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Quantifying the gap between equilibrium and optimum under monopolistic competition*

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Abstract

Equilibria and optima generally differ in imperfectly competitive markets. While this is well understood theoretically, it is unclear how large the welfare distortions are in the aggregate economy. Do they matter quantitatively? To answer this question, we develop a multi-sector monopolistic competition model with endogenous firm entry and selection, productivity, and markups. Using French and British data, we quantify the gap between the equilibrium and optimal allocations. In our preferred specification, inefficiencies in the labor allocation and entry between sectors, as well as inefficient selection and output per firm within sectors, generate welfare losses of about 6–10% of GDP.

Keywords: monopolistic competition; welfare distortions; equilibrium versus optimum; inefficient entry and selection; inter- and intra-sectoral allocations

JEL Classification: D43; D50; L13.

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1 Introduction

In imperfectly competitive markets, equilibria and optima generally differ in many respects such as the number of firms and firm-level outputs. While this is well understood theoretically, it is unclear how large the welfare losses from these distortions are in the aggregate economy. Do they matter quantitatively? To answer this question, we develop a multi-sector model of monopolistic competition with endogenous firm entry and selection, productivity, and markups. Using data from France and the United Kingdom (UK), we quantify the gap between the equilibrium and optimal allocations, and document patterns of inter- and intra-sectoral distortions that translate into welfare losses of about 6–10% of GDP. The welfare costs of monopolistic competition are hence sizable.

The theoretical literature on equilibrium versus optimum allocations under monopolistic competition dates back at least to Dixit and Stiglitz (1977). They analyze the tradeoff between product diversity and output per firm as a source of inefficiencies in general equilibrium models with unspecified utility functions. More recently, Zhelobodko, Kokovin, Parenti, and Thisse (2012) introduce heterogeneous firms into those models, and Dhingra and Morrow (2017) show that markets generally deliver a socially inefficient selection of firms.

While these insights are valuable, they are derived from models with a single monopolistically competitive sector. Extant studies thus abstract from a first-order feature of the data: sectors are highly heterogeneous. In France in 2008, for example, there are 4,889 textile and footwear producers, which compete for an expenditure share of 2% by French consumers. Those firms operate, arguably, in a different market and face different demands than the 4,607 manufacturers of wood products or the 124,202 health and personal service providers, on which French consumers spend less than 0.1% and almost 20% of aggregate income, respectively.[1]

Therefore, to answer the basic 'so-what' question—how great the overall welfare losses from imperfectly competitive markets are—we need to enrich existing general equilibrium models to have both between-sector and within-sector heterogeneity. For instance, the textile industry may have some firms that produce too little and others that produce too much, and at the same time, it may also attract too many (or too few) firms and workers in equilibrium. This, in turn, means that some other industries may have fewer (or more) firms and workers than are socially optimal.

Quantifying the magnitude of the aggregate welfare distortion in such a general equilib-

[1]See Table 1 in Section 4 for more details about the data. The large number of firms in each sector suggests that monopolistic competition is a reasonable approximation of the market structure.

rium framework is important for at least two reasons. First, since the seminal contribution by Dixit and Stiglitz (1977), the existence of a gap between the equilibrium and optimal allocations has been one of the most influential theoretical results in various applied fields of economics. Yet, despite its importance, we are not aware of any attempt to put numbers on it while taking into account heterogeneity both between and within sectors. Second, the rationale for government interventions in a particular sector typically relies on a partial equilibrium analysis, thus ignoring the interdependencies between heterogeneous sectors. For example, the question of excess or insufficient entry into industries should be viewed from a general equilibrium perspective: limited resources imply that excessive entry in some industries is likely to go hand-in-hand with insufficient entry in others.

Assessing the welfare costs of imperfect competition in a general equilibrium setting is also difficult for at least for two reasons. First, to capture misallocations within and between sectors, we need a model with heterogeneous firms and sectors. Developing such a model is challenging, especially with general utility functions and productivity distributions that can accommodate various specifications used in the literature. Second, we need to compare the equilibrium and optimal allocations. While the former is observable from the data, the latter is not. It is thus not obvious how we can measure the gap between the equilibrium and the optimum—quantifying something unobservable is not an easy task.

We address the first problem by building on Zhelobodko et al. (2012) and Dhingra and Morrow (2017), who study the positive and normative aspects of a single monopolistically competitive industry. We extend their approach to incorporate multiple sectors and allow the sectors to differ in many respects such as utility functions and productivity distributions. Imposing standard assumptions on the upper-tier utility function, we establish existence and uniqueness of the equilibrium and optimal allocations. Comparing those two allocations enables us to characterize various distortions, which include inefficiencies in the labor allocation and the masses of entrants between sectors, as well as inefficient firm selection and output per firm within sectors.

We cope with the second problem by using a novel way to quantify the gap between the equilibrium and the optimum: the Allais surplus (Allais, 1943, 1977). Roughly speaking, we consider a planner who minimizes the resource cost while achieving the equilibrium utility level. Since by definition the planner can do better than the market economy, it requires less resources, thus generating a surplus. The advantage of the Allais surplus is that it can be used for comparing the equilibrium with the first-best allocation, i.e., even in contexts where equivalent or compensating variations—or other related criteria to compare different equilibria—cannot be readily applied due to a lack of prices.

Previewing our theoretical findings, we show that the distortion in the labor allocation depends on the interaction between two types of elasticities: *the elasticities of the upper-tier utility* that govern the inter-sectoral allocation; and *the elasticities of the subutilities* that shape the intra-sectoral allocation. These elasticities are related to the equilibrium and optimal entry conditions. In equilibrium, entry in each sector occurs until the expected revenue equals the expected cost of labor allocated to that sector. Since each firm takes consumers' demands as given, its expected revenue depends on both types of elasticities that govern consumers' expenditure allocations between and within sectors. In contrast, the planner equates the marginal social benefit of entry with the marginal social cost of labor required for entry. Since the former equals the expected sectoral utility, it depends only on the upper-tier elasticities and does not require information on the elasticities of the subutilities. In fact, the planner does not face consumers' demand functions when determining entry in each sector, whereas firms do. This difference creates distortions in the sectoral labor allocation.

One key message of multi-sector general equilibrium models is that, contrary to the conventional approach that has studied single industries in partial equilibrium, distortions in one sector depend on the characteristics of all sectors in the economy. Indeed, sectors are interdependent, so that an excessive labor allocation to some sectors, for example, implies an insufficient labor allocation to others. The inefficient labor allocation across sectors, in turn, causes distortions in entry patterns. In particular, sectors with an excessive labor share tend to feature an excessive number of entrants. Thus, too many entrants in some sectors are accompanied by too few entrants in others, though this inter-sectoral prediction on entry needs to be adjusted by standard intra-sectoral business stealing and limited appropriability effects as in Mankiw and Whinston (1986).

Our general framework nests many specifications—in terms of utility functions and productivity distributions—that are used in the literature. We take two of those specifications to data. The first one builds on Cobb-Douglas upper-tier utility functions and constant elasticity of substitution (CES) subutility functions. Dhingra and Morrow (2017) show that selection and firm-level outputs are efficient in a single-sector economy if and only if the subutility function is of the CES form. This result is shown to hold in our multi-sector setting, thus implying that there are no intra-sectoral distortions. However, with multiple sectors, distortions in the labor allocation and firm entry still arise in general. Both disappear if and only if the elasticities of the CES subutility functions are identical across all sectors. Otherwise, the labor allocation and firm entry are efficient *within* but not *between* sectors. In particular, sectors with a higher elasticity of the subutility attract too many workers and firms, regardless of productivity distributions.

Our second example is a tractable model with variable elasticity of substitution (VES), where demands exhibit smaller price elasticities at higher consumption levels. Unlike the CES model, this VES model can account for variable markups and incomplete pass-through (e.g., Weyl and Fabinger, 2013; Mrázová and Neary, 2017). It features all the kinds of distortions that we highlight in the general framework. We show that high-productivity firms always produce too little and low-productivity firms too much, and that the market delivers too little selection compared to the social optimum. Entry and the labor allocation are also inefficient, and with Pareto distributions the market allocates too many firms and workers to sectors where a larger mass of the productivity distribution is concentrated on low-productivity firms.

Previewing our empirical findings, we establish four key results using data from France and the UK. First, there are substantial aggregate welfare distortions. In the multi-sector VES model, they equal 6–10% of the total labor input in either country. Second, inter-sectoral misallocations are crucial for these aggregate distortions. When we constrain the economy to consist of a single sector, thereby shutting down inefficiencies in entry and the labor allocation, the aggregate distortion can be 30% lower than the one predicted in the multi-sector case. Put differently, a single-sector model yields downward-biased predictions for the total welfare loss. Third, the multi-sector CES model predicts an aggregate distortion of 0.3–2.5%, which is much smaller than the VES model. The intuition is that this model displays by construction efficient selection and firm-level outputs, thereby missing distortions within sectors. Last, we find similar patterns of inefficient entry and selection between France and the UK. Insufficient entry arises almost exclusively for services, while manufacturing sectors tend to exhibit excessive entry. Equilibrium firm selection is generally closer to optimal one in manufacturing sectors. These results are robust to using different measures of firm size, e.g., employment or revenue, and different strategies to deal with fixed costs.

Our paper is closely related to the recent literature on the equilibrium and optimal allocations in models with a single monopolistically competitive sector, most notably Zhelobodko et al. (2012), Nocco, Ottaviano, and Salto (2014), Dhingra and Morrow (2017), and Parenti, Ushchev, and Thisse (2017). Relative to this recent strand of literature, we make two contributions. First, we characterize both the equilibrium and optimal allocations in a multi-sector monopolistic competition model. Second, while those papers focus exclusively on theory, we take our model with heterogeneous sectors and firms to data to assess the quantitative importance of the distortions under monopolistic competition—a question that remains unanswered since Spence (1976) and Dixit and Stiglitz (1977). Our work is further related to the classic literature in industrial organization that studies welfare implications of market power and inefficient entry for single industries in partial equilibrium. Harberger (1954) is a seminal reference for the former, and Mankiw and Whinston (1986) for the latter. Our monopolistic competition model is complementary to this line of research, and recognizes general equilibrium interdependencies between sectors.

The rest of the paper is organized as follows. Section 2 presents our general model, while Section 3 turns to the specific solvable examples. The quantification procedure and results are discussed in Section 4. Section 5 concludes.

2 General model

Consider an economy with a mass L of agents. Each agent is both a consumer and a worker, and supplies inelastically one unit of labor, which is the only factor of production. There are $j = 1, 2, \ldots, J$ sectors producing final consumption goods. Each good is supplied as a continuum of differentiated varieties, and each variety is produced by a single firm under monopolistic competition. Firms can differ by productivity, both within and between sectors. We denote by G_j the continuously differentiable cumulative distribution function, from which firms draw their marginal labor requirement, m, after entering sector j. An entrant need not operate and only firms with high productivity $1/m$ survive. Let N_j^E and m_j^d be the mass of entrants and the marginal labor requirement of the least productive firm in sector j, respectively. Given N_j^E, a mass $N_j^E G_j(m_j^d)$ of varieties are then supplied by firms with $m \leq m_j^d$.

2.1 Equilibrium allocation

The utility maximization problem of a representative consumer is given by:

$$\max_{\{q_j(m),\ \forall j,m\}} \quad U \equiv U\big(U_1, U_2, \ldots, U_J\big)$$

$$U_j \equiv N_j^E \int_0^{m_j^d} u_j\big(q_j(m)\big) \mathrm{d}G_j(m)$$

$$\text{s.t.} \quad \sum_{j=1}^{J} N_j^E \int_0^{m_j^d} p_j(m) q_j(m) \mathrm{d}G_j(m) = w, \tag{1}$$

where U is a strictly increasing and strictly concave *upper-tier utility* function that is twice continuously differentiable in all its arguments; u_j is a strictly increasing, strictly concave, and thrice continuously differentiable sector-specific *subutility* function satisfying $u_j(0) =$

0; $p_j(m)$ and $q_j(m)$ are the price and consumption of a sector-j variety produced with marginal labor requirement m; and w denotes a consumer's income. We assume that $\lim_{U_j \to 0}(\partial U / \partial U_j) = \infty$ for all sectors to be active in equilibrium.

Let λ denote the Lagrange multiplier associated with (1). The utility-maximizing consumptions satisfy the following first-order conditions:

$$u_j'\big(q_j(m)\big) = \lambda_j p_j(m), \quad \text{where} \quad \lambda_j \equiv \frac{\lambda}{\partial U / \partial U_j}. \tag{2}$$

To alleviate notation, let $p_j^d \equiv p_j(m_j^d)$ and $q_j^d \equiv q_j(m_j^d)$ denote the price set and quantity sold by the least productive firm operating in sector j, respectively. From the first-order conditions (2), which hold for any sector j and any firm with $m \le m_j^d$, we then have

$$\frac{u_j'(q_j^d)}{u_j'\big(q_j(m)\big)} = \frac{p_j^d}{p_j(m)} \quad \text{and} \quad \frac{u_j'(q_j^d)}{u_\ell'(q_\ell^d)} = \frac{\lambda_j}{\lambda_\ell}\frac{p_j^d}{p_\ell^d}, \tag{3}$$

which determine the equilibrium intra- and intersectoral consumption patterns, respectively.

We assume that the labor market is competitive, and that workers are mobile across sectors. All firms hence take the common wage w as given. Turning to technology, entry into each sector j requires to hire a sunk amount F_j of labor paid at the market wage. After paying the sunk cost, $F_j w > 0$, each firm draws its marginal labor requirement m from G_j, which is known to all firms. Conditional on survival, production takes place with constant marginal cost, mw, and sector-specific fixed cost, $f_j w \ge 0$.

Let $\pi_j(m)$ denote the operating profit of a firm with productivity $1/m$, divided by the wage rate w. Making use of condition (2), and of the equivalence between price and quantity as the firm's choice variable under monopolistic competition with a continuum of firms (Vives, 1999), the firm maximizes

$$\pi_j(m) = L\left[\frac{u_j'\big(q_j(m)\big)}{\lambda_j w} - m\right] q_j(m) - f_j \tag{4}$$

with respect to quantity $q_j(m)$. Although $\lambda_j w$ contains the information of all the other sectors by (2), each firm takes this market aggregate as given because there is a continuum of firms. From (4), the profit-maximizing price satisfies

$$p_j(m) = \frac{mw}{1 - r_{u_j}\big(q_j(m)\big)}, \tag{5}$$

where $r_{u_j}(x) \equiv -xu_j''(x)/u_j'(x)$ denotes the 'relative risk aversion' or the 'relative love for variety' (Behrens and Murata, 2007; Zhelobodko et al., 2012).[2] In what follows, we refer to $1/[1 - r_{u_j}(q_j(m))]$ as the *private markup* charged by a firm that produces output $q_j(m)$.

To establish the existence and uniqueness of an equilibrium cutoff, $(m_j^d)^{\text{eqm}}$, and equilibrium quantities, $q_j^{\text{eqm}}(m)$ for all $m \in [0, m_j^d]$, we consider the *zero cutoff profit* (ZCP) condition, given by $\pi_j(m_j^d) = 0$, and the *zero expected profit* (ZEP) condition, defined as $\int_0^{m_j^d} \pi_j(m)\mathrm{d}G_j(m) = F_j$. Using (2), (4), and (5), the ZCP and ZEP conditions can be expressed respectively as follows:

$$\left[\frac{1}{1 - r_{u_j}\left(q_j^d\right)} - 1\right] m_j^d q_j^d = \frac{f_j}{L}, \tag{6}$$

$$L \int_0^{m_j^d} \left[\frac{1}{1 - r_{u_j}(q_j(m))} - 1\right] m q_j(m) \mathrm{d}G_j(m) = f_j G_j(m_j^d) + F_j, \tag{7}$$

which—even in our multi-sector economy—allow us to prove the existence and uniqueness of the sectoral cutoff and quantities. Formally, we have the following result.

Proposition 1 (Equilibrium cutoff and quantities) *Assume that the fixed costs, f_j, and sunk costs, F_j, are not too large. Then, the equilibrium cutoff and quantities $\{(m_j^d)^{\text{eqm}}, q_j^{\text{eqm}}(m), \forall m \in [0, (m_j^d)^{\text{eqm}}]\}$ in each sector j are uniquely determined.*

Proof *See Appendix A.1.* □

Turning to the labor allocation, $L_j = N_j^E[L \int_0^{m_j^d} m q_j(m)\mathrm{d}G_j(m) + f_j G_j(m_j^d) + F_j]$, and the mass of entrants, N_j^E, in each sector j, we first provide two important expressions that must hold in equilibrium.[3] We then establish the existence and uniqueness of the equilibrium labor allocation and entry. To this end, we introduce the following notation. Let

$$\mathcal{E}_{U,U_j} \equiv \frac{\partial U}{\partial U_j}\frac{U_j}{U} \quad \text{and} \quad \mathcal{E}_{u_j,q_j(m)} \equiv \frac{u_j'(q_j(m))q_j(m)}{u_j(q_j(m))} \tag{8}$$

denote the elasticities of the upper-tier utility and of the subutility, respectively. Let further

$$\zeta_j(q_j(m)) \equiv \frac{u_j(q_j(m))}{\int_0^{m_j^d} u_j(q_j(m))\mathrm{d}G_j(m)} \quad \text{and} \quad \nu_j(q_j(m)) \equiv \frac{u_j'(q_j(m))q_j(m)}{\int_0^{m_j^d} u_j'(q_j(m))q_j(m)\mathrm{d}G_j(m)}$$

[2] We assume that the second-order conditions for profit maximization, $r_{u_j'}(x) \equiv -xu_j'''(x)/u_j''(x) < 2$ for all $j = 1, 2, \ldots, J$, hold (Zhelobodko et al., 2012, p.2771).

[3] To alleviate notation, we henceforth suppress the 'eqm' superscript when there is no possible confusion.

denote the shares that a variety produced with marginal labor requirement m in sector j contributes to the lower-tier utility U_j and to sectoral expenditure, respectively. Using these expressions, we obtain the following result.

Lemma 1 (Labor allocation and firm entry) *Any equilibrium labor allocation in sector $j = 1, 2, \ldots, J$ satisfies*

$$L_j = \frac{\mathcal{E}_{U,U_j}\overline{\mathcal{E}}_{u_j,q_j(m)}}{\sum_{\ell=1}^{J}\mathcal{E}_{U,U_\ell}\overline{\mathcal{E}}_{u_\ell,q_\ell(m)}}L, \tag{9}$$

where $\overline{\mathcal{E}}_{u_j,q_j(m)} \equiv \int_0^{m_j^d}\mathcal{E}_{u_j,q_j(m)}\zeta_j(q_j(m))\mathrm{d}G_j(m)$ is a weighted average of the elasticities of the subutility functions, where the weights are given by the contribution of each variety to the sectoral utility. Furthermore, any equilibrium mass of entrants satisfies

$$N_j^E = L_j\left\{\frac{1-\int_0^{m_j^d}[1-r_{u_j}(q_j(m))]\nu_j(q_j(m))\mathrm{d}G_j(m)}{f_jG_j(m_j^d)+F_j}\right\}. \tag{10}$$

Proof *See Appendix B.1.* □

Lemma 1 shows that, in any equilibrium, the labor allocation L_j can be expressed by the elasticities $\mathcal{E}_{U,U_j}$ of the upper-tier utility function and the weighted average $\overline{\mathcal{E}}_{u_j,q_j(m)}$ of the elasticities of the subutility functions. We will discuss the intuition for those terms in Section 2.3. The mass of entrants is affected not only by L_j, but also by effective entry cost $f_jG_j(m_j^d)+F_j$, the distribution of the markup terms $1-r_{u_j}(q_j(m))$, and the expenditure shares $\nu_j(q_j(m))$. It is worth emphasizing that we have not specified functional forms for either utility or productivity distributions to derive those results.

Note that Lemma 1 does not yet imply existence and uniqueness of the equilibrium labor allocation and the equilibrium mass of entrants. The reason is that, while the expression in the braces in (10) is uniquely determined by Proposition 1, the labor allocation L_j can depend on $\{N_j^E\}_{j=1,2,\ldots,J}$ via $\mathcal{E}_{U,U_j}$. Thus, to establish those properties, we impose some separability on the upper-tier utility function. More specifically, assume that the derivative of the upper-tier utility function with respect to the lower-tier utility in each sector can be divided into an own-sector and an economy-wide component as follows:

$$\frac{\partial U}{\partial U_j} = \gamma_j U_j^{\xi_j}U^{\xi}, \tag{11}$$

where $\gamma_j > 0$, $\xi_j < 0$, and $\xi \geq 0$ are parameters.[4] Specification (11) includes, for example, the

[4]The crucial points are that, under condition (11), the ratio of the derivatives in (2) with respect to j and ℓ

cases where the upper-tier utility function is of the Cobb-Douglas or the CES form. When condition (11) holds, we can prove the following result:

Proposition 2 (Equilibrium labor allocation and firm entry) *Assume that* (11) *holds. Then, the equilibrium labor allocation and masses of entrants* $\{L_j^{\text{eqm}}, (N_j^E)^{\text{eqm}}\}_{j=1,2,\ldots,J}$ *are uniquely determined by* (9) *and* (10)*.*

Proof *See Appendix A.2.* □

2.2 Optimal allocation

Having analyzed the equilibrium allocation, we now turn to the optimal allocation.[5] Assume that the planner chooses the quantities, cutoffs, and masses of entrants to maximize welfare subject to the resource constraint of the economy as follows:

$$\begin{aligned} \max_{\{q_j(m), m_j^d, N_j^E, \forall j, m\}} \quad & L \cdot U\big(U_1, U_2, \ldots, U_J\big) \\ & U_j \equiv N_j^E \int_0^{m_j^d} u_j(q_j(m)) \mathrm{d}G_j(m) \\ \text{s.t.} \quad & \sum_{j=1}^{J} N_j^E \left\{ \int_0^{m_j^d} \left[Lmq_j(m) + f_j \right] \mathrm{d}G_j(m) + F_j \right\} = L. \end{aligned} \tag{12}$$

The planner has no control over the uncertainty of the draws of m, but knows the underlying distributions G_j. Let δ denote the Lagrange multiplier associated with (12). The first-order conditions with respect to quantities, cutoffs, and the masses of entrants are given by:

$$u_j'(q_j(m)) = \delta_j m, \quad \delta_j \equiv \frac{\delta}{\partial U / \partial U_j} \tag{13}$$

$$L \frac{u_j(q_j^d)}{\delta_j} = Lm_j^d q_j^d + f_j \tag{14}$$

$$L \int_0^{m_j^d} \frac{u_j(q_j(m))}{\delta_j} \mathrm{d}G_j(m) = \int_0^{m_j^d} \left[Lmq_j(m) + f_j \right] \mathrm{d}G_j(m) + F_j. \tag{15}$$

is independent of N_k^E for $k \neq j, \ell$, and that it satisfies some monotonicity properties. Otherwise, the resulting system of equations becomes generally intractable.

[5] In the main text, we consider the 'primal' first-best problem where the planner maximizes utility subject to the economy's resource constraint. When quantifying the gap between the equilibrium and the optimum in Section 4, we will analyze a 'dual' problem where the planner minimizes the resource cost subject to a utility level. The latter allows us to derive the Allais surplus (Allais, 1943, 1977) that can be used for comparing the equilibrium with the first-best allocation, i.e., even in contexts where equivalent or compensating variations—or other related criteria to compare different equilibria—cannot be readily applied due to a lack of prices. More details are relegated to Appendices D and F.

From the first-order conditions (13), which hold for any sector j and any firm with $m \leq m_j^d$, we then have

$$\frac{u_j'(q_j^d)}{u_j'\big(q_j(m)\big)} = \frac{m_j^d}{m} \quad \text{and} \quad \frac{u_j'(q_j^d)}{u_\ell'(q_\ell^d)} = \frac{\delta_j}{\delta_\ell}\frac{m_j^d}{m_\ell^d}, \tag{16}$$

which determine the optimal intra- and intersectoral consumption patterns, respectively.

We start again with the cutoff and quantities. Noting that $\delta_j = u_j'(q_j(m))/m$ for any value of m from (13), we can rewrite condition (15) as follows:

$$L\int_0^{m_j^d}\left[\frac{1}{\mathcal{E}_{u_j,q_j(m)}} - 1\right] m q_j(m)\mathrm{d}G_j(m) = f_j G_j(m_j^d) + F_j, \tag{17}$$

where $\mathcal{E}_{u_j,q_j(m)}$ is defined in (8). We refer to $1/\mathcal{E}_{u_j,q_j(m)}$ as the *social markup* that a firm with marginal labor requirement m should optimally charge, and to $m/\mathcal{E}_{u_j,q_j(m)}$ as the *shadow price* of a variety produced by a firm with m in sector j.[6] Condition (17)—which equates the marginal social benefit of entry in sector j with its marginal social cost—may then be understood as the *zero expected social profit* (ZESP) condition, which is analogous to the ZEP condition (7). Furthermore, evaluating (13) at m_j^d and plugging the resulting expression into (14), we obtain an expression similar to the ZCP condition (6) as follows:

$$\left(\frac{1}{\mathcal{E}_{u_j,q_j^d}} - 1\right) m_j^d q_j^d = \frac{f_j}{L}, \tag{18}$$

which we call the *zero cutoff social profit* (ZCSP) condition. Using (17) and (18), we can establish the existence and uniqueness of the sectoral cutoff and quantities.

Proposition 3 (Optimal cutoff and quantities) *Assume that the fixed costs, f_j, and the sunk costs, F_j, are not too large. Then, the optimal cutoff and quantities $\{(m_j^d)^{\text{opt}}, q_j^{\text{opt}}(m),\ \forall m \in [0,(m_j^d)^{\text{opt}}]\}$ in each sector j are uniquely determined.*

Proof *See Appendix A.3.* □

Turning next to the optimal labor allocation, L_j, and the optimal masses of entrants, N_j^E, we proceed in the same way as for the equilibrium case, and provide the following two expressions.

[6] Dhingra and Morrow (2017) refer to $1 - \mathcal{E}_{u_j,q_j(m)} = [u_j(q_j(m)) - \delta_j m q_j(m)]/u_j(q_j(m))$ as the social markup, which captures the utility from consumption of a variety net of its resource costs. Moreover, they label $[p_j(m) - mw]/p_j(m) = r_{u_j}(q_j(m))$ as the private markup. We adopt their terminology but redefine the two markups in a slightly different way.

Lemma 2 (Labor allocation and firm entry) *Any optimal labor allocation in sector $j = 1, 2, \ldots, J$ satisfies*

$$L_j = \frac{\mathcal{E}_{U,U_j}}{\sum_{\ell=1}^{J} \mathcal{E}_{U,U_\ell}} L. \tag{19}$$

Furthermore, any optimal mass of entrants satisfies

$$N_j^E = L_j \left\{ \frac{1 - \int_0^{m_j^d} \mathcal{E}_{u_j,q_j(m)} \zeta_j(q_j(m)) \mathrm{d}G_j(m)}{f_j G_j(m_j^d) + F_j} \right\}. \tag{20}$$

Proof *See Appendix B.2.* □

Lemma 2 shows that, in any optimum, the labor allocation L_j can be expressed by the elasticities $\mathcal{E}_{U,U_j}$ of the upper-tier utility. The mass of entrants is affected not only by L_j, but also by effective entry costs $f_j G_j(m_j^d) + F_j$, the distribution of the social markup terms $\mathcal{E}_{u_j,q_j(m)}$, and the shares $\zeta_j(q_j(m))$ that capture the relative contribution of a variety produced with marginal labor requirement m to utility in sector j.

Finally, similarly to the equilibrium analysis, Lemma 2 does not yet imply the existence and uniqueness of the optimal labor allocation and the optimal masses of entrants. We thus impose again the separability condition (11) to establish those properties as follows:

Proposition 4 (Optimal labor allocation and firm entry) *Assume that* (11) *holds. Then, the optimal labor allocation and masses of entrants* $\{L_j^{\text{opt}}, (N_j^E)^{\text{opt}}\}_{j=1,2,\ldots,J}$ *are uniquely determined by* (19) *and* (20)*.*

Proof *See Appendix A.4.* □

2.3 Equilibrium versus optimum

Having established existence and uniqueness of the equilibrium and optimal allocations in Propositions 1–4, we now investigate the difference between these two allocations.

