NATIONAL RESEARCH UNIVERSITY
HIGHER SCHOOL OF ECONOMICS

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GOVERNANCE BY DEPOSITORS, BANK RUNS AND AMBIGUITY AVERSION: A THEORETICAL APPROACH

BASIC RESEARCH PROGRAM
WORKING PAPERS

SERIES: FINANCIAL ECONOMICS
WP BRP 68/FE/2018

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GOVERNANCE BY DEPOSITORS, BANK RUNS AND AMBIGUITY AVERSION: A THEORETICAL APPROACH[2,3]

We investigate the theoretical relationship between ambiguity aversion and the decision to withdraw early from a deposit contract. We first document and define the concepts to illustrate our results. Then we extend the theoretical framework of Gorton (1985) to implement a model of maxmin expected utility to match the ambiguity aversion hypothesis. We observe that the most ambiguous depositors are more likely to mistakenly withdraw their deposits, reducing bank stability and leading to inefficient bank runs. We also show higher ambiguity levels negatively impact bank equity levels.

JEL Classification: G02, G18

Keywords: Banking governance, ambiguity aversion, depositor's behaviour, bank runs

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[2] The paper was prepared within the framework of the Basic Research Program at the National Research University Higher School of Economics (HSE) and supported within the framework of a subsidy granted to the HSE by the Government of the Russian Federation for the implementation of the Global Competitiveness Program. The financial support from the Government of the Russian Federation within the framework of the implementation of the 5-100 Program Roadmap of the National Research University Higher School of Economics is acknowledged.

[3] The author acknowledges the comments and remarks by the participants of the PhD. defense on November 2016, and the participants 7th Annual CInSt Banking Workshop on October 6th, 2017

1. Introduction

Banking regulation over the last century has worked towards avoiding depositor initiated bank runs. The evolution of the literature dedicated to bank runs and their relationship with banking governance has been widely analysed both theoretically and empirically. Chen and Hasan (2006, 2008), Wu and Bowe (2012) and Diamond and Dybvig (1983) are among many examples available. Bank run research has contributed directly or indirectly to the evolution of the banking regulation.

In this article, we introduce the concept of ambiguity and ambiguity aversion[4] as a third state of the decision-making process in a deposit contract. Knight (1921) first introduced the concept of *“Knightian uncertainty”* where a third state of the decision-making process is likely: some risks are immeasurable, and it is, therefore, impossible to define an objective distribution of the probability of a potential event. The author’s conclusion allows for the definition of a subjective distribution of probability, useful for ambiguity computation. Ambiguity is considered as another state of the decision-making process under a horse-roulette preference scheme, which differs from certain and risky environments; while releasing the hypothesis of perfect rationality (Machina and Viscusi, 2014). It differs from the two classic states of the decision-making process by allowing economic agents to have multiple subjective distributions of probability for each particular event. In this setup, and applied to our research questions, depositors have a variety of visions of the value of a bank's investment’s portfolio, and are influenced by the degree of private information they obtained previously. In this situation, ambiguity is similar to a degree of optimism or pessimism, based on non-probabilistic beliefs.

This article introduces ambiguity aversion to the context of a potential bank run with uninsured depositors, in an asymmetric information scheme. To our knowledge, this article is the first attempt at theorizing ambiguity aversion in a deposit contract. To do so, we extended Gorton's (1985) asymmetric information framework between bank managers and depositors. It illustrates the real banking sector and is common in the literature regarding this issue (Chen 1999). Introducing ambiguity aversion in a deposit model is particularly innovative: it answers new concerns about depositors who may act without being fully rational. We do not focus on the suspension of convertibility as in the original model but the framework allowed us to introduce ambiguity aversion concept than Diamond and Dybvig (1983) or Cordella and Yeyati (1998). Diamond and Dybvig (1983) did not take into account incomplete information, which is a key component of ambiguity

[4] As of Ellsberg (1961)

aversion research while Cordella and Yeyati (1998) use a monopoly banking sector which significantly limits the overall value of the model for modern banking sectors.

The literature concerning ambiguity is not as flourishing as the depositor decision-making literature, such as Gorton (1985) or Diamond and Dybvig (1983). The main research question here concerns the impact of the degrees of ambiguity on the decision to withdraw early in an asymmetric information scheme. The principal results of the article are the following: depositors' decision to withdraw early is positively influenced by the degree of the ambiguity of the depositors. The more pessimistic the depositors, the more likely they are to mistakenly withdraw their deposit early. This leads to an inefficient bank run. The second result is the impact of the degree of ambiguity on the levels of equity which a bank needs to raise to continue its activity. The results obtained in this article are innovative and provide significant policy implications for implementing a quality disclosure policy to reduce the negative impact of rumours and to emphasize the stability of the banking sector while depositors are ambiguous. The article is organised as follows: first, we review the literature useful to the model, then we introduce the general framework of Gorton (1985), we describe the hypothesis used in our model and finally, we present the results and policy implications.

