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Armin Seibert, Andrei Sirchenko,
Gernot Müller

A MODEL FOR POLICY INTEREST RATES

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Armin Seibert
Augsburg University

Andrei Sirchenko[*†]
Higher School of Economics

Gernot Müller
Augsburg University

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Abstract

This paper introduces a model that addresses the key worldwide features of modern monetary policy making: the discreteness of policy interest rates both in magnitude and in timing, the preponderance of status quo decisions, policy inertia and regime switching. We capture them by developing a new dynamic discrete-choice model with switching among three latent policy regimes (dovish, neutral and hawkish), estimated via the Gibbs sampler with data augmentation. The simulations and an application to federal funds rate target demonstrate that ignoring these features leads to biased estimates, worse in- and out-of-sample fit, and qualitatively different inference. Using all Federal Open Market Committee's (FOMC) decisions made both at scheduled and unscheduled meetings as sample observations, we model the Federal Reserve's response to real-time data available right before each meeting, and control for the endogeneity of monetary policy shocks. The new model, fitted for Greenspan's tenure, correctly predicts the directions of about 90% of the next decisions on the target rate (hike, no change, or cut) out of sample during Bernanke's term including the status quo decisions after reaching the zero lower bound, while the conventional linear model fails to adequately tackle the zero bound and wrongly predicts further cuts.

KEYWORDS. Federal funds rate target, FOMC, discrete ordered choice, regime switching, endogeneity, MCMC, Gibbs sampler, data augmentation, autoregressive ordered probit, real-time data.

JEL CLASSIFICATION. C11, C34, C35, E52.

[*]Corresponding author. E-mail address: andrei.sirchenko@gmail.com.

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1 Introduction

We develop a dynamic model of discrete ordered choice with lagged latent dependent variables among regressors and with time-varying transition probabilities of switching among three latent regimes interpreted in the interest-rate-setting context as dovish, neutral and hawkish monetary policy stances. The new methodology synthesizes and extends the existing models of ordered choice and avoids important common misspecifications and distortions of the data-generating process (DGP) in the empirical identification of monetary policy rules. The new model assumes three implicit decisions: the policy regime decision and two decisions (conditional on the dovish or hawkish regime) on the amount of rate change. All three decisions are modeled jointly by the autoregressive ordered probit (AOP) models of Müller and Czado (2005).

The policy interest rates are currently a critical instrument and a principal measure of monetary policy in many countries. They are an anchor for other short-term market interest rates, and are closely watched and anticipated by many economic agents. If we can quantitatively formalize how monetary authorities set the policy rates, and "if practitioners in financial markets gain a better understanding of how policy is likely to respond to incoming information, asset prices and bond yields will tend to respond to economic data in ways that further the central bank's policy objectives" (Bernanke 2007). To forecast the state of the economy, or to evaluate the effects of economic shocks, monetary and fiscal policy actions, we need to understand central bank's systematic response to economic data. To improve the monetary policy, we also need a clear empirical description of what is going to be improved. It is really difficult to describe monetary policy without using an econometric model. Since the policy rates are set administratively by the monetary authorities, and are neither the outcomes of market interaction of supply and demand nor subject to technical fluctuations or extraneous sources of noise, it makes them of special interest for econometric modeling (Hamilton and Jorda, 2002).

We implement an econometric framework that addresses the main worldwide features of modern monetary policy making: the discreteness of policy interest rates, both in magnitude and in timing; the preponderance and heterogeneity of status quo (no change to the rate) decisions; interest rate smoothing and policy inertia; and policy regime switching and different nature of positive and negative interest rate movements. None of the existing models is able to adequately capture all these stylized facts. More specifically, we address the following issues.

The discreteness of policy rates. Monetary policy modeling has been historically implemented in a continuous framework, either by estimating a 'simple' linear policy rule such as the Taylor rule (Taylor 1993) or by estimating a monetary policy reaction function as a linear equation of a vector autoregressive model.[1] Nowadays, many central banks (and all the major ones) change policy rates by discrete amounts, typically by multiples of 25 basis points (bp), at special meetings of monetary policy committees, which are held 6–12 times a year. The notable examples are the Federal Open Market Committee of the U.S. Federal Reserve System (Fed), the Governing Council of the European Central Bank, the Monetary Policy Committee of the Bank of England, and the Policy Board of the Bank of

[1]See Rudebusch (1998a,b) for a critique of monetary policy identification in the context of vector autoregressive models.

Japan.

Despite the extensive literature on monetary policy modeling, most studies use monthly or longer-period data averages and employ regression techniques for a continuous dependent variable, and thus do not address the discreteness of interest rates. Ignoring the limited (discrete) nature of the dependent variable may lead to serious biases. While using regressions for continuous outcomes is appropriate with aggregated data, it raises the problems of distortions caused by the use of an incorrect information set and of simultaneity caused by disregarding possible interactions between the policy rates and the other variables that can happen during a period of aggregation. In recent years, discrete-choice approaches such as the ordered probit models (e.g., Vanderhart 2000; Gerlach 2007; Hayo and Neuenkirch 2010) have been employed. There have been also attempts to use the discrete-choice models with FOMC-meeting data frequency (Piazzesi 2005; Hu and Phillips 2004; Kim et al. 2009; Kauppi 2012); they, however, restricted their attention to the decisions made at the scheduled FOMC meetings only, thereby not reflecting the entire policy-making process. In fact, from 1983 to 1993 only about 15% of non-zero changes to the U.S. federal funds rate target (*target* henceforth) have been made at the scheduled FOMC meetings.

We address the discrete nature of policy rates by an ordered choice approach. The empirical results show that discreteness does matter in the estimation of monetary policy rules. We compare the in-sample (under Greenspan's tenure) and out-of-sample (under Bernanke's term) performance of the linear, the AOP and new model estimated using the same set of regressors. The performance of the linear model, especially out of sample, is remarkably inferior to that of the discrete-choice competitors: for example, the mean absolute error is twice as large as in the discrete-choice models, and the percentage of correct predictions is only 49% versus 90%. The new model fitted for Greenspan's tenure correctly predicts status quo decisions after reaching the zero lower bound during Bernanke's term, while the linear model fails to adequately deal with the zero lower bound and incorrectly predicts further cuts.

We carefully mimic the policy-making process by using all interest rate decisions (made both at the scheduled and unscheduled FOMC meetings) as sample observations. We match the FOMC decisions with the latest real-time vintages of macroeconomic and non-aggregated daily financial data truly available immediately before each meeting and not revised later on. Information about the exact timing of FOMC meetings together with the use of daily financial data (such as spreads between the long- and short-term market interest rates) allows us to substantially improve the identification of the Fed policy rule, which would not be possible to do using monthly aggregates of financial data. In addition, the market interest rates encapsulate the huge volume of data available to market participants, and can help avoid the omitted variables and time-varying parameter problems.

The preponderance of status quo decisions. The central banks often prefer to wait and see. Many of them leave the rates unchanged at more than a half of policy meetings in different macroeconomic circumstances: between rate hikes, between cuts and also between rate reversals. We address the heterogeneity of status quo decisions by a three-part modelling approach that allows status quo outcomes to be generated by three different latent regimes interpreted as dovish, neutral and hawkish policy stances. We extend the zero-inflated model of Harris and Zhao (2007) and the middle-inflated model of Brooks et al. (2012) by making them suitable for ordinal outcomes that take on negative, zero and positive values, and are past-dependent and autocorrelated. The Monte Carlo experiments suggest that the single-equation AOP model, ignoring the heterogeneity of status quo outcomes, delivers asymptotically biased estimates of the probabilities, whereas the proposed Bayesian estimator of the new model is consistent and performs well in small samples.

The empirical results suggest that less than one third of status quo decisions are generated by the neutral policy stance. During the Greenspan era, the average probabilities of dovish, neutral and hawkish regimes are 0.58, 0.15 and 0.27, respectively, whereas the frequencies of observed cuts, no changes and hikes are 0.24, 0.54 and 0.22, respectively.

Regime switching. The central banks may have asymmetric responses to incoming data, so the rate increases and decreases may be generated by different decision-making processes as well as the numerous status quo decisions may be also driven by distinct mechanisms. In other words, the policy actions may be generated in different regimes. We model this possibility by a regime-switching approach with time-varying transition probabilities of switching among three latent regimes. It allows the probabilities of positive, negative and no changes to be treated differently and to be asymmetrically affected by the economic data. The structure of the new model shares some similar features with the Markov switching model with constant probabilities of a transition from one regime to another introduced by Hamilton (1989) in the framework of linear regressions, and later extended by Diebold et al. (1994) and Filardo (1994) to the case where the transition probabilities may change over time. The existing applications of regime switching to monetary policy employ exclusively models for a continuous dependent variable, usually in the context of vector autoregressions (Sims and Zha 2006; Bikbov and Chernov 2013).

Interest rate smoothing. The policy interest rates are persistent. This stylized fact is usually referred to as interest rate smoothing. If central banks decide to change the rate they will most likely move it again in the same direction several times at the next meetings, avoiding frequent rate reversals. This feature is referred to as monetary policy inertia. We address these two stylized facts by the lagged latent dependent variable introduced to the covariates. It allows for a partial adjustment of interest rates and together with the lagged covariates can capture the autocorrelation of latent monetary shocks.

The estimated autoregressive (AR) coefficients on the lagged dependent variable in both the linear and AOP models are, however, small and not significant. By contrast, the estimated AR coefficients in the new model are large and significant. Moreover, they have the opposite signs in the regime and amount equations. It implies different dynamics of the regime and amount-of-change decisions. Positive autocorrelation in the regime equation leads to the persistency of regime decisions, whereas negative autocorrelations in the amount equations implies that the larger the desired change (a cut or a hike) at the previous meeting, the more likely is no change at the next meeting. Such inference is impossible if we estimate a linear or an AOP model.

The dynamic single-equation ordered probit models have been developed and applied to policy interest rates, among others, by Eichengreen et al. (1985), Davutyan and Parke (1995), Dueker (1999b), Hu and Phillips (2004), Kim et al. (2009), Monokroussos (2011), and Van den Hauwe et al. (2013). Hamilton and Jorda (2002) developed the dynamic two-equation ordered probit model, in which the first-stage binary decision (change or no change) is determined by the autoregressive conditional hazard model, and the magnitude of rate changes is determined by the ordered probit model conditional on a change at the first stage. Grammig and Kehrle (2008) modified this model by implementing the autoregressive conditional multinomial model of Russell and Engle (2005) at the second stage. Each stage in these models is estimated separately, and status quo observations are not included in the estimation of the second stage. We further extend these two-part models by implementing at the first stage a trichotomous decision on the latent policy regime (dovish, neutral or hawkish) that seems to be more realistic than a binary decision (change or no change): the policymakers, who are determined to change the rate, have already decided in which direction they are going to do it. Furthermore, we estimate both stages simultaneously and do not exclude no-change outcomes from the second stage. It allows us to discriminate among different types of the status quo decisions.

Endogeneity. The forward-looking behavior of central banks and financial markets as well as the autocorrelation of monetary shocks can lead to the endogeneity problem in the estimation of monetary policy rules due to correlation among shocks and explanatory variables (de Vries and Li 2014). We control for the endogeneity of policy shocks and explanatory variables by introducing bias correction terms as additional regressors. Following Kuttner (2001) and Bernanke and Kuttner (2005), we compute the market-based proxies for unknown monetary shocks as one-day surprises unanticipated by the federal funds futures. The null of exogeneity of monetary policy shocks is rejected.

In the next section we describe the proposed **Cro**ss-**n**ested **A**utoregressive **O**rdered **P**robit (CronAOP) model. We discuss the estimation and inference in Section 3. The estimation of such a dynamic three-equation ordered probit model is a daunting computational challenge — it requires the evaluation of multiple integrals with no closed form solution. We opt for a Bayesian approach using Markov chain Monte Carlo (MCMC) methods and a Gibbs sampler with data augmentation, which makes estimation computationally feasible and requires no numerical optimization and no high-dimensional integration. We study the finite sample performance of the CronAOP estimator under both its own DGP and the AOP one. In Section 4 we present the design and main results of the Monte Carlo experiments, which demonstrate a good performance of the proposed estimator. In Section 5 we discuss the empirical application to the FOMC decisions on the target. Supporting material is provided in the Online Appendix. Section 6 concludes.

2 Model

We introduce the CronAOP model in the context of monetary authority decisions on the policy interest rate. The observed dependent variable is a change to the policy rate $\Delta y_t = y_t - y_{t-1}$, where y_t is the level of the rate set at the meeting $t = 1, 2, .., T$; Δy_t takes on a finite number of discrete ordered values j coded as $J^-, ..., -1, 0, 1, ..., J^+$. The model assumes three latent regimes and three implicit decisions.

The regimes are determined by the continuous latent variable r_t^{0*} representing the degree of central bank policy stance according to a latent *regime decision*

$$r_t^{0*} = \phi^0 r_{t-1}^{0*} + \boldsymbol{\beta}^0 \mathbf{x}_t^0 + \varepsilon_t^0, \tag{1}$$

where $\boldsymbol{\beta}^0$ is a vector of k^0 unknown slope parameters, $\mathbf{x}_t^0$ is the t^{th} column of the observed $k^0 \times T$ data matrix $\boldsymbol{X}^0$, ϕ^0 is an unknown AR parameter, and ε_t^0 is an error term that is independently and identically distributed (IID) according to the standard normal cumulative distribution function (CDF) Φ.