The novel feature of our model lies in labor and entry distortions between sectors. It is important to notice that characterizing labor and entry distortions for one sector requires information on all sectors. Put differently, the labor allocation and, thus, entry are interdependent when there are multiple sectors. Hence, entry distortions in our multi-sector model generally differ from those in models with a single imperfectly competitive sector such as Mankiw and Whinston (1986) and Dhingra and Morrow (2017).

To characterize the labor distortions, we compare expressions (9) from Lemma 1 with (19) from Lemma 2. We then obtain the following proposition.

Proposition 5 (Distortions in the labor allocation) *The equilibrium and optimal labor allocations satisfy $L_j^{\text{eqm}} \gtreqless L_j^{\text{opt}}$ if and only if*

$$\Upsilon_j \equiv \frac{\mathcal{E}_{U,U_j}^{\text{eqm}} \overline{\mathcal{E}}_{u_j,q_j(m)}^{\text{eqm}}}{\mathcal{E}_{U,U_j}^{\text{opt}}} \gtreqless \frac{\sum_{\ell=1}^{J} \mathcal{E}_{U,U_\ell}^{\text{eqm}} \overline{\mathcal{E}}_{u_\ell,q_\ell(m)}^{\text{eqm}}}{\sum_{\ell=1}^{J} \mathcal{E}_{U,U_\ell}^{\text{opt}}}. \tag{21}$$

Assume, without loss of generality, that sectors are ordered such that Υ_j is non-decreasing in j. If there are at least two different Υ_j's, then there exists a unique threshold $j^* \in \{1, 2, \ldots, J-1\}$ such that the equilibrium labor allocation is not excessive for sectors $j \leq j^*$, whereas it is excessive for sectors $j > j^*$. The equilibrium labor allocation is optimal if and only if all Υ_j terms are the same.

Proof *See Appendix A.5.* □

As can be seen from (21), the interdependence of heterogeneous sectors is important for distortions in the labor allocation. Which sectors have an excessive labor allocation depends on two types of statistics: the elasticities $\mathcal{E}_{U,U_j}$ of the upper-tier utility function, evaluated at the equilibrium and the optimum; and the weighted averages $\overline{\mathcal{E}}_{u_j,q_j(m)}$ of the elasticities of the subutility functions, evaluated at the equilibrium.

To build intuition for these statistics we focus on expressions (9) and (19) from Lemmas 1 and 2. The former comes from the equilibrium free entry condition (7) and the latter from the optimal entry condition (15). Multiplying (7) and (15) by N_j^E and rearranging, we have

$$L_j^{\text{eqm}} = \frac{L(N_j^E)^{\text{eqm}}}{\lambda_j w} \int_0^{m_j^d} u_j'(q_j(m)) q_j(m) \mathrm{d}G_j(m) \tag{22}$$

$$L_j^{\text{opt}} = \frac{L(N_j^E)^{\text{opt}}}{\delta_j} \int_0^{m_j^d} u_j(q_j(m)) \mathrm{d}G_j(m). \tag{23}$$

The key difference is that the former reflects zero expected profit by each firm, whereas the latter equates the marginal social benefit and the marginal social cost of entry for the planner to maximize social welfare. Since firms and the planner have different objectives, the two allocations differ in general. Using (22) and (23) we discuss them in terms of $\mathcal{E}_{U,U_j}$ for equilibrium and optimum and of $\overline{\mathcal{E}}_{u_j,q_j(m)}$ for equilibrium.

Elasticities of the upper-tier utility function. Expressions (22) and (23), together with the definition of λ_j in (2) and the definition of δ_j in (13), reveal why L_j^{eqm} and L_j^{opt} involve the elasticities of the upper-tier utility function, $\mathcal{E}^{\text{eqm}}_{U,U_j}$ and $\mathcal{E}^{\text{opt}}_{U,U_j}$.

Intuitively, at the free entry equilibrium, the private cost $wL_j^{\text{eqm}}/(N_j^E)^{\text{eqm}}$ of an entrant in equation (22) is just offset by the private benefit, i.e., firms' expected revenue $(L/\lambda_j)\int_0^{m_j^d} u_j'(q_j(m))q_j(m)\mathrm{d}G_j(m)$. Since the latter depends on the inverse demand functions (2), the equilibrium labor allocation L_j^{eqm} is affected by the elasticities of the upper-tier utility via λ_j.

By contrast, the social cost of an additional entrant is proportional to $L_j^{\text{opt}}/(N_j^E)^{\text{opt}}$, which by equation (23) must be equal to $(L/\delta_j)\int_0^{m_j^d} u_j(q_j(m))\mathrm{d}G_j(m)$ at optimum. Note that the latter reflects the (expected) marginal social benefit generated by the additional entrant. Thus, the optimal labor allocation L_j^{opt} depends on the elasticities of the upper-tier utility via δ_j.

It is worth emphasizing that even when the equilibrium and optimal elasticities of upper-tier utility in each sector are the same, i.e., $\mathcal{E}^{\text{eqm}}_{U,U_j} = \mathcal{E}^{\text{opt}}_{U,U_j}$, their sectoral heterogeneity plays a crucial role in the labor distortions as long as there is sectoral heterogeneity in $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}$. Indeed, although $\mathcal{E}^{\text{eqm}}_{U,U_j}$ and $\mathcal{E}^{\text{opt}}_{U,U_j}$ in the left-hand side of (21) cancel out when they are identical, the elasticities in the right-hand side remain. We will elaborate on this point in the next section where we illustrate some examples.

Weighted average of the elasticities of the subutility functions. Expressions (22) and (23) reveal why L_j^{eqm} depends on the weighted averages of the elasticities of the subutility functions, $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}$, whereas L_j^{opt} does not.

To understand this difference, recall that the private benefit of an entrant is given by $(L/\lambda_j)\int_0^{m_j^d} u_j'(q_j(m))q_j(m)\mathrm{d}G_j(m)$. Since this expected revenue for the entrant involves the consumers' inverse demand functions, the equilibrium labor allocation L_j^{eqm} depends not only on $\mathcal{E}^{\text{eqm}}_{U,U_j}$ via λ_j but also on $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}$ via $u_j'(q_j(m))$.[7]

While the equilibrium labor allocation is determined by the firms that care about zero expected profit conditional on the consumers' demand, the optimal labor allocation is determined by the planner who maximizes social welfare with respect to the mass of entrants. Since the latter does not involve the inverse demand functions, it is independent of $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}$.

Other things equal, the higher $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}$ the more labor is allocated to sector j by (9) because consumers allocate a large share of their budget to that sector by (22). Furthermore, a sector with higher $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}$ relative to the other sectors tends to display an excessive labor

[7]The weighted average satisfies $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)} < 1$ because $\mathcal{E}_{u_j,q_j(m)} < 1$ for all $m \in [0, m_j^d]$ by concavity of u_j, and because $\int_0^{m_j^d} \mathcal{E}_{u_j,q_j(m)}\zeta_j(q_j(m))\mathrm{d}G_j(m) < \int_0^{m_j^d} \zeta_j(q_j(m))\mathrm{d}G_j(m) = 1$ by the definition of $\zeta_j(q_j(m))$.

allocation by (21).

To see why a sector with higher $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}$ tends to display an excessive labor allocation, assume that $\mathcal{E}^{\text{eqm}}_{U,U_j} = \mathcal{E}^{\text{opt}}_{U,U_j}$ for all j, which is the case with the Cobb-Douglas upper-tier utility. If $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)} = \overline{\mathcal{E}}^{\text{eqm}}_{u_1,q_1(m)}$ holds for all $j \neq 1$, then the equilibrium labor allocation is optimal by (9) and (19). However, if $\overline{\mathcal{E}}^{\text{eqm}}_{u_J,q_J(m)} > \overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)} = \overline{\mathcal{E}}^{\text{eqm}}_{u_1,q_1(m)}$ for $j = 2, 3, ..., J-1$, then expenditure on—and thus the labor allocation to—sector J gets larger at the expense of the other sectors, whereas the optimal labor allocation does not change. Thus, sector J displays an excessive labor allocation, whereas the other sectors $j = 1, 2, ..., J-1$ exhibit an insufficient labor allocation. The latter is a general equilibrium effect: an excessive allocation to one sector must go hand in hand with an insufficient allocation to the other sectors.

Note that what matters is the relative magnitude of $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}$. Indeed, it is easy to see that a proportionate increase in $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}$ for all j does not affect the equilibrium allocation by (9) and, hence, excess or insufficient labor allocation by (21).

Turning to entry distortions, we compare expression (10) from Lemma 1 with (20) from Lemma 2 to obtain the following proposition.

Proposition 6 (Distortions in firm entry) *The equilibrium and optimum masses of entrants satisfy* $(N_j^E)^{\text{eqm}}/(N_j^E)^{\text{opt}} \gtreqless 1$, *if and only if*

$$\frac{L_j^{\text{eqm}}}{L_j^{\text{opt}}} \cdot \frac{f_j G_j((m_j^d)^{\text{opt}}) + F_j}{f_j G_j((m_j^d)^{\text{eqm}}) + F_j} \cdot \frac{1 - \int_0^{(m_j^d)^{\text{eqm}}} [1 - r_{u_j}(q_j^{\text{eqm}}(m))] \nu_j(q_j^{\text{eqm}}(m)) \mathrm{d}G_j(m)}{1 - \int_0^{(m_j^d)^{\text{opt}}} \mathcal{E}_{u_j, q_j^{\text{opt}}(m)} \zeta_j^{\text{opt}}(q_j(m)) \mathrm{d}G_j(m)} \gtreqless 1. \quad (24)$$

Proof *Expression* (24) *directly follows from* (10) *and* (20). □

Expression (24) shows that $(N_j^E)^{\text{eqm}}/(N_j^E)^{\text{opt}}$ depends on three terms. The first term $L_j^{\text{eqm}}/L_j^{\text{opt}}$ vanishes in a single-sector model, because $L_j^{\text{eqm}} = L_j^{\text{opt}} = L$. In a multi-sector model, however, the gap between L_j^{eqm} and L_j^{opt} plays a crucial role, as mentioned above.

The second and third terms in (24) capture two additional margins, namely 'effective fixed costs' and 'private and social markups', which depend on the cutoffs and quantities both at equilibrium and optimum. Recall that by the proofs of Propositions 1 and 3, $\lambda_j w$ and δ_j are uniquely determined without any information on the other sectors. Hence, even in our multi-sector framework, the analysis of cutoff and quantity distortions in each sector j turns out to work as in the single-sector model by Dhingra and Morrow (2017). We shall not repeat their theoretical analysis here, but we first briefly discuss them, and then provide specific examples in the next section. Those examples will be taken to the data in Section 4.

Effective fixed costs. The second term in (24) shows that if the market delivers too little selection, $(m_j^d)^{\text{eqm}} > (m_j^d)^{\text{opt}}$, entry tends to be insufficient. The reason is that the higher survival probability in equilibrium, as compared to the optimum, increases the expected fixed costs that entrants have to pay. This reduces expected profitability and discourages entry more in equilibrium than in optimum. In contrast, other things equal, too much equilibrium selection, $(m_j^d)^{\text{eqm}} < (m_j^d)^{\text{opt}}$, leads to excessive entry.

Private and social markups. The last term in (24) shows that the gap between equilibrium and optimal entry depends on the private and social markup terms, which may exacerbate or attenuate excess entry (Mankiw and Whinston, 1986; Dhingra and Morrow, 2017). The numerator can be related to the *business stealing effect*: the higher the private markups $1/[1 - r_{u_j}(q_j(m))]$, the more excessive the entry. The denominator, in turn, captures the *limited appropriability effect*: the greater the social markups $1/\mathcal{E}_{u_j,q_j(m)}$, the more insufficient the entry. Thus, the last term in (24) depends on the relative strength of these two effects, as well as on the weighting schemes $\nu_j(q_j(m))$ and $\zeta_j(q_j(m))$ that are determined by the properties the subutility function u_j and the productivity distribution function G_j.

To sum up, the difference between market equilibrium and social optimum in terms of the labor allocation and firm entry across heterogeneous sectors depends, in general, on four key ingredients: the elasticities of the upper-tier utility; the weighted averages of the elasticities of the subutilities; effective fixed costs; and private and social markups. While distortions in a single-sector model are characterized solely by u_j and G_j *for that sector* (Dhingra and Morrow, 2017), in a multi-sector setting characterizing distortions *for one sector* requires additional information on the elasticities of the upper-tier utility, $\mathcal{E}_{U,U_j}$, and the weighted averages of the elasticities of the subutilities, $\overline{\mathcal{E}}_{u_j,q_j(m)}$, *for all sectors*. Hence, when assessing distortions we need to take into account the interdependence between heterogeneous sectors.

3 Examples

Our results in the Propositions and Lemmas presented so far are general enough to encompass various specifications of utility functions and productivity distributions used in the literature. We now consider specific upper-tier utility and subutility functions that enable us to express the two types of elasticities, $\mathcal{E}_{U,U_j}$ and $\overline{\mathcal{E}}_{u_j,q_j(m)}$, in simple parametric forms. We then take the parametric models to data in Section 4.

Concerning the subutility function u_j, we first analyze in Section 3.1 the ubiquitous CES case that has dominated much of the literature on monopolistic competition. We then turn to a tractable 'variable elasticity of substitution' (VES) model in Section 3.2

In doing so, notice that the lower-tier utility U_j in specification (1) does not nest the standard homothetic CES aggregator. To nest it, we consider a simple monotonic transformation of the lower-tier utility in (1) as $\widetilde{U}_j(U_j)$. In Section 3.1 we assume that $\widetilde{U}_j(U_j) = U_j^{1/\rho_j} = [N_j^E \int_0^{m_j^d} q_j(m)^{\rho_j} \mathrm{d}G_j(m)]^{1/\rho_j}$, whereas we retain $\widetilde{U}_j(U_j) = U_j$ in Section 3.2.

Even with the transformation $\widetilde{U}_j$ of the lower-tier utility, we can re-establish the general results shown in Section 2, as long as we let $\widetilde{U}_j(0) = 0$, $\widetilde{U}_j' > 0$, and $\lim_{U_j \to \infty} \widetilde{U}_j(U_j) = \infty$, while replacing the condition in (11) with

$$\frac{\partial U}{\partial \widetilde{U}_j} \frac{\partial \widetilde{U}_j}{\partial U_j} = \gamma_j \widetilde{U}_j^{\xi_j} U^{\xi}, \tag{25}$$

where $\gamma_j > 0$, $\xi_j < 0$, and $\xi \geq 0$ are parameters.[8]

Turning to the upper-tier utility function, we consider in the remainder of this paper the standard CES form: $U = \{\sum_{j=1}^J \beta_j [\widetilde{U}_j(U_j)]^{(\sigma-1)/\sigma}\}^{\sigma/(\sigma-1)}$, where $\sigma \geq 1$, $\beta_j > 0$ for all j, and $\sum_{j=1}^J \beta_j = 1$. Thus, the elasticity of the upper-tier utility function is given by $\mathcal{E}_{U,U_j} \equiv (\partial U/\partial \widetilde{U}_j)(\partial \widetilde{U}_j/\partial U_j)(U_j/U) = \beta_j(\partial \widetilde{U}_j/\partial U_j)(U_j/\widetilde{U}_j)(\widetilde{U}_j/U)^{(\sigma-1)/\sigma}$. When $\sigma \to 1$, the upper-tier utility reduces to the Cobb-Douglas form, $U = \prod_{j=1}^J [\widetilde{U}_j(U_j)]^{\beta_j}$, so that

$$\mathcal{E}_{U,U_j} = \beta_j \left(\frac{\partial \widetilde{U}_j}{\partial U_j} \frac{U_j}{\widetilde{U}_j} \right). \tag{26}$$

The Cobb-Douglas upper-tier utility function always satisfies condition (25) that guarantees the existence and uniqueness of the equilibrium and optimal allocations. When the upper-tier utility function is of the CES form, whereas the lower-tier utility is of the homothetic CES form with $\widetilde{U}_j(U_j) = U_j^{1/\rho_j}$, we have $(\partial U/\partial \widetilde{U}_j)(\partial \widetilde{U}_j/\partial U_j) = (\beta_j/\rho_j)\widetilde{U}_j^{\frac{\sigma-1}{\sigma} - \rho_j} U^{1/\sigma}$. Hence, in that case, it is required that $(\sigma-1)/\sigma < \rho_j$ for condition (25) to hold with $\xi_j < 0$.[9]

Retaining $\sigma \to 1$ for now, we consider two specific forms for the subutility functions for which the weighted averages of the elasticities of the subutility functions display a simple

[8] The proofs are virtually identical to the ones in Appendices A and B, except that $\partial U/\partial U_j$ needs to be replaced with $(\partial U/\partial \widetilde{U}_j)(\partial \widetilde{U}_j/\partial U_j)$. Observe that in a single-sector model, the choice of $\widetilde{U}_j$ does not affect distortions because it is a monotonic transformation of the overall utility in that case. In a multi-sector model, however, sectoral allocations and thus aggregate distortions are affected by $\widetilde{U}_j$.

[9] Should $(\sigma-1)/\sigma > \rho_j$ hold, goods are Hicks-Allen complements (see, e.g., Matsuyama, 1995) so that multiple equilibria with some inactive sectors may arise. Since $\xi_j < 0$ is not satisfied in that case, we exclude it from our analysis.

behavior. We will return to the case with $\sigma > 1$ in Section 4 where we quantify the model.

3.1 CES subutility

We first discuss the case of the CES subutility that has been widely used in the literature. Assume that $u_j(q_j(m)) = q_j(m)^{\rho_j}$ and $\widetilde{U}_j(U_j) = U_j^{1/\rho_j}$, where $\rho_j \in (0,1)$ for all sectors j.

The labor distortion in Proposition 5, and thus the first term of (24) that characterizes entry distortion in Proposition 6, depend on $\mathcal{E}^{\text{eqm}}_{U,U_j}$, $\mathcal{E}^{\text{opt}}_{U,U_j}$, and $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}$. Using (26), the elasticity of the upper-tier utility function can be rewritten as $\mathcal{E}^{\text{eqm}}_{U,U_j} = \mathcal{E}^{\text{opt}}_{U,U_j} = \beta_j/\rho_j$. Furthermore, when the subutility function is of the CES form, we know that $\mathcal{E}_{u_j,q_j(m)} = \rho_j$ for all m, so that $\overline{\mathcal{E}}^{\text{eqm}}_{U,U_j} = \rho_j$ by the definition in Lemma 1.

The entry distortion in Proposition 6 depends also on the cutoffs and quantities. Since we have shown that the cutoff and quantity distortions can be studied on a sector-by-sector basis even in our multi-sector model, we can apply the single-sector result by Dhingra and Morrow (2017), i.e., in the CES case $(m_j^d)^{\text{eqm}} = (m_j^d)^{\text{opt}}$ and $q_j^{\text{eqm}}(m) = q_j^{\text{opt}}(m)$ for all m irrespective of the underlying productivity distribution G_j. Furthermore, since $\mathcal{E}_{u_j,q_j(m)} = 1 - r_j(q_j(m))$ and $\nu_j(q_j(m)) = \zeta_j(q_j(m))$ hold for all m, the second and the third terms in (24) vanish, so that $(N_j^E)^{\text{eqm}}/(N_j^E)^{\text{opt}} = L_j^{\text{eqm}}/L_j^{\text{opt}}$. Hence, we can restate Propositions 5 and 6 for this specific example as follows:

Corollary 1 (Distortions in the labor allocation and firm entry with CES subutility) *Assume that the subutility function in each sector is of the CES form, $u_j(q_j(m)) = q_j(m)^{\rho_j}$. Then, the labor allocation and the masses of entrants satisfy $L_j^{\text{eqm}} \gtreqqless L_j^{\text{opt}}$ and $(N_j^E)^{\text{eqm}} \gtreqqless (N_j^E)^{\text{opt}}$, respectively, if and only if*

$$\rho_j \gtreqqless \frac{1}{\sum_{\ell=1}^{J} \beta_\ell/\rho_\ell}. \tag{27}$$

Assume, without loss of generality, that sectors are ordered such that ρ_j is non-decreasing in j. If there are at least two different ρ_j's, there exists a unique threshold $j^* \in \{1,2,\ldots,J-1\}$ such that the equilibrium labor allocation and firm entry are not excessive for sectors $j \leq j^*$, whereas they are excessive for sectors $j > j^*$. The equilibrium labor allocation and firm entry in the CES case are optimal if and only if all ρ_j's are the same.

Proof *See above.* □

Several comments are in order. First, since there are no cutoff and quantity distortions in the case of CES subutility functions, the market equilibrium is fully efficient if and only if the ρ_j's are the same across all sectors. However, there are distortions in the labor allocation and in the masses of entrants when the ρ_j's vary across sectors.[10]

Second, ρ_j in the CES model can be related not only to the inverse of the markup, but also to $\mathcal{E}_{U,U_j}$ and to $\overline{\mathcal{E}}_{u_j,q_j(m)}$. It is the latter two elasticities that matter for the labor and entry distortions. The reason is that the difference between the equilibrium and optimal labor allocations comes from $\mathcal{E}_{U,U_j} = \beta_j/\rho_j$ and $\overline{\mathcal{E}}_{u_j,q_j(m)} = \rho_j$, which are determined by the first derivatives of $\widetilde{U}_j$ and u_j as seen from (26) and the definition of $\overline{\mathcal{E}}_{u_j,q_j(m)}$. In contrast, the markup depends on r_{u_j}, which involves the second derivative of u_j. Thus, in the case of the Cobb-Douglas upper-tier utility and CES subutility functions, markup heterogeneity is not a determinant of labor and entry distortions.

Third, Corollary 1 holds irrespective of the functional form for G_j. Hence, productivity distributions play no role in the optimality of the market outcome for the standard case with the Cobb-Douglas upper-tier utility and CES subutility functions.

Last, since Corollary 1 only pertains to the class of CES subutility functions, it must not be read as a general 'if and only if' result for any subutility function. Indeed, as we show in the next subsection, the labor allocation and entry can be efficient even when the subutility function is *not* of the CES form.

3.2 VES subutility

We have so far examined the case of CES subutility functions without cutoff and quantity distortions. We now turn to our VES example where all types of distortions—cutoff, quantity, labor, and entry distortions—can operate. Specifically, we consider the 'constant absolute risk aversion' (CARA) subutility as in Behrens and Murata (2007), $u_j(q_j(m)) = 1 - \mathrm{e}^{-\alpha_j q_j(m)}$, where α_j is a strictly positive parameter.

This specification can be viewed as an example of the VES subutility analyzed in the seminal paper by Krugman (1979). It is analytically tractable, and generates demand functions exhibiting smaller price elasticities at higher consumption levels. Unlike the CES model, this VES case can therefore account for the empirically well-documented facts of incomplete pass-through and higher markups charged by more productive firms within each sector.

In what follows, we assume that $\widetilde{U}_j(U_j) = U_j$, so that $\mathcal{E}^{\mathrm{eqm}}_{U,U_j} = \mathcal{E}^{\mathrm{opt}}_{U,U_j} = \beta_j$ by (26). To express $\overline{\mathcal{E}}^{\mathrm{eqm}}_{u_j,q_j(m)}$ in a parametric form, we also assume that G_j follows a Pareto distribution $G_j(m) = \left(m/m_j^{\max}\right)^{k_j}$, where both the upper bounds $m_j^{\max} > 0$ and the shape parameters

[10] Hsieh and Klenow (2009) consider a heterogeneous firms model where the mass of firms is either fixed or invariant, and where the ρ_j's are the same across all sectors. In contrast, Epifani and Gancia (2011) allow for heterogeneity in the ρ_j's across sectors, yet consider homogeneous firms within sectors.

$k_j \geq 1$ may differ across sectors. We relegate most analytical details for the case with CARA subutilities and Pareto productivity distributions to Appendix E. We show there that the equilibrium and optimal cutoffs are given as follows:

$$(m_j^d)^{\text{eqm}} = \left[\frac{\alpha_j F_j (m_j^{\max})^{k_j}}{\kappa_j L}\right]^{\frac{1}{k_j+1}} \quad \text{and} \quad (m_j^d)^{\text{opt}} = \left[\frac{\alpha_j F_j (m_j^{\max})^{k_j}(k_j+1)^2}{L}\right]^{\frac{1}{k_j+1}}, \tag{28}$$

where $\kappa_j \equiv k_j \mathrm{e}^{-(k_j+1)} \int_0^1 (1+z)\left(z^{-1}+z-2\right)(z\mathrm{e}^z)^{k_j} \mathrm{e}^z \mathrm{d}z > 0$ is a function of the shape parameter k_j only. Using expressions (28), we can establish the following result:

Proposition 7 (Distortions in the cutoff and quantities with CARA subutility) *Assume that the subutility function in each sector is of the CARA form $u_j\big(q_j(m)\big) = 1 - \mathrm{e}^{-\alpha_j q_j(m)}$, and that the productivity distribution follows a Pareto distribution, $G_j(m) = (m/m_j^{\max})^{k_j}$. Then, the equilibrium cutoff exceeds the optimal cutoff in each sector, i.e., $(m_j^d)^{\text{eqm}} > (m_j^d)^{\text{opt}}$. Furthermore, there exists a unique threshold $m_j^* \in (0, (m_j^d)^{\text{opt}})$ such that $q_j^{\text{eqm}}(m) < q_j^{\text{opt}}(m)$ for all $m \in [0, m_j^*)$ and $q_j^{\text{eqm}}(m) > q_j^{\text{opt}}(m)$ for all $m \in (m_j^*, (m_j^d)^{\text{eqm}})$.*

Proof *See Appendix A.6.* □

Three comments are in order. First, in this model, more productive firms with $m < m_j^*$ underproduce, whereas less productive firms with $m > m_j^*$ overproduce in equilibrium as compared to the optimum in each sector j. Notice that both types of firms coexist in equilibrium since the threshold m_j^* satisfies the inequalities $0 < m_j^* < (m_j^d)^{\text{opt}} < (m_j^d)^{\text{eqm}}$.[11]

Second, using (28), the gap between the equilibrium and optimal selection can be expressed as a simple function of the sectoral shape parameter only: $(m_j^d)^{\text{opt}}/(m_j^d)^{\text{eqm}} = [\kappa_j(k_j+1)^2]^{1/(k_j+1)} < 1$. Since this expression increases with k_j, the larger the value of k_j (i.e., a larger mass of the productivity distribution is concentrated on low-productivity firms) the smaller is the magnitude of insufficient selection in sector j.

Finally, Proposition 7 holds on a sector-by-sector basis, regardless of the labor allocation and the masses of entrants. Thus, our results on cutoff and quantity distortions would also apply to a single-sector version of the CARA model.