2. Literature, historical approaches and definition

Despite the large variety of protocol to test paradox and theoretical frameworks Ellsberg (1961), there is no stable behaviour regarding ambiguity (Hilton, 2006). Nonetheless, agent preferences are a complex set of incentives which are significantly influenced by economical, sociological and psychological factors and characteristics (Payne et al., 1993). Analysing this concept, and therefore ambiguity aversion, has been on-going since the beginning of the 20th century. The economic and applied mathematics literature provides abundant work on the early notion, even if not named as such. This section will introduce some results regarding subjective beliefs and ambiguity to allow us to continue our analysis. We also provide a technical review of the axiomatization of ambiguity and ambiguity aversion in the appendix *a* and *b*. The last subsection will be devoted to a review of the literature regarding depositor governance.

2.1. Subjective probability in early literature

To understand the following section and the rest of the article, it is important to define the difference between objective probability and subjective probability. The difference comes from the relative randomness of an event: when you flip a perfectly balanced coin, there is no doubt concerning the probability outcome, but when you bet on a horse the probability is not as easy to determine. In objective probability the gain is known and a function of explicit probabilities. The gamble with a game, G, is as follows: $G = (x_1, p_1; \dots ; x_n, p_n)$. The probability p_n is explicitly known and common knowledge for a perfectly balanced coin or roulette wheel. For subjective probability, the game and gains differ: $G = (x_1, E_1; \dots; x_n, E_n)$. The probability of occurrence of the gain is not explicit but is based on expectation. Expectations open the calculation to beliefs and interpretation. This kind of gamble is typically used for horse races where E_n is the expected performance of a horse, based on past performance, type of soil, weather conditions, or any other parameters which influence the performance of the horse.

Historically speaking, Knight (1921) was the first to point out the difference between probabilistic beliefs and non-probabilistic beliefs. The author used the notion of “risk” to qualify a situation where no objective probabilities are known. Knight’s notion of risk refers to an *ex-ante* probability, theoretically deduced, or empirically observed while “uncertainty refers to situations that do not provide objective probabilities.” The author then suggested the existence of “estimates” which will be considered as subjective probabilities. Keynes (1921) also defined a “logical relation” to explain probabilities based on a rational degree of belief. The author allowed for different types of probabilities: numerical, or not, and probabilities which cannot be ordinally compared[5]. To measure and analyse the degree of subjective probabilities, Ramsey (1926) observed the existence of probabilistic beliefs through the strong and bizarre bets of agents in a lottery. The author studied the measurements of subjective probability and used the term “degree of beliefs” to capture the attitude of agents towards risk. Using the Bernoullian principle of expected utility maximization, the author says subjective probability is the consequence of mathematical presuppositions: “behaviour is governed by what is called the mathematical expectation of utility, [...] sufficient conditions enter into his calculations multiplied by a fraction called degree of belief.” Similarly, Shackle (1949a, 1949b) introduced the concept of “potential surprise” to materialize the expectations of experiences upon learning a particular event has occurred.

[5] To estimate the probability of rain, he used the terminology *less likely* and *more likely*.

With the development of applied mathematics in economic analysis, more authors have contributed to the field and proposed different definitions of subjective probabilities, leading to the ambiguity aversion model. Both objective and subjective probabilities can be resolved or modelled. Objective expected utility was suggested by Bernouilli in the mid-18th century and later formalized by von Neumann, Morgenstern, Kuhn and Rubinstein (1944). The ordinal preference function $V(x_1, p_1; \dots ; x_n, p_n) = \sum_{i=1}^{n} U(x_i).p_i$ associated with classical utility expectation axioms, has become the cornerstone of the modern economic analysis. The independence axiom of classical economic analysis allows agents to rank their preferences. With the introduction of ambiguity and ambiguity aversion, the independence axiom is rejected to enable specific mechanisms.

Savage and Wiley (1954) proposed an alternative to the classical expected utility model which contributed to the literature regarding subjective probability. They proposed a model of subjective expected utility with a preference system composed of subjective acts. The following ordinal function takes into account the subjective act: $W(x_1, p_1; \dots ; x_n, p_n) = \sum_{j=1}^{n} U\left(x_j\right).\mu(E_j)$ where $\mu(.)$ is the subjective probability measure and $E(j)$ the expected probability of an event. It represents the beliefs of the likelihood of the different states of nature. The beliefs depend on their knowledge and information acquired. The “sure-thing Principle” suggested by the authors is close to the Independence axiom of the classic objective expected utility model. Preference schemes over subjective acts are separable across mutually exclusive events, allowing for preference rankings.

2.2. Ambiguity: Ellsberg Urn and Paradox

Ellsberg (1961) shows the existence of a paradox in decision theory in which an agent’s choice violates the axiom and postulates of the subjective expected utility of Savage and Wiley (1954) and Anscombe and Aumann (1963). The following example of Ellsberg urns, from Coleman (2011), presents the Ellsberg paradox. The setup involves a single urn composed of red, black and yellow balls.