The observed discrete change Δy_t is conditional on a latent discrete variable s_t^* (coded as -1, 0, or 1 if the central bank policy regime is dovish, neutral or hawkish, respectively). Both Δy_t and s_t^* are determined in an ordered probit fashion as

$$\begin{aligned} &\Delta y_t|(s_t^*=-1)=j \quad \text{if} \quad c_{j-1}^- < r_t^{-*} \leq c_j^- \text{ for } j\leq 0, \\ &\Delta y_t|(s_t^*=0)=0, \\ &\Delta y_t|(s_t^*=1)=j \quad \text{if} \quad c_j^+ < r_t^{+*} \leq c_{j+1}^+ \text{ for } j\geq 0, \end{aligned} \qquad s_t^* = \begin{cases} -1 & \text{if} \quad r_t^{0*}\leq c_1^0, \\ 0 & \text{if} \quad c_1^0 < r_t^{0*} \leq c_2^0, \\ 1 & \text{if} \quad c_2^0 < r_t^{0*}, \end{cases}$$

where $-\infty < c_1^0 \leq c_2^0 < \infty$, $-\infty \equiv c_{J^--1}^- \leq c_{J^-}^- \leq ... \leq c_0^- \equiv \infty$ and $-\infty \equiv c_0^+ \leq c_1^+ \leq ... \leq c_{J^++1}^+ \equiv \infty$ are the unobserved cutpoint parameters; and r_t^{-*} and r_t^{+*} are the continuous latent variables, representing the desired amount of the rate change in the dovish and hawkish regimes, respectively. They are driven by the latent *amount decisions*

$$r_t^{-*} = \phi^- r_{t-1}^{-*} + \boldsymbol{\beta}^- \mathbf{x}_t^- + \varepsilon_t^- \qquad \text{and} \qquad r_t^{+*} = \phi^+ r_{t-1}^{+*} + \boldsymbol{\beta}^+ \mathbf{x}_t^+ + \varepsilon_t^+, \tag{2}$$

where $\boldsymbol{\beta}^-$and $\boldsymbol{\beta}^+$ are the vectors of k^- and k^+ unknown slope parameters, $\mathbf{x}_t^-$ and $\mathbf{x}_t^+$ are the t^{th} columns of the observed $k^- \times T$ and $k^+ \times T$ data matrices $\boldsymbol{X}^-$ and $\boldsymbol{X}^+$, ϕ^- and ϕ^+ are the unknown AR parameters, and ε_t^- and ε_t^+ are the IID error terms with the standard normal CDF Φ. The errors ε_t^0, ε_t^- and ε_t^+ are mutually independent.

Henceforth, the superscript indexes '0', '$-$' and '$+$' refer to the regime decision (1) and two amount decisions (2) conditional on the dovish and hawkish regimes, respectively.

The CronAOP model can be summarized as

$$\Delta y_t = \begin{cases} j \ (j<0) & \text{if} \quad r_t^{0*} \leq c_1^0 \text{ and } c_{j-1}^- < r_t^{-*} \leq c_j^-, \\ 0 & \text{if} \quad \begin{cases} c_1^0 < r_t^{0*} \leq c_2^0, \\ \text{or } (r_t^{0*} \leq c_1^0 \text{ and } c_{-1}^- < r_t^{-*}), \\ \text{or } (c_2^0 < r_t^{0*} \text{ and } r_t^{+*} \leq c_1^+) \end{cases}, \\ j \ (0<j) & \text{if} \quad c_2^0 < r_t^{0*} \text{ and } c_j^+ < r_t^{+*} \leq c_{j+1}^+. \end{cases}$$

To simplify the notation we let $e_{t,m}^i := c_m^i - \phi^i r_{t-1}^{i*} - \boldsymbol{\beta}^i \mathbf{x}_t^i$ for $i \in \{0,-,+\}$. The probabilities to observe the outcome j are then given by

$$\begin{aligned} \Pr(\Delta y_t = j) &= I_{j\leq 0}\Phi(e_{t,1}^0)[\Phi(e_{t,j}^-) - \Phi(e_{t,j-1}^-)] \\ &\quad + I_{j=0}\left[\Phi(e_{t,2}^0) - \Phi(e_{t,1}^0)\right] + I_{j\geq 0}\left[1-\Phi(e_{t,2}^0)\right][\Phi(e_{t,j+1}^+) - \Phi(e_{t,j}^+)], \end{aligned} \tag{3}$$

where $I_{j\leq 0}$, $I_{j=0}$ and $I_{j\geq 0}$ are the indicator functions such that: $I_{j\leq 0}=1$ if $j\leq 0$ and $I_{j\leq 0}=0$ otherwise; $I_{j=0}=1$ if $j=0$ and $I_{j=0}=0$ otherwise; and $I_{j\geq 0}=1$ if $j\geq 0$ and $I_{j\geq 0}=0$ otherwise.

To identify the model parameters we fix the variances of ε_t^0, ε_t^- and ε_t^+ to one, and the intercept components of $\boldsymbol{\beta}^0$, $\boldsymbol{\beta}^-$and $\boldsymbol{\beta}^+$ to zero. These identifying assumptions are arbitrary and standard in the discrete-choice modeling. The probabilities in (3) are absolutely identifiable and invariant to the choice of parameter-identifying assumptions.

The marginal effect (ME) of the covariates on the probabilities is the partial derivative of the probabilities with respect to one of the covariates x_m, $ME_{m,j,t} := \partial_{x_{t,m}} \Pr(\Delta y_t = j)$

keeping all other parameters and covariates constant:

$$
\begin{aligned}
ME_{m,j,t} &= I_{j=0}\left[f(e^0_{t,2})-f(e^0_{t,1})\right]\beta^{0,all}_m \\
&+ I_{j\le 0}\left\{f(e^0_{t,1})\left[\Phi(e^-_{t,j})-\Phi(e^-_{t,j-1})\right]\beta^{0,all}_m+\Phi(e^0_{t,1})\left[f(e^-_{t,j})-f(e^-_{t,j-1})\right]\beta^{-,all}_m\right\} \\
&+ I_{j\ge 0}\left\{f(e^0_{t,2})\left[\Phi(e^+_{t,j+1})-\Phi(e^+_{t,j})\right]\beta^{0,all}_m+\Phi(-e^0_{t,2})\left[f(e^+_{t,j+1})-f(e^+_{t,j})\right]\beta^{+,all}_m\right\},
\end{aligned}
$$

where f is the normal probability density function (PDF), and $\boldsymbol{\beta}^{i,all}$ is a vector of the slope coefficients in the latent equation i on all covariates in the model (in $\boldsymbol{X}^0$, $\boldsymbol{X}^-$ and $\boldsymbol{X}^+$): $\beta^{i,all}_m = 0$ if the covariate m does not appear in the equation i. We compute the MEs and probabilities at the empirical medians of the covariates and the theoretical medians of the lagged dependent latent variables.

Joint Distribution

For parameter estimation through a Gibbs sampler we need to derive the joint distribution of our model. Let $\boldsymbol{R}^*$ denote the $(T+1)\times 3$ matrix, the t^{th} row of which is $\boldsymbol{r}^*_t = (r^{0*}_t, r^{-*}_t, r^{+*}_t)$, $t = 0, 1, ., T$ and the three columns of which are $\boldsymbol{r}^{0*}$, $\boldsymbol{r}^{-*}$ and $\boldsymbol{r}^{+*}$; $\boldsymbol{\Delta}y$ denote the vector of dependent variables $(\Delta y_1, ..., \Delta y_T)$; $\boldsymbol{\theta}$ denote the vector of model parameters $(\boldsymbol{\beta}, \boldsymbol{\phi}, \boldsymbol{c})$, where $\boldsymbol{\beta} = (\boldsymbol{\beta}^0, \boldsymbol{\beta}^0, \boldsymbol{\beta}^+)$, $\boldsymbol{\phi} = (\boldsymbol{\phi}^0, \boldsymbol{\phi}^-, \boldsymbol{\phi}^+)$, $\boldsymbol{c} = (\boldsymbol{c}^0, \boldsymbol{c}^-, \boldsymbol{c}^+)$, $\boldsymbol{c}^0 = (c^0_1, c^0_2)$, $\boldsymbol{c}^- = (c^-_{J^-}, ..., c^-_{-1})$, and $\boldsymbol{c}^+ = (c^+_1, ..., c^+_{J^+})$. The joint density for $\boldsymbol{R}^*$, $\boldsymbol{\Delta}y$ and $\boldsymbol{\theta}$ can be factorized as

$$
f(\boldsymbol{\Delta}y, \boldsymbol{R}^*, \boldsymbol{\theta}) = \left[\prod_{t=1}^{T} f(\Delta y_t, \boldsymbol{r}^*_t|\Delta y_1, ..., \Delta y_{t-1}, \boldsymbol{r}^*_0, ..., \boldsymbol{r}^*_{t-1}, \boldsymbol{\theta})\right]\pi(\boldsymbol{r}^*_0, \boldsymbol{\theta}),
$$

where π is the prior distribution. By the Markov property it reduces to

$$
f(\boldsymbol{\Delta}y, \boldsymbol{R}^*, \boldsymbol{\theta}) = \left[\prod_{t=1}^{T} f(\Delta y_t, \boldsymbol{r}^*_t|\Delta y_{t-1}, r^*_{t-1}, \boldsymbol{\theta})\right]\pi(\boldsymbol{r}^*_0, \boldsymbol{\theta}).
$$

Δy_t is fully determined by the latent variables $\boldsymbol{r}^*_t$ and cutpoints $\boldsymbol{c}$; and $\boldsymbol{r}^*_t$ in turn is determined by $\boldsymbol{r}^*_{t-1}, \boldsymbol{\beta}$ and $\boldsymbol{\phi}$. Hence, the joint density factorizes further into

$$
f(\boldsymbol{\Delta}y, \boldsymbol{R}^*, \boldsymbol{\theta}) = \left[\prod_{t=1}^{T} f(\Delta y_t|\boldsymbol{r}^*_t, \boldsymbol{c}) f(\boldsymbol{r}^*_t|\boldsymbol{r}^*_{t-1}, \boldsymbol{\beta}, \boldsymbol{\phi})\right]\pi(\boldsymbol{r}^*_0, \boldsymbol{\theta}).
$$

So we have to determine the factors $f(\Delta y_t|\boldsymbol{r}^*_t, \boldsymbol{c})$ and $f(\boldsymbol{r}^*_t|\boldsymbol{r}^*_{t-1}, \boldsymbol{\beta}, \boldsymbol{\phi})$. The first one is a sum of indicators of whether $\boldsymbol{r}^*_t$ lie between the corresponding cutpoints:

$$
f(\Delta y_t|\boldsymbol{r}^*_t, \boldsymbol{c}) = I_{r^{0*}_t < c^0_1, r^{-*}_t \in [c^-_{\Delta y_t - 1}, c^-_{\Delta y_t}]} + I_{r^{0*}_t \in [c^0_1, c^0_2], \Delta y_t = 0} + I_{r^{0*}_t > c^0_2, r^{+*}_t \in [c^+_{\Delta y_t}, c^+_{\Delta y_t + 1}]}. \tag{4}
$$

The probability density of $\boldsymbol{r}^*_t$ is given by

$$
f(\boldsymbol{r}^*_t|\boldsymbol{r}^*_{t-1}, \boldsymbol{\beta}, \boldsymbol{\phi}) \propto \prod_{i=0,+,-} \exp\left[-\frac{1}{2}\left(r^{i*}_t - \boldsymbol{x}^{i\prime}_t \boldsymbol{\beta}^i - \phi^i r^{i*}_{t-1}\right)^2\right].
$$

Finally, we have to specify a prior π. We choose a uniform prior on the real numbers $\mathbb{R}$, i.e. an improper prior, for almost all parameters. The only restriction we have to consider is $\phi^i \in (-1, 1)$ to ensure stationarity of the latent AR equations (1) and (2). Besides, we choose the initial latent variables to be distributed as $r_0^{i*} \sim \mathcal{N}(0, (\tau)^2 = 10^2)$ for numerical reasons that will be apparent later. Finally, each pair of cutpoints is uniformly distributed only on the subset of $\mathbb{R}^2$, where the ordering $c_m^i < c_{m+1}^i$ is satisfied.

3 Estimation and inference

While the classical statistical approaches, such as maximum likelihood or method of moments, will not work for this model we can estimate the parameters and choice probabilities using a Gibbs sampler based on the MCMC algorithm for the AOP model with a grouped move step that drastically accelerates the convergence of the Gibbs sampler (Müller and Czado 2005). The structure of the Gibbs sampler is as follows:

- Update for the latent variables: Sample the latent dependent variables $\boldsymbol{R}^*$ from the corresponding truncated normal distributions.
- Update for the cutpoint parameters: Sample the cutpoint parameters $\boldsymbol{c}$ from the uniform distributions.
- Update for the slope parameters: Sample the slope parameters $\boldsymbol{\beta}$ and the AR parameters $\boldsymbol{\phi}$ from the multivariate normal distributions.
- Grouped move step: Sample a scale parameter from a Gamma distribution to rescale the parameters $\boldsymbol{c}$, $\boldsymbol{\beta}$ and $\boldsymbol{\phi}$ and the latent variables $\boldsymbol{R}^*$.

We use no assumption on the signs of the slope or AR parameters. Without loss of generality and to match the data on the changes to the target in our empirical application, we restrict our inference to the case of five outcome categories $j \in \{-2, -1, 0, 1, 2\}$ of the dependent variable Δy_t with two unknown cutpoints in each latent decision: $\boldsymbol{c}^0 = (c_1^0, c_2^0)$, $\boldsymbol{c}^- = (c_{-2}^-, c_{-1}^-)$ and $\boldsymbol{c}^+ = (c_1^+, c_2^+)$. The initial values for the Gibbs sampler for all three latent decisions $i \in \{0, -, +\}$ are $\phi_{\text{ini}}^i = 0$, $\boldsymbol{\beta}_{\text{ini}}^i = \boldsymbol{0}$ and $(c_{\text{l, ini}}^i, c_{\text{u, ini}}^i) = (-0.5, 0.5)$.

We will now derive all full conditional densities from the joint distribution exploiting the fact that each full conditional density is proportional to the joint distribution.

3.1 Update for the latent variables

For convenience we use the abbreviation rem for all other remaining parameters and latent variables. For instance, the full conditional for the latent variable r_t^{0*} will be written as

$$f(r_t^{0*}|rem) := f(r_t^{0*}|\boldsymbol{\Delta}y, \boldsymbol{r}^{+*}, \boldsymbol{r}^{-*}, r_0^{0*}, ..., r_{t-1}^{0*}, r_{t+1}^{0*}, ..., r_T^{0*}, \boldsymbol{\theta}).$$

The full conditional of the latent variable r_t^{i*}, $t \in \{1, 2, ..., T-1\}$, $i \in \{0, -, +\}$ is

$$\begin{aligned} f(r_t^{i*}|rem) &\propto f(\Delta y_t|\boldsymbol{r}_t^*, \boldsymbol{c}) f(\boldsymbol{r}_t^*|\boldsymbol{r}_{t-1}^*, \boldsymbol{\beta}, \boldsymbol{\phi}) f(r_{t+1}^*|\boldsymbol{r}_t^*, \boldsymbol{\beta}, \boldsymbol{\phi}) \\ &\propto f(\Delta y_t|\boldsymbol{r}_t^*, \boldsymbol{c}) \exp\left[-\frac{1}{2}\left(r_t^{i*} - \boldsymbol{\beta}^i \boldsymbol{x}_t^i - \phi^i r_{t-1}^{i*}\right)^2 - \frac{1}{2}\left(r_{t+1}^{i*} - \boldsymbol{\beta}^i \boldsymbol{x}_{t+1}^i - \phi^i r_t^{i*}\right)^2\right] \\ &\propto f(\Delta y_t|\boldsymbol{r}_t^*, \boldsymbol{c}) \exp\left[-\frac{1}{2}\left(\frac{r_t^{i*} - \mu_t^i}{\sigma^i}\right)^2\right], \end{aligned}$$

where variance $(\sigma^i)^2 := \left[1 + (\phi^i)^2\right]^{-1}$, mean $\mu_t^i := (\sigma^i)^2 \left[\boldsymbol{\beta}^i \boldsymbol{x}_t^i + \phi^i (r_{t-1}^{i*} + r_{t+1}^{i*} - \boldsymbol{\beta}^i \boldsymbol{x}_{t+1}^i)\right]$. Thus, r_t^{i*} must be sampled from a normal distribution $r_t^{i*}|rem \sim \mathcal{N}_{\text{trunc}_t^i}\left(\mu_t^i, (\sigma^i)^2\right)$, which is truncated by $f(\Delta y_t|\mathbf{r}_t^*, \mathbf{c})$ if the observation is influenced by r_t^{i*} (this is not always the case: e.g., if $\Delta y_t = 1$ or 2, r_t^{-*} can be arbitrary).