Turning to the labor and entry distortions, the combination of CARA subutility functions and Pareto productivity distributions yields the equilibrium and optimal masses of entrants

[11] This need not always be the case, however. For example, Dhingra and Morrow (2017) derive general conditions for cutoff and quantity distortions in a single-sector framework. In their model with an arbitrary subutility function and an arbitrary productivity distribution, it is possible that m_j^* exceeds $(m_j^d)^{\text{eqm}}$. In that case, all firms (even the least productive ones) would underproduce, whereas in our model some firms (the least productive ones) always overproduce from a social welfare point of view.

as follows (see expressions (E-15)–(E-16) and (E-30)–(E-31) in Appendix E):

$$(N_j^E)^{\text{eqm}} = \frac{L_j^{\text{eqm}}}{(k_j+1)F_j} \quad \text{and} \quad (N_j^E)^{\text{opt}} = \frac{L_j^{\text{opt}}}{(k_j+1)F_j}. \tag{29}$$

Thus, as in the CES case, we have $(N_j^E)^{\text{eqm}}/(N_j^E)^{\text{opt}} = L_j^{\text{eqm}}/L_j^{\text{opt}}$. From Proposition 5 we know that distortions in the labor allocation are determined by $\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}$, together with $\mathcal{E}^{\text{eqm}}_{U,U_j} = \mathcal{E}^{\text{opt}}_{U,U_j} = \beta_j$. When the subutility function is of the CARA form and the productivity distribution follows a Pareto distribution, we can show that $\overline{\mathcal{E}}_{u_j,q_j(m)}$ depends solely on the sectoral shape parameter k_j as follows:

Lemma 3 (Weighted average of the elasticities of the CARA subutility functions) *Assume that the subutility function in each sector is of the* CARA *form, $u_j\big(q_j(m)\big) = 1 - \mathrm{e}^{-\alpha_j q_j(m)}$, and that the productivity distribution follows a Pareto distribution, $G_j(m) = (m/m_j^{\max})^{k_j}$. Then, the weighted average $\overline{\mathcal{E}}_{u_j,q_j(m)}$ of the elasticities of the subutility functions in each sector can be rewritten as*

$$\theta_j \equiv \frac{\int_0^1 (1-z)\mathrm{e}^{z-1}(1+z)\mathrm{e}^{z-1}\left(z\mathrm{e}^{z-1}\right)^{k_j-1}\mathrm{d}z}{\int_0^1 (1-\mathrm{e}^{z-1})(1+z)\mathrm{e}^{z-1}\left(z\mathrm{e}^{z-1}\right)^{k_j-1}\mathrm{d}z}. \tag{30}$$

Proof *See Appendix B.3.* □

To characterize the labor and entry distortions, we rank sectors such that $\theta_1 \leq \theta_2 \leq \ldots \leq \theta_J$. Since θ_j is increasing in k_j, ranking sectors by θ_j is equivalent to ranking them by k_j. Plugging (30) into (21), using $\mathcal{E}_{U,U_j} = \beta_j$ from the upper-tier Cobb-Douglas specification, and noting that $(N_j^E)^{\text{eqm}}/(N_j^E)^{\text{opt}} = L_j^{\text{eqm}}/L_j^{\text{opt}}$ by (29), we can restate Propositions 5 and 6 for this example as follows:

Corollary 2 (Distortions in the labor allocation and firm entry with CARA subutility) *Assume that the subutility function in each sector is of the* CARA *form, $u_j\big(q_j(m)\big) = 1 - \mathrm{e}^{-\alpha_j q_j(m)}$, and that the productivity distribution follows a Pareto distribution, $G_j(m) = (m/m_j^{\max})^{k_j}$. Then, the labor allocation and the masses of entrants satisfy $L_j^{\text{eqm}} \gtreqless L_j^{\text{opt}}$ and $(N_j^E)^{\text{eqm}} \gtreqless (N_j^E)^{\text{opt}}$, respectively, if and only if*

$$\theta_j \gtreqless \sum_{\ell=1}^{J} \beta_\ell \theta_\ell. \tag{31}$$

Assume, without loss of generality, that sectors are ordered such that θ_j is non-decreasing in j. If there are at least two different θ_j's, there exists a unique threshold $j^* \in \{1, 2, \ldots, J-1\}$ such that the equilibrium labor allocation and firm entry are not excessive for sectors $j \leq j^*$, whereas they are *excessive for sectors $j > j^*$. The equilibrium labor allocation and firm entry in the* CARA *case are optimal if and only if all θ_j's, and thus all k_j's, are the same.*

Proof *See above.* □

Corollary 2 states that sectors with larger values of k_j (i.e., sectors where a larger mass of the productivity distribution is concentrated on low-productivity firms) are more likely to display excess entry and excess labor allocation in equilibrium. As mentioned after Proposition 7, sectors with larger values of k_j also display smaller cutoff distortions. Thus, more excessive entry comes with more efficient selection. Furthermore, Corollary 2 shows that all sectors with θ_j above the weighted average $\sum_{\ell=1}^{J} \beta_\ell \theta_\ell$ display excess entry and labor allocation, whereas the opposite is true for all sectors with θ_j below that threshold. Hence, interdependence of heterogeneous sectors matters for those distortions. If there is no heterogeneity in k_j, then the labor allocation and entry are efficient although the cutoffs and quantities are inefficient in all sectors.

4 Quantification

In this section, we take our model to the data in order to quantify the gap between the equilibrium and optimal allocations.[12] Our approach only requires data that is accessible for many countries. In particular, we need the expenditure shares across sectors, and some aggregate statistics of the firm-size distribution within sectors. We make use of firm-level data from France in 2008 and from the UK in 2005. Using two different countries enables us to assess the robustness of our quantification approach, and to compare the distortions in those two different cases. We show that our results are robust to the use of two alternative measures of firm size (employment and revenue). In the employment case, we further consider two alternative measures of fixed costs (R&D expenditure and aggregate profits). As explained below, the revenue case does not require information on fixed costs.

We first focus on the VES model from Section 3.2 that captures all types of distortions. We then quantify the CES model from Section 3.1, where cutoff and output distortions are absent. Finally, we put the quantitative predictions of the two models into perspective.

[12]Our paper differs from the literature that uses various equilibria to quantify the impact of resource misallocation on aggregate TFP. Hsieh and Klenow (2009), for example, compare observed equilibria in China and India with counterfactual equilibria in which those countries would attain the "U.S. efficiency" level. Unlike this literature, we compare the observed market equilibrium and the optimal allocation that the social planner would choose.

4.1 Data

Our quantification procedure requires expenditure shares at the sectoral level and a firm-level measure of size (employment or revenue). The employment case further requires R&D outlays or aggregate profits at the sectoral level.[13] Data on sectoral expenditure shares and R&D outlays are rather standard and available for many countries, while at the same time information on firm-level revenue and/or employment is becoming increasingly accessible. In this respect, France and the UK are two ideal countries for our study because in both instances data on firm-level employment and revenue are available for virtually the whole firm population.

For France, the firm-level data come from the 'Élaboration des Statistiques Annuelles d'Entreprises' (ESANE) database, which combines administrative and survey data to produce structural business statistics. We use the administrative part of the dataset that contains revenue and employment figures for almost all business organizations in France. It is compiled from annual tax returns that companies file to the tax authorities and from annual social security data that supply additional information on the employees. We focus on the year 2008, for which there are 1,100,220 firms with positive employment records.[14] For each firm, we also have information about its sectoral affiliation. The French input-output tables contain information on 35 sectors, the public sector plus 34 private sectors, roughly corresponding to 2-digit NACE (revision 1.1) codes. This dictates the level of aggregation in our analysis. We discard the public sector (12.12% of expenditure) and focus on the remaining 34 private sectors. For those sectors, we obtain expenditure shares $\widehat{e}_j$ by re-scaling total expenditure such that the shares sum up to one. These observed expenditure shares are reported in Table 1.

The data for the UK have the same structure. We use the 'Business Structure Database' (BSD), which contains a small number of variables, including employment, revenue, and sectoral affiliation for almost all business organizations in the UK. The BSD is derived primarily from the 'Inter-Departmental Business Register' (IDBR), which is a live register of data collected by 'Her Majesty's Revenue and Customs' (HMRC) via VAT and 'Pay As You Earn' (PAYE) records. We focus on the year 2005 for which there are 1,704,543 firms with positive employment records.[15] We can distinguish the exact same 34 sectors as for France for the sectoral affiliation of those firms, for which we obtain expenditure shares from the British

[13]Further details concerning the datasets can be found in Appendix C.1.

[14]The dataset contains 3 employment variables. We use employment on December 31st from the French Business Register (OCSANE) source.

[15]The dataset contains 2 employment variables. We use employment count excluding the firm owners.

input-output tables. These observed expenditure shares, $\widehat{e}_j$, re-scaled again to sum to one, are reported in Table 2.

In order to assess the robustness of our quantification exercise we consider several variants of our procedure. In the baseline quantification procedure we use firm employment in order to back out heterogeneity in productivity across firms and employ a measure of industry-level fixed costs based on R&D outlays. More specifically, we measure fixed costs as the industry-level ratio of R&D expenditure to gross output (see below for more details).

As a first robustness check, we use firm revenue in order to back out heterogeneity in productivity across firms. In this case, we do not need a measure of fixed costs to quantify the gap between the equilibrium and optimal allocations. Despite being less demanding in terms of assumptions on the measurement of fixed costs, the analysis based on firm-level revenue is more vulnerable than the analysis based on firm-level employment to the presence of measurement error. This is why we see them as complementary.[16] Revenue data are in fact more likely to be measured with error than employment because the latter are cross-validated, for both France and the UK, from information coming from different sources (social security, tax returns, balance sheets, etc.), while the former is sometimes estimated/imputed for small firms.

As a second robustness check, we use firm employment as in the baseline case to back out heterogeneity in productivity across firms but employ a different measure of industry-level fixed costs. More specifically, we use the profits-to-revenue ratio, where data on industry-wide profits and revenue are obtained via aggregation of firm-level profits and revenue. Firm-level profits for the whole firm population are available only for France from the ESANE database. Thus, this procedure is not readily applicable to the UK.[17]

4.2 Quantifying distortions: the CARA subutility case

To quantify the VES model, we first match a theory-based moment of the sector-specific firm-size distribution to its empirical counterpart.

In our baseline case we derive an analytical expression for the standard deviation of (log) firm-level employment in sector j, excluding the labor input F_j that all firms have to bear as a sunk entry cost. The resulting expression depends only on the shape parameter k_j of the

[16] Part of the variation in revenue data is thus attributable to measurement error rather than to differences in underlying productivity across firms. This is less of a problem in the case of employment data.

[17] The BSD dataset for the UK contains information on firm revenue but not profits. Some UK datasets, like the Annual Respondents Database, do contain information on both variables but cover only a small portion (roughly 70,000 firms) of the UK firm population.

sector-specific Pareto productivity distribution (see equation (C-1) in Appendix C.2).

To construct its empirical counterpart, we compute for each sector j the ratio of R&D expenditure (our proxy for sectoral sunk entry costs) to gross output and then multiply the ratio by total employment in that sector. Dividing this by the number of firms yields a measure for F_j, which we then subtract from the employment of each firm in the respective sector (see Appendix C.1 for more details). Finally, we calculate for each sector j the standard deviation of the resulting (log) number of employees. This data moment and the number of firms in each sector are reported in Table 1 for France, and in Table 2 for the UK.

With the standard deviation of the (log) number of employees at hand, we can then uniquely back out $\widehat{k}_j$ for each sector and compute $\widehat{\theta}_j$ and $\widehat{\kappa}_j$, which depend solely on $\widehat{k}_j$. Using $\widehat{\theta}_j$ and the observed expenditure shares $\widehat{e}_j$, we obtain $\widehat{\beta}_j$ by solving $\sum_{\ell=1}^{J} \widehat{\beta}_\ell = 1$ and $\widehat{e}_j = \widehat{\beta}_j \widehat{\theta}_j / \sum_{\ell=1}^{J} \widehat{\beta}_\ell \widehat{\theta}_\ell$.[18] We can proceed in a similar way in the case of CES upper-tier utility, and the details are provided in Appendix F.

We summarize the structural parameters that we obtain for the two countries in Tables 1 and 2. Observe the substantial heterogeneity across French sectors: the shape parameters $\widehat{k}_j$ of the sectoral Pareto distributions range from 2.0 to 24.3, with an (unweighted) average of 5.7. In the UK, the differences are even larger, as the values of $\widehat{k}_j$ range from 1.5 to 41.3, with an (unweighted) average of 7.4.

Cutoff distortions. Given the values of $\widehat{k}_j$, $\widehat{\theta}_j$, $\widehat{\kappa}_j$, and $\widehat{\beta}_j$, we can now quantify the distortions in France and in the UK. We first compare the equilibrium and optimal cutoffs in each sector. Using the expressions in (28), we compute for each sector j the following measure of cutoff distortions:

$$\frac{(m_j^d)^{\text{eqm}} - (m_j^d)^{\text{opt}}}{(m_j^d)^{\text{opt}}} \times 100 = \left\{ \left[\kappa_j (k_j + 1)^2 \right]^{-\frac{1}{k_j+1}} - 1 \right\} \times 100, \tag{32}$$

which depends only on k_j as κ_j is a function of k_j only. Since there is too little selection by Proposition 7, $(m_j^d)^{\text{eqm}} > (m_j^d)^{\text{opt}}$ holds, so that expression (32) is always positive. The gap between the equilibrium and optimal cutoffs is smaller the larger is the sectoral shape parameter k_j, i.e., a larger mass of the productivity distribution is concentrated on low-productivity firms.

Tables 1 and 2 report the magnitudes of cutoff distortions for all sectors in France and the UK, which we illustrate in Figures 1 and 2 for those two countries. We find substantial

[18] The latter equality can be obtained by noting that the total revenue equals the total wage in each sector, i.e., $Le_j w = wL_j$, which implies $e_j = L_j/L$. We then evaluate (9) for the case of Cobb-Douglas upper-tier utility and CARA subutility functions, where $\mathcal{E}_{U,U_j} = \beta_j$ and $\overline{\mathcal{E}}_{u_j,q_j(m)} = \theta_j$.

Table 1: Sectoral data, parameter values, and distortions for France in 2008.

					Cobb-Douglas - CARA & Pareto						**Cobb-Douglas - CES & Pareto**			
				Std. dev.					Cutoff	Entry			Cutoff	Entry
Sector	Description	Firms	$\widehat{e}_j$	log emp	$\widehat{k}_j$	$\widehat{\theta}_j$	$\widehat{\kappa}_j$	$\widehat{\beta}_j$	distortions		$\widehat{\rho}_j$	$\widehat{\beta}_j$	distortions	
1	Agriculture	5551	0.0188	1.0038	2.8670	0.8721	0.0312	0.0188	21.8406	-0.1886	0.7421	0.0188	0	0.3470
2	Mining and quarrying	1132	0.0002	1.0523	3.5570	0.8911	0.0227	0.0002	17.9533	1.9848	0.7892	0.0002	0	6.7070
3	Food products, beverages, tobacco	38582	0.0697	0.9858	2.6642	0.8653	0.0346	0.0704	23.3225	-0.9765	0.7242	0.0697	0	-2.0711
4	Textiles, leather and footwear	4889	0.0205	1.0354	3.2891	0.8845	0.0255	0.0203	19.2867	1.2213	0.7730	0.0205	0	4.5251
5	Wood products	4607	0.0008	1.1811	8.4447	0.9471	0.0055	0.0007	7.9290	8.3958	0.9089	0.0008	0	22.8950
6	Pulp, paper, printing and publishing	12136	0.0086	1.1805	8.3928	0.9469	0.0055	0.0079	7.9764	8.3625	0.9083	0.0086	0	22.8198
7	Coke, refined petroleum, nuclear fuel	27	0.0168	1.1447	6.1501	0.9303	0.0094	0.0158	10.7480	6.4650	0.8756	0.0168	0	18.3985
8	Chemicals and chemical products	1194	0.0285	1.1688	7.5071	0.9413	0.0067	0.0264	8.8810	7.7318	0.8977	0.0285	0	21.3827
9	Rubber and plastics products	2760	0.0037	1.0332	3.2565	0.8836	0.0259	0.0037	19.4626	1.1220	0.7709	0.0037	0	4.2374
10	Other non-metallic mineral products	3426	0.0020	1.0428	3.4013	0.8873	0.0243	0.0019	18.7050	1.5521	0.7801	0.0020	0	5.4774
11	Basic metals	602	0.0001	1.2166	13.1203	0.9646	0.0025	0.0001	5.1666	10.3951	0.9410	0.0001	0	27.2453
12	Fabricated metal products	17249	0.0021	1.1442	6.1290	0.9301	0.0095	0.0020	10.7833	6.4415	0.8752	0.0021	0	18.3419
13	Machinery and equipment	8227	0.0053	1.1003	4.5835	0.9109	0.0153	0.0050	14.1902	4.2470	0.8345	0.0053	0	12.8416
14	Office, accounting, computing mach.	160	0.0033	1.0684	3.8519	0.8976	0.0201	0.0032	16.6828	2.7305	0.8045	0.0033	0	8.7831
15	Electrical machinery and apparatus	1656	0.0034	1.2466	24.2501	0.9802	0.0008	0.0030	2.8241	12.1791	0.9680	0.0034	0	30.8871
16	Radio, TV, communication equip.	786	0.0042	1.1439	6.1119	0.9299	0.0095	0.0040	10.8121	6.4223	0.8749	0.0042	0	18.2957
17	Medical, precision, optical instr.	3753	0.0050	1.0383	3.3327	0.8856	0.0250	0.0049	19.0565	1.3517	0.7758	0.0050	0	4.9020
18	Motor vehicles and (semi-)trailers	835	0.0326	1.1046	4.7020	0.9127	0.0147	0.0312	13.8546	4.4568	0.8386	0.0326	0	13.3862
19	Other transport equipment	452	0.0028	1.1128	4.9432	0.9162	0.0135	0.0026	13.2186	4.8581	0.8462	0.0028	0	14.4165
20	Manufacturing n.e.c; recycling	9802	0.0130	1.1760	8.0324	0.9447	0.0060	0.0120	8.3212	8.1207	0.9043	0.0130	0	22.2727
21	Electricity, gas and water supply	1279	0.0225	0.9745	2.5480	0.8610	0.0368	0.0228	24.2650	-1.4664	0.7129	0.0225	0	-3.6039
22	Construction	188513	0.0082	0.9992	2.8127	0.8704	0.0320	0.0083	22.2182	-0.3915	0.7376	0.0082	0	-0.2700
23	Wholesale and retail trade; repairs	274437	0.1377	1.0151	3.0067	0.8765	0.0291	0.1373	20.9236	0.3099	0.7532	0.1377	0	1.8463
24	Hotels and restaurants	113317	0.0489	0.9489	2.3083	0.8512	0.0420	0.0502	26.4702	-2.5803	0.6866	0.0489	0	-7.1669
25	Transport and storage	26847	0.0291	0.9962	2.7783	0.8692	0.0326	0.0292	22.4649	-0.5232	0.7346	0.0291	0	-0.6727
26	Post and telecommunications	1144	0.0191	1.0374	3.3186	0.8852	0.0252	0.0188	19.1303	1.3099	0.7749	0.0191	0	4.7813
27	Finance and insurance	12383	0.0376	0.9141	2.0264	0.8379	0.0498	0.0393	29.6331	-4.1024	0.6494	0.0376	0	-12.1881
28	Real estate activities	36902	0.1649	0.9517	2.3334	0.8523	0.0414	0.1691	26.2215	-2.4570	0.6895	0.1649	0	-6.7672
29	Renting of machinery and equipment	4815	0.0022	1.1101	4.8613	0.9151	0.0139	0.0021	13.4279	4.7255	0.8437	0.0022	0	14.0777
30	Computer and related activities	16355	0.0010	1.1944	9.7504	0.9535	0.0042	0.0010	6.8991	9.1285	0.9209	0.0010	0	24.5238
31	Research and development	1562	0.0074	1.2375	19.2934	0.9754	0.0012	0.0067	3.5386	11.6260	0.9598	0.0074	0	29.7810
32	Other Business Activities	132159	0.0073	1.0964	4.4803	0.9092	0.0159	0.0070	14.4958	4.0571	0.8309	0.0073	0	12.3453
33	Education	11401	0.0799	1.0726	3.9371	0.8994	0.0194	0.0776	16.3484	2.9297	0.8085	0.0799	0	9.3287
34	Health, social work, personal services	124202	0.1930	0.9659	2.4642	0.8577	0.0385	0.1966	24.9935	-1.8394	0.7042	0.1930	0	-4.7851

Notes: Column 1 reports the number of firms in each sector in the ESANE database for France in 2008 after trimming, column 2 the observed (re-scaled) expenditure shares from the French input-output table, and column 3 the observed standard deviation of the log number of employees across firms, where data are constructed as described in Appendix C.1. Column 4 reports the values of $\widehat{k}_j$ that we obtain by matching the numbers from column 3 to expression (C-1) in Appendix C.2. Columns 5 and 6 report the values of $\widehat{\theta}_j$ and $\widehat{\kappa}_j$ which are transformations of $\widehat{k}_j$. Column 7 reports the value $\widehat{\beta}_j$ obtained as described in Section 4.1. In columns 8 and 9 we report the magnitudes of cutoff and entry distortions at the sectoral level obtained from (32) and (33), respectively. Column 10 reports the value of $\widehat{\rho}_j$ obtained by matching the numbers from column 3 to expression (C-3) in Appendix C.2 while using $\widehat{k}_j$ from column 4. Column 11 reports the values $\widehat{\beta}_j$ which correspond to the expenditure shares from column 2. Finally, column 12 reports only zeroes as the CES model does not exhibit cutoff distortions, and column 13 reports the magnitudes of entry distortions as computed in (36).

Table 2: Sectoral data, parameter values, and distortions for the United Kingdom in 2005.

					Cobb-Douglas - CARA & Pareto						**Cobb-Douglas - CES & Pareto**			
				Std. dev.					Cutoff	Entry			Cutoff	Entry
Sector	Description	Firms	$\widehat{e}_j$	log emp	$\widehat{k}_j$	$\widehat{\theta}_j$	$\widehat{\kappa}_j$	$\widehat{\beta}_j$	distortions		$\widehat{\rho}_j$	$\widehat{\beta}_j$	distortions	
1	Agriculture	57969	0.0127	0.8424	1.5152	0.8069	0.0706	0.0138	37.7850	-8.1349	0.5607	0.0127	0	-24.3233
2	Mining and quarrying	1124	0.0008	1.2580	35.5036	0.9863	0.0004	0.0007	1.9363	12.2922	0.9781	0.0008	0	32.0111
3	Food products, beverages. tobacco	4606	0.0442	1.1260	5.3830	0.9220	0.0118	0.0421	12.1970	4.9662	0.8584	0.0442	0	15.8529
4	Textiles, leather and footwear	9041	0.0213	1.1829	8.6063	0.9480	0.0053	0.0198	7.7852	7.9348	0.9106	0.0213	0	22.8954
5	Wood products	7301	0.0014	1.1079	4.7949	0.9141	0.0142	0.0013	13.6024	4.0730	0.8416	0.0014	0	13.5846
6	Pulp, paper, printing and publishing	24882	0.0112	1.1142	4.9862	0.9168	0.0133	0.0108	13.1112	4.3825	0.8475	0.0112	0	14.3787
7	Coke, refined petroleum, nuclear fuel	122	0.0104	1.1442	6.1295	0.9301	0.0095	0.0098	10.7826	5.8902	0.8752	0.0104	0	18.1245
8	Chemicals and chemical products	1989	0.0088	1.2614	41.2898	0.9882	0.0003	0.0079	1.6669	12.5055	0.9812	0.0088	0	32.4242
9	Rubber and plastics products	5152	0.0035	1.1077	4.7899	0.9140	0.0142	0.0034	13.6159	4.0646	0.8414	0.0035	0	13.5630
10	Other non-metallic mineral products	3412	0.0017	1.0171	3.0332	0.8773	0.0287	0.0017	20.7588	-0.1201	0.7552	0.0017	0	1.9273
11	Basic metals	1203	0.0003	1.1800	8.3555	0.9466	0.0056	0.0002	8.0108	7.7767	0.9079	0.0003	0	22.5386
12	Fabricated metal products	24116	0.0019	1.2025	10.7654	0.9575	0.0035	0.0017	6.2663	9.0180	0.9283	0.0019	0	25.2887
13	Machinery and equipment	8719	0.0064	1.1206	5.1953	0.9196	0.0125	0.0061	12.6131	4.6993	0.8534	0.0064	0	15.1825
14	Office, accounting, computing mach.	898	0.0006	1.0715	3.9145	0.8989	0.0196	0.0006	16.4360	2.3441	0.8075	0.0006	0	8.9841
15	Electrical machinery and apparatus	2694	0.0015	1.0675	3.8347	0.8973	0.0202	0.0014	16.7521	2.1569	0.8037	0.0015	0	8.4690
16	Radio, TV, communication equip.	1004	0.0057	1.2070	11.4206	0.9598	0.0032	0.0052	5.9160	9.2724	0.9324	0.0057	0	25.8380
17	Medical, precision, optical instr.	2443	0.0016	1.0956	4.4595	0.9089	0.0160	0.0016	14.5590	3.4788	0.8301	0.0016	0	12.0353
18	Motor vehicles and (semi-)trailers	2059	0.0272	1.1459	6.2088	0.9308	0.0093	0.0256	10.6513	5.9773	0.8768	0.0272	0	18.3347
19	Other transport equipment	1012	0.0036	1.2551	31.7979	0.9848	0.0005	0.0032	2.1599	12.1161	0.9756	0.0036	0	31.6677
20	Manufacturing n.e.c; recycling	16028	0.0109	1.0735	3.9535	0.8997	0.0193	0.0107	16.2857	2.4335	0.8093	0.0109	0	9.2289
21	Electricity, gas and water supply	428	0.0261	1.1854	8.8336	0.9492	0.0050	0.0241	7.5915	8.0711	0.9128	0.0261	0	23.2015
22	Construction	156266	0.0085	0.9638	2.4443	0.8569	0.0389	0.0087	25.1733	-2.4391	0.7020	0.0085	0	-5.2515
23	Wholesale and retail trade; repairs	306437	0.1850	0.9788	2.5911	0.8626	0.0359	0.1884	23.9071	-1.7932	0.7172	0.1850	0	-3.2016
24	Hotels and restaurants	130213	0.0781	0.9975	2.7940	0.8697	0.0323	0.0789	22.3519	-0.9789	0.7359	0.0781	0	-0.6720
25	Transport and storage	31912	0.0392	0.9289	2.1417	0.8436	0.0464	0.0408	28.2533	-3.9495	0.6655	0.0392	0	-10.1799
26	Post and telecommunications	4654	0.0181	0.9526	2.3417	0.8527	0.0412	0.0186	26.1401	-2.9224	0.6905	0.0181	0	-6.8087
27	Finance and insurance	15890	0.0807	0.9190	2.0638	0.8398	0.0486	0.0844	29.1713	-4.3838	0.6548	0.0807	0	-11.6270
28	Real estate activities	80146	0.1104	0.8570	1.6199	0.8141	0.0654	0.1192	35.7739	-7.3083	0.5813	0.1104	0	-21.5440
29	Renting of machinery and equipment	13615	0.0061	1.0636	3.7599	0.8957	0.0209	0.0059	17.0596	1.9760	0.8000	0.0061	0	7.9678
30	Computer and related activities	102580	0.0010	0.8645	1.6720	0.8176	0.0630	0.0010	34.8511	-6.9194	0.5911	0.0010	0	-20.2240
31	Research and development	1603	0.0001	1.0575	3.6486	0.8932	0.0218	0.0001	17.5386	1.6963	0.7942	0.0001	0	7.1867
32	Other Business Activities	371014	0.0041	0.9100	1.9952	0.8363	0.0508	0.0043	30.0287	-4.7829	0.6448	0.0041	0	-12.9669
34	Education	23494	0.0625	1.1440	6.1179	0.9300	0.0095	0.0591	10.8019	5.8774	0.8750	0.0625	0	18.0934
35	Health, social work, personal services	215336	0.2044	1.0816	4.1275	0.9031	0.0180	0.1988	15.6477	2.8157	0.8170	0.2044	0	10.2671

Notes: Column 1 reports the number of firms in each sector in the BSD database for the UK in 2005 after trimming, column 2 the observed (re-scaled) expenditure shares from the UK input-output table, and column 3 the observed standard deviation of the log number of employees across firms, where data are constructed as described in Appendix C.1. Column 4 reports the values of $\widehat{k}_j$ that we obtain by matching the numbers from column 3 to expression (C-1) in Appendix C.2. Columns 5 and 6 report the values of $\widehat{\theta}_j$ and $\widehat{\kappa}_j$ which are transformations of $\widehat{k}_j$. Column 7 reports the value $\widehat{\beta}_j$ obtained as described in Section 4.1. In columns 8 and 9 we report the magnitudes of cutoff and entry distortions at the sectoral level obtained from (32) and (33), respectively. Column 10 reports the value of $\widehat{\rho}_j$ obtained by matching the numbers from column 3 to expression (C-3) in Appendix C.2 while using $\widehat{k}_j$ from column 4. Column 11 reports the values $\widehat{\beta}_j$ which correspond to the expenditure shares from column 2. Finally, column 12 reports only zeroes as the CES model does not exhibit cutoff distortions, and column 13 reports the magnitudes of entry distortions as computed in (36).

distortions due to insufficient selection. For France, the simple average across sectors is 15.9%, but with huge sectoral variation from only 2.8% to almost 30%. In the UK, the average is 16.7% and the range goes from 1.7% to 37.8%. The correlation of those distortions between the two countries is 0.356, while the Spearman rank correlation is 0.328. Thus, the model makes roughly similar predictions on which sectors in France and the UK exhibit greater cutoff distortions. We discuss this point in more detail below.

