *Econometrica* 83, 2127-2189.

[2] Allen, T., C. Arkolakis. 2014. Trade and topography of the spatial economy. *Quarterly Journal of Economics* 129, 1085-1140.

[3] Anderson, S., A. de Palma, J.-F. Thisse. 1992. *Discrete Choice Theory of Product Differentiation.* MIT Press.

[4] Armington, P.S. 1969. A theory of demand for products distinguished by place of production. *International Monetary Fund Staff Papers* 16, 159-178.

[5] Behrens, K., G. Mion, Y. Murata, J. Südekum. 2017. Spatial frictions. *Journal of Urban Economics* 97, 40-70.

[6] Brülhart, M., C. Carrère, F. Trionfetti. 2012. How wages and employment adjust to trade liberalization: Quasi-experimental evidence from Austria, *Journal of International Economics* 86, 68-81.

[7] Davis, M.A., F. Ortalo-Magné. 2011. Housing expenditures, wages, rents. *Review of Economic Dynamics* 14, 248-261.

[8] Desmet, K., D.K. Nagy, E. Rossi-Hansberg. 2018. The geography of development. *Journal of Political Economy*, forthcoming.

[9] Diamond, R. 2016. The determinants and welfare implications of US workers' diverging location choices by skill: 1980-2000. *American Economic Review* 106, 479-524.

[10] Hanson, G.H. 2005. Market potential, increasing returns, and geographic concentration. *Journal of International Economics* 67, 1-24.

[11] Helpman, E. 1998. The size of regions. In: D. Pines, E. Sadka, I. Zilcha (eds.), Topics in Public Economics. Theoretical and Empirical Analysis, Cambridge University Press, pp.33-54.

[12] Kline, P. and E. Moretti. 2014. People, places, and public policy: Some simple welfare economics of local economic development programs, *Annual Review of Economics* 6, 629-62.

[13] McFadden, D. 1974. Conditional logit analysis of qualitative choice behavior. In: P. Zarembka (ed.), Frontiers in Econometrics. New York: Academic Press, pp.105-42.

[14] Murata, Y. 2003. Product diversity, taste heterogeneity, and geographic distribution of economic activities: Market vs. non-market interactions, *Journal of Urban Economics* 53, 126-144.

[15] Redding S.J. 2016. Goods trade, factor mobility and welfare. *Journal of International Economics* 101, 148-167.

[16] Redding, S.J., E. Rossi-Hansberg. 2017. Quantitative spatial economics. *Annual Review of Economics* 9, 21-58.

[17] Redding, S.J., D.M. Sturm. 2008. The costs of remoteness: Evidence from German division and reunification. *American Economic Review* 98,1766-1797.

[18] Robert-Nicoud, F. 2005. The structure of simple 'New Economic Geography' models (or, On identical twins). *Journal of Economic Geography* 5, 201-234.

[19] Small, K.A., H.S. Rosen. 1981. Applied welfare economics with discrete choice models. *Econometrica* 49, 105-130.

[20] Tabuchi, T., J.-F. Thisse. 2002. Taste heterogeneity, labor mobility and economic geography, *Journal of Development Economics* 69, 155-177.

[21] Thisse, J.-F., P. Ushchev. 2016. When can a demand system be described by a multinomial logit with income effect? *HSE Working papers WP BRP 139/EC/2016*, National Research University Higher School of Economics.