The truncation intervals for the regime decision r_t^{0*} are $\text{trunc}_t^0 = (a_t^0, b_t^0)$ with

$$a_t^0 = \begin{cases} c_2^0, & \text{if } \Delta y_t > 0, \\ c_1^0 & \text{if } \Delta y_t = 0, r_t^{-*} < c_{-1}^- , \\ -\infty & \text{else,} \end{cases} \qquad b_t^0 = \begin{cases} c_1^0 & \text{if } \Delta y_t < 0, \\ c_2^0 & \text{if } \Delta y_t = 0, r_t^{+*} > c_1^+, \\ \infty & \text{else.} \end{cases}$$

The truncations for the amount decisions in the dovish and hawkish regimes are

$$\text{trunc}_t^- = \begin{cases} (c_{\Delta y_t - 1}^-, c_{\Delta y_t}^-) & \text{if } r_t^{0*} < c_1^{0*}, \\ (-\infty, \infty) & \text{else,} \end{cases} \qquad \text{trunc}_t^+ = \begin{cases} (c_{\Delta y_t}^+, c_{\Delta y_t + 1}^+) & \text{if } r_t^{0*} > c_2^{0*}, \\ (-\infty, \infty) & \text{else.} \end{cases}$$

The updates for r_0^{i*} and r_T^{i*} slightly differ due to a missing predecessor or a successor. In addition, the density of r_0^{i*} is never truncated since there are no observations for it, and is given by

$$\begin{aligned} f(r_0^{i*}|rem) &\propto f(\boldsymbol{r}_1^*|\boldsymbol{r}_0^*, \boldsymbol{\beta}, \boldsymbol{\phi}) \pi(\boldsymbol{r}_0^*) \\ &\propto \exp\left[-\frac{1}{2}\left(r_1^{i*} - \boldsymbol{\beta}^i \boldsymbol{x}_1^i - \phi^i r_0^{i*}\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{r_0^{i*}}{\tau}\right)^2\right] \\ &\propto \exp\left[-\frac{1}{2}\left(\frac{r_0^{i*} - \mu_0^i}{\sigma_0^i}\right)^2\right], \end{aligned}$$

where mean $\mu_0^i := \phi^i (r_1^{i*} - \beta^i x_1^i)(\sigma_0^i)^2$, variance $(\sigma_0^i)^2 := \left[(\phi^i)^2 + (\tau)^{-2}\right]^{-1}$, and the prior hyperparameter $\tau = 10$ is chosen to avoid a big variance $(\sigma_0^i)^2$ for parameters ϕ^i close to zero. In other words, r_0^{i*} must be sampled from $r_0^{i*}|rem \sim \mathcal{N}(\mu_0^i, (\sigma_0^i)^2)$.

Finally, r_T^{i*} has the following full conditional density:

$$\begin{aligned} f(r_T^{i*}|rem) &\propto f(\Delta y_T|\boldsymbol{r}_T^*, \boldsymbol{c}) f(\boldsymbol{r}_T^*|\boldsymbol{r}_{T-1}^*, \boldsymbol{\beta}, \boldsymbol{\phi}) \\ &\propto (\Delta y_T|\boldsymbol{r}_T^*, \boldsymbol{c}) \exp\left[-\frac{1}{2}\left(r_T^{i*} - \boldsymbol{\beta}^i \boldsymbol{x}_T^i - \phi^i r_{T-1}^{i*}\right)^2\right] \\ &= (\Delta y_T|\boldsymbol{r}_T^*, \boldsymbol{c}) \exp\left[-\frac{1}{2}\left(r_T^{i*} - \mu_T^i\right)^2\right] \end{aligned}$$

with mean $\mu_T^i := \boldsymbol{\beta}^i \boldsymbol{x}_T^i + \phi^i r_{T-1}^{i*}$; hence, r_T^{i*} must be sampled from $r_T^{i*}|rem \sim \mathcal{N}_{\text{trunc}_T^i}(\mu_T^i, 1)$. The interval trunc_T^i is the same as trunc_t^i, $t \in \{1, 2, ..., T-1\}$.

The sampling of the $3(T+1)$ latent variables $\boldsymbol{R}^*$ in this section constitutes the bottleneck of the whole Gibbs sampler. To speed up the sampler, every second step can be sampled simultaneously since r_t^* depends only on r_{t-1}^* and r_{t+1}^*.

3.2 Update for the slope parameters

We denote the parameters on all regressors in each latent decision $i \in \{0, -, +\}$ by $\boldsymbol{B}^i := (\boldsymbol{\beta}^i, \phi^i)$ and all regressors in each latent equation by a $T \times (k^i + 1)$ matrix $\boldsymbol{Z}^i := (\boldsymbol{X}^{i\prime}, \boldsymbol{r}_{-T}^{i*})$, the t^{th} row of which is $\boldsymbol{z}_t^i := (\boldsymbol{x}_t^{i\prime}, r_{t-1}^{i*})$, $t \in \{1, 2, ..., T\}$, where $\boldsymbol{r}_{-T}^{i*} := (r_0^{i*}, r_1^{i*}, ..., r_{T-1}^{i*})'$. The full conditional density for $\boldsymbol{B}^i$ is

$$\begin{aligned}
f(\boldsymbol{B}^i|rem) &\propto \left[\prod_{t=1}^T f(r_t^{i*}|r_{t-1}^{i*}, \boldsymbol{B}^i)\right] \times \pi(\boldsymbol{B}^i) \\
&\propto \exp\left\{-\tfrac{1}{2}\left[\sum_{t=1}^T \left(r_t^{i*} - \boldsymbol{\beta}^i \boldsymbol{x}_t^i - \phi^i r_{t-1}^{i*}\right)^2\right]\right\} \times I_{|\phi^i|<1} \\
&\propto \exp\left\{2\left[\sum_{t=1}^T r_t^{i*} \boldsymbol{z}_t^{i\prime}\right] \boldsymbol{B}^i - \tfrac{1}{2}\boldsymbol{B}^{i\prime}\left[\sum_{t=1}^T \boldsymbol{z}_t^i \boldsymbol{z}_t^{i\prime}\right] \boldsymbol{B}^i\right\} \times I_{|\phi^i|<1}.
\end{aligned}$$

Note that $\boldsymbol{z}_t^i \boldsymbol{z}_t^{i\prime}$ is not a scalar but a dyadic product. So all $\boldsymbol{B}^i$ must be sampled independently from multivariate normal distributions

$$\boldsymbol{B}^i|rem \sim \mathcal{N}_{k^i+1}[(\boldsymbol{Z}^{i\prime}\boldsymbol{Z}^i)^{-1}\boldsymbol{Z}^{i\prime}\boldsymbol{r}_{-0}^{i*}, (\boldsymbol{Z}^{i\prime}\boldsymbol{Z}^i)^{-1}] \times I_{|\phi^i|<1},$$

which is truncated by $|\phi^i| < 1$ in one dimension, and where $\boldsymbol{r}_{-0}^{i*} := (r_1^{i*}, r_2^{i*}, ..., r_T^{i*})'$.

3.3 Update for the cutpoint parameters

The full conditional density $f(c_m^i|rem) \propto \prod_{t=1}^T f(\Delta y_t|\boldsymbol{r}_t^*, \boldsymbol{c})\pi(\boldsymbol{c})$ for each cutpoint parameter in each latent decision i is uniform over an interval (l_m^i, u_m^i), i.e. $c_m^i|rem \sim \mathcal{U}(l_m^i, u_m^i)$, where the lower l_m^i and upper u_m^i bounds are determined by:

$$\begin{array}{lll}
c_1^0: & l_1^0 = \max\{r_t^{0*}|\Delta y_t = -1, -2\}, & u_1^0 = \min\left\{c_2^0, r_t^{0*}|\Delta y_t = 0, r_t^{-*} < c_{-1}^-\right\}, \\
c_2^0: & l_2^0 = \max\left\{c_1^0, r_t^{0*}|\Delta y_t = 0, r_t^{+*} > c_1^+\right\}, & u_2^0 = \min\{r_t^{0*}|\Delta y_t = 1, 2\}, \\
c_{-2}^-: & l_{-2}^- = \max\left\{r_t^{-*}|\Delta y_t = -2\right\}, & u_{-2}^- = \min\left\{c_{-1}^-, r_t^{-*}|\Delta y_t = -1\right\}, \\
c_{-1}^-: & l_{-1}^- = \max\left\{c_{-2}^-, r_t^{-*}|\Delta y_t = -1\right\}, & u_{-1}^- = \min\left\{r_t^{-*}|\Delta y_t = 0, r_t^{0*} < c_1^0\right\}, \\
c_1^+: & l_1^+ = \max\left\{r_t^{+*}|\Delta y_t = 0, r_t^{0*} > c_2^0\right\}, & u_1^+ = \min\left\{c_2^+, r_t^{+*}|\Delta y_t = 1\right\}, \\
c_2^+: & l_2^+ = \max\left\{c_1^+, r_t^{+*}|\Delta y_t = 1\right\}, & u_2^+ = \min\left\{r_t^{+*}|\Delta y_t = 2\right\}.
\end{array}$$

These restrictions do not allow the cutpoint parameters to move substantially within each Gibbs step and slow down the convergence of the sampler.

3.4 Grouped move step

The problem with the slowly moving cutpoints was solved for the AOP model by using grouped move steps (Müller and Czado 2005). As it turns out, it is also possible to develop a suitable grouped move step for the CronAOP model, employing a theorem from Liu and Sabatti (2000) that states:

If Γ is a locally compact group of transformations defined on the sample space S, L its left Haar measure with density l, $\boldsymbol{\omega} \in S$ follows a distribution with density f, and $\gamma \in \Gamma$ is drawn from the density $f_{\boldsymbol{\omega}}(\gamma) = f(\gamma(\boldsymbol{\omega}))|J_\gamma(\boldsymbol{\omega})|l(\gamma)$, where $|J|$ is the Jacobian determinant of this transformation. Then $\omega^* := \gamma(\boldsymbol{\omega})$ has the density f too.[2]

To apply the theorem to our case (where the invariant density in the theorem f is the posterior) we write all latent variables and parameters into one vector $\boldsymbol{\omega} = (\boldsymbol{r}^{0*\prime}, \boldsymbol{r}^{-*\prime}, \boldsymbol{r}^{+*\prime}, \boldsymbol{\theta})$ and exploit the fact that the joint density is a product of normal densities $f(r_t^*|r_{t-1}^*, \boldsymbol{\beta}, \boldsymbol{\phi}) \propto \prod_{i=0,+,-} \exp\left[-\frac{1}{2}\left(\varepsilon_t^i\right)^2\right] \times I$, where I is the product of the indicator functions in (4), which is $I = 1$ if $\boldsymbol{R}^*$, $\boldsymbol{c}$ and $\boldsymbol{\Delta y}$ match each other and $I = 0$ otherwise, and the error terms are $\varepsilon_t^i = r_t^{i*} - \boldsymbol{\beta}^i \boldsymbol{x}_t^i - \phi^i r_{t-1}^{i*}$. If we rescale the latent variables and the cutpoint parameters with the same value $g > 0$, the value of the indicator function I is unaffected (e.g., if $r_t^{0*} < c_1^0 \Leftrightarrow g r_t^{0*} < g c_1^0$). To get a simple distribution for the sampling of g we must also rescale the slope parameters to $g\boldsymbol{\beta}$. Therefore, we use the transformation

$$\gamma_g(\boldsymbol{\omega}) = (g\boldsymbol{r}^{0*\prime}, g\boldsymbol{r}^{-*\prime}, g\boldsymbol{r}^{+*\prime}, g\boldsymbol{\beta}, g\boldsymbol{c}, \boldsymbol{\phi}),$$

in which we rescale everything except for the AR parameters $\boldsymbol{\phi}$. The group Γ is, hence, the group of positive real numbers with the multiplication $\{\mathbb{R}_{>0}, \cdot\}$.

The left Haar measure of this group has the density $l(\gamma_g) = 1/g$ and Jacobian determinant $|J_\gamma(\boldsymbol{\omega})| = |\partial\gamma(\boldsymbol{\omega})/\partial\omega| = g^p$, where $p = 3T + k^0 + k^- + k^+ + C$ is the number of scaled parameters ($C = 6$ is the number of cutpoint parameters). Then the density for the scaling parameter $g > 0$ is

$$f_g(g) = g^{p-1}\pi(\gamma_g(\boldsymbol{\omega})) \propto g^{p-1} \exp\left[-\frac{1}{2}g^2 \sum_i \sum_t \left(\varepsilon_t^i\right)^2\right].$$

We abbreviate $q := \frac{1}{2}\sum_i \sum_t \left(\varepsilon_t^i\right)^2$ and compute the density of g^2 as

$$f_{g^2}(g^2) = f_g(g)\frac{\partial g}{\partial g^2} \propto (g^2)^{\frac{p}{2}-1} \exp\left[-qg^2\right],$$

which is the density of a Gamma distribution $\Gamma(a, q)$ for g^2 with a shape parameter $a = \frac{p}{2}$ and a rate parameter q. The accelerating grouped move step consists of drawing a number g^2 from this distribution and multiplying the corresponding parameters with g. In fact, because the joint distribution factorizes into three parts from the three decisions $i \in \{0, -, +\}$, it is even possible to perform three independent grouped moves for the parameters of each decision separately. This will lead to an even higher variation and a faster convergence of the Gibbs sampler.