Entry distortions. Turning to the gap between the equilibrium and optimal entry, or equivalently the gap between the equilibrium and optimal labor allocations in our examples, we use expressions (29) and Proposition 5, together with (30), to compute the following measure of intersectoral distortions for each sector j:

$$\frac{(N_j^E)^{\text{eqm}} - (N_j^E)^{\text{opt}}}{(N_j^E)^{\text{opt}}} \times 100 = \frac{(L_j)^{\text{eqm}} - (L_j)^{\text{opt}}}{(L_j)^{\text{opt}}} \times 100 = \left(\frac{\theta_j}{\sum_{\ell=1}^{J} \beta_\ell \theta_\ell} - 1 \right) \times 100. \tag{33}$$

Based on (33), our model predicts that 25 sectors in the French economy exhibit excess entry by up to 12.2%. The remaining 9 sectors display insufficient entry by up to -4.1%. In the UK, excess entry arises in 23 sectors, whereas insufficient entry occurs in 11 sectors, with a range of entry distortions from -8.1% to 12.5%. See Tables 1 and 2 for the detailed numbers, and Figures 1 and 2 for a graphical illustration of those distortions.

Digging deeper into these patterns, we find some similarities between France and the UK. In both countries, excess entry typically occurs in manufacturing. See, for example, [11] 'Basic metals' and [15] 'Electrical machinery and apparatus' in France, or [8] 'Chemical products' and [19] 'Transport equipment' in the UK, where it is particularly strong. By contrast, insufficient entry is almost exclusively a phenomenon of service sectors.[19] See, for example, [24] 'Hotels and restaurants' and [27] 'Finance and insurance' in France, or [28] 'Real estate' and [32] 'Other business services' in the UK, where we find strongly negative values. Overall, the correlation of entry distortions across sectors in the two countries is 0.330 and the Spearman rank correlation is 0.328. Furthermore, the direction or 'sign' of inefficient entry is the same in 26 out of 34 sectors, i.e., in more than three-quarter of the sectors. Put differently, the model makes similar predictions as to which sectors in the two countries tend to display excessive or insufficient entry.

[19]The sector [1] 'Agriculture' also exhibits insufficient entry in both countries, and particularly so in the UK, but hardly any manufacturing sector in either country has too few entrants. Notice that these findings do, of course, *not* imply that the mass of entrants in manufacturing is larger than that in services in equilibrium, since they refer to a sector-by-sector comparison of the equilibrium and the optimal entry.

Figure 1: Cutoff and entry distortions, Cobb-Douglas - CARA model for France in 2008.

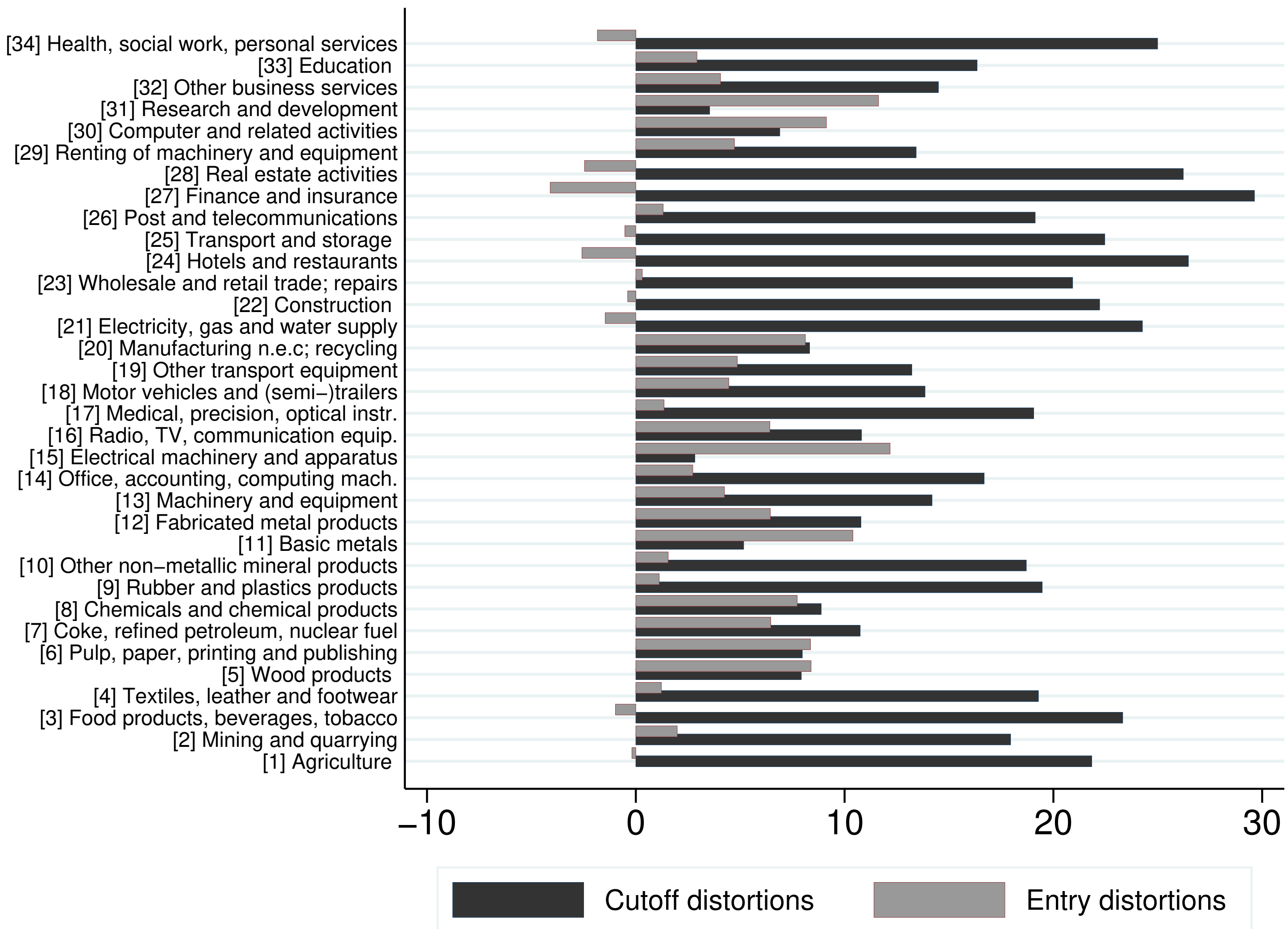
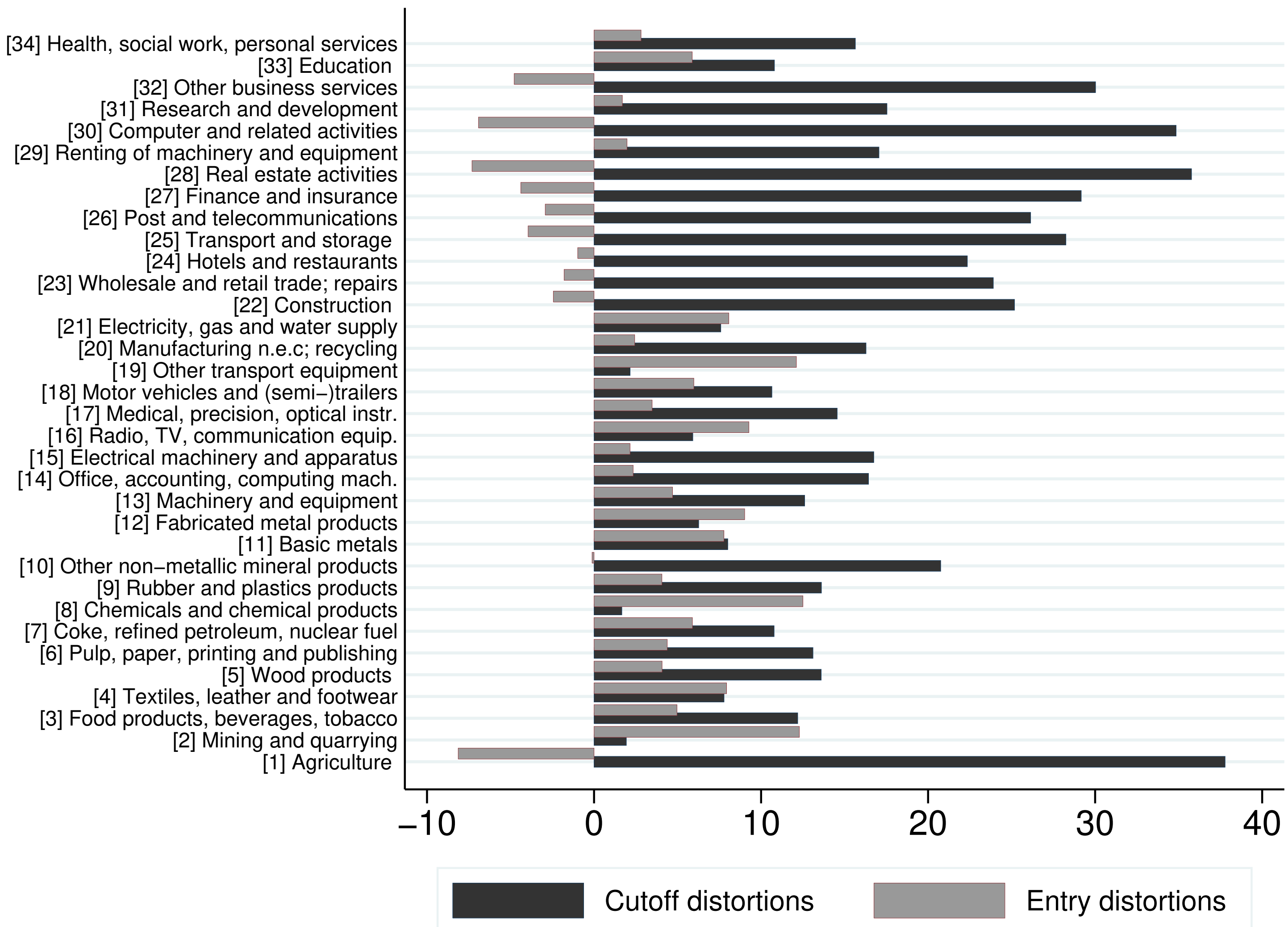


Figure 2: Cutoff and entry distortions, Cobb-Douglas - CARA model for the United Kingdom in 2005.



Recall that in the CARA model the larger the value of k_j, the more excessive are the firm entry and the labor allocation, but the smaller is the magnitude of insufficient selection. In other words, manufacturing sectors in both countries not only tend to attract an excessive number of firms and workers, but also display relatively smaller cutoff distortions, i.e., equilibrium firm selection relatively closer to the optimum. By contrast, there are too few entrants in many service sectors, and firm selection is far less severe than it should be from a social point of view. It is worth emphasizing that those predictions are based on a general equilibrium model that recognizes all interdependencies across sectors in the economy. Thus, our analysis differs from the conventional approach in industrial organization that has typically studied entry and selection for a single industry in partial equilibrium.

Aggregate welfare distortion. Having analyzed cutoff and entry distortions in each sector, we now consider the aggregate welfare distortion in the economy. To this end, we use the concept of the *Allais surplus* (Allais, 1943, 1977) since compensating and equivalent variations, which are used to analyze the welfare change *between two equilibria*, are not readily applicable to measuring the welfare gap *between the equilibrium and optimum*. Intuitively, we measure the amount of labor—which is taken as the numeraire—that can be saved when the planner minimizes the resource cost of attaining the equilibrium utility level.

Let $L^A(U^{\text{eqm}})$ denote the minimum amount of labor that the social planner requires to attain the equilibrium utility level. By construction, $L^A(U^{\text{eqm}})$ is not greater than the amount of labor L that the market economy requires to reach the equilibrium utility level because the labor market clears in equilibrium and because their may be distortions. As shown in Appendix D, we can define a measure of the aggregate welfare distortion based on the Allais surplus as follows:

$$-\frac{L^A(U^{\text{eqm}}) - L}{L} \times 100 = \left\{ 1 - \frac{\prod_{j=1}^{J} \left[(k_j+1)^2 \kappa_j \right]^{\frac{\beta_j}{k_j+1}}}{\sum_{\ell=1}^{J} \beta_\ell \theta_\ell} \right\} \times 100. \tag{34}$$

Plugging the values of $\widehat{k}_j$, $\widehat{\theta}_j$, $\widehat{\kappa}_j$, and $\widehat{\beta}_j$ from Tables 1 and 2 into (34), we can compute the magnitude of the aggregate welfare distortion in France and in the UK, respectively.

Table 3 summarizes our results. For France, the aggregate welfare distortion is 5.93%, and for the UK it is 5.85%. In words, to achieve the equilibrium utility level in each of the two countries, the social planner requires almost 6% less aggregate labor input when compared to the case with utility maximizing consumers and profit maximizing firms.

Disentangling the relative contribution of the cutoff and entry distortions is difficult,

Table 3: Aggregate welfare distortions as measured by the Allais surplus.

	France		UK	
	CARA	CES	CARA	CES
Aggregate distortion (% of aggregate labor input saved)	5.93	0.34	5.85	0.99
Cutoff and quantity distortion	81.81	0	95.11	0
Entry and labor distortion (as % of aggregate distortion)	18.19	100	4.89	100

since it is generally not possible to shut down one without affecting the other.[20] To gauge the potential importance of within and between sector distortions, we hence proceed as follows. We pool our data across all sectors and proceed *as if* there were only a single sector. Distortions in the labor allocation cannot arise in this single-sector case—since by definition $L_j^{\mathrm{eqm}} = L_j^{\mathrm{opt}} = L$—and entry is efficient by (29). Therefore, the welfare gap between the equilibrium and optimum depends only on cutoff and output distortions. We then estimate the value of k for that single sector in the same way as before, by matching the standard deviation of the (log) employment distribution across all firms. This yields $\widehat{k} = 3.5687$ for France and $\widehat{k} = 3.0598$ for the UK. Plugging that common value into (34), we compute the associated Allais surplus for the single-sector economy and compare it with the Allais surplus in the multi-sector case. The results are summarized in the bottom part of Table 3. As can be seen, the distortions in the single-sector case are 18.19% smaller for France, and 4.89% smaller for the UK. Put differently, disregarding entry and labor distortions would lead to an underestimation of the aggregate welfare distortion by 5%–18% in our CARA example with a Cobb-Douglas upper-tier utility function.

CES upper-tier utility. We have also considered the case of an alternative upper-tier utility function. In particular, we have replaced the Cobb-Douglas upper-tier function with the CES function $U = \{\sum_{j=1}^J \beta_j [\widetilde{U}_j(U_j)]^{(\sigma-1)/\sigma}\}^{\sigma/(\sigma-1)}$. The Allais surplus for that case with CES upper-tier utility and CARA subutility is given by (see Appendix F for details):

$$-\frac{L^A(U^{\mathrm{eqm}}) - L}{L} \times 100 = \left[1 - \frac{1}{\sum_{\ell=1}^J \beta_\ell \theta_\ell} \cdot \left\{ \sum_{j=1}^J \beta_j \left[(k_j+1)^2 \kappa_j\right]^{\frac{1-\sigma}{k_j+1}} \right\}^{\frac{1}{1-\sigma}} \right] \times 100. \qquad (35)$$

[20]We know from the results in Corollary 2 that entry in the CARA case is efficient if and only if all k_j's are the same. Hence, one could think of setting all k_j's to same common value to shut down entry distortions. However, the common value of k that is chosen has an effect on the magnitude of cutoff distortions.

Notice that, once we choose a value of σ, this expression for the aggregate welfare distortion can be computed using $\widehat{k}_j$, $\widehat{\theta}_j$, $\widehat{\kappa}_j$, and $\widehat{\beta}_j$ from Tables 1 and 2, respectively.

Figure 3: Aggregate welfare distortions in the CES - CARA model as a function of σ.

(a) France. (b) UK.

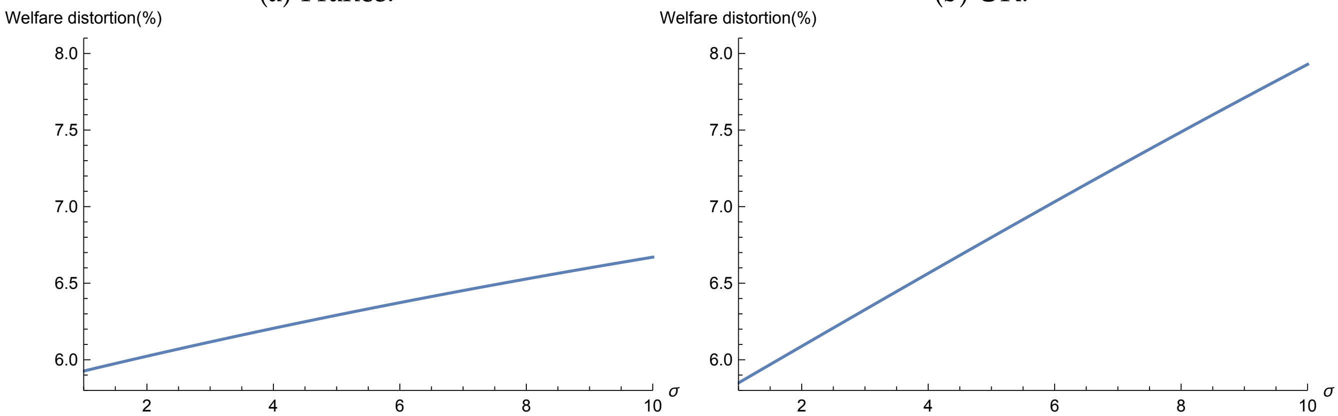


Figure 3 illustrates the magnitude of the aggregate welfare distortion given by (35) as a function of σ for France (panel (a)) and the UK (panel (b)). We find that the higher is the elasticity of substitution between sectors, the stronger is the aggregate welfare distortion in both countries. It ranges between 6% and 7% in France, and between 6% and 8% in the UK. Treating the economy *as if* it consisted of a single sector, as before, we re-quantify the entry and labor distortions as a percentage of the aggregate welfare distortion. For $\sigma \in (1, 10)$, it ranges between 18% and 27% in France, and between 5% and 29% in the UK. Thus, the higher the elasticity of substitution for the upper-tier utility function, the stronger the underestimation of the aggregate welfare distortion from disregarding inefficient entry and labor allocation. It can reach almost 30% for reasonable parameter values.

4.3 Quantifying distortions: the CES subutility case

Finally, we quantify the workhorse model with Cobb-Douglas upper-tier and CES subutility functions. Recall that there are no cutoff distortions with CES subutility functions. However, by Corollary 1, there are still labor and entry distortions due to heterogeneity in the elasticity $\mathcal{E}_{U,U_j}$ of the upper-tier utility function and in the weighted average $\overline{\mathcal{E}}_{u_j,q_j(m)}$ of the elasticities of the subutility functions when the ρ_j terms differ across sectors. How large are the welfare distortions for France and the UK predicted by the CES model?

To quantify this model, we use the same sector-specific statistics as before: the standard deviation of (log) firm-level employment, not including the labor input for R&D which we use as a proxy for sunk entry and fixed costs. To match this observed data moment, we also assume sector-specific Pareto distributions for productivity draws, and then derive the corresponding theoretical expression for the CES case. As can be seen from equation (C-3) in Appendix C.2, this expression now depends on two parameters: ρ_j and k_j. Since the k_j's are technology parameters that do not depend on consumer preferences, we keep the same values of $\widehat{k}_j$ from the VES model above. We can then uniquely back out the corresponding values for $\widehat{\rho}_j$. Since the equilibrium expenditure share is β_j for this case, the value of $\widehat{\beta}_j$ for each sector can be obtained by setting $\widehat{\beta}_j = \widehat{e}_j$, where $\sum_{j=1}^{J} \widehat{\beta}_j = \sum_{j=1}^{J} \widehat{e}_j = 1$ by definition of the observed expenditure share.

The parameter values thus obtained for France and the UK are reported in Tables 1 and 2. Equipped with those numbers, we can quantify the magnitude of entry distortions for each sector j as follows:

$$\frac{(N_j^E)^{\text{eqm}} - (N_j^E)^{\text{opt}}}{(N_j^E)^{\text{opt}}} \times 100 = \frac{L_j^{\text{eqm}} - L_j^{\text{opt}}}{L_j^{\text{opt}}} \times 100 = \left(\rho_j \sum_{\ell=1}^{J} \frac{\beta_\ell}{\rho_\ell} - 1 \right) \times 100. \tag{36}$$

As can be seen from Tables 1 and 2, in both countries the CES and VES models make very similar predictions as to which sectors display excess or insufficient entry. Yet, the CES model implies larger magnitudes than the VES model. In France, the range of inefficient entry and labor allocation goes from -12.2% to 30.9%, and in the UK from -24.3% to 32.4%.

To quantify the aggregate welfare distortion, we again rely on the Allais surplus and compute the following expression (see Appendix D for details):

$$-\frac{L^A(U^{\text{eqm}}) - L}{L} \times 100 = \left[1 - \prod_{j=1}^{J} \left(\rho_j \sum_{\ell=1}^{J} \frac{\beta_\ell}{\rho_\ell} \right)^{\frac{\beta_j/\rho_j}{\sum_{\ell=1}^{J}(\beta_\ell/\rho_\ell)}} \right] \times 100. \tag{37}$$

The results are 0.34% for France, and 0.99% for the UK, as summarized in Table 3. In other words, less than 1% of the aggregate labor input could be saved if the social planner minimized the resource cost to attain the equilibrium utility level. Compared to the VES model, where the corresponding number is roughly 6%, it appears that the aggregate welfare distortion in the CES model is much smaller than that in the VES model. However, correcting the inefficiencies between sectors would still lead to substantial changes in entry patterns and sectoral employment shares.

4.4 Quantifying distortions: robustness checks

Tables G-1 to G-3 in Appendix G provide a set of results for two additional robustness checks. Details on the data we use and the various expressions required to compute the numbers are contained in Appendix C.

In the first robustness check, we back out heterogeneity in productivity across firms from revenue data as compared to the employment data used in the baseline quantification. Tables G-1 and G-2 in Appendix G provide detailed results. In this instance we do not need a measure of fixed costs but, as explained above, revenue data are more likely to contain measurement error. The correlation between the standard deviation of log employment and log revenue is 0.67 for the UK and 0.39 for France. Patterns of excess/insufficient entry are broadly in line with the baseline cases, with manufacturing (service) industries being characterized by excess (insufficient) entry. In terms of aggregate welfare distortions, we get 10.29% with CARA subutilites and 2.48% with CES subutilities for France; and 9.85% for CARA subutilities and 1.06% for CES subutilities for the UK. These numbers are again similar between the UK and France and close to those obtained in the baseline specification.

In the second robustness check, we employ an alternative measure of fixed costs based on industry-level profits rather than R&D outlays. Such an exercise can be performed for France only and results are reported in Table G-3. In terms of the correlation between the standard deviation of log employment from Tables 1 and G-3 the value stands at 0.56. Patterns of excess/insufficient entry are similar in those tables. More specifically, excess entry is a manufacturing-industry phenomenon (the only common exception being the "Food products, beverages and tobacco" industry) while service industries are often characterized by insufficient entry. Crucially, as far as aggregate welfare distortions are concerned, we get 6.62% for the CARA case and 0.95% for the CES case, which are again in line with our baseline quantification results (5.93% and 0.34%, respectively).

To summarize, throughout our quantification analyses we find consistent patterns and numbers using different data to back out productivity differences across firms (and different proxies for fixed costs). These findings suggest that our key results are robust and we may conclude that the aggregate welfare distortions for France in 2008 and the UK in 2005 are in the 6-10% range.

5 Conclusions

We have developed a general equilibrium model of monopolistic competition with multiple sectors and heterogeneous firms. Comparing the equilibrium and optimal allocations in our general framework with unspecified utility functions and productivity distributions, we have characterized the various distortions that operate in our economy. We have considered two examples that can be readily taken to the data. Using French data for 2008 and UK data for 2005, we have quantified the aggregate welfare distortions while uncovering substantial sectoral heterogeneity and assessing the contribution of each type of distortions to the overall welfare losses.

Our preferred specification implies substantial aggregate welfare distortions for France and for the UK, each of which amounts to almost 6% of the respective economy's aggregate labor input. Our results suggest that inefficiencies within and between sectors both matter in practice. Removing those distortions would presumably require rather different interventions: industry policies to address the latter problem, combined with policies targeted at specific firms to address the former. A general lesson that one can deduce from our analysis is that interdependencies are important for the design of such programs: the optimal policy for one sector is not only influenced by conditions of that particular sector, but it depends on the characteristics of all sectors in the economy. We leave it to future work to explore the details of feasible policy schemes that alleviate misallocations.

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Appendix

A. Proofs of the propositions

This appendix provides all the proofs of the propositions. To alleviate notation, we suppress indices for sectors and arguments wherever possible.

A.1. Proof of Proposition 1. This result can be established using a similar method as in Zhelobodko et al. (2012). However, we provide an alternative proof that can be readily applied to the optimal cutoff and quantities (see Appendix A.3). Using the profit-maximizing price (5) for the marginal variety, we can rewrite the ZCP condition (6) as

$$\frac{r_{u_j}(q_j^d)}{1-r_{u_j}(q_j^d)}m_j^d q_j^d = r_{u_j}(q_j^d)\frac{p_j^d}{w}q_j^d = \frac{f_j}{L},$$

which, together with the first-order condition (2) for the marginal variety, $u_j'(q_j^d) = \lambda_j p_j^d$, yields

$$r_{u_j}(q_j^d)u_j'(q_j^d)q_j^d = -(q_j^d)^2 u_j''(q_j^d) = \frac{f_j}{L}\lambda_j w.$$

The left-hand side is increasing in q_j^d since

$$\frac{\partial}{\partial q_j^d}\left(-(q_j^d)^2 u_j''(q_j^d)\right) = -q_j^d u_j''(q_j^d)\left[2-\left(-\frac{q_j^d u_j'''(q_j^d)}{u_j''(q_j^d)}\right)\right] = -q_j^d u_j''(q_j^d)\left[2-r_{u_j'}(q_j^d)\right] > 0,$$

where we use the second-order condition $r_{u_j'}(q_j(m)) < 2$. Thus, we know that q_j^d is increasing in the market aggregate $\lambda_j w$.

Furthermore, using the first-order condition (2) and the profit-maximizing price (5) for the marginal variety, we have

$$\left[1-r_{u_j}(q_j^d)\right]u_j'(q_j^d) = (\lambda_j w)m_j^d. \tag{A-1}$$

The left-hand side is decreasing in q_j^d since

$$\frac{\partial}{\partial q_j^d}\left\{\left[1-r_{u_j}(q_j^d)\right]u_j'(q_j^d)\right\} = u_j''(q_j^d)\left[2-r_{u_j'}(q_j^d)\right] < 0.$$

Hence, since we have shown above that $\partial q_j^d/\partial(\lambda_j w) > 0$, the left-hand side in (A-1) decreases as $\lambda_j w$ on the right-hand side of (A-1) increases. It then follows that m_j^d is decreasing in $\lambda_j w$.

Similarly, using the first-order conditions (2) and the profit-maximizing prices (5) for other varieties, we have

$$\left[1 - r_{u_j}(q_j(m))\right] u_j'(q_j(m)) = (\lambda_j w)m.$$

Since the left-hand side is decreasing in $q_j(m)$, we know that $q_j(m)$ is decreasing in $\lambda_j w$.