The efficiency of the grouped move can be explained as follows. Since the expectation and variance of the Gamma distribution are $\mathbb{E}_{\Gamma(a,q)}[g^2] = \frac{a}{q}$ and $\mathrm{Var}_{\Gamma(a,q)}[g^2] = \frac{a}{q^2}$, both Gamma distribution parameters are huge numbers in our case. The summands

[2] A measure L on a group Γ is called a left Haar measure, if for all elements γ and all measureable subsets A of this group the measure is invariant under application of γ, i.e. $L(A) = L(\gamma A)$.

of q are the squares of the error terms ϵ_t^i. Assuming both our model and the current Gibbs parameters are true, these increments are standard normally distributed and their square is χ^2 distributed with expectation value 1. Thus, close to the true parameters, we have $a \approx 3T/2 \approx \mathbb{E}[q]$. The expectation value for g is then $E_{\Gamma(a,q)}[g^2] \approx 1$ and the variance $\text{Var}_{\Gamma(a,q)}[g^2] = \frac{a}{q^2} \approx \frac{2}{3T}$ is small. This means that almost no rescaling happens (all sampled values for g will be close to 1) if the algorithm has almost converged. Far from convergence, e.g. $\boldsymbol{\omega} \approx (\lambda \boldsymbol{r}^{0*\prime}, \lambda \boldsymbol{r}^{-*\prime}, \lambda \boldsymbol{r}^{+*\prime}, \lambda \boldsymbol{\beta}, \lambda \boldsymbol{c}, \boldsymbol{\phi}) = \gamma_\lambda(\boldsymbol{\omega}_{\text{likely}})$, we have $\mathbb{E}[q] \approx 3\lambda^2 T/2 \Rightarrow E_{\Gamma(a,q)}[g^2] \approx \frac{1}{\lambda^2}$. Thus, the variance is still small and g close to $\mathbb{E}[g] \approx 1/\lambda$ will be sampled, so the next (scaled) parameters will be closer to the true parameters $\gamma(\boldsymbol{\omega}) = g\boldsymbol{\omega} \approx \boldsymbol{\omega}_{\text{likely}}$.

4 Finite sample performance

4.1 Monte Carlo design

The goal of the experiments is to assess the finite sample bias and uncertainty of the estimates of the parameters, choice probabilities and MEs of covariates on choice probabilities (and their asymptotic standard errors) in the proposed CronAOP estimator, and to compare the performance of the AOP and CronAOP estimators under each of the true DGPs.

The dependent variable is generated with five ordered choices. The values of parameters are calibrated to yield on average the following frequencies of the observed choices: 7%, 14%, 58%, 14% and 7%. The number of replications is 10,000 in each experiment. Three vectors of covariates $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ are drawn at each replication as $v_{1,t} \sim 0.41 v_{1,t-1} + \mathcal{N}(0, 0.3^2)$, $v_{2,t} \sim 0.17 v_{2,t-1} + \mathcal{N}(0, 0.4^2)$ and $v_{3,t} \sim 0.01 + 0.87 v_{3,t-1} + \mathcal{N}(0, 0.8^2)$. Since the dependent variable represents changes to policy interest rates, the simulated artificial covariates $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ mimic macroeconomic variables that are of interest to central banks such as changes to the inflation rate (personal consumption expenditures: chain-type price index, percent change from year ago), changes to the output gap, and the interest rate spread (one-year treasury constant maturity minus federal funds rate), respectively. The vectors of error terms in the latent equations (1) and (2) are repeatedly generated as IID standard normal random variables.

Two competing models are simulated: the AOP and the CronAOP. For each true DGP, the repeated samples with 200, 500, and 1,000 observations are generated as follows: (i) for the AOP DGP — with the covariates $\boldsymbol{v}_1, \boldsymbol{v}_2$ and $\boldsymbol{v}_3$, the vector of slope parameters $(0.6, 0.8, 1)$, the vector of cut-point parameters $(-4.61, -2.45, 2.77, 4.95)$, and an AR coefficient 0.5; (ii) and for the CronAOP DGP — with $\boldsymbol{X} = (\boldsymbol{v}_1, \boldsymbol{v}_2)'$, $\boldsymbol{X}^- = \boldsymbol{X}^+ = \boldsymbol{v}_3$, $\boldsymbol{\beta} = (0.6, 0.8)$, $\boldsymbol{\beta}^- = 1.5$, $\boldsymbol{\beta}^+ = 2.4$, $\boldsymbol{c}^0 = (-0.31, 0.31)$, $\boldsymbol{c}^- = (-2.8, 0.31)$, $\boldsymbol{c}^+ = (-0.02, 4.79)$, $\phi^0 = 0.5$, $\phi^- = 0.2$, and $\phi^+ = 0.2$.

Under each true DGP, the competing models are estimated using the same set of covariates: (i) under the AOP DGP — the AOP model is estimated with the covariates $\boldsymbol{v}_1, \boldsymbol{v}_2$ and $\boldsymbol{v}_3$, and CronAOP model is estimated with $\boldsymbol{X} = \boldsymbol{X}^- = \boldsymbol{X}^+ = (\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3)'$; (ii) and under the CronAOP DGP — the AOP model is estimated with $\boldsymbol{v}_1, \boldsymbol{v}_2$ and $\boldsymbol{v}_3$, and the CronAOP model is estimated with $\boldsymbol{X} = (\boldsymbol{v}_1, \boldsymbol{v}_2)'$ and $\boldsymbol{X}^- = \boldsymbol{X}^+ = \boldsymbol{v}_3$. The simulations in (ii) are expected to provide some evidence that the AOP estimator deliver asymptotically biased estimates under the CronAOP DGP, whereas the simulations in (i) are expected to show that the CronAOP estimator provides asymptotically unbiased estimates under its own DGP and also even if the true DGP is the AOP model.

For each repeated estimation, we initialize the parameters at the starting point described in Section 3. We run the Gibbs sampler for a burn-in period with 80,000 iterations. We found that the Gibbs sampler converges much faster in most runs, but sometimes takes this long. Subsequently we produce 30,000 more iterations to approximate the posterior distribution. From this estimation period we compute the posterior means and all other estimates.

4.2 Monte Carlo results

First, we study the finite sample performance of the CronAOP estimator, when the data are generated by its own DGP with 200, 500 and 1,000 observations. As Table 1 shows, the accuracy of the estimates improves with the number of observations. The small true values of ϕ^- and ϕ^+ imply a small positive AR effect, which, however, is detected even for only 200 observations. The root mean square errors (RMSE) of the parameters shrinks roughly with the root of the sample size. The biases do so as well, with an exception for a few cutpoint parameters. The results support the asymptotic consistency of the employed Gibbs sampler.

Table 1. Monte Carlo results: the accuracy of parameter estimates in the CronAOP model

Para-meter	True value	Sample size:	Bias 200	Bias 500	Bias 1000	RMSE 200	RMSE 500	RMSE 1000
c_1^0	-0.31		0.005	0.016	0.019	0.226	0.148	0.108
c_2^0	0.31		-0.022	-0.015	-0.014	0.212	0.141	0.101
c_{-2}^-	-2.80		-0.774	-0.264	-0.106	1.193	0.578	0.324
c_{-1}^-	0.31		0.274	0.090	0.029	0.647	0.344	0.214
c_1^+	-0.02		0.024	-0.032	-0.022	0.549	0.327	0.227
c_2^+	4.79		0.914	0.564	0.311	1.321	0.974	0.699
ϕ	0.50		-0.012	-0.005	-0.002	0.097	0.059	0.042
ϕ^-	0.20		-0.027	-0.010	-0.005	0.136	0.083	0.058
ϕ^+	0.20		-0.022	-0.008	-0.003	0.110	0.067	0.046
β_1^0	0.60		0.046	0.015	0.009	0.323	0.190	0.133
β_2^0	0.80		0.062	0.020	0.011	0.287	0.167	0.116
β_3^-	1.50		0.563	0.198	0.087	0.836	0.377	0.208
β_3^+	2.40		0.624	0.342	0.186	0.897	0.565	0.388

Next, we compare the performance of the AOP and CronAOP models under each DGP. The parameter estimates are not compatible due to the different structure of the two models. To compare them we also estimate the choice probabilities, the MEs of each covariate on each choice probability (the matrix of the MEs has $3 \times 5 = 15$ elements; their values, which depend on the values of the covariates, are computed at the population medians of the covariates and lagged latent dependent variables), and the standard errors of ME estimates.

Table 2 shows the measures of accuracy of the ME estimates in both models under two alternative DGPs. If the simulated and estimated models are identical, the empirical coverage probabilities are quite close to 95% nominal level even for 200 observations and are getting even closer as sample size increases. The bias and RMSE are larger for the CronAOP model since it has more parameters than the AOP model. The biases and RMSE of both estimators seem to converge to zero. The biases of the standard deviation estimates also shrink with growing number of observations, which means the standard deviation of all posterior means is close to the average posterior standard deviation, fostering the reliability of the posterior.

Table 2. Monte Carlo results: the accuracy of ME estimates in the AOP and CronAOP models under each DGP

Sample	Simulated model:	AOP		CronAOP	
size	Estimated model:	AOP	CronAOP	AOP	CronAOP
200	Bias (the absolute difference between the estimated and true values of MEs)	0.03	0.12	5.41	0.47
500		0.01	0.06	6.07	0.22
1000		0.00	0.04	6.37	0.17
200	RMSE (the root mean square error of ME estimates relative to their true values)	3.19	4.09	6.74	7.76
500		1.59	2.18	6.71	5.31
1000		0.92	1.39	6.73	3.79
200	Coverage probability (the percentage of times the estimated 95% credibility intervals cover the true values of MEs)	90.1	68.6	34.4	92.4
500		93.0	73.0	18.1	93.5
1000		93.9	75.7	10.5	94.1
200	Standard deviation bias (the difference between the average of standard deviation estimates and the standard deviation of ME estimates)	2.02	1.79	0.61	2.28
500		1.04	1.01	0.44	1.88
1000		0.59	0.62	0.31	1.41

Notes. All reported measures of accuracy are averaged across all five choices and all three covariates. Bias, RMSE and standard deviation bias are multiplied by 100.

The CronAOP estimator behaves much better under the AOP true DGP than the AOP estimator under the data generated using the CronAOP model (although the AOP model is not nested in the CronAOP model). When the CronAOP model is fitted to data from the AOP model, as sample size grows, the biases and RMSE decrease sharply, the coverage probability slowly approaches the nominal level and reaches 75% with 1,000 observations. However, when the AOP model is fitted to data from the CronAOP model, as sample size grows, the bias increases, the RMSE remains almost the same, and the coverage probability deteriorates further from an already very bad value of 34% with 200 observations toward about 10% with 1,000 observations.

5 Empirical application

We apply the CronAOP model to explain the FOMC decisions on the target during the 7/1987 – 1/2006 period under Greenspan's chairmanship, and to predict the target decisions out-of-sample for the 3/2006 – 12/2011 period under Bernanke's chairmanship. We contrast the in- and out-of-sample performance of the CronAOP model with that of the AOP model and linear model estimated by ordinary least squares (OLS). We also address the problem of endogeneity in the estimation of the policy rule.

5.1 Data and model specification

The target, a principal measure of U.S. monetary policy during the entire sample (*de facto* since the fall of 1982; see Thornton 2005), is set by the FOMC either at eight prescheduled meetings per year or sometimes at the extra (unscheduled) meetings. To model the target, instead of using the quarterly or monthly averages of the federal funds rate and economic variables as is common practice in the literature, we employ the data at the frequency of FOMC decisions. We use the historical dates of all FOMC decisions (made either at scheduled or unscheduled meetings, or occasionally at the discretion of the chairman during intermeeting periods) as sample observations with the following advantages.

First, we avoid the problem of reverse causation typical for time-aggregated data. Second, we avoid noise from periods with no movements in the target, when the observed policy inactions may not reflect the actual Fed response to economic developments, but rather the Fed reluctance to change the target between scheduled meetings, especially in the weeks prior to them. Third, information about the exact timing of FOMC meetings together with the use of daily financial data allows us to substantially improve the identification of the Fed policy rule. We match the FOMC decisions with the latest real-time vintages of monthly macroeconomic and non-aggregated daily financial data truly available before each meeting and not revised later on.

The dates of FOMC decisions and the original (unconsolidated) target changes are reported in Table A1 in the Online Appendix, and are based on information available at ALFRED.[3] We classify the target decisions into five categories of the dependent variable Δy_t: 'large cut' – a decrease more than 25 basis points (bp), 'small cut' – a decrease 25 bp or less but more than 6.25 bp, 'no change' – either no change or a change no more than 6.25 bp, 'small hike' – an increase 25 bp or less but more than 6.25 bp, and 'large hike' – an increase more than 25 bp. The sample consists of 190 observations with 14, 32, 102, 32 and 10 observations in the above categories, respectively.

The relationship between economic developments and the Fed response to them is often modeled using a simple policy rule such as the Taylor rule (Taylor 1993), which establishes a simple linear relation between the policy interest rate, inflation and the output gap. Recent studies, using discrete-choice models for the target, document that financial indicators, such as the spread between the long- and short-term interest rates, explain the Fed policy decisions better than the Taylor-rule variables (Hamilton and Jorda 2002; Piazzesi 2005; Kauppi 2012; Van den Hauwe et al. 2013). The FOMC always starts its meetings with a review of the 'financial outlook'. The spread can be interpreted as a market-based proxy of

[3] ALFRED (ArchivaL Federal Reserve Economic Data) is available at https://alfred.stlouisfed.org/.

future inflation and real activity (Mishkin 1990; Estrella and Hardouvelis 1991; Frankel and Lown 1994; Estrella and Mishkin 1998). In addition to the spread, we employ two forward-looking indicators of the economic situation computed by Fed staff in the Greenbook for each FOMC meeting: the projection of housing starts (one of the leading indicators of economic activity, frequently mentioned in the FOMC documents) and the projection of growth in the gross domestic product (the economic growth is one of the four goals of the Fed's monetary policy). Finally, we construct a Fed-based proxy of future inflation and real activity, derived from the FOMC documents. Together with its decision on the target, the FOMC issued (before 2000) a statement about its expectations of the future stance of monetary policy and (since 2000) a statement on the balance of risks for inflation and economic growth. These post-meeting statements have been widely seen as the indicators of the next FOMC actions, and are shown to contain predictive content for forecasting target even after controlling for macroeconomic variables (Lapp and Pearce 2000; Pakko 2005; Hayo and Neuenkirch 2010). FOMC statements and interest rate spreads summarize the vast amount of forward-looking information available both to the Fed in setting the policy rate and to Fed watchers in anticipating the Fed decisions, and can help surmounting the omitted variables and time-varying parameter problems (Cochrane and Piazzesi 2002).

Thus, our explanatory variables include: (i) $spread_t$ — the difference between the one-year treasury constant maturity rate and the effective federal funds rate, five-business-day moving average (data source: ALFRED); (ii) $houstart_t$ — the Greenbook projection for the current quarter of the total number of new privately owned housing units started (data source: RTDSM[4]); the projection for the current quarter provides a better fit than for one, two, three and four quarters ahead; (iii) Δgdp_t – the Greenbook projection for the current quarter of quarter-over-quarter growth in nominal gross domestic product (before 1992: nominal gross national product), annualized percentage points (data source: RTDSM); the projection for the current quarter provides a better fit than for one, two, three and four quarters ahead; and (iv) $tbias_{t-1}$ and (v) $ebias_{t-1}$ — the two binary indicators that we constructed from the 'policy bias' or 'balance-of-risks' statements at the previous FOMC decision announcement: $tbias_{t-1}$ is equal to one if the statement at the previous FOMC meeting was tightening, and zero otherwise; and $ebias_{t-1}$ is equal to one if the statement was easing, and zero otherwise (data source: FOMC statements and minutes[5]).

The values of dependent and explanatory variables are reported in Table A1 in the Online Appendix. Sample descriptive statistics are shown in Table A2 in the Online Appendix. The covariate first-order autocorrelation coefficients are between 0.63 and 0.98; the observed dependent variable autocorrelation coefficient is only 0.50. According to the augmented Dickey-Fuller unit root test, the null hypothesis of a unit root is rejected for all employed variables but one at the 0.0001 significance level (see Table A3 in the Online Appendix); only $houstart_t$ seems to be nonstationary in our sample with 241 observations. However, if we test the actual housing starts series $houstart\_act_t$, which is highly correlated with the Greenbook projections (the correlation coefficient is 0.98), but available for a far longer period (we used a sample with 680 monthly observations), we reject the null of a unit root at the 0.01 level.

[4] RTDSM (Real-Time Data Set for Macroeconomists) is available at https://www.philadelphiafed.org.

[5] https://www.federalreserve.gov/monetarypolicy/fomc_historical.htm.

5.2 Estimation results

The parameter estimates in the CronAOP model are reported in Table 3 (see the right three columns).

Table 3. Parameter estimates in the linear OLS, AOP and CronAOP models

Variables	Linear model (OLS)	AOP model	CronAOP model		
			Regime equation	Amount equations	
				Dovish regime	Hawkish regime
			Slope parameters		
spread $_t$	0.20 (0.02)	1.38 (0.21)	2.48 (0.58)	1.25 (0.34)	
houstart $_t$	0.17 (0.04)	1.46 (0.37)	5.08 (1.29)		
Δgdp $_t$	0.04 (0.01)	0.30 (0.07)	0.31 (0.13)	0.36 (0.11)	1.41 (0.48)
tbias $_{t-1}$	0.13 (0.03)	0.65 (0.24)	0.95 (0.41)		
ebias $_{t-1}$	-0.07 (0.03)	-0.53 (0.24)	-7.50 (3.41)		
			Autoregressive parameters		
	-0.12 (0.06)	0.04 (0.08)	0.30 (0.09)	-0.26 (0.16)	-0.89 (0.08)
	Intercept parameter		Cutpoint parameters		
	-0.47 (0.08)	1.27 (0.66)	12.44 (3.42)	-0.35 (0.31)	0.98 (1.17)
		2.56 (0.66)	15.52 (3.56)	1.02 (0.33)	6.61 (1.94)
		5.39 (0.77)			
		6.71 (0.81)			

Notes. Sample period: 7/1987–1/2006 (190 observations). Standard deviations of parameters are in parentheses. Each FOMC decision in the sample is matched by the real-time values of covariates (described in Section 5.1) as they were known at the end of the previous day.

The coefficients on the covariates have the expected signs and all but one are statistically different from zero at the 0.05 significance level (the coefficient on r_{t-1}^{-*} is significant at the 0.1 level) according to the empirical confidence intervals from the Gibbs sampling and assuming the asymptotic normality of the posterior distribution. The latent continuous dependent variable r_t^{0*}, representing the degree of the Fed policy stance and determining the regime decision, is driven by $spread_t$, $houstart_t$, Δgdp_t (the larger the covariate values, the larger the probability of a hawkish regime, and the smaller the probability of a dovish regime), $tbias_{t-1}$ (if the 'policy bias' was tightening at the previous meeting, the probability of a hawkish regime at the next meeting is larger), $ebias_{t-1}$ (if the 'policy bias' was easing, the probability of a hawkish regime is smaller), and its lagged value r_{t-1}^{0*} (the higher the degree of the policy stance at the previous meeting, the larger the probability of a hawkish regime at the next meting). The continuous latent dependent variables r_t^{-*} and r_t^{+*}, representing the desired amount of the target change in the dovish and hawkish regimes, respectively, are driven by $spread_t$ (only in the dovish regime), Δgdp_t (the larger the covariate values, the larger the probability of a higher target level), and their lagged values r_{t-1}^{-*} and r_{t-1}^{+*}, respectively (the larger their values, the smaller the probability of a higher target level). The coefficient on $spread_t$ is not significant in the hawkish regime. The coefficients on $houstart_t$, $tbias_{t-1}$ and $ebias_{t-1}$ are not significant in both amount equations.

The responses to the easing and tightening 'policy biases' are asymmetrical: the Fed seems to be much more eager to cut the target rate under the easing policy directive rather than to hike it under the tightening directive. The most striking finding is the opposite sign of the coefficients on the lagged dependent variables: the sign is positive in the regime equation (the value of the coefficient is significant at the 0.001 level), but negative in the amount equations (the value of the coefficient is significant at the 0.001 level in the hawkish regime and at the 0.1 level in the dovish regime). It implies that the regime and amount decisions have different dynamics. The positive autocorrelation in the regime equation leads to the persistency of regime decisions, whereas the negative autocorrelations in the amount equations mean that the larger the desired change (a cut or a hike) at the previous meeting, the more likely a status quo decision at the next meeting. The Fed seems to deliberately smooth the path of its target — it prefers to wait and see and to avoid making consecutive changes. Such inference is impossible if we estimate a single-equation AOP or a linear model with the same set of covariates as in the CronAOP model (see the left two columns in Table 3). The AR coefficients on the lagged dependent variable in the linear and the AOP models are small and not significant (at the 0.05 level in the linear model, and at the 0.63 level in the AOP model).

5.3 Comparison of competing models

Three competing models, estimated with the same set of covariates, are contrasted in Table 4 (see the left three columns). We compare the in-sample fit for the Greenspan era and the out-of-sample one-step-ahead forecasting performance with recursive re-estimation with an increasing window for the next 51 observations during Bernanke's term[6], using hit rates (the percentage of correct predictions), mean absolute errors (MAE) and two strictly proper scoring rules: the probability, or Brier, score (Brier 1950) and ranked probability score (Epstein 1969). Both scores measure the accuracy of the probabilistic forecast (contrary to the hit rates that do not distinguish between cases in which the estimated probability of a particular choice is, for example, 0.51 or 0.99). Both scores have a minimum value of zero when all the observed choices are forecasted with a unit probability. In contrast to the Brier score, the ranked probability score punishes forecasts more severely for non-zero predicted probabilities of choices that are further from the observed choice.

To estimate the probabilities of discrete choices in the linear OLS model we assume that the errors are distributed according to normal CDF Φ with mean $\widehat{\boldsymbol{\beta}}_{ols}\boldsymbol{X}$ and variance $(\Delta\boldsymbol{y} - \widehat{\boldsymbol{\beta}}_{ols}\boldsymbol{X})'(\Delta\boldsymbol{y} - \widehat{\boldsymbol{\beta}}_{ols}\boldsymbol{X})/(T-k-1)$, where $\boldsymbol{X}$ is a data matrix and $\widehat{\boldsymbol{\beta}}_{ols}$ is an OLS estimate of k slope parameters. We use the midpoints between the nearest unconsolidated target changes on the border between two adjacent consolidated categories as thresholds; for example, 0.09375 is a midpoint between the 0.0625 hike (which is classified as 'no change') and the nearest 0.125 hike (which is classified as a 'small hike'):

[6] The Greenbook projections are currently not available after 2011.

$$
\begin{aligned}
\Pr_{ols}(\Delta y_t = \text{'large cut'}) &= \Phi(-0.28125 - \widehat{\boldsymbol{\beta}}_{ols}\mathbf{x}_t), \\
\Pr_{ols}(\Delta y_t = \text{'small cut'}) &= \Phi(-0.09375 - \widehat{\boldsymbol{\beta}}_{ols}\mathbf{x}_t) - \Phi(-0.28125 - \widehat{\boldsymbol{\beta}}_{ols}\mathbf{x}_t), \\
\Pr_{ols}(\Delta y_t = \text{'no change'}) &= \Phi(0.09375 - \widehat{\boldsymbol{\beta}}_{ols}\mathbf{x}_t) - \Phi(-0.09375 - \widehat{\boldsymbol{\beta}}_{ols}\mathbf{x}_t), \\
\Pr_{ols}(\Delta y_t = \text{'small hike'}) &= \Phi(0.28125 - \widehat{\boldsymbol{\beta}}_{ols}\mathbf{x}_t) - \Phi(0.09375 - \widehat{\boldsymbol{\beta}}_{ols}\mathbf{x}_t), \\
\Pr_{ols}(\Delta y_t = \text{'large hike'}) &= 1 - \Phi(0.28125 - \widehat{\boldsymbol{\beta}}_{ols}\mathbf{x}_t).
\end{aligned}
$$

Table 4. Performance of competing models: the FOMC decisions on federal funds rate target favor the CronAOP model

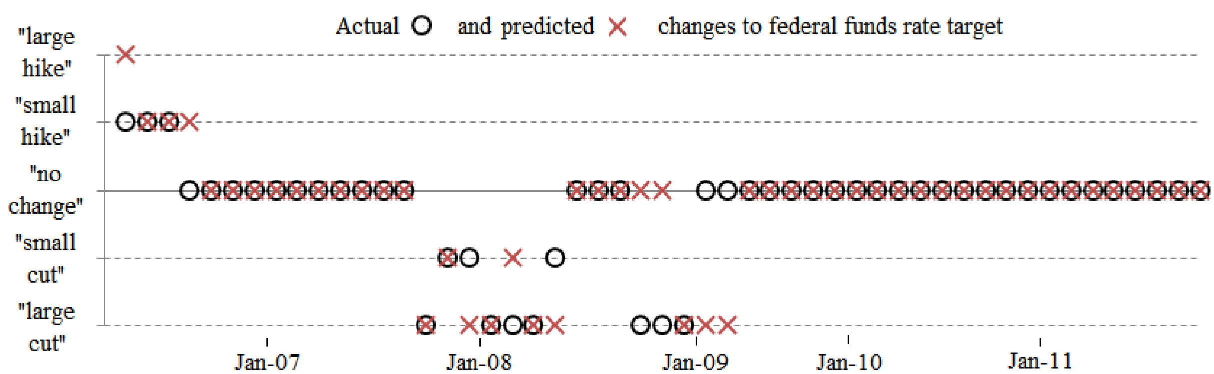
Model:	Linear OLS	AOP	CronAOP	With ex-post controls for endogeneity: Linear OLS	With ex-post controls for endogeneity: CronAOP
In-sample fit (in 7/1987 - 1/2006 period, 190 observations)					
Hit rate (among five choices)	0.65	0.63	**0.73**	0.70	**0.82**
Hit rate (cut, no change or hike)	0.75	0.73	**0.78**	0.79	**0.89**
Mean absolute error, bp	10.6	10.3	**7.2**	9.0	**4.7**
Brier probability score	0.55	0.49	**0.37**	0.46	**0.28**
Ranked probability score	0.30	0.29	**0.21**	0.23	**0.15**
Out-of-sample forecast (for 3/2006 - 12/2011 period, 51 observations)					
Hit rate (among five choices)	0.39	0.78	**0.82**	0.65	**0.90**
Hit rate (cut, no change or hike)	0.49	0.90	**0.90**	0.76	**0.94**
Mean absolute error, bp	15.6	9.3	**7.8**	12.0	**4.9**
Brier probability score	1.19	0.39	**0.35**	0.64	**0.21**
Ranked probability score	0.68	0.28	**0.25**	0.37	**0.14**

Notes. Specifications in the first three columns are those reported in Table 3. Each FOMC decision is matched by the real-time covariates as they were known at the end of the previous day. The out-of-sample forecast of the next FOMC decision is performed using a recursive re-estimation with an increasing window. Specifications in the two last columns are those reported in Table 5 with the ex-post controls for endogeneity using the difference in the federal funds futures rate at the end of the day of Fed action and at the end of the previous day (see Section 5.4).

Hit rate is the percentage of correct predictions. Predicted choice is that with the highest predicted probability. Probabilities for the linear OLS model are constructed assuming normally distributed error terms as explained in Section 5.2. Probabilities for the AOP and CronAOP models are evaluated numerically as $\Pr(\Delta y_t = j|\boldsymbol{\Omega}) = \int \Pr(\Delta y_t = j|\boldsymbol{\Omega}, \boldsymbol{R}^*, \boldsymbol{\theta}) d\Pr(\boldsymbol{R}^*, \boldsymbol{\theta}|\boldsymbol{\Omega})$, where $\boldsymbol{\Omega}$ is the available data. Brier probability score is computed as $\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{5}[\Pr(\Delta y_t = j) - d_{jt}]^2$, where indicator $d_{jt} = 1$ if $\Delta y_t = j$ and $d_{jt} = 0$ otherwise. Ranked probability score is computed as $\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{5}[P_{jt} - D_{jt}]^2$, where $P_{jt} = \sum_{i=1}^{j}\Pr(\Delta y_t = i)$ and $D_{jt} = \sum_{i=1}^{j} d_{it}$. The better the prediction, the smaller both score values. Mean absolute error is the average absolute difference between the observed and predicted discrete choice (for the AOP and CronAOP models), or between the observed outcome and continuous value of the dependent variable predicted using the OLS estimation (for the linear model).