Next, we rewrite the ZEP condition (7) as

$$L \int_0^{m_j^d} \left\{ \left[\frac{1}{1 - r_{u_j}(q_j(m))} - 1 \right] m q_j(m) - \frac{f_j}{L} \right\} \mathrm{d}G_j(m) = F_j. \tag{A-2}$$

Given that m_j^d and $q_j(m)$ are decreasing in $\lambda_j w$, we differentiate the left-hand side of this expression with respect to $\lambda_j w$ as follows:

$$\begin{aligned} &L \left\{ \left[\frac{1}{1 - r_{u_j}(q_j^d)} - 1 \right] m_j^d q_j(m_j^d) - \frac{f_j}{L} \right\} g_j(m_j^d) \frac{\partial m_j^d}{\partial(\lambda_j w)} \\ &\qquad + L \int_0^{m_j^d} \left\{ \frac{r_{u_j}'(q_j(m))}{[1 - r_{u_j}(q_j(m))]^2} q_j(m) + \frac{r_{u_j}(q_j(m))}{1 - r_{u_j}(q_j(m))} \right\} m \frac{\partial q_j(m)}{\partial(\lambda_j w)} \mathrm{d}G_j(m). \end{aligned}$$

The first-term is zero by the ZCP condition (6). Noting that

$$\begin{aligned} r_{u_j}(q_j(m)) &= -\frac{q_j(m) u_j''(q_j(m))}{u_j'(q_j(m))} \\ r_{u_j}'(q_j(m)) &= -\frac{[u_j''(q_j(m)) + u_j'''(q_j(m)) q_j(m)] u_j'(q_j(m)) - q_j(m)[u_j''(q_j(m))]^2}{[u_j'(q_j(m))]^2}, \end{aligned}$$

the second term can be expressed as:

$$L \int_0^{m_j^d} \left\{ \frac{[2 - r_{u_j'}(q_j(m))] r_{u_j}(q_j(m))}{[1 - r_{u_j}(q_j(m))]^2} \right\} m \frac{\partial q_j(m)}{\partial(\lambda_j w)} \mathrm{d}G_j(m) < 0,$$

where we use the second-order condition $r_{u_j'}(q_j(m)) < 2$. Hence, the left-hand side of the ZEP condition (A-2) is decreasing in $\lambda_j w$.

Assume that fixed costs, f_j, and sunk costs, F_j, are not too large. The former ensures that profits are non-negative (see the ZCP condition in (6)). The latter ensures existence. The left-hand side of the ZEP condition is strictly decreasing in $\lambda_j w$, whereas the right-hand side is constant. Hence, if fixed costs, f_j, and sunk costs, F_j, are not too large, then there exists a unique solution for $\lambda_j w$. Using the unique $\lambda_j w$ thus obtained, we can establish the existence and uniqueness of m_j^d and $q_j(m)$ since both are decreasing in $\lambda_j w$. □

A.2. Proof of Proposition 2. The first-order conditions (2) and (3), when combined with equation (11), imply that

$$\frac{\left[N_j^E \int_0^{m_j^d} u_j(q_j(m))\mathrm{d}G_j(m)\right]^{\xi_j}}{\left[N_\ell^E \int_0^{m_\ell^d} u_\ell(q_\ell(m))\mathrm{d}G_\ell(m)\right]^{\xi_\ell}} = \frac{p_j^d}{p_\ell^d}\frac{\gamma_\ell}{\gamma_j}\frac{u_\ell'(q_\ell^d)}{u_j'(q_j^d)}. \tag{A-3}$$

When f_j and F_j are not too large, the market aggregate $\lambda_j w$ is uniquely determined by the ZEP condition and so are sector-specific cutoffs m_j^d and the associated prices p_j^d and quantities q_j^d and $q_j(m)$ (see Appendix A.1). Since the ZEP condition does not include N_j^E, those variables are independent of N_j^E. Thus, the integrals in (A-3) are independent of N_j^E and N_ℓ^E. The right-hand side of equation (A-3) is strictly positive and finite. By monotonicity, there clearly exists a unique $N_j^E(N_\ell^E)$. This relationship satisfies $(N_j^E)' > 0$, $N_j^E(0) = 0$ and $\lim_{N_\ell^E \to \infty} N_j^E(N_\ell^E) = \infty$.

In each sector j, labor supply L_j equals labor demand $N_j^E\left\{\int_0^{m_j^d} \left[Lmq_j(m) + f_j\right]\mathrm{d}G_j(m) + F_j\right\}$, so that

$$\frac{L_j}{N_j^E} - L\int_0^{m_j^d} mq_j(m)\mathrm{d}G_j(m) = f_j G_j(m_j^d) + F_j. \tag{A-4}$$

Plugging expression (A-4) into (7) yields

$$N_j^E \int_0^{m_j^d} \frac{mq_j(m)}{1 - r_{u_j}\big(q_j(m)\big)}\mathrm{d}G_j(m) = \frac{L_j}{L}. \tag{A-5}$$

Substituting $N_j^E(N_\ell^E)$ obtained from (A-3) into (A-5), summing over j, and using the overall labor market clearing condition $L = \sum_{j=1}^J L_j$, we then have the following equilibrium condition:

$$\sum_{j=1}^J N_j^E(N_\ell^E)\int_0^{m_j^d} \frac{mq_j(m)}{1 - r_{u_j}(q_j(m))}\mathrm{d}G_j(m) = 1. \tag{A-6}$$

Observe that all integral terms on the left-hand side of (A-6) are positive and independent of the masses of entrants, whereas the right-hand side equals one. Since the limit of the left-hand side is zero when N_ℓ^E goes to zero, and infinity when N_ℓ^E goes to infinity, the existence and uniqueness of a solution for N_ℓ^E follows directly by the properties of $N_j^E(\cdot)$. Since the terms in braces of the right-hand side of (10) are uniquely determined by Proposition 1, the existence and uniqueness of N_j^E implies those of L_j, which proves Proposition 2. □

A.3. Proof of Proposition 3. Plugging the first-order condition for the marginal variety $m_j^d = u_j'(q_j^d)/\delta_j$ into (18), we have

$$u_j(q_j^d) - u_j'(q_j^d)q_j^d = \frac{f_j}{L}\delta_j. \tag{A-7}$$

The left-hand side is increasing in q_j^d (since $u_j'' < 0$), which establishes that q_j^d is increasing in δ_j. Thus, $u'(q_j^d)$ is decreasing in δ_j. Then, from the first-order condition for the marginal variety, we see that when δ_j increases, m_j^d must decrease because $u_j'(q_j^d)/\delta_j$ decreases. Hence, m_j^d is a decreasing function of δ_j. From the first-order conditions for the other varieties, $u'(q_j(m)) = \delta_j m$, we know that $q_j(m)$ is decreasing in δ_j.

Next, we rewrite the ZESP condition (17) as

$$L\int_0^{m_j^d}\left[\left(\frac{1}{\mathcal{E}_{u_j,q_j(m)}} - 1\right)mq_j(m) - \frac{f_j}{L}\right]g_j(m)\mathrm{d}m = F_j.$$

Given that m_j^d and $q_j(m)$ are decreasing in δ_j, we differentiate the left-hand side of this expression with respect to δ_j as follows:

$$L\left[\left(\frac{1}{\mathcal{E}_{u_j,q_j^d}} - 1\right)m_j^d q_j^d - \frac{f_j}{L}\right]g_j(m_j^d)\frac{\partial m_j^d}{\partial \delta_j}$$
$$+L\int_0^{m_j^d}\left[-\frac{1}{\mathcal{E}_{u_j,q_j(m)}}\frac{\partial \mathcal{E}_{u_j,q_j(m)}}{\partial q_j(m)}\frac{q_j(m)}{\mathcal{E}_{u_j,q_j(m)}} + \frac{1-\mathcal{E}_{u_j,q_j(m)}}{\mathcal{E}_{u_j,q_j(m)}}\right]m\frac{\partial q_j(m)}{\partial \delta_j}g_j(m)\mathrm{d}m,$$

where the first term is zero by (18). Using

$$\frac{\partial \mathcal{E}_{u_j,q_j(m)}}{\partial q_j(m)}\frac{q_j(m)}{\mathcal{E}_{u_j,q_j(m)}} = 1 - r_{u_j}(q_j(m)) - \mathcal{E}_{u_j,q_j(m)},$$

we finally have

$$L\int_0^{m_j^d}\frac{r_{u_j}(q_j(m))}{\mathcal{E}_{u_j,q_j(m)}}m\frac{\partial q_j(m)}{\partial \delta_j}g_j(m)\mathrm{d}m < 0,$$

where the inequality comes from $\partial q_j(m)/\partial \delta_j < 0$.

Assume that fixed costs, f_j, and sunk costs, F_j, are not too large. The former ensures that social profits are non-negative (see the ZCSP condition (18)), and the latter ensures existence. The left-hand side of the ZESP condition is strictly decreasing in δ_j, whereas the right-hand side is constant. Hence, if fixed costs, f_j, and sunk costs, F_j, are not too large, then there exists a unique solution for δ_j. Using the unique δ_j thus obtained, we can establish the existence and uniqueness of m_j^d and $q_j(m)$ since both are decreasing in δ_j. □

A.4. Proof of Proposition 4. The first-order conditions (13) and (16), when combined with equation (11), imply that

$$\frac{\left[N_j^E \int_0^{m_j^d} u_j(q_j(m))\mathrm{d}G_j(m)\right]^{\xi_j}}{\left[N_\ell^E \int_0^{m_\ell^d} u_\ell(q_\ell(m))\mathrm{d}G_\ell(m)\right]^{\xi_\ell}} = \frac{m_j^d}{m_\ell^d}\frac{\gamma_\ell}{\gamma_j}\frac{u_\ell'(q_\ell^d)}{u_j'(q_j^d)}. \tag{A-8}$$

When f_j and F_j are not too large, δ_j is uniquely determined by the ZESP condition, and so are the sector-specific cutoffs m_j^d and the associated quantities q_j^d and $q_j(m)$. Since the ZESP condition does not include N_j^E, those variables are independent of N_j^E. Thus, the integrals in (A-8) are independent of N_j^E and N_ℓ^E. The right-hand side of equation (A-8) is strictly positive and finite. By monotonicity, there clearly exists a unique $N_j^E(N_\ell^E)$. This relationship satisfies $(N_j^E)' > 0$, $N_j^E(0) = 0$ and $\lim_{N_\ell^E \to \infty} N_j^E(N_\ell^E) = \infty$.

Plugging expression (A-4) for the optimal allocation into (17) yields

$$N_j^E \int_0^{m_j^d} \frac{mq_j(m)}{\mathcal{E}_{u_j,q_j(m)}}\mathrm{d}G_j(m) = \frac{L_j}{L}. \tag{A-9}$$

Substituting $N_j^E(N_\ell^E)$ obtained from (A-8) into (A-9), summing over j, and using the overall labor market clearing condition $L = \sum_{j=1}^J L_j$, we then have the following equilibrium condition:

$$\sum_{j=1}^J N_j^E(N_\ell^E) \int_0^{m_j^d} \frac{mq_j(m)}{\mathcal{E}_{u_j,q_j(m)}}\mathrm{d}G_j(m) = 1. \tag{A-10}$$

Observe that all integral terms on the left-hand side of (A-10) are positive and independent of the masses of entrants, whereas the right-hand side equals one. Since the limit of the left-hand side is zero when N_ℓ^E goes to zero, and infinity when N_ℓ^E goes to infinity, the existence and uniqueness of a solution for N_ℓ^E follows directly by the properties of $N_j^E(\cdot)$. Since the terms in braces of the right-hand side of (20) are uniquely determined by Proposition 3, the existence and uniqueness of N_j^E implies those of L_j, which proves Proposition 4. □

A.5. Proof of Proposition 5. The former claim—substantiated by equation (21)—can readily be obtained from (9) and (19). The latter claim can be shown as follows. Without loss of generality, we order sectors by non-decreasing values of Υ_j such that $\Upsilon_1 \leq \Upsilon_2 \leq \ldots \leq \Upsilon_J$.

Then, by that ranking, we must have

$$\Upsilon_1 \leq \Upsilon_j, \quad \forall j \quad \Rightarrow \quad \mathcal{E}^{\text{eqm}}_{U,U_1}\overline{\mathcal{E}}^{\text{eqm}}_{u_1,q_1(m)}\mathcal{E}^{\text{opt}}_{U,U_j} \leq \mathcal{E}^{\text{opt}}_{U,U_1}\mathcal{E}^{\text{eqm}}_{U,U_j}\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}, \quad \forall j.$$

Taking the sum of each side with respect to j and rearranging yield

$$\Upsilon_1 \equiv \frac{\mathcal{E}^{\text{eqm}}_{U,U_1}\overline{\mathcal{E}}^{\text{eqm}}_{u_1,q_1(m)}}{\mathcal{E}^{\text{opt}}_{U,U_1}} \leq \frac{\sum_{j=1}^{J}\mathcal{E}^{\text{eqm}}_{U,U_j}\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}}{\sum_{j=1}^{J}\mathcal{E}^{\text{opt}}_{U,U_j}}.$$

Conversely, we must have

$$\Upsilon_J \geq \Upsilon_j, \quad \forall j \quad \Rightarrow \quad \mathcal{E}^{\text{eqm}}_{U,U_J}\overline{\mathcal{E}}^{\text{eqm}}_{u_J,q_J(m)}\mathcal{E}^{\text{opt}}_{U,U_j} \geq \mathcal{E}^{\text{opt}}_{U,U_J}\mathcal{E}^{\text{eqm}}_{U,U_j}\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}, \quad \forall j.$$

Taking the sum of each side with respect to j and rearranging yield

$$\Upsilon_J \equiv \frac{\mathcal{E}^{\text{eqm}}_{U,U_J}\overline{\mathcal{E}}^{\text{eqm}}_{u_J,q_J(m)}}{\mathcal{E}^{\text{opt}}_{U,U_J}} \geq \frac{\sum_{j=1}^{J}\mathcal{E}^{\text{eqm}}_{U,U_j}\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)}}{\sum_{j=1}^{J}\mathcal{E}^{\text{opt}}_{U,U_j}}.$$

Since the Υ_j are non-decreasing in j, we have $\Upsilon_1 < \Upsilon_J$ if there are at least two different Υ_j's. In that case, there exists a unique threshold $j^* \in \{1, 2, \ldots, J-1\}$ such that no sector with $j \leq j^*$ attracts too much labor in equilibrium, whereas all sectors with $j > j^*$ attract too much labor in equilibrium.

To see that the intersectoral allocation is optimal if and only if all Υ_j's are the same, we proceed as follows. First, assume that $\Upsilon_j = c$ for all j, where c is independent of j. Then, $\mathcal{E}^{\text{eqm}}_{U,U_j}\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)} = c \times \mathcal{E}^{\text{opt}}_{U,U_j}$ for all j, so that summing over j we have $\sum_{j=1}^{J}\mathcal{E}^{\text{eqm}}_{U,U_j}\overline{\mathcal{E}}^{\text{eqm}}_{u_j,q_j(m)} = c \times \sum_{j=1}^{J}\mathcal{E}^{\text{opt}}_{U,U_j}$, which implies that the right-hand side of (21) equals c. Since all Υ_j's equal c by assumption and are equal to the right-hand side, this proves the if part. To see the only if part, assume that $L^{\text{eqm}}_j = L^{\text{opt}}_j$ for all j. Equating (9) and (19) for all j, it can be readily verified that this is only possible if $\Upsilon_j = \sum_{\ell=1}^{J}\mathcal{E}^{\text{eqm}}_{U,U_\ell}\overline{\mathcal{E}}^{\text{eqm}}_{u_\ell,q_\ell(m)} / \sum_{\ell=1}^{J}\mathcal{E}^{\text{opt}}_{U,U_\ell}$ for all j. This completes the proof of Proposition 5. □

A.6. Proof of Proposition 7. Taking the ratio of $(m_j^d)^{\text{opt}}$ and $(m_j^d)^{\text{eqm}}$ from (28) yields

$$\left[\frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}}\right]^{k_j+1} = \kappa_j(k_j+1)^2. \tag{A-11}$$

Since $\kappa_j(k_j+1)^2 < 1$ (see the discussion below (E-22) in Appendix E.1), this immediately implies that $(m_j^d)^{\text{opt}}/(m_j^d)^{\text{eqm}} < 1$.

Next, taking the difference between the optimal quantity (E-23) and the equilibrium quantity (E-2), evaluated at the equilibrium price (E-3), we have the following two cases. First, when $0 \leq m \leq (m_j^d)^{\text{opt}}$, we obtain

$$q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m) = \frac{1}{\alpha_j} \ln\left[\frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}}\right] - \frac{1}{\alpha_j} \ln W\left(\mathrm{e}\frac{m}{(m_j^d)^{\text{eqm}}}\right). \tag{A-12}$$

Recalling that $W(0) = 0$ by the property of the Lambert W function, we know that $\lim_{m\to+0}[q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)] > 0$. Second, when $(m_j^d)^{\text{opt}} < m < (m_j^d)^{\text{eqm}}$, we know that $q_j^{\text{opt}}(m) = 0$, and that $q_j^{\text{eqm}}(m) > 0$, so that

$$q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m) = \frac{1}{\alpha_j} \ln\left[\frac{m}{(m_j^d)^{\text{eqm}}}\right] - \frac{1}{\alpha_j} \ln W\left(\mathrm{e}\frac{m}{(m_j^d)^{\text{eqm}}}\right) < 0. \tag{A-13}$$

Recalling that $W(\mathrm{e}) = 1$ by the property of the Lambert W function, we know that $\lim_{m\to(m_j^d)^{\text{eqm}}-0}[q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)] = 0$. Noting that (A-13) is strictly increasing in m,[21] and that $(m_j^d)^{\text{opt}} < (m_j^d)^{\text{eqm}}$, it is verified that $\lim_{m\to(m_j^d)^{\text{opt}}+0}[q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)] < 0$.

Finally, since $q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)$ is continuous at $(m_j^d)^{\text{opt}}$ by expressions (A-12) and (A-13), $\lim_{m\to(m_j^d)^{\text{opt}}-0}[q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)] < 0$ must hold in (A-12). Noting that expression (A-12) is strictly decreasing in m, and that $\lim_{m\to+0}[q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)] > 0$, we know that there exists a unique $m_j^* \in (0, (m_j^d)^{\text{opt}})$ such that $q_j^{\text{opt}}(m) > q_j^{\text{eqm}}(m)$ for $m \in (0, m_j^*)$ and $q_j^{\text{opt}}(m) < q_j^{\text{eqm}}(m)$ for $m \in (m_j^*, (m_j^d)^{\text{opt}}]$. This, together with the inequality in (A-13) for $m \in ((m_j^d)^{\text{opt}}, (m_j^d)^{\text{eqm}})$ proves our claim. □

B. Proofs of the lemmas

B.1. Proof of Lemma 1. Multiplying the ZEP condition (7) by N_j^E and using (5), we get

$$N_j^E L \int_0^{m_j^d} \frac{p_j(m)}{w} q_j(m)\mathrm{d}G_j(m) = N_j^E \left[L \int_0^{m_j^d} m q_j(m)\mathrm{d}G_j(m) + f_j G_j(m_j^d) + F_j\right] = L_j. \tag{B-1}$$

[21] To derive this property, we use $W'(x) = W(x)/\{x[1+W(x)]\}$.

Eliminating $p_j(m)$ in the left-hand side by using (2), we get

$$\frac{N_j^E}{\lambda_j w}\int_0^{m_j^d} u_j'(q_j(m))q_j(m)\mathrm{d}G_j(m) = \frac{L_j}{L}. \tag{B-2}$$

Using the definition of λ_j from (2), noting that $N_j^E u_j(q_j(m))/U_j = \zeta_j(q_j(m))$, and setting $\overline{\mathcal{E}}_{u_j,q_j(m)} = \int_0^{m_j^d} \mathcal{E}_{u_j,q_j(m)}\zeta_j(q_j(m))\mathrm{d}G_j(m)$, equation (B-2) can be rewritten as

$$\frac{U}{\lambda w}\mathcal{E}_{U,U_j}\overline{\mathcal{E}}_{u_j,q_j(m)} = \frac{L_j}{L}.$$

Summing over j and noting that $\sum_{j=1}^J L_j = L$, we obtain

$$\frac{U}{\lambda w}\sum_{j=1}^{J}\mathcal{E}_{U,U_j}\overline{\mathcal{E}}_{u_j,q_j(m)} = 1 \quad \Rightarrow \quad \frac{U}{\lambda w} = \frac{1}{\sum_{j=1}^J \mathcal{E}_{U,U_j}\overline{\mathcal{E}}_{u_j,q_j(m)}}.$$

Hence, any equilibrium labor allocation in sector $j = 1, 2, \dots, J$ satisfies

$$L_j = \frac{U}{\lambda w}\mathcal{E}_{U,U_j}\overline{\mathcal{E}}_{u_j,q_j(m)}L = \frac{\mathcal{E}_{U,U_j}\overline{\mathcal{E}}_{u_j,q_j(m)}}{\sum_{\ell=1}^J \mathcal{E}_{U,U_\ell}\overline{\mathcal{E}}_{u_\ell,q_\ell(m)}}L. \tag{B-3}$$

Turning to the mass of entrants, from (B-1) we obtain

$$N_j^E = \frac{L_j}{L\int_0^{m_j^d} mq_j(m)\mathrm{d}G_j(m) + f_j G_j(m_j^d) + F_j}. \tag{B-4}$$

Plugging $mq_j(m) = q_j(m)p_j(m)[1 - r_{u_j}(q_j(m))]/w = q_j(m)[1 - r_{u_j}(q_j(m))]u_j'(q_j(m))/(\lambda_j w)$, which is obtained from profit maximization and the consumer's first-order conditions, into (B-4), using $\lambda_j w = (LN_j^E/L_j)\int_0^{m_j^d} u_j'(q_j(m))q_j(m)\mathrm{d}G_j(m)$ from (B-2), and noting the definition of $\nu_j(q_j(m))$, we can solve the resulting equation for N_j^E, which yields (10). This completes the proof of Lemma 1. □

B.2. Proof of Lemma 2. Multiplying (15) by N_j^E, we get

$$N_j^E L\int_0^{m_j^d}\frac{u_j(q_j(m))}{\delta_j}\mathrm{d}G_j(m) = N_j^E\left\{\int_0^{m_j^d}\left[Lmq_j(m) + f_j\right]\mathrm{d}G_j(m) + F_j\right\} = L_j. \tag{B-5}$$

Since the left-hand side equals LU_j/δ_j by definition of U_j, we have $L_j/L = U_j/\delta_j$. Using the definition of δ_j from (13) we thus have

$$\frac{L_j}{L} = \frac{U}{\delta}\mathcal{E}_{U,U_j}. \tag{B-6}$$

Since $\sum_{j=1}^{J} L_j = L$, it then follows using (B-6) that

$$\frac{U}{\delta}\sum_{j=1}^{J}\mathcal{E}_{U,U_j} = 1 \quad \Rightarrow \quad \frac{U}{\delta} = \frac{1}{\sum_{j=1}^{J}\mathcal{E}_{U,U_j}}.$$

Hence, any optimal labor allocation in sector $j = 1, 2, \ldots, J$ satisfies

$$L_j = \frac{U}{\delta}\mathcal{E}_{U,U_j}L = \frac{\mathcal{E}_{U,U_j}}{\sum_{\ell=1}^{J}\mathcal{E}_{U,U_\ell}}L. \tag{B-7}$$

Finally, turning to the mass of entrants, from (B-5) we can obtain (B-4) for the optimal allocation. We know from (13) that $mq_j(m) = q_j(m)u_j'(q_j(m))/\delta_j$. Using the definitions of δ_j, $\mathcal{E}_{U,U_j}$, and $\mathcal{E}_{u_j,q_j(m)}$, and (B-6) then yields $mq_j(m) = (L_j/L)q_j(m)u_j'(q_j(m))/U_j = (L_j/L)\mathcal{E}_{u_j,q_j(m)}u_j(q_j(m))/U_j$ for the optimal allocation. Plugging this into (B-4), noting that U_j depends on N_j^E, and using the definition of $\zeta_j(q_j(m))$, we can solve the resulting equation for N_j^E, which yields (20). This completes the proof of Lemma 2. $\square$

B.3. Proof of Lemma 3. By definition, the weighted average of the elasticities of the subutility functions is given by

$$\overline{\mathcal{E}}_{u_j,q_j(m)} = \int_0^{m_j^d}\frac{\mathcal{E}_{u_j,q_j(m)}u_j(q_j(m))}{\int_0^{m_j^d}u_j(q_j(m))\mathrm{d}G_j(m)}\mathrm{d}G_j(m) = \frac{\int_0^{m_j^d}u_j'(q_j(m))q_j(m)\mathrm{d}G_j(m)}{\int_0^{m_j^d}u_j(q_j(m))\mathrm{d}G_j(m)}. \tag{B-8}$$

In what follows, we rewrite the numerator and the denominator of (B-8) by using CARA subutilities with Pareto productivity distributions. As shown in Appendix E, the equilibrium quantities are given by $q_j(m) = (1/\alpha_j)[1 - W(\mathrm{e}\,m/m_j^d)]$, where W is the Lambert W function defined as $\varphi = W(\varphi)\mathrm{e}^{W(\varphi)}$. To integrate the foregoing expressions, we use the change in variables suggested by Corless et al. (1996, p.341). Let

$$z \equiv W\left(\mathrm{e}\,\frac{m}{m_j^d}\right), \quad \text{so that} \quad \mathrm{e}\,\frac{m}{m_j^d} = z\mathrm{e}^z.$$

This change in variables then yields $\mathrm{d}m = (1+z)\mathrm{e}^{z-1}m_j^d\mathrm{d}z$, with the new integration bounds given by 0 and 1. Substituting the expressions for quantities into the numerator of (B-8), using the definition of W, and making the above change in variables, we have:

$$\begin{aligned}\int_0^{m_j^d} u_j'(q_j(m))q_j(m)\mathrm{d}G_j(m) &= \int_0^{m_j^d}[1-W(\mathrm{e}\,m/m_j^d)]\mathrm{e}^{W(\mathrm{e}\,m/m_j^d)-1}g_j(m)\mathrm{d}m \\ &= m_j^d\int_0^1(1-z)\mathrm{e}^{z-1}(1+z)\mathrm{e}^{z-1}g_j(z\mathrm{e}^{z-1}m_j^d)\mathrm{d}z. \quad \text{(B-9)}\end{aligned}$$

Applying the same technique to the denominator of (B-8), we obtain

$$\begin{aligned}\int_0^{m_j^d} u_j(q_j(m))\mathrm{d}G_j(m) &= \int_0^{m_j^d}[1-\mathrm{e}^{W(\mathrm{e}\,m/m_j^d)-1}]g_j(m)\mathrm{d}m \\ &= m_j^d\int_0^1(1-\mathrm{e}^{z-1})(1+z)\mathrm{e}^{z-1}g_j(z\mathrm{e}^{z-1}m_j^d)\mathrm{d}z. \quad \text{(B-10)}\end{aligned}$$

Dividing (B-9) by (B-10), we then obtain:

$$\overline{\mathcal{E}}_{u_j,q_j(m)} = \frac{\int_0^1(1-z)\mathrm{e}^{z-1}(1+z)\mathrm{e}^{z-1}g_j(z\mathrm{e}^{z-1}m_j^d)\mathrm{d}z}{\int_0^1(1-\mathrm{e}^{z-1})(1+z)\mathrm{e}^{z-1}g_j(z\mathrm{e}^{z-1}m_j^d)\mathrm{d}z}, \quad \text{(B-11)}$$

where $(1-z)\mathrm{e}^{z-1} < 1-\mathrm{e}^{z-1}$ for all $z \in [0,1)$. With a Pareto distribution, we have $g_j(z\mathrm{e}^{z-1}m_j^d) = k_j(z\mathrm{e}^{z-1}m_j^d)^{k_j-1}(m_j^{\max})^{-k_j}$, so that expression (B-11) can be written as (30). □

C. Additional details for the quantification procedure

This appendix provides details on the data that we use and derives additional expressions required for the different variants of the quantification procedure.

C.1. Data. Besides the firm-level ESANE dataset for France and the BSD dataset for the UK, we build on industry-level information from the OECD STAN database for both countries. More specifically, we obtain sectoral expenditure shares and R&D expenditure data by ISIC Rev. 3 from the French and UK input-output tables. These input-output tables contain information on 35 sectors and dictate the level of aggregation in our analysis. We discard the 'Public Administration and Defense' aggregate (12.12% of expenditure for France and 11.29% for the UK). Expenditure for each sector is computed as the sum of 'Households Final Consumption' (code C39) and 'General Government Final Consumption' (code C41).