The CronAOP model overwhelmingly outperforms the competitors both in and out of sample according to all the employed criteria. The MAE in the CronAOP model is lower by more than 40% in sample and by more than 30% out of sample, and both probability scores are lower by more than 30% in sample and by more than 10% out of sample compared to that in the AOP model. The performance of the linear OLS model in sample is similar but slightly worse than that of the AOP model. The out-of-sample forecast of the linear model is remarkably inferior to that of the discrete-choice competitors: the MAE is bigger by 67% and 99%, both probabilities scores are bigger by 148% and 177%, and the hit rate is half, or less, of those in the AOP and CronAOP models, respectively. In terms of the direction of the target change (hike, no change, cut), the CronAOP model out of sample misses only one hike and incorrectly predicts the timing (with three-meeting lags) of two cuts, correctly predicting 22 out of 24 status quo decisions after reaching the zero lower bound (see Figure 1). By contrast, the linear OLS model after reaching the zero lower bound correctly predicts only two no-change decisions, wrongly predicting 22 cuts.

Figure 1. Out-of-sample forecast of the CronAOP model: it correctly predicts 82 percent of FOMC decisions while the linear OLS model correctly predicts only 39 percent



Notes. Forecasting period: 3/2006–12/2011 (51 observations). The estimates are obtained from the CronAOP model (see Table 3). Forecasts of the next FOMC decisions are performed using recursive re-estimation with an increasing window and the values of covariates as they were known at the end of the preceding day.

The hit rates of the CronAOP model (in terms of three choices: hike, no change, cut), which are 0.78 in sample and 0.90 out of sample, exceed the hit rates of the existing single-equation dynamic discrete-choice models for the FOMC decisions during Greenspan's tenure such as Hu and Phillips (2004), Piazzesi (2005) and Kim et al. (2009). They model target changes in the 2/1994–12/2001, 2/1994–12/1998 and 2/1994–12/2006 periods, and respectively, with 64, 40 and 96 FOMC decisions made at the scheduled meetings only. The scheduled meetings are easier to predict, and their samples are much smaller. They do not report the MAE and the probability scores.

5.4 Controlling for endogeneity

Although we predict the next FOMC decision using the predetermined values of the explanatory variables observed before each FOMC meeting and thus avoid the reverse causal effects from the error terms to the covariates, we should be concerned with the possible correlation between the error terms and the covariates (see de Vries and Li (2014) for a discussion of the endogeneity problem and the bias in the conventional estimates of central bank reaction functions). The covariates $houstart_t$, Δgdp_t, $tbias_{t-1}$ and $ebias_{t-1}$, which are forward-looking by construction, may contain internal FOMC anticipations of a monetary policy shock at the next policy meeting. Besides, the covariate $spread_t$ may contain financial market anticipations of the next FOMC decision. Thus, all our explanatory variables may be endogenous to the monetary shocks.

Table 5. Parameter estimates in the linear OLS and CronAOP models with bias correction terms: endogeneity does matter

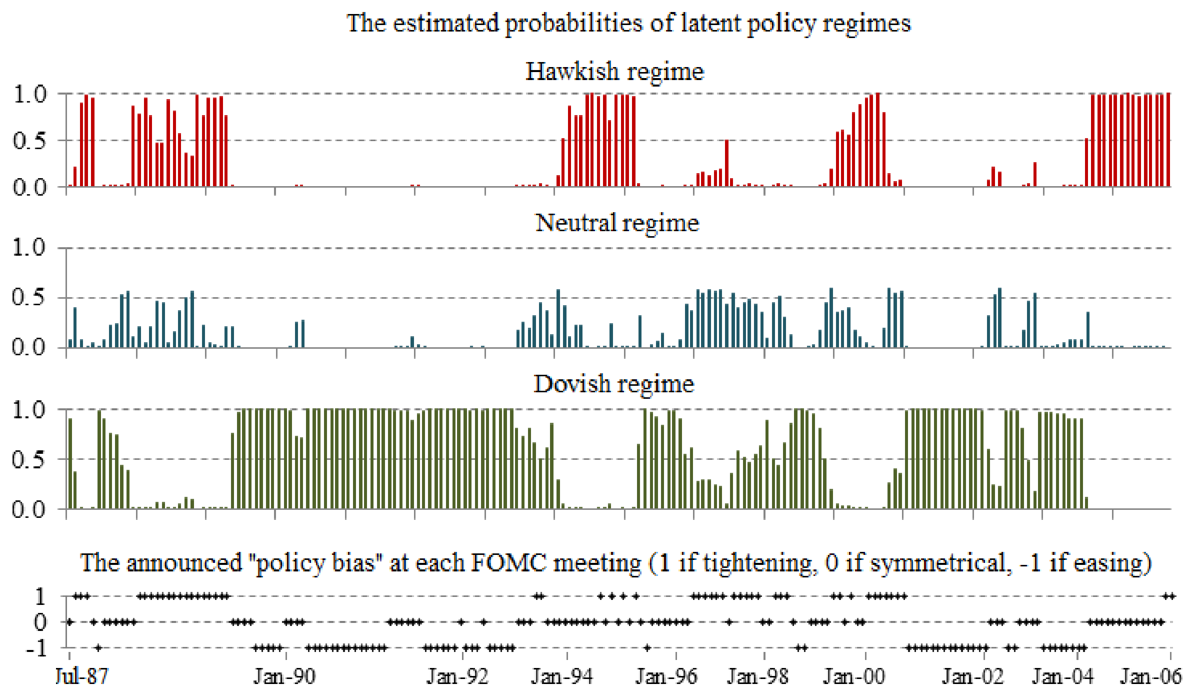
Variables	Linear model (OLS)	CronAOP model		
		Regime equation	Amount equations	
			Dovish regime	Hawkish regime
		Slope parameters		
$spread_t$	0.17 (0.02)	4.10 (1.53)	1.50 (0.34)	
$houstart_t$	0.10 (0.04)	7.34 (2.66)		
Δgdp_t	0.03 (0.01)	0.60 (0.30)	0.28 (0.10)	1.11 (0.48)
$tbias_{t-1}$	0.10 (0.02)	1.75 (0.73)		
$ebias_{t-1}$	-0.02 (0.02)	-8.35 (3.34)		
$surprise_t$	0.10 (0.01)	1.78 (0.86)	1.06 (0.17)	2.69 (0.72)
		Autoregressive parameters		
	0.01 (0.05)	0.32 (0.11)	0.02 (0.10)	-0.67 (0.13)
	Intercept parameter	Cutpoint parameters		
	-0.30 (0.07)	22.63 (10.65)	-1.79 (0.50)	1.95 (1.35)
		25.26 (10.21)	0.31 (0.47)	6.50 (2.21)

Notes. Sample period: 7/1987–1/2006 (190 observations). Standard deviations of parameters are in parentheses. Each FOMC decision in the sample is matched by the real-time values of covariates (described in Section 5.1) as they were known at the end of the previous day. The endogeneity is controlled by the bias correction terms as explained in Section 5.4.

To control for endogeneity we introduce bias correction terms as additional regressors into all three latent equations. The bias correction terms are, hopefully, those parts of the endogenous covariates that are correlated with any unknown shocks. Cochrane and Piazzesi (2002) conclude that monetary shocks computed from daily financial data are almost the perfect measures of unanticipated FOMC decisions on the target. Following Kuttner (2001) and Bernanke and Kuttner (2005), we compute the market-based proxies for unknown monetary shocks as one-day surprises unanticipated by the federal funds futures using the difference in the current-month futures rate at the end of the day of Fed action and at the end of the previous day in order to use them as controls for endogeneity. Krueger and Kuttner (1996) and Gürkaynak et al. (2007) report that the federal funds futures outperform the other securities in predicting Fed monetary policy. The 30-day federal funds futures are traded on the Chicago Mercantile Exchange. The raw data are obtained from Quandl.[7] The calculated monetary policy surprises are shown in Table A1 in the Online Appendix.

We first test for the exogeneity of the constructed policy surprises by running an OLS regression of them on all covariates. We reject the null hypothesis of exogeneity (F-statistic is 7.12, the p-value of F-test is 0.0000). Table 5 reports the CronAOP estimates with policy surprises introduced as controls for endogeneity into each latent equation. The coefficient on $surprise_t$ is significant at the 0.05 level in the regime decision, and at the 0.001 level in both amount decisions. Conditional on the correct model specification, we again reject the null of exogeneity. Endogeneity does matter.

Figure 2. Estimated probabilities of latent regimes and announced policy bias at each FOMC meeting during the Greenspan tenure



Notes. Sample period: 7/1987–1/2006 (190 observations). The estimates are obtained from the CronAOP model with controls for endogeneity (see Table 5).

The policy surprises, of course, may not be used to forecast the FOMC decisions since the surprises are constructed using information available after an FOMC decision, at the end of the day when this decision was implemented. Therefore, the out-of-sample estimations with surprises, reported in the last two columns of Table 4, can be seen only as an ex-post bias-correction exercise. The addition of policy surprises makes the model fit considerably better both in and out of sample (see the fourth column of Table 4): the MAEs decrease by

[7] https://www.quandl.com

at least 34%, the probabilities scores decrease by at least 25% in sample and at least 39% out of sample, the hit rate reaches 0.82 in sample and 0.90 out of sample (after reaching the zero lower bound the predicted probability of a dovish regime is equal to one, and the model correctly predicts 23 out of 24 status quo decisions). The CronAOP model with bias correction terms overwhelmingly outperforms the linear OLS model (see the fifth column of Table 4): for example, the out-of-sample hit rate is by 39% higher and the MAE is 59% lower than in the linear model (the linear model correctly predicts only 13 out of 24 status quo decisions after reaching the zero lower bound, wrongly predicting eleven cuts).

The estimated probabilities of the three policy regimes are shown in Figure 2 for each FOMC decision during the Greenspan era. The average probabilities of dovish, neutral and hawkish regimes are 0.58, 0.15 and 0.27, respectively, though the frequencies of observed cuts, status quo decisions and hikes are 0.24, 0.54 and 0.22, respectively. Apparently, less than one third of the predicted status quo decisions are generated by the neutral policy stance. The average estimated probability of the neutral policy regime during the observed status quo decisions is only 0.23. The decomposition of the probability of no change $\Pr(\Delta y_t = 0)$ into three components $\Pr(\Delta y_t = 0|s_t = -1)$, $\Pr(\Delta y_t = 0|s_t = 0)$ and $\Pr(\Delta y_t = 0|s_t = 1)$ conditional on the dovish, neutral and hawkish regimes is on average 0.62, 0.28 and 0.10, respectively. The amount decisions tend to smooth the target and moderate or offset the vast majority of the dovish and hawkish policy stances.

6 Concluding remarks

We develop a new dynamic discrete-choice model for monetary policy interest rates. Central banks typically adjust policy rates by discrete increments and often leave them unchanged in different economic circumstances; if central banks make a change, it is usually followed by further changes in the same direction. To address these stylized facts the new cross-nested autoregressive ordered probit allows (i) the status quo outcomes to be heterogeneous and generated in three latent regimes, which can be interpreted as monetary policy stances (dovish, neutral and hawkish); (ii) the probabilities of positive and negative changes to the rate to be driven by different processes; and (iii) the persistency of policy rates and monetary policy inertia to be captured by the lagged dependent latent variables among the regressors.

The simulations demonstrate that the proposed Bayesian estimator performs well in small samples. The application to the FOMC decisions on the federal funds rate target shows that the discrete-choice approach, regime switching and endogeneity do matter in the empirical estimation of monetary policy rules. The proposed model overwhelmingly outperforms the linear OLS model, the existing discrete-choice models and the single-equation AOP model both in sample and out of sample. The new model fitted for Greenspan's tenure correctly predicts the direction of 90% of the next decisions on the target rate out of sample including status quo decisions after reaching the zero lower bound during Bernanke's term.

The new methodology can be employed to model the policy rate decisions of many central banks, changes to rankings, tick-by-tick stock price changes, and other ordinal data.

References

Bernanke, B. S., and Kuttner, K. N. (2005), "What explains the stock market's reaction to Federal Reserve policy?" *Journal of Finance,* 60 (3), 1221–1257.

Bikbov, R., and Chernov, M. (2013), "Monetary policy regimes and the term structure of interest rates." *Journal of Econometrics*, 174 (1), 27–43.

Brier, G. W. (1950), "Verification of forecasts expressed in terms of probability." *Monthly Weather Review,* 78 (1), 1–3.

Brooks, R., Harris, M. N., and Spencer, C. (2012), "Inflated ordered outcomes." *Economics Letters,* 117 (3), 683–686.

Cochrane, J. H., and Piazzesi, M. (2002), "The Fed and interest rates: A high-frequency identification." *American Economic Review,* 92 (2), 90–95.

Davutyan, N., and Parke, W. R. (1995), "The operations of the Bank of England, 1890-1908: a dynamic probit approach." *Journal of Money, Credit, and Banking,* 27, 1099–1112.

De Vries, C. G., and Li, W. (2014), "Identifying monetary policy rules with serially correlated monetary policy shocks." Working paper, Erasmus University Rotterdam and Tinbergen Institute.

Dueker, M. J. (1999a), "Measuring monetary policy inertia in target Fed funds rate changes." Federal Reserve Bank of St. Louis *Review,* 81 (5), 3–9.

Dueker, M. J. (1999b), "Conditional heteroscedasticity in qualitative response models of time series: a Gibbs-sampling approach to the bank prime rate", *Journal of Business and Economic Statistics,* 17 (4), 466–472.

Eichengreen, B., Watson, M., and Grossman, R. (1985), "Bank rate policy under the interwar gold standard: a dynamic probit model." *Economic Journal,* 95, 725–745.

Epstein, E. S. (1969), "A scoring system for probability forecasts of ranked categories." *Journal of Applied Meteorology*, 8, 985–987.

Estrella, A., and Hardouvelis, G. A. (1991), "The term structure as a predictor of real economic activity." *Journal of Finance,* 46 (2), 555–576.

Estrella, A., and Mishkin, F. S. (1998), "Predicting U.S. recessions: Financial variables as leading indicators." *Review of Economics and Statistics,* 80 (1), 45–61.

Frankel, J. A., and Lown, C. S. (1994), "An indicator of future inflation expected from the steepness of the interest rate yield curve along its entire length." *Quarterly Journal of Economics,* 109 (2), 517–530.

Gerlach, S. (2007), "Interest rate setting by the ECB, 1999-2006: Words and deeds." *International Journal of Central Banking*, 3 (3), 1–46.

Grammig, J., and Kehrle, K. (2008), "A new marked point process model for the federal funds rate target: Methodology and forecast evaluation." *Journal of Economic Dynamics and Control*, 32 (7), 2370–2396.

Gürkaynak, R. S., Sack, B. P., and Swanson, E. T. (2007), "Market-based measures of monetary policy expectations." *Journal of Business and Economic Statistics,* 25 (2), 201–212.

Hamilton, J. D., and Jorda, O. (2002), "A model for the federal funds rate target." *Journal of Political Economy,* 110 (5), 1135–1167.

Harris, M. N., and Zhao, X. (2007), "A zero-inflated ordered probit model, with an application to modelling tobacco consumption." *Journal of Econometrics,* 141 (2), 1073–1099.

Hayo, B., and Neuenkirch, M. (2010), "Do Federal Reserve communications help predict federal funds target rate decisions?" *Journal of Macroeconomics*, 32 (4), 1014–1024.
Hu, L., and Phillips, P. C. B. (2004), "Dynamics of the federal funds target rate: a nonstationary discrete choice approach." *Journal of Applied Econometrics*, 19, 851–867.
Kauppi, H. (2012), "Predicting the direction of the Fed's target rate." *Journal of Forecasting,* 31, 47–67.
Kim, H., Jackson, J., and Saba, R. (2009), "Forecasting the FOMC's interest rate setting behavior: a further analysis." *Journal of Forecasting,* 28, 145–165.
Krueger, J. T., and Kuttner, K. N. (1996), "The Fed funds futures rate as a predictor of federal reserve policy." *Journal of Futures Markets*, 16, 865–879.
Kuttner, K. N. (2003), "Dating changes in the federal funds rate, 1989–92." Manuscript, Federal Reserve Bank of New York.
Lapp, J. S., and Pearce, D. K. (2000), "Does a bias in FOMC policy directives help predict intermeeting policy changes?" *Journal of Money, Credit and Banking,* 32 (3), 435–441.
Liu, J. S., and Sabatti, C. (2000), "Generalized Gibbs sampler and multigrid Monte Carlo for Bayesian computation." *Biometrika,* 87 (2), 353–369.
Mishkin, F. S. (1990), "The information in the longer-maturity term structure about future inflation." *Quarterly Journal of Economics,* 105 (3), 815–28.
Monokroussos, G. (2011), "Dynamic limited dependent variable modeling and U.S. monetary policy." *Journal of Money, Credit and Banking*, 43 (2-3), 519–534.
Müller, G., and Czado, C. (2005), "An autoregressive ordered probit model with application to high-frequency financial data." *Journal of Computational and Graphical Statistics,* 14 (2), 320–338.
Pakko, M. R. (2005), "On the information content of asymmetric FOMC policy statements: evidence from a Taylor-rule perspective." *Economic Inquiry,* 43 (3), 558–569.
Piazzesi, M. (2005), "Bond yields and the Federal Reserve." *Journal of Political Economy,* 113 (2), 311–344.
Rudebusch, Glenn D. (1998a), "Do measures of monetary policy in a VAR make sense?" *International Economic Review,* 39 (4), 907–931.
Rudebusch, Glenn D. (1998b), "Do measures of monetary policy in a VAR make sense? A reply to Christopher A. Sims." *International Economic Review,* 39 (4), 943–948.
Sims, C. A., and Zha, T. (2006), "Were there regime switches in U.S. monetary policy?" *American Economic Review*, 96 (1), 54–81.
Taylor, J. B. (1993), "Discretion versus policy rules in practice." *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.
Thornton, D. L. (2005), "When did the FOMC begin targeting the federal funds rate? What the verbatim transcripts tell us.", Federal Reserve Bank of St. Louis Working Paper No. 2004-015B.
Vanderhart, P. G. (2000), "The Federal Reserve's reaction function under Greenspan: An ordinal probit analysis." *Journal of Macroeconomics*, 22 (4), 631–644.
Van den Hauwe, S., Paap, R., and van Dijk, D. J. C. (2013), "Bayesian forecasting of federal funds target rate decisions." *Journal of Macroeconomics,* 37, 19–40.

Online Appendix
for
"A model for policy interest rates"

By Armin Seibert, Andrei Sirchenko and Gernot Müller

Table A1. Data

Date of FOMC decision	Change to federal funds rate target	Dependent variable Δy_t	$spread_t$	$houstart_t$	Δgdp_t	$tbias_{t-1}$	$ebias_{t-1}$	$surprise_t$
7-Jul-87	0	no change	0.164	1.61	5.9	0	0	-0.040
18-Aug-87	0	no change	0.228	1.61	6.4	0	0	-0.020
27-Aug-87	0.125	small hike	0.260	1.61	6.4	1	0	0.040
3-Sep-87	0.5	large hike	0.426	1.61	6.4	1	0	0.090
22-Sep-87	0.0625	no change	0.426	1.60	6.8	1	0	0.010
3-Nov-87	-0.5	large cut	0.008	1.55	5.0	0	0	-0.130
16-Dec-87	0	no change	0.534	1.54	5.4	0	1	-0.140
5-Jan-88	0	no change	0.130	1.54	5.4	0	0	-0.050
28-Jan-88	-0.1875	small cut	0.134	1.55	5.1	0	0	-0.060
10-Feb-88	-0.125	small cut	0.146	1.55	4.9	0	0	0.030
29-Mar-88	0.25	small hike	0.144	1.46	6.1	0	0	0.010
9-May-88	0.25	small hike	0.540	1.53	5.7	0	0	0.050
17-May-88	0	no change	0.110	1.51	7.1	0	0	-0.040
25-May-88	0.25	small hike	0.482	1.51	7.1	1	0	-0.020
22-Jun-88	0.1875	small hike	-0.108	1.51	7.1	1	0	-0.010
30-Jun-88	0.0625	no change	-0.136	1.49	8.2	1	0	-0.070
19-Jul-88	0.1875	small hike	0.028	1.47	6.3	1	0	0.000
8-Aug-88	0.4375	large hike	0.100	1.47	6.3	1	0	0.090
16-Aug-88	0	no change	0.166	1.47	7.3	1	0	-0.040
20-Sep-88	0	no change	-0.110	1.47	6.5	1	0	0.020
1-Nov-88	0	no change	-0.192	1.46	6.7	1	0	0.032
17-Nov-88	0.25	small hike	0.208	1.46	6.7	1	0	-0.078
14-Dec-88	0.3125	large hike	0.450	1.51	7.0	1	0	0.018
5-Jan-89	0.3125	large hike	-0.270	1.48	8.8	1	0	-0.024
8-Feb-89	0.125	small hike	0.058	1.50	8.9	1	0	0.000
14-Feb-89	0.1875	small hike	-0.070	1.50	8.9	1	0	0.020
24-Feb-89	0.4375	large hike	-0.158	1.50	8.9	1	0	0.070

Table A1 (contd). Data

Date of FOMC decision	Change to federal funds rate target	Dependent variable Δy_t	$spread_t$	$houstart_t$	Δgdp_t	$tbias_{t-1}$	$ebias_{t-1}$	$surprise_t$
28-Mar-89	0	no change	-0.194	1.56	9.4	1	0	-0.070
16-May-89	0.0625	no change	-0.744	1.44	7.3	1	0	-0.022
5-Jun-89	-0.25	small cut	-1.086	1.44	7.3	0	0	-0.036
6-Jul-89	-0.25	small cut	-1.446	1.40	5.1	0	0	-0.026
26-Jul-89	-0.25	small cut	-1.224	1.40	5.1	0	0	-0.062
22-Aug-89	0	no change	-0.708	1.43	5.8	0	0	0.000
3-Oct-89	0	no change	-0.720	1.43	5.3	0	1	0.034
16-Oct-89	-0.3125	large cut	-0.854	1.43	5.3	0	1	-0.207
6-Nov-89	-0.25	small cut	-0.940	1.43	5.3	0	1	-0.104
14-Nov-89	0	no change	-0.652	1.38	4.9	0	1	-0.020
19-Dec-89	-0.25	small cut	-0.828	1.39	4.5	0	1	-0.169
7-Feb-90	0	no change	-0.110	1.37	5.2	0	0	-0.014
27-Mar-90	0	no change	0.034	1.47	7.6	0	0	0.000
15-May-90	0	no change	0.084	1.30	6.8	0	0	0.000
3-Jul-90	0	no change	-0.248	1.21	5.7	0	0	0.000
13-Jul-90	-0.25	small cut	-0.118	1.21	5.7	0	1	-0.138
21-Aug-90	0	no change	-0.464	1.19	5.7	0	1	0.000
7-Sep-90	0	no change	-0.594	1.19	5.7	0	1	0.039
17-Sep-90	0	no change	-0.286	1.19	5.7	0	1	-0.023
2-Oct-90	0	no change	-0.480	1.09	2.6	0	1	0.021
29-Oct-90	-0.25	small cut	-0.472	1.09	2.6	0	1	-0.020
13-Nov-90	-0.25	small cut	-0.484	1.10	1.3	0	1	0.000
7-Dec-90	-0.25	small cut	-0.274	1.10	1.3	0	1	-0.271
18-Dec-90	-0.25	small cut	-0.206	1.02	0.9	0	1	-0.233
8-Jan-91	-0.25	small cut	0.122	1.04	4.3	0	1	-0.175
1-Feb-91	-0.5	large cut	-0.918	1.00	3.3	0	1	-0.259
6-Feb-91	0	no change	-0.244	1.00	3.3	0	1	0.000
8-Mar-91	-0.25	small cut	0.096	1.00	3.3	0	1	-0.162
26-Mar-91	0	no change	0.170	0.95	2.1	0	1	0.000
12-Apr-91	0	no change	0.452	1.04	5.7	0	0	-0.100
30-Apr-91	-0.25	small cut	0.310	1.04	5.7	0	0	-0.170
14-May-91	0	no change	0.374	1.01	3.0	0	0	0.019
3-Jul-91	0	no change	0.198	1.05	8.1	0	0	0.000
5-Aug-91	-0.25	small cut	0.384	1.05	8.1	0	0	-0.149
20-Aug-91	0	no change	0.052	1.05	4.9	0	0	0.124

Table A1 (contd). Data

Date of FOMC decision	Change to federal funds rate target	Dependent variable Δy_t	$spread_t$	$houstart_t$	Δgdp_t	$tbias_{t-1}$	$ebias_{t-1}$	$surprise_t$
13-Sep-91	-0.25	small cut	0.070	1.05	4.9	0	1	-0.053
1-Oct-91	0	no change	0.102	1.06	4.5	0	1	-0.011
30-Oct-91	-0.25	small cut	0.104	1.08	5.8	0	1	-0.060
5-Nov-91	-0.25	small cut	0.114	1.02	3.5	0	1	-0.125
6-Dec-91	-0.25	small cut	-0.190	1.02	3.5	0	1	-0.087
17-Dec-91	-0.5	large cut	-0.086	1.08	2.7	0	1	-0.238
5-Feb-92	0	no change	0.190	1.14	4.3	0	0	-0.013
31-Mar-92	0	no change	0.674	1.25	4.7	0	1	0.010
9-Apr-92	-0.25	small cut	0.418	1.27	5.6	0	1	-0.243
19-May-92	0	no change	0.120	1.21	4.9	0	1	0.000
1-Jul-92	-0.5	large cut	0.144	1.17	4.8	0	0	-0.363
18-Aug-92	0	no change	0.122	1.23	3.4	0	1	0.026
4-Sep-92	-0.25	small cut	0.106	1.23	3.4	0	1	-0.219
6-Oct-92	0	no change	-0.528	1.24	3.4	0	1	0.052
17-Nov-92	0	no change	0.648	1.23	4.5	0	1	-0.100
22-Dec-92	0	no change	0.732	1.24	6.2	0	1	0.039
3-Feb-93	0	no change	0.330	1.30	6.2	0	0	-0.012
23-Mar-93	0	no change	0.290	1.22	6.7	0	0	-0.044
18-May-93	0	no change	0.270	1.26	4.2	0	0	-0.026
7-Jul-93	0	no change	0.146	1.28	4.4	1	0	0.027
17-Aug-93	0	no change	0.428	1.28	4.8	1	0	0.000
21-Sep-93	0	no change	0.228	1.28	3.5	0	0	0.000
16-Nov-93	0	no change	0.546	1.36	6.6	0	0	0.023
21-Dec-93	0	no change	0.604	1.40	7.4	0	0	0.000
4-Feb-94	0.25	small hike	0.324	1.44	7.2	0	0	0.117
28-Feb-94	0	no change	0.736	1.44	7.2	0	0	-0.050
22-Mar-94	0.25	small hike	1.104	1.37	5.7	0	0	-0.034
18-Apr-94	0.25	small hike	1.282	1.52	5.0	0	0	0.100
17-May-94	0.5	large hike	1.678	1.41	6.2	0	0	0.133
6-Jul-94	0	no change	0.986	1.38	5.2	0	0	-0.050
16-Aug-94	0.5	large hike	1.340	1.33	4.4	1	0	0.145
27-Sep-94	0	no change	1.112	1.41	4.8	0	0	-0.200
15-Nov-94	0.75	large hike	1.444	1.41	6.1	1	0	0.140
20-Dec-94	0	no change	1.758	1.42	6.9	0	0	-0.169
1-Feb-95	0.5	large hike	1.208	1.49	6.4	1	0	0.052