In the baseline quantification we use the ratio of R&D expenditure to gross output at basic prices to proxy for sunk entry costs and fixed costs. We also trim the employment data by getting rid of the top and bottom 1.5% of the firm-level employment distribution across all sectors.[22]

In the first robustness check, we do not need to construct a proxy for sunk entry costs and fixed costs and directly use revenue data. We trim the data by focusing on firms with revenue higher or equal to 50,000 GBP/EUR for the UK/France and trim the top 2.5% of the revenue distribution across all sectors.

In our second robustness check, we use the industry-level profits-to-revenue ratio as a proxy for sunk entry costs and fixed costs. Industry-level revenue and profits are obtained by summing firm-level revenue and profits. We trim the data by getting rid of the top and bottom 1.5% of the firm-level employment distribution across all sectors.[23]

C.2. Additional expressions. We derive the expressions needed to back out the structural parameters of the model for the different variants of our quantification procedure.

CARA subutility. In the CARA case, firm variable employment used for production in the market equilibrium with Pareto productivity distribution is given by:

$$\text{emp}_j^{\text{CARA}}(m) = \frac{m}{\alpha_j}\left(1 - W_j\right),$$

where $W_j \equiv W(\mathrm{e}\, m/m_j^d)$ denotes the Lambert W function. Using $z \equiv W(\mathrm{e}m/m_j^d)$, $\mathrm{e}m/m_j^d = z\mathrm{e}^z$ and $\mathrm{d}m = (1+z)\mathrm{e}^{z-1}m_j^d\mathrm{d}z$, the conditional mean of $\ln[\text{emp}_j^{\text{CARA}}(m)]$ is given by:

$$\text{mean\_lnemp}_j^{\text{CARA}} = \frac{1}{G_j(m_j^d)}\int_0^{m_j^d} \ln\left[\frac{m}{\alpha_j}(1 - W_j)\right]\mathrm{d}G_j(m) = M_j + \ln m_j^d - \ln \alpha_j,$$

[22]We first match the R&D expenditure data with our 34 sectors and compute, for each sector, the ratio of R&D expenditure to gross output at basic prices (code R49) with the latter information coming from input-output tables. We then multiply the ratio by total employment in that sector, divide it by the number of firms to get a proxy measure of F_j and f_j, and subtract it from the employment of each firm. We ignore those firms ending up with a non-positive employment.

[23]We multiply the industry-level profits-to-revenue ratio by total employment in that sector, divide it by the number of firms to get a proxy measure of F_j and f_j, and subtract it from the employment of each firm. We ignore those firms ending up with a non-positive employment.

where $M_j \equiv -1/k_j + k_j \int_0^1 (z\mathrm{e}^{z-1})^{k_j-1}(1+z)\mathrm{e}^{z-1}\ln(1-z)\mathrm{d}z$ is a function of k_j only. In turn, the standard deviation of $\ln[\text{emp}_j^{\text{CARA}}(m)]$ becomes:

$$\begin{aligned}\text{sd\_lnemp}_j^{\text{CARA}} &= \sqrt{\frac{1}{G_j(m_j^d)}\int_0^{m_j^d}\left\{\ln\left[\frac{m}{\alpha_j}(1-W_j)\right] - \text{mean\_lnemp}_j^{\text{CARA}}\right\}^2 \mathrm{d}G_j(m)} \\ &= \sqrt{\frac{2}{k_j^2} - M_j^2 + k_j\int_0^1 \ln\left[(z\mathrm{e}^{z-1})^2(1-z)\right](z\mathrm{e}^{z-1})^{k_j-1}(1+z)\mathrm{e}^{z-1}\ln(1-z)\mathrm{d}z}. \quad \text{(C-1)}\end{aligned}$$

Moving to firm revenue, we have:

$$\text{rev}_j^{\text{CARA}}(m) = \frac{mw}{\alpha_j}(W_j^{-1} - 1).$$

Hence, the conditional mean of $\ln[\text{rev}_j^{\text{CARA}}(m)]$ is given by:

$$\text{mean\_lnrev}_j^{\text{CARA}} = \frac{1}{G_j(m_j^d)}\int_0^{m_j^d}\ln\left[\frac{mw}{\alpha_j}(W_j^{-1}-1)\right]\mathrm{d}G_j(m) = \widetilde{M}_j + \ln m_j^d + \ln(w/\alpha_j),$$

where $\widetilde{M}_j = -1/k_j + k_j\int_0^1 (z\mathrm{e}^{z-1})^{k_j-1}(1+z)\mathrm{e}^{z-1}\ln(z^{-1}-1)\mathrm{d}z$ is a function of k_j only. The standard deviation of $\ln[\text{rev}_j^{\text{CARA}}(m)]$ is then:

$$\begin{aligned}\text{sd\_lnrev}_j^{\text{CARA}} &= \sqrt{\frac{1}{G_j(m_j^d)}\int_0^{m_j^d}\left\{\ln\left[\frac{mw}{\alpha_j}(W_j^{-1}-1)\right] - \text{mean\_lnrev}_j^{\text{CARA}}\right\}^2\mathrm{d}G_j(m)} \\ &= \sqrt{\int_0^1\left\{\ln[z\mathrm{e}^{z-1}(z^{-1}-1)] - \widetilde{M}_j\right\}^2 k_j\,(z\mathrm{e}^{z-1})^{k_j-1}(1+z)\mathrm{e}^{z-1}\mathrm{d}z}. \quad \text{(C-2)}\end{aligned}$$

CES **subutility.** Turning to the CES case, firm variable employment used for production in the market equilibrium with Pareto productivity distribution is given by:

$$\text{emp}_j^{\text{CES}}(m) = \frac{f_j\rho_j}{1-\rho_j}\left(\frac{m_j^d}{m}\right)^{\frac{\rho_j}{1-\rho_j}}.$$

The conditional mean of $\ln[\text{emp}_j^{\text{CES}}(m)]$ is given by:

$$\text{mean\_lnemp}_j^{\text{CES}} = \frac{1}{G_j(m_j^d)}\int_0^{m_j^d}\ln\left[\frac{f_j\rho_j}{1-\rho_j}\left(\frac{m_j^d}{m}\right)^{\frac{\rho_j}{1-\rho_j}}\right]\mathrm{d}G_j(m) = \ln\left(\frac{f_j\rho_j}{1-\rho_j}\right) + \frac{\rho_j}{k_j(1-\rho_j)}.$$

Using the same approach than in the CARA case, one can obtain the standard deviation of $\ln[\text{emp}_j^{\text{CES}}(m)]$, which depends on k_j and ρ_j, as follows:

$$\text{sd\_lnemp}_j^{\text{CES}} = \frac{\rho_j}{k_j(1-\rho_j)}. \tag{C-3}$$

Moving to firm log-revenue, its standard deviation is identical to that of log-employment:

$$\text{sd\_lnrev}_j^{\text{CES}} = \frac{\rho_j}{k_j(1-\rho_j)}. \tag{C-4}$$

Indeed, both firm revenue and variable employment in the CES case are given by a bundle of parameters multiplied by $m^{-\rho_j/(1-\rho_j)}$. When taking the log, the standard deviation of the whole expression is thus simply the standard deviation of $\ln[m^{-\rho_j/(1-\rho_j)}]$ in both cases.

D. Allais surplus

This appendix derives the *Allais surplus* (Allais, 1943, 1977), which is the welfare measure we use when quantifying aggregate welfare distortions. In our context, the Allais surplus is defined as the maximum amount of the numeraire that can be saved when the social planner minimizes the resource cost of providing the agents with the equilibrium utility. We thus consider the following optimization problem:

$$\begin{aligned} \min_{\{N_j^E, m_j^d, q_j(m)\}} \quad & L^A \equiv \sum_{j=1}^{J} N_j^E \left\{ \int_0^{m_j^d} [Lmq_j(m) + f_j]\, \mathrm{d}G_j(m) + F_j \right\} \\ \text{s.t.} \quad & U\big(\widetilde{U}_1(U_1), \widetilde{U}_2(U_2), \ldots, \widetilde{U}_J(U_J)\big) \geq \overline{U}, \end{aligned} \tag{D-1}$$

where $\overline{U}$ is a fixed target utility level that needs to be provided to each agent. The solution to this problem yields the minimum resource cost, $L^A(\overline{U})$, required to achieve the target utility level. Setting $\overline{U} = U^{\text{eqm}}$, the Allais surplus is formally defined as:

$$A \equiv L - L^A(U^{\text{eqm}}), \tag{D-2}$$

where the first term L is the amount of labor needed for the market economy to attain the equilibrium utility since the labor market clears in equilibrium. If there are distortions, the planner requires, by definition, less labor to attain the equilibrium utility than the market economy does. Thus, the minimum resource cost must satisfy $L^A(U^{\text{eqm}}) \leq L$, so that $A \geq 0$.

Let μ denote the Lagrange multiplier associated with the utility constraint. From (D-1), the first-order conditions with respect to $q_j(m)$, m_j^d, and N_j^E are given by

$$
\begin{aligned}
u_j'(q_j(m)) &= \frac{L}{\mu_j} m, \qquad \mu_j \equiv \mu \frac{\partial U}{\partial \widetilde{U}_j} \frac{\partial \widetilde{U}_j}{\partial U_j} && \text{(D-3)} \\
\mu_j u_j(q_j^d) &= L m_j^d q_j^d + f_j && \text{(D-4)} \\
\mu_j \int_0^{m_j^d} u_j(q_j(m)) \mathrm{d}G_j(m) &= \int_0^{m_j^d} [Lmq_j(m) + f_j] \mathrm{d}G_j(m) + F_j && \text{(D-5)}
\end{aligned}
$$

as well as the constraint $\overline{U} = U\big(\widetilde{U}_1(U_1), \widetilde{U}_2(U_2), \ldots, \widetilde{U}_J(U_J)\big)$. Comparing (D-3)–(D-5) with (13)–(15) reveals that the first-order conditions are isomorphic. Thus, we can conclude that the optimal cutoffs and quantities are the same in the Allais surplus problem and the 'primal' optimal problem in Section 2.2. In what follows, we focus on the optimal labor allocation and entry.

D.1. CARA subutility. Assume that the subutility function is of the CARA form $u_j(q_j(m)) = 1 - \mathrm{e}^{-\alpha_j q_j(m)}$, that the upper-tier utility function U is of the CES form as in (E-1), that $\widetilde{U}_j(U_j) = U_j$, and that G_j follows a Pareto distribution. We also assume that $f_j = 0$ in the CARA subutility case.

To derive the optimal masses of entrants, we use the multipliers $\mu_j \equiv \mu \mathcal{E}_{U,U_j} \frac{U}{U_j}$. Given the CES upper-tier utility, the ratio of multipliers in sectors j and ℓ is

$$
\frac{\mu_j}{\mu_\ell} = \frac{\beta_j}{\beta_\ell} \left(\frac{U_\ell}{U_j} \right)^{\frac{1}{\sigma}} = \frac{\alpha_\ell}{\alpha_j} \frac{m_j^d}{m_\ell^d}, \tag{D-6}
$$

where we have used (D-3) evaluated at $m = m_j^d$ to get the last equality. It follows from (D-6) that

$$
U_\ell = \left(\frac{\alpha_\ell \beta_\ell}{\alpha_j \beta_j} \frac{m_j^d}{m_\ell^d} \right)^{\sigma} U_j,
$$

which, together with the utility constraint $\overline{U} = [\sum_{\ell=1}^J \beta_\ell U_\ell^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}$, yields

$$
\overline{U} = U_j \cdot \left[\beta_j^{1-\sigma} \left(\frac{m_j^d}{\alpha_j} \right)^{\sigma-1} \sum_{\ell=1}^{J} \beta_\ell^{\sigma} \left(\frac{m_\ell^d}{\alpha_\ell} \right)^{1-\sigma} \right]^{\frac{\sigma}{\sigma-1}}. \tag{D-7}
$$

Since the optimal quantities and cutoffs are the same in the 'primal' and 'dual' problems, we can plug (E-24) into (D-7) to eliminate U_j. We can then use $G_j(m_j^d) = \alpha_j F_j (k_j+1)^2/(L m_j^d)$

from the expression of the optimal cutoff (E-28) to solve for N_j^E as follows

$$N_j^E = \frac{\alpha_j^{\sigma-1}\beta_j^{\sigma}[(m_j^d)^{\text{opt}}]^{1-\sigma}}{\left[\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}[(m_\ell^d)^{\text{opt}}]^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}}\frac{L\overline{U}}{F_j(k_j+1)} = (N_j^E)^{\text{opt}}\frac{\overline{U}}{U^{\text{opt}}}, \tag{D-8}$$

where $U^{\text{opt}} = \{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}[(m_\ell^d)^{\text{opt}}]^{1-\sigma}\}^{1/(\sigma-1)}$ as given by (E-33) and where $(N_j^E)^{\text{opt}}$ is given by (E-30). As can be seen from (D-8), the mass of entrants in sector j needed to achieve $\overline{U}$ is proportional to this target utility level.

Summing up, to achieve the target utility $\overline{U}$ in the resource minimization problem, the planner imposes the socially optimal cutoffs $(m_j^d)^{\text{opt}}$ and firm-specific quantities $q_j^{\text{opt}}(m) = (1/\alpha_j)\ln(m_j^{\text{opt}}/m)$, and chooses the mass of entrants (D-8) that is proportional to $\overline{U}$. Thus, to achieve a higher $\overline{U}$ the planner would allow more entrants, but always choose the same level of selection. The associated resource cost $L^A(\overline{U})$ can be obtained by plugging this solution back into the objective function as follows

$$L^A(\overline{U}) = \sum_{j=1}^{J}(N_j^E)^{\text{opt}}\frac{\overline{U}}{U^{\text{opt}}}\left[\int_0^{(m_j^d)^{\text{opt}}} Lmq_j^{\text{opt}}(m)\mathrm{d}G_j(m) + F_j\right] = \frac{\overline{U}}{U^{\text{opt}}}L.$$

The last equality holds because the optimal allocation in Appendix E, by definition, clears the labor market. Setting $\overline{U} = U^{\text{eqm}}$ yields

$$\frac{L^A(U^{\text{eqm}})}{L} = \frac{U^{\text{eqm}}}{U^{\text{opt}}} < 1, \quad \text{i.e.,} \quad \frac{L - L^A(U^{\text{eqm}})}{L} = \frac{U^{\text{opt}} - U^{\text{eqm}}}{U^{\text{opt}}}, \tag{D-9}$$

where the numerator of the left-hand side is the Allais surplus. This expression provides a measure of the aggregate welfare distortion in the economy. Note that we may use the welfare measure based on utility and the measure based on the Allais surplus interchangeably.

D.2. CES subutility. Assume that the subutility function is of the CES form $u_j(q_j(m)) = q_j(m)^{\rho_j}$, that the upper-tier utility function U is of the CES form as in (E-1), that $\widetilde{U}_j(U_j) = U_j^{1/\rho_j}$, and that G_j follows a Pareto distribution. We also assume that $f_j > 0$.

To derive the optimal masses of entrants, we use the multipliers $\mu_j \equiv \mu\frac{\partial U}{\partial \widetilde{U}_j}\frac{\partial \widetilde{U}_j}{\partial U_j}$. Given the CES upper-tier utility, the ratio of multipliers in sectors j and ℓ is

$$\frac{\mu_j}{\mu_\ell} = \frac{\beta_j/\rho_j}{\beta_\ell/\rho_\ell}\frac{U_\ell^{\frac{1-\sigma(1-\rho_\ell)}{\sigma\rho_\ell}}}{U_j^{\frac{1-\sigma(1-\rho_j)}{\sigma\rho_j}}} = \frac{\rho_\ell(q_j^d)^{1-\rho_j}m_j^d}{\rho_j(q_\ell^d)^{1-\rho_\ell}m_\ell^d}, \tag{D-10}$$

where we have used (D-3) evaluated at $m = m_j^d$ in the second equality. Since the optimal cutoffs and quantities are as in Appendix E, using (E-35) allows us to rewrite expression (D-10) as follows:

$$\frac{U_j^{\frac{1-\sigma(1-\rho_j)}{\sigma\rho_j}}}{U_\ell^{\frac{1-\sigma(1-\rho_\ell)}{\sigma\rho_\ell}}} = \left(\frac{\beta_j}{\beta_\ell}\right)\left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j-1}\left[\frac{f_\ell\rho_\ell}{L(1-\rho_\ell)}\right]^{1-\rho_\ell}\frac{(m_j^d)^{-\rho_j}}{(m_\ell^d)^{-\rho_\ell}}. \tag{D-11}$$

Since the right-hand side of (D-11) is the same as that of (E-40), we obtain

$$\frac{U_j^{\frac{1-\sigma(1-\rho_j)}{\sigma\rho_j}}}{U_\ell^{\frac{1-\sigma(1-\rho_\ell)}{\sigma\rho_\ell}}} = \frac{(U_j^{\text{opt}})^{\frac{1-\sigma(1-\rho_j)}{\sigma\rho_j}}}{(U_\ell^{\text{opt}})^{\frac{1-\sigma(1-\rho_\ell)}{\sigma\rho_\ell}}}.$$

As in Appendix E, we now consider that $\sigma \to 1$ in order to derive closed-form solutions. We then have $U_j/U_\ell = U_j^{\text{opt}}/U_\ell^{\text{opt}}$ and from the definition of $\overline{U}$ we obtain:

$$\begin{aligned}\overline{U} &= \prod_{\ell=1}^{J} U_j^{\frac{\beta_\ell}{\rho_\ell}}\left(\frac{U_\ell}{U_j}\right)^{\frac{\beta_\ell}{\rho_\ell}} = \prod_{\ell=1}^{J} U_j^{\frac{\beta_\ell}{\rho_\ell}}\left(\frac{U_\ell^{\text{opt}}}{U_j^{\text{opt}}}\right)^{\frac{\beta_\ell}{\rho_\ell}} \\ &= \prod_{\ell=1}^{J}(U_\ell^{\text{opt}})^{\frac{\beta_\ell}{\rho_\ell}}\prod_{\ell=1}^{J}\left(\frac{U_j}{U_j^{\text{opt}}}\right)^{\frac{\beta_\ell}{\rho_\ell}} = U^{\text{opt}}\left(\frac{U_j}{U_j^{\text{opt}}}\right)^{\sum_{\ell=1}^{J}\frac{\beta_\ell}{\rho_\ell}}.\end{aligned} \tag{D-12}$$

Using (E-38), and because $m_j^d = (m_j^d)^{\text{opt}}$, we know that $U_j/U_j^{\text{opt}} = N_j^E/(N_j^E)^{\text{opt}}$. Plugging this expression into (D-12), we obtain

$$N_j^E = (N_j^E)^{\text{opt}}\left(\frac{\overline{U}}{U^{\text{opt}}}\right)^{\frac{1}{\sum_{\ell=1}^{J}\frac{\beta_\ell}{\rho_\ell}}}.$$

Thus, we have

$$\begin{aligned}L^A(\overline{U}) &= \sum_{j=1}^{J}(N_j^E)^{\text{opt}}\left(\frac{\overline{U}}{U^{\text{opt}}}\right)^{\frac{1}{\sum_{\ell=1}^{J}\frac{\beta_\ell}{\rho_\ell}}}\left\{\int_0^{(m_j^d)^{\text{opt}}}\left[Lmq_j^{\text{opt}}(m)+f_j\right]\mathrm{d}G_j(m)+F_j\right\} \\ &= \left(\frac{\overline{U}}{U^{\text{opt}}}\right)^{\frac{1}{\sum_{\ell=1}^{J}\frac{\beta_\ell}{\rho_\ell}}} L,\end{aligned}$$

where the last equality holds because the optimal allocation clears the labor market. Hence, evaluating $\overline{U}$ at U^{eqm}, we obtain

$$\frac{L - L^A(U^{\text{eqm}})}{L} = 1 - \left(\frac{U^{\text{eqm}}}{U^{\text{opt}}}\right)^{\frac{1}{\Sigma_{\ell=1}^{J} \frac{\beta_\ell}{\rho_\ell}}}. \tag{D-13}$$

This expression provides a measure of the aggregate welfare distortion in the economy. Note that we may not use the welfare measure based on utility and the measure based on the Allais surplus interchangeably in this case, as we could in the CARA case in Appendix D.1. The reason is the presence of $\widetilde{U}_j$, which is a transformation of the lower-tier utility. Without that transformation, which in the CES case would amount to setting all ρ_ℓ's that appear in the power of (D-13) equal to one, the foregoing result that utility and the Allais surplus can be used interchangeably would still hold.

E. Analytical expressions

We assume that the upper-tier utility is of the CES form:

$$U = \left\{ \sum_{j=1}^{J} \beta_j \left[\widetilde{U}_j(U_j)\right]^{(\sigma-1)/\sigma} \right\}^{\sigma/(\sigma-1)}, \tag{E-1}$$

where $\sigma > 1$ is the intersectoral elasticity of substitution, and where the β_j are strictly positive parameters that sum to one. The lower-tier utility is $U_j \equiv N_j^E \int_0^{m_j^d} u_j\big(q_j(m)\big)\mathrm{d}G_j(m)$. In what follows, we focus on cases in which the CES form in (E-1) satisfies condition (25), so that there exist unique intersectoral equilibrium and optimal allocations. As explained in the main text, this is always the case for CARA subutility functions and $\widetilde{U}_j(U_j) = U_j$, and it is the case for homothetic lower-tier CES utility functions with $\widetilde{U}_j(U_j) = U_j^{1/\rho_j}$ when the lower-tier elasticity of substitution exceeds the upper-tier elasticity of substitution. Observe that (E-1) includes the Cobb-Douglas form as a limit case. All results based on the Cobb-Douglas specification, as given in the main text, can be retrieved from the following expressions by letting $\sigma \to 1$.

E.1. CARA subutility. We provide detailed derivations of the equilibrium and optimal allocations in the CARA case.

Equilibrium allocation. We first derive the equilibrium cutoffs and quantities.[24] Assume that $\widetilde{U}_j(U_j) = U_j$, and that $u_j(q_j(m)) = 1 - \mathrm{e}^{-\alpha_j q_j(m)}$, so that $u_j'(q_j(m)) = \alpha_j \mathrm{e}^{-\alpha_j q_j(m)}$, $u_j''(q_j(m)) = -\alpha_j^2 \mathrm{e}^{-\alpha_j q_j(m)}$, and $r_u(q_j(m)) = \alpha_j q_j(m)$. We assume in what follows that there are no fixed costs for production, i.e., $f_j = 0$ for all sectors j. We can do so since, as in Melitz and Ottaviano (2008) but contrary to Melitz (2003), the marginal utility of each variety is bounded at zero consumption so that demand for a variety drops to zero when its price exceeds some threshold. Since for the least productive firm, which is indifferent between producing and not producing, we have $q_j^d \equiv q_j(m_j^d) = 0$, the first-order conditions (2) evaluated for any m and at the cutoff m_j^d imply the following demand functions:

$$q_j(m) = \frac{1}{\alpha_j} \ln \left[\frac{p_j^d}{p_j(m)} \right] \quad \text{for} \quad 0 \le m \le m_j^d, \tag{E-2}$$

where $p_j^d \equiv p_j(m_j^d)$. Making use of the profit maximizing prices (5), $r_u(q_j(m)) = \alpha_j q_j(m)$, and $q_j^d = 0$, we have

$$q_j(m) = \frac{1}{\alpha_j} \ln \left[\frac{m_j^d}{1 - r_{u_j}(q_j^d)} \frac{1 - r_u(q_j(m))}{m} \right] = \frac{1}{\alpha_j} \ln \left\{ \frac{m_j^d}{m} [1 - \alpha_j q_j(m)] \right\}.$$

This implicit equation can be solved for $q_j(m) = (1 - W_j)/\alpha_j$, where $W_j \equiv W(\mathrm{e}\, m/m_j^d)$ denotes the Lambert W function, defined as $\varphi = W(\varphi)\mathrm{e}^{W(\varphi)}$ (see Corless et al., 1996). We suppress its argument to alleviate notation whenever there is no possible confusion. Since $r_{u_j} = 1 - W_j$, we then also have the following profit maximizing prices, quantities, and operating profits divided by the wage rate:

$$p_j(m) = \frac{mw}{W_j}, \quad q_j(m) = \frac{1}{\alpha_j}(1 - W_j), \quad \pi_j(m) = \frac{Lm}{\alpha_j}(W_j^{-1} + W_j - 2). \tag{E-3}$$

By definition of the Lambert W function, we have $W(\varphi) \ge 0$ for all $\varphi \ge 0$. Taking logarithms on both sides of $\varphi = W(\varphi)\mathrm{e}^{W(\varphi)}$ and differentiating yields

$$W'(\varphi) = \frac{W(\varphi)}{\varphi[W(\varphi) + 1]} > 0$$

for all $\varphi > 0$. Finally, we have: $0 = W(0)\mathrm{e}^{W(0)}$, which implies $W(0) = 0$; and $\mathrm{e} = W(\mathrm{e})\mathrm{e}^{W(\mathrm{e})}$, which implies $W(\mathrm{e}) = 1$. Hence, we have $0 \le W_j \le 1$ if $0 \le m \le m_j^d$. The expressions

[24] Additional information on the equilibrium cutoffs and quantities can be found in Behrens and Murata (2007) and in Behrens et al. (2014).

in (E-3) show that a firm with a draw m_j^d charges a price equal to marginal cost, faces zero demand, and earns zero operating profits. Furthermore, using the properties of W', we readily obtain $\partial p_j(m)/\partial m > 0$, $\partial q_j(m)/\partial m < 0$, and $\partial \pi_j(m)/\partial m < 0$. In words, firms with higher productivity $1/m$ charge lower prices, produce larger quantities, and earn higher operating profits. Our specification with variable demand elasticity also features higher markups for more productive firms. Indeed, the markup

$$\Lambda_j(m) \equiv \frac{p_j(m)}{mw} = \frac{1}{W_j} \tag{E-4}$$

is such that $\partial \Lambda_j(m)/\partial m < 0$.

Using (E-3) and $r_{u_j} = 1 - W_j$, and recalling that $f_j = 0$, the zero expected profit condition (7) can be expressed as

$$\int_0^{m_j^d} m \left(W_j^{-1} + W_j - 2 \right) \mathrm{d}G_j(m) = \frac{\alpha_j F_j}{L}. \tag{E-5}$$

To derive closed-form solutions for various expressions with CARA subutility functions, we need to compute integrals involving the Lambert W function. This can be done by using the change in variables suggested by Corless et al. (1996, p.341). Let

$$z \equiv W\left(\mathrm{e}\,\frac{m}{m_j^d} \right), \quad \text{so that} \quad \mathrm{e}\,\frac{m}{m_j^d} = z\mathrm{e}^z.$$

The change in variables then yields $\mathrm{d}m = (1+z)\mathrm{e}^{z-1} m_j^d \mathrm{d}z$, with the new integration bounds given by 0 and 1. Using the change in variables, the LHS of (E-5) can be expressed as follows:

$$\int_0^{m_j^d} m \left(W_j^{-1} + W_j - 2 \right) \mathrm{d}G_j(m) = (m_j^d)^2 \int_0^1 z(1+z)\mathrm{e}^{2(z-1)} (z^{-1} + z - 2) g_j\left(z\mathrm{e}^{z-1} m_j^d\right) \mathrm{d}z$$

for an arbitrary distribution $g_j(\cdot)$ of draws.