Table A1 (contd). Data

Date of FOMC decision	Change to federal funds rate target	Dependent variable Δy_t	$spread_t$	$houstart_t$	Δgdp_t	$tbias_{t-1}$	$ebias_{t-1}$	$surprise_t$
28-Mar-95	0	no change	0.326	1.34	5.8	0	0	0.103
23-May-95	0	no change	0.012	1.27	3.5	1	0	0.000
6-Jul-95	-0.25	small cut	-0.610	1.33	3.9	0	0	-0.012
22-Aug-95	0	no change	0.080	1.37	4.6	0	1	0.000
26-Sep-95	0	no change	-0.122	1.40	5.1	0	0	0.000
15-Nov-95	0	no change	-0.324	1.45	4.8	0	0	0.060
19-Dec-95	-0.25	small cut	-0.394	1.38	3.9	0	0	-0.103
31-Jan-96	-0.25	small cut	-0.466	1.39	4.3	0	0	-0.070
26-Mar-96	0	no change	0.198	1.47	4.5	0	0	-0.031
21-May-96	0	no change	0.316	1.48	5.6	0	0	0.000
3-Jul-96	0	no change	0.060	1.39	4.5	0	0	-0.050
20-Aug-96	0	no change	0.448	1.43	4.5	1	0	-0.042
24-Sep-96	0	no change	0.666	1.43	4.2	1	0	-0.125
13-Nov-96	0	no change	0.128	1.45	4.0	1	0	0.000
17-Dec-96	0	no change	0.182	1.41	4.5	1	0	0.011
5-Feb-97	0	no change	0.328	1.41	4.6	1	0	-0.030
25-Mar-97	0.25	small hike	0.458	1.45	6.4	1	0	0.026
20-May-97	0	no change	0.354	1.43	3.9	0	0	-0.113
2-Jul-97	0	no change	-0.272	1.44	4.8	1	0	-0.016
19-Aug-97	0	no change	0.008	1.44	3.9	1	0	-0.013
30-Sep-97	0	no change	-0.070	1.43	4.5	1	0	0.000
12-Nov-97	0	no change	-0.126	1.43	5.4	1	0	-0.042
16-Dec-97	0	no change	-0.058	1.47	6.1	1	0	-0.010
4-Feb-98	0	no change	-0.310	1.48	4.4	0	0	0.000
31-Mar-98	0	no change	-0.150	1.56	4.5	0	0	0.000
19-May-98	0	no change	-0.150	1.60	4.2	1	0	-0.026
1-Jul-98	0	no change	-0.480	1.55	3.7	1	0	-0.005
18-Aug-98	0	no change	-0.370	1.58	3.6	1	0	0.012
29-Sep-98	-0.25	small cut	-0.896	1.63	4.2	0	0	0.060
15-Oct-98	-0.25	small cut	-0.906	1.57	3.6	0	1	-0.262
17-Nov-98	-0.25	small cut	-0.542	1.57	3.2	0	1	-0.058
22-Dec-98	0	no change	-0.362	1.69	4.2	0	0	-0.017
3-Feb-99	0	no change	-0.182	1.68	4.4	0	0	0.000
30-Mar-99	0	no change	-0.116	1.75	5.1	0	0	0.000
18-May-99	0	no change	0.018	1.66	4.8	0	0	-0.036

Table A1 (contd). Data

Date of FOMC decision	Change to federal funds rate target	Dependent variable Δy_t	$spread_t$	$houstart_t$	Δgdp_t	$tbias_{t-1}$	$ebias_{t-1}$	$surprise_t$
30-Jun-99	0.25	small hike	0.284	1.64	4.7	1	0	-0.040
24-Aug-99	0.25	small hike	0.232	1.64	5.0	1	0	0.022
5-Oct-99	0	no change	-0.064	1.61	6.1	0	0	-0.042
16-Nov-99	0.25	small hike	0.138	1.64	5.9	1	0	0.086
21-Dec-99	0	no change	0.426	1.62	6.4	0	0	0.016
2-Feb-00	0.25	small hike	0.564	1.64	6.1	0	0	-0.054
21-Mar-00	0.25	small hike	0.398	1.74	7.3	1	0	-0.031
16-May-00	0.5	large hike	0.322	1.65	7.9	1	0	0.052
28-Jun-00	0	no change	-0.372	1.61	6.8	1	0	-0.020
22-Aug-00	0	no change	-0.294	1.53	4.8	1	0	-0.017
3-Oct-00	0	no change	-0.500	1.57	5.7	1	0	0.000
15-Nov-00	0	no change	-0.394	1.56	5.9	1	0	0.000
19-Dec-00	0	no change	-0.810	1.52	4.7	1	0	0.052
3-Jan-01	-0.5	large cut	-1.052	1.55	4.9	0	1	-0.382
31-Jan-01	-0.5	large cut	-1.208	1.59	2.4	0	1	0.005
20-Mar-01	-0.5	large cut	-1.186	1.65	4.2	0	1	0.056
11-Apr-01	0	no change	-1.014	1.64	2.7	0	1	0.016
18-Apr-01	-0.5	large cut	-0.852	1.64	2.7	0	1	-0.425
15-May-01	-0.5	large cut	-0.680	1.62	3.8	0	1	-0.078
27-Jun-01	-0.25	small cut	-0.472	1.62	3.5	0	1	0.050
21-Aug-01	-0.25	small cut	-0.290	1.64	2.5	0	1	0.016
17-Sep-01	-0.5	large cut	-0.306	1.64	2.5	0	1	-0.323
2-Oct-01	-0.5	large cut	-0.508	1.56	-0.5	0	1	-0.069
6-Nov-01	-0.5	large cut	-0.460	1.51	-2.0	0	1	-0.100
11-Dec-01	-0.25	small cut	0.322	1.54	-1.8	0	1	0.000
30-Jan-02	0	no change	0.434	1.57	3.3	0	1	0.015
19-Mar-02	0	no change	0.846	1.65	5.4	0	1	-0.026
7-May-02	0	no change	0.528	1.65	4.1	0	0	0.000
26-Jun-02	0	no change	0.378	1.65	3.2	0	0	0.000
13-Aug-02	0	no change	-0.032	1.65	3.5	0	0	0.034
24-Sep-02	0	no change	0.006	1.67	4.3	0	1	0.025
6-Nov-02	-0.5	large cut	-0.252	1.68	2.9	0	1	-0.194
10-Dec-02	0	no change	0.282	1.66	3.1	0	0	0.000
29-Jan-03	0	no change	0.066	1.77	3.8	0	0	0.000
18-Mar-03	0	no change	-0.080	1.82	4.3	0	0	0.048

Table A1 (contd). Data

Date of FOMC decision	Change to federal funds rate target	Dependent variable Δy_t	$spread_t$	$houstart_t$	Δgdp_t	$tbias_{t-1}$	$ebias_{t-1}$	$surprise_t$
6-May-03	0	no change	-0.028	1.76	3.1	0	0	0.037
25-Jun-03	-0.25	small cut	-0.280	1.70	2.4	0	1	0.150
12-Aug-03	0	no change	0.342	1.74	4.6	0	1	0.000
16-Sep-03	0	no change	0.204	1.80	5.9	0	1	0.000
28-Oct-03	0	no change	0.288	1.84	5.2	0	1	0.000
9-Dec-03	0	no change	0.374	1.93	5.5	0	1	0.000
28-Jan-04	0	no change	0.182	1.92	6.6	0	1	0.000
16-Mar-04	0	no change	0.158	1.90	6.6	0	1	0.000
4-May-04	0	no change	0.536	1.89	6.2	0	1	-0.006
30-Jun-04	0.25	small hike	1.116	1.97	7.4	0	0	-0.010
10-Aug-04	0.25	small hike	0.752	1.93	4.9	0	0	0.022
21-Sep-04	0.25	small hike	0.528	1.98	4.5	0	0	0.017
10-Nov-04	0.25	small hike	0.636	1.98	5.2	0	0	0.000
14-Dec-04	0.25	small hike	0.546	1.98	5.6	0	0	0.000
2-Feb-05	0.25	small hike	0.502	1.97	5.1	0	0	0.000
22-Mar-05	0.25	small hike	0.658	2.15	7.2	0	0	0.000
3-May-05	0.25	small hike	0.480	2.02	6.3	0	0	0.000
30-Jun-05	0.25	small hike	0.286	2.00	5.6	0	0	0.000
9-Aug-05	0.25	small hike	0.462	2.01	5.9	0	0	0.000
20-Sep-05	0.25	small hike	0.252	2.00	5.8	0	0	0.015
1-Nov-05	0.25	small hike	0.424	2.10	6.6	0	0	0.000
13-Dec-05	0.25	small hike	0.256	2.00	5.6	0	0	0.000
31-Jan-06	0.25	small hike	0.142	2.10	6.3	1	0	0.000
28-Mar-06	0.25	small hike	0.136	2.10	8.2	1	0	0.000
10-May-06	0.25	small hike	0.170	2.00	7.0	1	0	-0.007
29-Jun-06	0.25	small hike	0.264	1.90	5.9	1	0	-0.015
8-Aug-06	0	no change	-0.150	1.80	5.3	1	0	-0.040
20-Sep-06	0	no change	-0.228	1.70	4.0	1	0	0.000
25-Oct-06	0	no change	-0.170	1.60	4.0	1	0	0.000
12-Dec-06	0	no change	-0.326	1.50	2.9	1	0	0.000
31-Jan-07	0	no change	-0.156	1.50	5.7	1	0	0.000
21-Mar-07	0	no change	-0.328	1.40	5.5	1	0	0.000
9-May-07	0	no change	-0.314	1.40	5.5	1	0	0.000
28-Jun-07	0	no change	-0.308	1.50	6.0	1	0	0.000
7-Aug-07	0	no change	-0.460	1.30	3.6	1	0	0.026

Table A1 (contd). Data

Date of FOMC decision	Change to federal funds rate target	Dependent variable Δy_t	$spread_t$	$houstart_t$	Δgdp_t	$tbias_{t-1}$	$ebias_{t-1}$	$surprise_t$
10-Aug-07	0	no change	-0.490	1.30	3.6	1	0	0.000
17-Aug-07	0	no change	-0.202	1.30	3.6	1	0	0.155
18-Sep-07	-0.5	large cut	-1.008	1.30	3.5	0	0	-0.150
31-Oct-07	-0.25	small cut	-0.848	1.20	2.3	0	0	-0.020
11-Dec-07	-0.25	small cut	-1.246	1.20	1.9	0	0	0.008
22-Jan-08	-0.75	large cut	-1.394	1.20	1.9	0	0	-0.741
30-Jan-08	-0.5	large cut	-1.182	1.00	3.3	0	1	-0.095
18-Mar-08	-0.75	large cut	-1.420	1.00	2.7	0	1	0.167
30-Apr-08	-0.25	small cut	-0.324	0.90	-0.6	0	1	-0.055
25-Jun-08	0	no change	0.604	1.00	1.9	0	0	-0.030
5-Aug-08	0	no change	0.260	0.90	4.3	0	0	-0.006
16-Sep-08	0	no change	-0.202	0.90	5.5	0	0	0.059
8-Oct-08	-0.5	large cut	-0.154	0.90	5.5	0	0	-0.142
29-Oct-08	-0.5	large cut	0.734	0.80	2.9	0	1	-0.060
16-Dec-08	-0.75	large cut	0.356	0.70	-2.4	0	1	-0.119
28-Jan-09	0	no change	0.252	0.50	-4.3	0	1	0.000
18-Mar-09	0	no change	0.508	0.40	-3.3	0	1	-0.006
29-Apr-09	0	no change	0.342	0.50	-1.0	0	1	-0.005
24-Jun-09	0	no change	0.262	0.50	-1.6	0	1	-0.025
12-Aug-09	0	no change	0.324	0.60	1.6	0	1	-0.008
23-Sep-09	0	no change	0.244	0.60	3.1	0	1	0.000
4-Nov-09	0	no change	0.270	0.70	3.1	0	1	0.000
16-Dec-09	0	no change	0.230	0.60	4.6	0	1	-0.010
27-Jan-10	0	no change	0.188	0.60	4.7	0	1	-0.019
16-Mar-10	0	no change	0.234	0.60	4.2	0	1	0.000
28-Apr-10	0	no change	0.248	0.60	4.6	0	1	0.000
23-Jun-10	0	no change	0.110	0.60	4.8	0	1	0.000
10-Aug-10	0	no change	0.080	0.60	3.8	0	1	0.000
21-Sep-10	0	no change	0.052	0.60	3.6	0	1	0.000
3-Nov-10	0	no change	0.028	0.60	2.8	0	1	0.003
14-Dec-10	0	no change	0.126	0.50	2.8	0	1	0.000
26-Jan-11	0	no change	0.096	0.60	5.7	0	1	0.000
15-Mar-11	0	no change	0.108	0.60	4.8	0	1	0.000
27-Apr-11	0	no change	0.132	0.60	6.1	0	1	0.000
22-Jun-11	0	no change	0.084	0.50	5.8	0	1	-0.009

Table A1 (contd). Data

Date of FOMC decision	Change to federal funds rate target	Dependent variable Δy_t	$spread_t$	$houstart_t$	Δgdp_t	$tbias_{t-1}$	$ebias_{t-1}$	$surprise_t$
9-Aug-11	0	no change	0.024	0.600	5.100	0	1	0.000
21-Sep-11	0	no change	0.002	0.600	5.300	0	1	0.008
2-Nov-11	0	no change	0.052	0.600	3.900	0	1	0.000
13-Dec-11	0	no change	0.032	0.600	4.300	0	1	-0.004

Table A2. Sample descriptive statistics

Variable	Mean	Median	Standard deviation	Minimum	Maximum	First-order autocorrelation coefficient
Δy_t	-0.04	0.00	0.92	-0.50	0.50	0.50
$spread_t$	0.05	0.11	0.54	-1.45	1.76	0.82
$houstart_t$	1.48	1.47	0.26	0.95	2.15	0.98
Δgdp_t	5.05	5.10	1.75	-2.00	9.40	0.79
$tbias_{t-1}$	0.27	0.00	0.44	0.00	1.00	0.63
$ebias_{t-1}$	0.32	0.00	0.47	0.00	1.00	0.73
$surprise_t$	-0.29	0.00	0.91	-4.25	1.50	-0.02

Notes. Sample period: 7/1987–1/2006 (190 observations).

Table A3. Tests for unit roots

Variable	The Augmented Dickey-Fuller (ADF) unit root tests					
	Sample period (and size)	Data frequency	Deterministic terms*	Lag length	t-statistic	P-value**
Δy_t	7/87-12/16 (281 obs.)	FOMC decisions	C	1	-6.01	0.0000
$spread_t$	1/62-2/17 (14341 obs.)	daily	C	41	-7.84	0.0000
$houstart\_act_t$	1/59-1/17 (680 obs.)	monthly	C, LT	16	-4.23	0.0043
Δgdp_t	7/87-12/11 (241 obs.)	FOMC decisions	C, LT	0	-5.73	0.0000
$tbias_{t-1}$	7/87-12/16 (281 obs.)	FOMC decisions	C	1	-4.82	0.0001
$ebias_{t-1}$	7/87-12/16 (281 obs.)	FOMC decisions	C	0	-5.22	0.0000
$surprise_t$	7/87-1/14 (258 obs.)	FOMC decisions	C, LT	0	-16.18	0.0000

Notes. * C - constant, LT - linear trend; ** MacKinnon (1996) one-sided p-values. The lag order of the lagged first differences of the dependent variable in the ADF tests is selected according to a criterion of no serial correlation among the ADF regression residuals.