We consider the Pareto distribution $G_j(m) = (m/m_j^{\max})^{k_j}$ with upper bound $m_j^{\max} > 0$ and shape parameter $k_j \geq 1$. Then, the integral reduces to

$$\int_0^{m_j^d} m \left(W_j^{-1} + W_j - 2 \right) \mathrm{d}G_j(m) = \kappa_j \left(m_j^{\max} \right)^{-k_j} (m_j^d)^{k_j+1}, \tag{E-6}$$

where $\kappa_j \equiv k_j \mathrm{e}^{-(k_j+1)} \int_0^1 (1+z) \left(z^{-1} + z - 2 \right) \left(z\mathrm{e}^z \right)^{k_j} \mathrm{e}^z \mathrm{d}z > 0$ is a constant term which solely depends on the shape parameter k_j. Plugging (E-6) into (E-5), we obtain the equilibrium cutoffs

$$(m_j^d)^{\text{eqm}} = \left[\frac{\alpha_j F_j (m_j^{\max})^{k_j}}{\kappa_j L}\right]^{\frac{1}{k_j+1}} \tag{E-7}$$

and quantities $q_j^{\text{eqm}}(m) = [1 - W_j(\mathrm{e}\, m/(m_j^d)^{\text{eqm}})]/\alpha_j$. Note that (E-7) implies that

$$\left[\frac{(m_j^d)^{\text{eqm}}}{m_j^{\max}}\right]^{k_j} = G_j((m_j^d)^{\text{eqm}}) = \frac{\alpha_j F_j}{\kappa_j L}\frac{1}{(m_j^d)^{\text{eqm}}}, \tag{E-8}$$

a relationship that we will use in what follows.

We now turn to the equilibrium labor allocation and masses of entrants. Using (E-3), labor market clearing in sector j can be written as

$$N_j^E\left[L\int_0^{m_j^d} m q_j(m)\mathrm{d}G_j(m) + F_j\right] = N_j^E\left[\frac{L}{\alpha_j}\int_0^{m_j^d} m\,(1-W_j)\,\mathrm{d}G_j(m) + F_j\right] = L_j. \tag{E-9}$$

Making use of the same change in variables for integration as before, and imposing the Pareto distribution, we have

$$\int_0^{m_j^d} m\,(1-W_j)\,\mathrm{d}G_j(m) = \kappa_{1j}\left(m_j^{\max}\right)^{-k_j}(m_j^d)^{k_j+1}, \tag{E-10}$$

where $\kappa_{1j} \equiv k_j \mathrm{e}^{-(k_j+1)}\int_0^1 (1-z^2)\,(z\mathrm{e}^z)^{k_j}\,\mathrm{e}^z \mathrm{d}z > 0$ is a constant term which solely depends on the shape parameter k_j. Plugging (E-10) into (E-9) and using (E-7), we have $L_j = N_j^E F_j[(\kappa_{1j}/\kappa_j) + 1]$. It can be verified that $\kappa_{1j}/\kappa_j = k_j$, so that

$$L_j = N_j^E F_j (k_j + 1). \tag{E-11}$$

To determine the masses of entrants, we insert the definition of λ_j into (3). Computing $\partial U/\partial U_j$ from (E-1) for $\widetilde{U}_j(U_j) = U_j$ and recalling that $q_j^d = 0$ and $p_j^d = m_j^d w$ for all j, we obtain

$$\frac{\alpha_j}{\alpha_\ell} = \frac{m_j^d}{m_\ell^d}\frac{\lambda_j}{\lambda_\ell} \quad \Rightarrow \quad \frac{U_j}{U_\ell} = \left(\frac{\alpha_j}{\alpha_\ell}\right)^{\sigma}\left(\frac{\beta_j}{\beta_\ell}\right)^{\sigma}\left[\frac{(m_j^d)^{\text{eqm}}}{(m_\ell^d)^{\text{eqm}}}\right]^{-\sigma}. \tag{E-12}$$

Using the definition of $\mathcal{E}_{U,U_j}$, (E-1), and (E-12), it is verified that

$$\mathcal{E}_{U,U_j} = \frac{\beta_j U_j^{\frac{\sigma-1}{\sigma}}}{\sum_{\ell=1}^J \beta_\ell U_\ell^{\frac{\sigma-1}{\sigma}}} = \frac{\beta_j}{\sum_{\ell=1}^J \beta_\ell\left(\frac{U_\ell}{U_j}\right)^{\frac{\sigma-1}{\sigma}}} = \frac{\beta_j^\sigma \alpha_j^{\sigma-1}[(m_j^d)^{\text{eqm}}]^{1-\sigma}}{\sum_{\ell=1}^J \beta_\ell^\sigma \alpha_\ell^{\sigma-1}[(m_\ell^d)^{\text{eqm}}]^{1-\sigma}}. \tag{E-13}$$

From Lemmas 1 and 3, we have

$$L_j = \frac{\mathcal{E}_{U,U_j}\theta_j}{\sum_{\ell=1}^{J}\mathcal{E}_{U,U_\ell}\theta_\ell}L. \tag{E-14}$$

Substituting (E-11) and (E-13) into (E-14) yields

$$(N_j^E)^{\text{eqm}} = \frac{\alpha_j^{\sigma-1}\beta_j^\sigma\theta_j[(m_j^d)^{\text{eqm}}]^{1-\sigma}}{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^\sigma\theta_\ell[(m_\ell^d)^{\text{eqm}}]^{1-\sigma}}\frac{L}{(k_j+1)F_j}, \tag{E-15}$$

so that from (E-11) we obtain

$$L_j^{\text{eqm}} = \frac{\alpha_j^{\sigma-1}\beta_j^\sigma\theta_j[(m_j^d)^{\text{eqm}}]^{1-\sigma}}{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^\sigma\theta_\ell[(m_\ell^d)^{\text{eqm}}]^{1-\sigma}}L. \tag{E-16}$$

Turning to the lower-tier utility, equation (B-8), the definition of U_j, and Lemma 3 imply that

$$U_j = \frac{\int_0^{m_j^d} u_j'(q_j(m))q_j(m)\mathrm{d}G_j(m)}{\theta_j/N_j^E}. \tag{E-17}$$

Making use of the same change in variables for integration as before, and imposing the Pareto distribution, the numerator of (E-17) can be rewritten as

$$\int_0^{m_j^d}(1-W_j)\mathrm{e}^{W_j-1}\mathrm{d}G_j(m) = \kappa_{2j}\left(m_j^{\max}\right)^{-k_j}(m_j^d)^{k_j}, \tag{E-18}$$

where $\kappa_{2j} \equiv k_j\mathrm{e}^{-(k_j+1)}\int_0^1(1-z^2)\left(z\mathrm{e}^z\right)^{k_j-1}(\mathrm{e}^z)^2\mathrm{d}z > 0$ is a constant term which solely depends on the shape parameter k_j. Using (E-18), (E-8), and (E-11), we can rewrite (E-17) as $U_j^{\text{eqm}} = [(\alpha_j/\theta_j)(L_j^{\text{eqm}}/L)/(m_j^d)^{\text{eqm}}][(\kappa_{2j}/\kappa_j)/(k_j+1)]$. It can be verified that $\kappa_{2j}/\kappa_j = k_j+1$, so that

$$U_j^{\text{eqm}} = \frac{\alpha_j}{\theta_j}\frac{(L_j^{\text{eqm}}/L)}{(m_j^d)^{\text{eqm}}}. \tag{E-19}$$

Making use of the upper-tier utility (E-1), (E-16), and (E-19), the utility U is then

$$U^{\text{eqm}} = \left\{\frac{\sum_{j=1}^{J}\alpha_j^{\sigma-1}\beta_j^\sigma[(m_j^d)^{\text{eqm}}]^{1-\sigma}}{\left[\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^\sigma\theta_\ell[(m_\ell^d)^{\text{eqm}}]^{1-\sigma}\right]^{\frac{\sigma-1}{\sigma}}}\right\}^{\frac{\sigma}{\sigma-1}} = \frac{\left\{\sum_{j=1}^{J}\alpha_j^{\sigma-1}\beta_j^\sigma[(m_j^d)^{\text{eqm}}]^{1-\sigma}\right\}^{\frac{\sigma}{\sigma-1}}}{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^\sigma\theta_\ell[(m_\ell^d)^{\text{eqm}}]^{1-\sigma}}. \tag{E-20}$$

When the upper-tier utility function is of the Cobb-Douglas form, $\sigma = 1$, so that (E-14) becomes $L_j^{\text{eqm}}/L = \beta_j\theta_j / \sum_{\ell=1}^{J}(\beta_\ell\theta_\ell)$. Hence, (E-20) reduces to

$$U^{\text{eqm}} = \prod_{j=1}^{J}\left[\frac{\alpha_j\beta_j}{\sum_{\ell=1}^{J}(\beta_\ell\theta_\ell)}\frac{1}{(m_j^d)^{\text{eqm}}}\right]^{\beta_j}.$$

Another way of deriving the equilibrium utility is useful for proving some analytical results. Using the demand functions (E-2) and the profit-maximizing prices in (E-3), the lower-tier utility is given by

$$U_j = N_j^E\left[G_j(m_j^d) - \frac{1}{m_j^d}\int_0^{m_j^d} mW_j^{-1}\mathrm{d}G_j(m)\right], \qquad \text{(E-21)}$$

which can be integrated (using again the same change in variables as before) to obtain:

$$\int_0^{m_j^d} mW_j^{-1}\mathrm{d}G_j(m) = \kappa_{3j}\left(m_j^{\max}\right)^{-k_j}(m_j^d)^{k_j+1},$$

where $\kappa_{3j} \equiv k_j\mathrm{e}^{-(k_j+1)}\int_0^1(z^{-1}+1)\left(z\mathrm{e}^z\right)^{k_j}\mathrm{e}^z\mathrm{d}z > 0$ is a constant term which solely depends on the shape parameter k_j. One can verify that $1-\kappa_{3j} = \frac{1}{k_j+1} - (\kappa_{1j}+\kappa_j)$, so that the lower-tier utility (E-21) becomes

$$U_j = \left[\frac{1}{k_j+1} - (\kappa_{1j}+\kappa_j)\right]N_j^E G_j(m_j^d). \qquad \text{(E-22)}$$

Since $U_j > 0$ by construction of the lower-tier utility, we have $(\kappa_{1j}+\kappa_j)(k_j+1) < 1$, which is equivalent to $\kappa_j(k_j+1)^2 < 1$ since $\kappa_{1j} = \kappa_j k_j$.

Optimal allocation. We next derive the expressions for the optimal cutoffs and quantities in the CARA case. From the first-order conditions (13), the optimal consumptions must satisfy

$$\frac{\alpha_j\mathrm{e}^{-\alpha_j q_j(m_j^d)}}{\alpha_j\mathrm{e}^{-\alpha_j q_j(m)}} = \frac{m_j^d}{m} \quad \text{and} \quad \frac{\alpha_j\mathrm{e}^{-\alpha_j q_j(m_j^d)}}{\alpha_\ell\mathrm{e}^{-\alpha_\ell q_\ell(m_\ell^d)}} = \frac{\delta_j}{\delta_\ell}\frac{m_j^d}{m_\ell^d}.$$

The first conditions, together with $q_j(m_j^d) = 0$, can be solved to yield:

$$q_j(m) = \frac{1}{\alpha_j}\ln\left(\frac{m_j^d}{m}\right) \quad \text{for} \quad 0 \le m \le m_j^d. \qquad \text{(E-23)}$$

Plugging (E-23) into U_j and letting $\overline{m}_j \equiv (1/G_j(m_j^d)] \int_0^{m_j^d} m \mathrm{d}G_j(m)$ denote the average value of m, we obtain:

$$U_j = \left(1 - \frac{\overline{m}_j}{m_j^d}\right) N_j^E G_j(m_j^d) = \frac{N_j^E G_j(m_j^d)}{k_j + 1}, \tag{E-24}$$

where we have used the property of the Pareto distribution that $\overline{m}_j = [k_j/(k_j+1)]m_j^d$ to obtain the second equality.

Assuming that the upper-tier utility function is given by (E-1), the planner's problem can be redefined using (E-24) as follows:

$$\max_{\{N_j^E, m_j^d\}} \quad \widehat{V} \equiv L \cdot \left\{ \sum_{j=1}^{J} \beta_j \left[\frac{N_j^E G_j(m_j^d)}{k_j + 1} \right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}} \tag{E-25}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} N_j^E \left[\frac{L}{\alpha_j} \frac{k_j}{(k_j+1)^2} m_j^d G_j(m_j^d) + F_j \right] = L, \tag{E-26}$$

where the resource constraint is obtained by plugging (E-23) into (12) and integrating the resulting expression. Denoting by $\widehat{\delta}$ the Lagrange multiplier of this redefined problem, the first-order conditions with respect to N_j^E and m_j^d are given by

$$\frac{\beta_j \widehat{V}}{N_j^E} \frac{\left[\frac{N_j^E G_j(m_j^d)}{k_j+1}\right]^{\frac{\sigma-1}{\sigma}}}{\sum_{\ell=1}^{J} \beta_\ell \left[\frac{N_\ell^E G_\ell(m_\ell^d)}{k_\ell+1}\right]^{\frac{\sigma-1}{\sigma}}} = \widehat{\delta} \left[\frac{L}{\alpha_j} \frac{k_j}{(k_j+1)^2} m_j^d G_j(m_j^d) + F_j \right] \tag{E-27}$$

$$\frac{\beta_j \widehat{V}}{N_j^E} \frac{\left[\frac{N_j^E G_j(m_j^d)}{k_j+1}\right]^{\frac{\sigma-1}{\sigma}}}{\sum_{\ell=1}^{J} \beta_\ell \left[\frac{N_\ell^E G_\ell(m_\ell^d)}{k_\ell+1}\right]^{\frac{\sigma-1}{\sigma}}} = \widehat{\delta} \frac{L}{\alpha_j} \frac{k_j}{(k_j+1)^2} \frac{G_j(m_j^d)}{G_j'(m_j^d)} \left[G_j(m_j^d) + m_j^d G_j'(m_j^d) \right].$$

Because the left-hand side is common, we obtain the optimal cutoffs

$$(m_j^d)^{\text{opt}} = \left[\frac{\alpha_j F_j (m_j^{\max})^{k_j} (k_j+1)^2}{L} \right]^{\frac{1}{k_j+1}} \tag{E-28}$$

and quantities $q_j^{\text{opt}}(m) = (1/\alpha_j)\ln[(m_j^d)^{\text{opt}}/m]$. Note that (E-28) implies that

$$\left[\frac{(m_j^d)^{\text{opt}}}{m_j^{\max}}\right]^{k_j} = G_j((m_j^d)^{\text{opt}}) = \frac{\alpha_j F_j (k_j+1)^2}{L}\frac{1}{(m_j^d)^{\text{opt}}}, \tag{E-29}$$

a relationship that we will use repeatedly in what follows.

Using (E-29), the right-hand side of (E-27) becomes $\widehat{\delta} F_j(k_j+1)$. Moreover, taking the ratio of (E-27) for sectors j and ℓ, we have

$$\frac{N_j^E}{N_\ell^E} = \left(\frac{\beta_j}{\beta_\ell}\right)^{\sigma}\left[\frac{G_j(m_j^d)}{k_j+1}\right]^{\sigma-1}\left[\frac{G_\ell(m_\ell^d)}{k_\ell+1}\right]^{1-\sigma}\left[\frac{(k_j+1)F_j}{(k_\ell+1)F_\ell}\right]^{-\sigma}$$

for all $j = 1, 2, \ldots, J$. Plugging this relationship into the resource constraint (E-26), and using (E-29), we readily obtain the optimal mass of entrants in sector j:

$$(N_j^E)^{\text{opt}} = \frac{\alpha_j^{\sigma-1}\beta_j^{\sigma}[(m_j^d)^{\text{opt}}]^{1-\sigma}}{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}[(m_\ell^d)^{\text{opt}}]^{1-\sigma}}\frac{L}{(k_j+1)F_j}. \tag{E-30}$$

Plugging (E-28) into (E-26), we have $L_j = N_j^E F_j(k_j+1)$, which implies the optimal labor allocation as follows:

$$L_j^{\text{opt}} = \frac{\alpha_j^{\sigma-1}\beta_j^{\sigma}[(m_j^d)^{\text{opt}}]^{1-\sigma}}{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}[(m_\ell^d)^{\text{opt}}]^{1-\sigma}}L. \tag{E-31}$$

Finally, plugging (E-29) and (E-30) into (E-24), the lower-tier utility from sector j at the optimal allocation can be expressed as

$$U_j^{\text{opt}} = \alpha_j \frac{(L_j^{\text{opt}}/L)}{(m_j^d)^{\text{opt}}}, \tag{E-32}$$

so that

$$U^{\text{opt}} = \left\{\sum_{j=1}^{J}\alpha_j^{\sigma-1}\beta_j^{\sigma}[(m_j^d)^{\text{opt}}]^{1-\sigma}\right\}^{\frac{1}{\sigma-1}}. \tag{E-33}$$

When the upper-tier utility function is of the Cobb-Douglas form, $\sigma = 1$, so that (E-31) reduces to $L_j^{\text{opt}}/L = \beta_j$. Expression (E-32) can then be rewritten as $U_j^{\text{opt}} = \alpha_j\beta_j/(m_j^d)^{\text{opt}}$. Hence, (E-33) reduces to

$$U^{\text{opt}} = \prod_{j=1}^{J}\left[\frac{\alpha_j\beta_j}{(m_j^d)^{\text{opt}}}\right]^{\beta_j}.$$

E.2. CES subutility. We briefly summarize the equilibrium and optimal allocations in the case with CES subutility functions, $u_j(q_j(m)) = q_j(m)^{\rho_j}$, where $0 < \rho_j < 1$, and Pareto distribution functions, $G_j(m) = (m/m_j^{\max})^{k_j}$. As in the existing literature, we also assume that $\widetilde{U}_j(U_j) = U_j^{1/\rho_j}$ and that $f_j > 0$.

First, with CES subutility functions, $1 - r_{u_j}(q_j(m)) = \mathcal{E}_{u_j,q_j(m)} = \rho_j$ holds for all m, and $q_j(m) = (m_j^d/m)^{1/(1-\rho_j)} q_j^d$ holds for both the equilibrium and optimal allocations. Thus, the ZEP and ZCP conditions, (7) and (6), are equivalent to the ZESP and ZCSP conditions, (17) and (18). The resulting equilibrium and optimum cutoffs are therefore the same and given by

$$(m_j^d)^{\text{eqm}} = (m_j^d)^{\text{opt}} = m_j^{\max} \left[\frac{F_j}{f_j} \frac{k_j(1-\rho_j) - \rho_j}{\rho_j} \right]^{\frac{1}{k_j}}, \tag{E-34}$$

which implies that the demand functions $q_j(m)$ are common between the equilibrium and the optimum for all $m \leq m_j^d$. In particular q_j^d can be obtained from (6) or (18) as follows:

$$q_j^d = \frac{f_j}{L} \frac{\rho_j}{1-\rho_j} \frac{1}{m_j^d}. \tag{E-35}$$

Second, given the foregoing results, $\nu_j(q_j(m)) = \zeta_j(q_j(m))$ holds for all $m \leq m_j^d$, so that the expressions in the braces of (10) and those of (20) are the same. Thus, the equilibrium and optimal masses of entrants satisfy

$$(N_j^E)^{\text{eqm}} = L_j^{\text{eqm}} \frac{\rho_j}{k_j F_j} \quad \text{and} \quad (N_j^E)^{\text{opt}} = L_j^{\text{opt}} \frac{\rho_j}{k_j F_j}. \tag{E-36}$$

Third, the conditions (3) for equilibrium intersectoral consumption can be rewritten as

$$\frac{U_j^{\frac{1-\sigma(1-\rho_j)}{\sigma\rho_j}}}{U_\ell^{\frac{1-\sigma(1-\rho_\ell)}{\sigma\rho_\ell}}} = \left(\frac{\beta_j \rho_j}{\beta_\ell \rho_\ell} \right) \left[\frac{f_j \rho_j}{L(1-\rho_j)} \right]^{\rho_j - 1} \left[\frac{f_\ell \rho_\ell}{L(1-\rho_\ell)} \right]^{1-\rho_\ell} \frac{(m_j^d)^{-\rho_j}}{(m_\ell^d)^{-\rho_\ell}}.$$

To obtain closed-form solutions, we assume that $\sigma = 1$, so that the above expression reduces to the Cobb-Douglas case:

$$\frac{U_j}{U_\ell} = \left(\frac{\beta_j \rho_j}{\beta_\ell \rho_\ell} \right) \left[\frac{f_j \rho_j}{L(1-\rho_j)} \right]^{\rho_j - 1} \left[\frac{f_\ell \rho_\ell}{L(1-\rho_\ell)} \right]^{1-\rho_\ell} \frac{(m_j^d)^{-\rho_j}}{(m_\ell^d)^{-\rho_\ell}}. \tag{E-37}$$

Using (E-34) and (E-35), together with $q_j(m) = (m_j^d/m)^{1/(1-\rho_j)} q_j^d$ and the Pareto distribution, the lower-tier utility is given by

$$U_j = \frac{N_j^E k_j F_j}{L} \left[\frac{f_j \rho_j}{L(1-\rho_j)}\right]^{\rho_j - 1} (m_j^d)^{-\rho_j}. \tag{E-38}$$

Plugging (E-38) into (E-37) and using (E-36), we then obtain

$$\frac{(N_j^E)^{\text{eqm}}}{(N_\ell^E)^{\text{eqm}}} = \frac{\beta_j \rho_j}{\beta_\ell \rho_\ell} \frac{k_\ell F_\ell}{k_j F_j} = \frac{L_j^{\text{eqm}}}{L_\ell^{\text{eqm}}} \frac{\rho_j}{\rho_\ell} \frac{k_\ell F_\ell}{k_j F_j} \quad \Rightarrow \quad L_\ell^{\text{eqm}} = \frac{\beta_\ell}{\beta_j} L_j^{\text{eqm}}.$$

Since $\sum_{\ell=1}^J L_\ell = L$, we finally obtain

$$L_j^{\text{eqm}} = \beta_j L. \tag{E-39}$$

Using (E-36) and (E-39), expression (E-38) can be rewritten as

$$U_j^{\text{eqm}} = \beta_j \rho_j \left[\frac{f_j \rho_j}{L(1-\rho_j)}\right]^{\rho_j - 1} \left[(m_j^d)^{\text{eqm}}\right]^{-\rho_j},$$

which yields

$$U^{\text{eqm}} = \prod_{j=1}^J \left\{ \beta_j \rho_j \left[\frac{f_j \rho_j}{L(1-\rho_j)}\right]^{\rho_j - 1} \left[(m_j^d)^{\text{eqm}}\right]^{-\rho_j} \right\}^{\frac{\beta_j}{\rho_j}}.$$

Turning to the optimal allocation, the conditions (16) for optimal intersectoral consumption can be rewritten as

$$\frac{U_j^{\frac{1-\sigma(1-\rho_j)}{\sigma \rho_j}}}{U_\ell^{\frac{1-\sigma(1-\rho_\ell)}{\sigma \rho_\ell}}} = \left(\frac{\beta_j}{\beta_\ell}\right) \left[\frac{f_j \rho_j}{L(1-\rho_j)}\right]^{\rho_j - 1} \left[\frac{f_\ell \rho_\ell}{L(1-\rho_\ell)}\right]^{1-\rho_\ell} \frac{(m_j^d)^{-\rho_j}}{(m_\ell^d)^{-\rho_\ell}}. \tag{E-40}$$

Assume again that the upper-tier utility is Cobb-Douglas, i.e., $\sigma \to 1$. In that case, we can use the same procedure as above to obtain

$$\frac{(N_j^E)^{\text{opt}}}{(N_\ell^E)^{\text{opt}}} = \frac{\beta_j}{\beta_\ell} \frac{k_\ell F_\ell}{k_j F_j} = \frac{L_j^{\text{opt}}}{L_\ell^{\text{opt}}} \frac{\rho_j}{\rho_\ell} \frac{k_\ell F_\ell}{k_j F_j} \quad \Rightarrow \quad L_\ell^{\text{opt}} = \frac{\beta_\ell / \rho_\ell}{\beta_j / \rho_j} L_j^{\text{opt}}$$

so that

$$L_j^{\text{opt}} = \frac{\beta_j / \rho_j}{\sum_{\ell=1}^J (\beta_\ell / \rho_\ell)} L.$$

Using (E-36), expression (E-38) can be rewritten as

$$U_j^{\text{opt}} = \frac{\beta_j}{\sum_{\ell=1}^J (\beta_\ell/\rho_\ell)} \left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j-1} \left[(m_j^d)^{\text{opt}}\right]^{-\rho_j},$$

which yields

$$U^{\text{opt}} = \prod_{j=1}^J \left\{ \frac{\beta_j}{\sum_{\ell=1}^J (\beta_\ell/\rho_\ell)} \left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j-1} \left[(m_j^d)^{\text{opt}}\right]^{-\rho_j} \right\}^{\frac{\beta_j}{\rho_j}}.$$

F. Expressions for quantifying the CES-CARA case.

Quantifying the Cobb-Douglas-CARA case is relatively easy because when $\sigma \to 1$ the equilibrium and optimal expenditure shares are independent of the α_j parameters and the cutoffs m_j^d (which subsume other parameters such as the sunk entry costs F_j). This no longer holds in the CES-CARA case, which makes the quantification more involved. However, we can proceed as follows.

Let $\{\widehat{e}_j^{\text{eqm}}\}_{j=1}^J$ be the equilibrium expenditure shares from the data, and let $\{\widehat{\theta}_j\}_{j=1}^J$ be the weighted averages of the elasticities of the subutility functions obtained from the standard deviation formula in Appendix C.2. Recall that in the Cobb-Douglas case those two pieces of information allow us to back out $\{\widehat{\beta}_j^{\text{eqm}}\}_{j=1}^J$ by solving

$$\widehat{e}_j^{\text{eqm}} = \frac{\widehat{\beta}_j^{\text{eqm}}\widehat{\theta}_j}{\sum_{\ell=1}^J \widehat{\beta}_\ell^{\text{eqm}}\widehat{\theta}_\ell}, \quad \sum_j \widehat{\beta}_j^{\text{eqm}} = 1.$$

In the CES case, using (E-16) and noting that the total revenue equals the total wage in each sector, i.e., $Le_jw = wL_j$, the equilibrium expenditure share can be rewritten as

$$\widehat{e}_j^{\text{eqm}} = \frac{\alpha_j^{\sigma-1}\beta_j^{\sigma}\widehat{\theta}_j[(m_j^d)^{\text{eqm}}]^{1-\sigma}}{\sum_{\ell=1}^J \alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}\widehat{\theta}_\ell[(m_\ell^d)^{\text{eqm}}]^{1-\sigma}} = \frac{\left[\frac{\alpha_j\beta_j}{(m_j^d)^{\text{eqm}}}\right]^{\sigma-1}\beta_j\widehat{\theta}_j}{\sum_{\ell=1}^J \left[\frac{\alpha_\ell\beta_\ell}{(m_\ell^d)^{\text{eqm}}}\right]^{\sigma-1}\beta_\ell\widehat{\theta}_\ell} = \frac{\widetilde{\beta}_j^{\text{eqm}}\widehat{\theta}_j}{\sum_{\ell=1}^J \widetilde{\beta}_\ell^{\text{eqm}}\widehat{\theta}_\ell},$$

where $\widetilde{\beta}_j^{\text{eqm}} \equiv \left[(\alpha_j\beta_j)/(m_j^d)^{\text{eqm}}\right]^{\sigma-1}\beta_j$, and where $\widehat{e}_j^{\text{eqm}}$ and $\widehat{\theta}_j$ come from the data. Clearly, $\widetilde{\beta}_j^{\text{eqm}} = \text{const.} \times \widehat{\beta}_j^{\text{eqm}}$ is a solution to the foregoing equation, i.e., the $\widetilde{\beta}_j^{\text{eqm}}$ parameters in the CES case are proportional to the $\widehat{\beta}_j^{\text{eqm}}$ parameters in the Cobb-Douglas case. The constant term is shown to disappear in the end.

Using the same transformation for the β terms as above, the equilibrium utility (E-20) can be rewritten as

$$U^{\text{eqm}} = \frac{\left\{\sum_{j=1}^{J} \alpha_j^{\sigma-1}\beta_j^{\sigma}[(m_j^d)^{\text{eqm}}]^{1-\sigma}\right\}^{\frac{\sigma}{\sigma-1}}}{\sum_{\ell=1}^{J} \alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}\widehat{\theta}_\ell[(m_\ell^d)^{\text{eqm}}]^{1-\sigma}} = \frac{\left(\sum_{j=1}^{J} \widetilde{\beta}_j^{\text{eqm}}\right)^{\frac{\sigma}{\sigma-1}}}{\sum_{\ell=1}^{J} \widetilde{\beta}_\ell^{\text{eqm}}\widehat{\theta}_\ell},$$

which, using $\widetilde{\beta}_j^{\text{eqm}} = \text{const.} \times \widehat{\beta}_j^{\text{eqm}}$, can be rewritten as

$$U^{\text{eqm}} = (\text{const.})^{\frac{\sigma}{\sigma-1}-1} \frac{\left(\sum_{j=1}^{J} \widehat{\beta}_j^{\text{eqm}}\right)^{\frac{\sigma}{\sigma-1}}}{\sum_{\ell=1}^{J} \widehat{\beta}_\ell^{\text{eqm}}\widehat{\theta}_\ell} = (\text{const.})^{\frac{\sigma}{\sigma-1}-1} \frac{1}{\sum_{\ell=1}^{J} \widehat{\beta}_\ell^{\text{eqm}}\widehat{\theta}_\ell}.$$

Turning to the optimal labor share, we have

$$\frac{L_j^{\text{opt}}}{L} = \frac{\alpha_j^{\sigma-1}\beta_j^{\sigma}[(m_j^d)^{\text{opt}}]^{1-\sigma}}{\sum_{\ell=1}^{J} \alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}[(m_\ell^d)^{\text{opt}}]^{1-\sigma}} = \frac{\left[\frac{\alpha_j\beta_j}{(m_j^d)^{\text{opt}}}\right]^{\sigma-1}\beta_j}{\sum_{\ell=1}^{J}\left[\frac{\alpha_\ell\beta_\ell}{(m_\ell^d)^{\text{opt}}}\right]^{\sigma-1}\beta_\ell} = \frac{\widetilde{\beta}_j^{\text{opt}}}{\sum_{\ell=1}^{J} \widetilde{\beta}_\ell^{\text{opt}}},$$

where $\widetilde{\beta}_j^{\text{opt}} \equiv \left[(\alpha_j\beta_j)/(m_j^d)^{\text{opt}}\right]^{\sigma-1}\beta_j$. We know that

$$\frac{\widetilde{\beta}_j^{\text{eqm}}}{\widetilde{\beta}_j^{\text{opt}}} = \left[\frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}}\right]^{\sigma-1} \quad \Rightarrow \quad \widetilde{\beta}_j^{\text{opt}} = \left[\frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}}\right]^{1-\sigma}\widetilde{\beta}_j^{\text{eqm}} = \text{const.} \times \left[\frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}}\right]^{1-\sigma}\widehat{\beta}_j^{\text{eqm}}. \tag{F-1}$$

The optimal utility in (E-33) can be rewritten as

$$U^{\text{opt}} = \left\{\sum_{j=1}^{J} \alpha_j^{\sigma-1}\beta_j^{\sigma}[(m_j^d)^{\text{opt}}]^{1-\sigma}\right\}^{\frac{1}{\sigma-1}} = \left\{\sum_{j=1}^{J}\left[\frac{\alpha_j\beta_j}{(m_j^d)^{\text{opt}}}\right]^{\sigma-1}\beta_j\right\}^{\frac{1}{\sigma-1}} = \left(\sum_{j=1}^{J}\widetilde{\beta}_j^{\text{opt}}\right)^{\frac{1}{\sigma-1}}.$$

Hence, using (F-1), we have

$$U^{\text{opt}} = (\text{const.})^{\frac{1}{\sigma-1}}\left(\sum_{j=1}^{J}\widehat{\beta}_j^{\text{opt}}\right)^{\frac{1}{\sigma-1}},$$

where $\widehat{\beta}_j^{\text{opt}} = \left[(m_j^d)^{\text{opt}}/(m_j^d)^{\text{eqm}}\right]^{1-\sigma}\widehat{\beta}_j^{\text{eqm}}$. Finally, taking the ratio of U^{eqm} and U^{opt}, we obtain

$$\frac{U^{\text{eqm}}}{U^{\text{opt}}} = \frac{(\text{const.})^{\frac{\sigma}{\sigma-1}-1}\frac{1}{\sum_{\ell=1}^{J}\widehat{\beta}_{\ell}^{\text{eqm}}\widehat{\theta}_{\ell}}}{(\text{const.})^{\frac{1}{\sigma-1}}\left(\sum_{j=1}^{J}\widehat{\beta}_{j}^{\text{opt}}\right)^{\frac{1}{\sigma-1}}} = \frac{\frac{1}{\sum_{\ell=1}^{J}\widehat{\beta}_{\ell}^{\text{eqm}}\widehat{\theta}_{\ell}}}{\left\{\sum_{j=1}^{J}\left[\frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}}\right]^{1-\sigma}\widehat{\beta}_{j}^{\text{eqm}}\right\}^{\frac{1}{\sigma-1}}}.$$

We already know $\widehat{\beta}_{j}^{\text{eqm}}$ and $\widehat{\theta}_j$. Since the cutoff ratio is a function of k_j only, the above expression can be quantified for any given value of σ. Then, using (D-9), we can compute the associated Allais surplus required to quantify the distortions.

G. Robustness checks

Table G-1: Sectoral data, parameter values, and distortions for France in 2008. Alternative firm size measure (revenue).

					Cobb-Douglas - CARA & Pareto						**Cobb-Douglas - CES & Pareto**			
				Std. dev.					Cutoff	Entry			Cutoff	Entry
Sector	Description	Firms	$\widehat{e}_j$	log emp	$\widehat{k}_j$	$\widehat{\theta}_j$	$\widehat{\kappa}_j$	$\widehat{\beta}_j$	distortions		$\widehat{\rho}_j$	$\widehat{\beta}_j$	distortions	
1	Agriculture	16225	0.0188	0.9490	1.0402	0.7658	0.1042	0.0200	50.6033	-6.3271	0.4968	0.0188	0	-17.0719
2	Mining and quarrying	1264	0.0002	1.2047	5.9058	0.9278	0.0101	0.0001	11.1708	13.4870	0.8768	0.0002	0	46.3639
3	Food products, beverages, tobacco	46279	0.0697	0.9535	1.0612	0.7680	0.1022	0.0742	49.8611	-6.0631	0.5030	0.0697	0	-16.0393
4	Textiles, leather and footwear	6672	0.0205	1.2566	18.4586	0.9743	0.0013	0.0172	3.6961	19.1735	0.9587	0.0205	0	60.0359
5	Wood products	5733	0.0008	1.2423	11.7605	0.9608	0.0030	0.0007	5.7492	17.5267	0.9359	0.0008	0	56.2412
6	Pulp, paper, printing and publishing	16433	0.0086	1.1744	4.1427	0.9034	0.0179	0.0078	15.5942	10.4938	0.8295	0.0086	0	38.4744
7	Coke, refined petroleum, nuclear fuel	32	0.0168	0.9853	1.2258	0.7837	0.0887	0.0175	44.7000	-4.1422	0.5471	0.0168	0	-8.6772
8	Chemicals and chemical products	1700	0.0285	1.2568	18.5717	0.9745	0.0013	0.0239	3.6740	19.1917	0.9589	0.0285	0	60.0770
9	Rubber and plastics products	3066	0.0037	1.2139	6.7456	0.9356	0.0081	0.0033	9.8403	14.4388	0.8912	0.0037	0	48.7676
10	Other non-metallic mineral products	4686	0.0020	1.2444	12.4217	0.9628	0.0027	0.0017	5.4504	17.7622	0.9392	0.0020	0	56.7918
11	Basic metals	567	0.0001	1.2570	18.7108	0.9746	0.0013	0.0001	3.6471	19.2137	0.9592	0.0001	0	60.1269
12	Fabricated metal products	19030	0.0021	1.2171	7.0925	0.9384	0.0074	0.0018	9.3788	14.7748	0.8962	0.0021	0	49.6046
13	Machinery and equipment	13954	0.0053	1.1780	4.2982	0.9062	0.0169	0.0047	15.0684	10.8363	0.8351	0.0053	0	39.4036
14	Office, accounting, computing mach.	255	0.0033	1.2315	9.2006	0.9510	0.0047	0.0028	7.2983	16.3280	0.9189	0.0033	0	53.3972
15	Electrical machinery and apparatus	2475	0.0034	1.2392	10.8874	0.9580	0.0035	0.0029	6.1979	17.1757	0.9310	0.0034	0	55.4157
16	Radio, TV, communication equip.	1728	0.0042	1.2380	10.5883	0.9569	0.0036	0.0036	6.3682	17.0433	0.9291	0.0042	0	55.1028
17	Medical, precision, optical instr.	8260	0.0050	1.0670	1.8662	0.8293	0.0552	0.0049	31.7869	1.4362	0.6657	0.0050	0	11.1253
18	Motor vehicles and (semi-)trailers	1411	0.0326	1.2365	10.2341	0.9555	0.0039	0.0279	6.5823	16.8775	0.9268	0.0326	0	54.7096
19	Other transport equipment	1846	0.0028	1.1541	3.4226	0.8879	0.0240	0.0025	18.5983	8.6012	0.7980	0.0028	0	33.2111
20	Manufacturing n.e.c; recycling	13789	0.0130	1.1481	3.2520	0.8835	0.0259	0.0120	19.4873	8.0615	0.7887	0.0130	0	31.6691
21	Electricity, gas and water supply	3444	0.0225	1.2325	9.3922	0.9519	0.0045	0.0193	7.1540	16.4381	0.9205	0.0225	0	53.6613
22	Construction	291286	0.0082	1.0048	1.3453	0.7939	0.0805	0.0085	41.5661	-2.8964	0.5748	0.0082	0	-4.0493
23	Wholesale and retail trade; repairs	403178	0.1377	1.1402	3.0476	0.8777	0.0285	0.1283	20.6695	7.3576	0.7765	0.1377	0	29.6296
24	Hotels and restaurants	156601	0.0489	0.9575	1.0801	0.7699	0.1005	0.0519	49.2101	-5.8293	0.5084	0.0489	0	-15.1287
25	Transport and storage	50914	0.0291	1.2573	18.9309	0.9749	0.0012	0.0244	3.6053	19.2479	0.9597	0.0291	0	60.2044
26	Post and telecommunications	2683	0.0191	1.1175	2.5707	0.8618	0.0363	0.0181	24.0753	5.4145	0.7418	0.0191	0	23.8287
27	Finance and insurance	18351	0.0376	0.9574	1.0794	0.7698	0.1006	0.0400	49.2333	-5.8377	0.5082	0.0376	0	-15.1616
28	Real estate activities	80723	0.1649	0.9748	1.1680	0.7784	0.0931	0.1732	46.3893	-4.7882	0.5324	0.1649	0	-11.1228
29	Renting of machinery and equipment	10616	0.0022	1.1887	4.8301	0.9146	0.0141	0.0020	13.5096	11.8725	0.8517	0.0022	0	42.1725
30	Computer and related activities	31426	0.0010	1.0817	2.0353	0.8384	0.0495	0.0010	29.5217	2.5484	0.6876	0.0010	0	14.7924
31	Research and development	3830	0.0074	0.9487	1.0386	0.7657	0.1043	0.0079	50.6588	-6.3465	0.4963	0.0074	0	-17.1482
32	Other Business Activities	228933	0.0073	1.0659	1.8543	0.8286	0.0556	0.0072	31.9586	1.3537	0.6640	0.0073	0	10.8497
34	Education	16908	0.0799	0.9907	1.2575	0.7865	0.0864	0.0830	43.8253	-3.8008	0.5547	0.0799	0	-7.3973
35	Health, social work, personal services	321184	0.1930	0.9529	1.0582	0.7677	0.1025	0.2056	49.9634	-6.0997	0.5021	0.1930	0	-16.1820

Notes: Column 1 reports the number of firms in each sector in the ESANE database for France in 2008 after trimming, column 2 the observed (re-scaled) expenditure shares from the French input-output table, and column 3 the observed standard deviation of log revenue across firms, where data are constructed as described in Appendix C.1. Column 4 reports the values of $\widehat{k}_j$ that we obtain by matching the numbers from column 3 to expression (C-2) in Appendix C.2. Columns 5 and 6 report the values of $\widehat{\theta}_j$ and $\widehat{\kappa}_j$ which are transformations of $\widehat{k}_j$. Column 7 reports the value $\widehat{\beta}_j$ obtained as described in Section 4.1. In columns 8 and 9 we report the magnitudes of cutoff and entry distortions at the sectoral level obtained from (32) and (33), respectively. Column 10 reports the value of $\widehat{\rho}_j$ obtained by matching the numbers from column 3 to expression (C-4) in Appendix C.2 while using $\widehat{k}_j$ from column 4. Column 11 reports the values $\widehat{\beta}_j$ which correspond to the expenditure shares from column 2. Finally, column 12 reports only zeroes as the CES model does not exhibit cutoff distortions, and column 13 reports the magnitudes of entry distortions as computed in (36).

Table G-2: Sectoral data, parameter values, and distortions for the United Kingdom in 2005. Alternative firm size measure (revenue).

					Cobb-Douglas - CARA & Pareto						**Cobb-Douglas - CES & Pareto**			
Sector	Description	Firms	$\widehat{e}_j$	Std. dev. log emp	$\widehat{k}_j$	$\widehat{\theta}_j$	$\widehat{\kappa}_j$	$\widehat{\beta}_j$	Cutoff distortions	Entry distortions	$\widehat{\rho}_j$	$\widehat{\beta}_j$	Cutoff distortions	Entry distortions
1	Agriculture	94598	0.0127	0.9421	1.0092	0.7626	0.1072	0.0135	51.7394	-6.4710	0.4874	0.0127	0	-20.8092
2	Mining and quarrying	952	0.0008	1.2471	13.4142	0.9653	0.0024	0.0007	5.0559	18.3969	0.9436	0.0008	0	53.3165
3	Food products, beverages, tobacco	5458	0.0442	1.1922	5.0361	0.9175	0.0131	0.0393	12.9889	12.5313	0.8572	0.0442	0	39.2834
4	Textiles, leather and footwear	8688	0.0213	1.0916	2.1641	0.8447	0.0457	0.0206	28.0000	3.6024	0.7026	0.0213	0	14.1567
5	Wood products	7705	0.0014	1.0615	1.8084	0.8260	0.0574	0.0013	32.6408	1.3035	0.6575	0.0014	0	6.8289
6	Pulp, paper, printing and publishing	23234	0.0112	1.0998	2.2814	0.8501	0.0427	0.0108	26.7435	4.2582	0.7150	0.0112	0	16.1788
7	Coke, refined petroleum, nuclear fuel	138	0.0104	1.1090	2.4246	0.8561	0.0393	0.0099	25.3527	5.0019	0.7289	0.0104	0	18.4355
8	Chemicals and chemical products	2922	0.0088	1.2107	6.4233	0.9328	0.0088	0.0077	10.3116	14.4096	0.8861	0.0088	0	43.9677
9	Rubber and plastics products	5827	0.0035	1.1825	4.5093	0.9097	0.0157	0.0032	14.4085	11.5742	0.8421	0.0035	0	36.8224
10	Other non-metallic mineral products	4483	0.0017	1.1184	2.5888	0.8625	0.0360	0.0016	23.9258	5.7846	0.7433	0.0017	0	20.7703
11	Basic metals	1334	0.0003	1.1972	5.3524	0.9216	0.0119	0.0002	12.2630	13.0309	0.8650	0.0003	0	40.5480
12	Fabricated metal products	23394	0.0019	1.1157	2.5388	0.8606	0.0369	0.0018	24.3427	5.5538	0.7391	0.0019	0	20.0860
13	Machinery and equipment	11103	0.0064	1.1943	5.1615	0.9192	0.0126	0.0057	12.6910	12.7355	0.8604	0.0064	0	39.8017
14	Office, accounting, computing mach.	1577	0.0006	1.0105	1.3831	0.7969	0.0781	0.0007	40.6609	-2.2591	0.5829	0.0006	0	-5.2874
15	Electrical machinery and apparatus	4226	0.0015	1.2221	7.7129	0.9427	0.0064	0.0013	8.6530	15.6235	0.9041	0.0015	0	46.8970
16	Radio, TV, communication equip.	2149	0.0057	1.2044	5.8778	0.9275	0.0102	0.0050	11.2213	13.7603	0.8762	0.0057	0	42.3699
17	Medical, precision, optical instr.	5103	0.0016	1.1738	4.1164	0.9029	0.0181	0.0015	15.6868	10.7346	0.8285	0.0016	0	34.6205
18	Motor vehicles and (semi-)trailers	2904	0.0272	1.1167	2.5579	0.8613	0.0366	0.0257	24.1821	5.6426	0.7407	0.0272	0	20.3493
19	Other transport equipment	2176	0.0036	1.1189	2.5971	0.8628	0.0358	0.0034	23.8579	5.8224	0.7440	0.0036	0	20.8817
20	Manufacturing n.e.c; recycling	16374	0.0109	1.0424	1.6284	0.8147	0.0650	0.0109	35.6211	-0.0774	0.6293	0.0109	0	2.2434
21	Electricity, gas and water supply	326	0.0261	1.2541	16.7661	0.9719	0.0016	0.0219	4.0628	19.1982	0.9546	0.0261	0	55.1042
22	Construction	197153	0.0085	0.9524	1.0560	0.7675	0.1027	0.0090	50.0410	-5.8718	0.5014	0.0085	0	-18.5242
23	Wholesale and retail trade; repairs	347165	0.1850	1.0275	1.5064	0.8063	0.0711	0.1871	37.9630	-1.1141	0.6075	0.1850	0	-1.2915
24	Hotels and restaurants	139080	0.0781	0.9460	1.0265	0.7644	0.1055	0.0833	51.0996	-6.2470	0.4927	0.0781	0	-19.9519
25	Transport and storage	54673	0.0392	1.0951	2.2126	0.8470	0.0444	0.0377	27.4668	3.8789	0.7079	0.0392	0	15.0129
26	Post and telecommunications	8826	0.0181	1.0508	1.7043	0.8196	0.0616	0.0180	34.3017	0.5252	0.6417	0.0181	0	4.2615
27	Finance and insurance	20825	0.0807	1.0435	1.6386	0.8154	0.0645	0.0807	35.4367	0.0060	0.6310	0.0807	0	2.5243
28	Real estate activities	75252	0.1104	1.0094	1.3759	0.7963	0.0785	0.1131	40.8295	-2.3290	0.5814	0.1104	0	-5.5344
29	Renting of machinery and equipment	13256	0.0061	1.0735	1.9379	0.8333	0.0527	0.0059	30.7852	2.2001	0.6754	0.0061	0	9.7318
30	Computer and related activities	79335	0.0010	1.0543	1.7374	0.8217	0.0602	0.0009	33.7554	0.7787	0.6469	0.0010	0	5.1027
31	Research and development	2079	0.0001	1.1567	3.5017	0.8898	0.0232	0.0001	18.2132	9.1343	0.8020	0.0001	0	30.3091
32	Other Business Activities	332122	0.0041	1.0520	1.7154	0.8203	0.0611	0.0040	34.1159	0.6112	0.6434	0.0041	0	4.5472
34	Education	23005	0.0625	1.0551	1.7448	0.8221	0.0599	0.0620	33.6366	0.8343	0.6480	0.0625	0	5.2861
35	Health, social work, personal services	209071	0.2044	0.9945	1.2801	0.7884	0.0848	0.2114	43.2211	-3.2995	0.5600	0.2044	0	-9.0025

Notes: Column 1 reports the number of firms in each sector in the BSD database for the UK in 2005 after trimming, column 2 the observed (re-scaled) expenditure shares from the UK input-output table, and column 3 the observed standard deviation of log revenue across firms, where data are constructed as described in Appendix C.1. Column 4 reports the values of $\widehat{k}_j$ that we obtain by matching the numbers from column 3 to expression (C-2) in Appendix C.2. Columns 5 and 6 report the values of $\widehat{\theta}_j$ and $\widehat{\kappa}_j$ which are transformations of $\widehat{k}_j$. Column 7 reports the value $\widehat{\beta}_j$ obtained as described in Section 4.1. In columns 8 and 9 we report the magnitudes of cutoff and entry distortions at the sectoral level obtained from (32) and (33), respectively. Column 10 reports the value of $\widehat{\rho}_j$ obtained by matching the numbers from column 3 to expression (C-4) in Appendix C.2 while using $\widehat{k}_j$ from column 4. Column 11 reports the values $\widehat{\beta}_j$ which correspond to the expenditure shares from column 2. Finally, column 12 reports only zeroes as the CES model does not exhibit cutoff distortions, and column 13 reports the magnitudes of entry distortions as computed in (36).

Table G-3: Sectoral data, parameter values, and distortions for France in 2008. Alternative fixed costs measure (profits).

Sector	Description	Firms	$\widehat{e}_j$	Std. dev. log emp	Cobb-Douglas - CARA & Pareto $\widehat{k}_j$	$\widehat{\theta}_j$	$\widehat{\kappa}_j$	$\widehat{\beta}_j$	Cutoff distortions	Entry distortions	Cobb-Douglas - CES & Pareto $\widehat{\rho}_j$	$\widehat{\beta}_j$	Cutoff distortions	Entry distortions
1	Agriculture	3842	0.0188	0.8891	1.8433	0.8280	0.0560	0.0196	32.1195	-4.2374	0.6211	0.0188	0	-12.0899
2	Mining and quarrying	854	0.0002	0.9922	2.7343	0.8677	0.0333	0.0002	22.7878	0.3562	0.7307	0.0002	0	3.4283
3	Food products, beverages, tobacco	31667	0.0697	0.9296	2.1471	0.8439	0.0462	0.0714	28.1916	-2.3974	0.6662	0.0697	0	-5.6964
4	Textiles, leather and footwear	4260	0.0205	1.0493	3.5065	0.8899	0.0232	0.0199	18.1903	2.9254	0.7863	0.0205	0	11.2992
5	Wood products	3828	0.0008	1.0996	4.5651	0.9106	0.0154	0.0008	14.2437	5.3165	0.8339	0.0008	0	18.0360
6	Pulp, paper, printing and publishing	9214	0.0086	1.1002	4.5819	0.9109	0.0153	0.0082	14.1948	5.3472	0.8345	0.0086	0	18.1191
7	Coke, refined petroleum, nuclear fuel	13	0.0168	1.2149	12.7673	0.9637	0.0026	0.0150	5.3062	11.4585	0.9394	0.0168	0	32.9766
8	Chemicals and chemical products	1084	0.0285	1.0949	4.4426	0.9086	0.0161	0.0271	14.6108	5.0863	0.8295	0.0285	0	17.4118
9	Rubber and plastics products	2834	0.0037	1.1573	6.7956	0.9360	0.0080	0.0035	9.7710	8.2557	0.8872	0.0037	0	25.5820
10	Other non-metallic mineral products	2465	0.0020	1.0980	4.5233	0.9099	0.0156	0.0019	14.3670	5.2390	0.8324	0.0020	0	17.8265
11	Basic metals	484	0.0001	1.1290	5.4942	0.9233	0.0114	0.0001	11.9633	6.7831	0.8612	0.0001	0	21.8983
12	Fabricated metal products	14200	0.0021	0.9881	2.6891	0.8661	0.0341	0.0021	23.1298	0.1737	0.7266	0.0021	0	2.8445
13	Machinery and equipment	6894	0.0053	1.2044	11.0308	0.9585	0.0034	0.0047	6.1195	10.8538	0.9300	0.0053	0	31.6414
14	Office, accounting, computing mach.	193	0.0033	1.1264	5.3971	0.9221	0.0117	0.0031	12.1670	6.6494	0.8587	0.0033	0	21.5544
15	Electrical machinery and apparatus	1467	0.0034	1.1876	9.0384	0.9503	0.0048	0.0031	7.4250	9.9033	0.9148	0.0034	0	29.4866
16	Radio, TV, communication equip.	1266	0.0042	1.1216	5.2292	0.9200	0.0123	0.0040	12.5360	6.4087	0.8543	0.0042	0	20.9311
17	Medical, precision, optical instr.	3780	0.0050	1.0524	3.5596	0.8912	0.0227	0.0048	17.9412	3.0711	0.7893	0.0050	0	11.7257
18	Motor vehicles and (semi-)trailers	1111	0.0326	1.2031	10.8443	0.9578	0.0035	0.0294	6.2219	10.7784	0.9288	0.0326	0	31.4728
19	Other transport equipment	517	0.0028	1.2064	11.3219	0.9595	0.0032	0.0025	5.9662	10.9670	0.9318	0.0028	0	31.8934
20	Manufacturing n.e.c; recycling	6750	0.0130	1.0681	3.8455	0.8975	0.0201	0.0125	16.7083	3.8022	0.8042	0.0130	0	13.8346
21	Electricity, gas and water supply	383	0.0225	1.2034	10.8938	0.9580	0.0035	0.0203	6.1944	10.7986	0.9291	0.0225	0	31.5181
22	Construction	134776	0.0082	0.8944	1.8810	0.8301	0.0546	0.0086	31.5745	-3.9897	0.6272	0.0082	0	-11.2209
23	Wholesale and retail trade; repairs	197330	0.1377	0.9079	1.9796	0.8355	0.0513	0.1425	30.2319	-3.3691	0.6425	0.1377	0	-9.0529
24	Hotels and restaurants	79898	0.0489	0.8493	1.5655	0.8104	0.0680	0.0521	36.7927	-6.2683	0.5707	0.0489	0	-19.2121
25	Transport and storage	22696	0.0291	1.0931	4.3980	0.9079	0.0163	0.0277	14.7491	5.0000	0.8278	0.0291	0	17.1764
26	Post and telecommunications	196	0.0191	1.0917	4.3623	0.9073	0.0165	0.0182	14.8618	4.9300	0.8265	0.0191	0	16.9848
27	Finance and insurance	5773	0.0376	0.8535	1.5952	0.8125	0.0666	0.0401	36.2304	-6.0324	0.5765	0.0376	0	-18.3915
28	Real estate activities	17452	0.1649	0.8937	1.8764	0.8299	0.0548	0.1718	31.6403	-4.0197	0.6264	0.1649	0	-11.3262
29	Renting of machinery and equipment	3399	0.0022	1.1475	6.2862	0.9316	0.0091	0.0021	10.5261	7.7413	0.8783	0.0022	0	24.3168
30	Computer and related activities	9088	0.0010	1.0515	3.5439	0.8908	0.0228	0.0010	18.0139	3.0285	0.7884	0.0010	0	11.6011
31	Research and development	905	0.0074	1.0744	3.9733	0.9001	0.0192	0.0071	16.2104	4.1024	0.8102	0.0074	0	14.6854
32	Other Business Activities	71871	0.0073	1.0110	2.9548	0.8749	0.0298	0.0072	21.2555	1.1880	0.7492	0.0073	0	6.0484
34	Education	8205	0.0799	1.0220	3.0983	0.8792	0.0279	0.0785	20.3633	1.6834	0.7600	0.0799	0	7.5763
35	Health, social work, personal services	70236	0.1930	1.0317	3.2343	0.8830	0.0262	0.1890	19.5840	2.1230	0.7694	0.1930	0	8.9117

Notes: Column 1 reports the number of firms in each sector in the ESANE database for France in 2008 after trimming, column 2 the observed (re-scaled) expenditure shares from the French input-output table, and column 3 the observed standard deviation of the log number of employees across firms, where data are constructed as described in Appendix C.1. Column 4 reports the values of $\widehat{k}_j$ that we obtain by matching the numbers from column 3 to expression (C-1) in Appendix C.2. Columns 5 and 6 report the values of $\widehat{\theta}_j$ and $\widehat{\kappa}_j$ which are transformations of $\widehat{k}_j$. Column 7 reports the value $\widehat{\beta}_j$ obtained as described in Section 4.1. In columns 8 and 9 we report the magnitudes of cutoff and entry distortions at the sectoral level obtained from (32) and (33), respectively. Column 10 reports the value of $\widehat{\rho}_j$ obtained by matching the numbers from column 3 to expression (C-3) in Appendix C.2 while using $\widehat{k}_j$ from column 4. Column 11 reports the values $\widehat{\beta}_j$ which correspond to the expenditure shares from column 2. Finally, column 12 reports only zeroes as the CES model does not exhibit cutoff distortions, and column 13 reports the magnitudes of entry distortions as computed in (36).

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