Table 1 Single urn example of the Ellsberg paradox

	30 Balls	60 Balls	
Gamble	Red	Black	Yellow
1	$1	$0	$0
2	$0	$1	$0
3	$1	$0	$1
4	$0	$1	$1

In Table 1, we consider a single urn containing 90 balls, 30 of which are known to be red and the rest are of an unknown distribution of black and yellow balls. The payoff of gamble 1 is *"receive $1 if red, and $0 if black or yellow."* The payoff of gamble 2 is *"received $0 if red, $1 if black and $0 if yellow."* The gamble 3 is *"receive $1 if red, $0 if black and $1 if yellow."* While the payoff of gamble 4 is *"receive $0 if red, $1 if black or yellow."* With such setup, the author proposes two questions: do you prefer payoff on red versus payoff on black? Do you prefer payoff on red/yellow versus black/yellow? The author reports a frequent pattern of response in favour of gamble 1 over gamble 2, while gamble 4 is preferred over gamble 3. The response pattern suggests a preference toward a known probability distribution (preference for payoff on red) while also rejecting potential gains from unknown distribution of probability (in gamble 3). The inconsistency in the preference scheme is the Ellsberg paradox.

Ellsberg's discovery illustrates the violation of Savage's "sure-thing Principle", while also violating the comparative probability property. These two assumptions are considered key principles of perfect rationality. Therefore, the result suggested by the author provoked discussions and reactions from decision theorists. Debreu's response, among others, eludes to the Principles of Insufficient Reason in which probabilities are relative frequencies rather than degrees of belief, conditional upon available information.

Several Ellsberg's urn experiments have been conducted to test the validity of Ellsberg (1961). Fellner (1961) confirms a 50:50 trend instead of preference for unknown odds. Maccrimmon (1968) and Curley and Yates (1989) were able to confirm Ellsberg's paradox, showing a preference for the unknown. Most of these studies were conducted with students but later work in the late 1980s and

1990s[6] show similar results with business owners, trade union leaders, managers and executives. For these reasons, Ellsberg (1961) has been the source of the most recent literature concerning limited rationality, ambiguity and ambiguity aversion.

2.3. Ambiguity aversion in modern behavioural finance literature

As in Machina and Siniscalchi (2014) there is no consensus on what ambiguity aversion exactly is, nonetheless a definition approaching consensus exists. Ambiguity aversion is a preference for known risks over unknown risks. The behavioural finance literature studying such aversion is not as large as the literature concerning depositor governance. Nonetheless, a recent development has found evidence for the impact of ambiguity on asset prices and portfolio choices.

Puri and Robinson (2007) suggested the creation of a measure of optimism which correlates positive beliefs and future economic conditions. The authors show that moderate optimists display prudent financial behaviour while, the most optimistic behave more carelessly. The portfolio choice literature provides evidence relating optimal portfolio choices and ambiguity-averse economic agents. Guidolin and Rinaldi (2013) survey the literature exploring the implications of decision-making under ambiguity for both portfolio choices and equilibrium asset prices. The authors suggest negative effects of ambiguity and ambiguity aversion covering several aspects such as home equity preference in international portfolio diversification, excessive volatility of asset returns, ambiguity pricing in premium, and occurrences of trading breakdowns.

Dow and Werlang (1992) analyse the problem of optimal investment decisions by seeking to distinguish between “quantifiable risk” and “unknown uncertainties”. While using non-additive subjective probability, to capture the “unknown uncertainty”, there is an interval of investment prices in which agents neither buy nor sell short the asset. Such an interval does not appear while only taking quantifiable risk into account. Routledge and Zin (2010) show that aversion to uncertainty increases the market-makers bid-ask spreads and reduces liquidity. Bossaerts, el al. (2010) and Gagliardini, Porchia and Trojani (2009) confirm the evidence of stock price and bond return adjustment in the presence of ambiguity-averse investors.

Dimmock et al. (2016) test the nature of the relation between ambiguity aversion and five household portfolio choice puzzles: nonparticipation in equities, low allocations of equity, home bias, own-company stock ownership, and portfolio under-diversification. The authors survey US households to

[6] See Kunreuther (1989)or Viscusi and Chesson (1999)

measure ambiguity aversion using custom-designed questions based on the Ellsberg urn. Evidence is found for ambiguity aversion being negatively associated with stock market participation, stock levels and foreign stock ownership. The authors emphasize the fact that ambiguity-averse households were more likely to sell short. Similarly, Ahn, et al. (2014) use experimental data to estimate models of attitude toward risk and ambiguity. The authors show evidence that portfolio choices lean toward unambiguous portfolios. Brenner and Izhakian (2017) study the relationship between ambiguity, risk and expected returns. They show that ambiguity, on average, has a significantly positive effect on expected returns which contrasts with prior puzzling results.

2.4. Bank runs and depositor withdrawal decisions

Bank governance is different from classic corporate governance for several reasons and depositor governance is one of them (Becht, et al., 2011). Depositors exert a pressure on bank managers and decision makers in order to satisfy their objective functions. Their governance differs from the two other types of governance, managerial and investor. They expect the bank to be able to hold deposits and be safe enough that they can withdraw their entire endowment plus potential interest. This is only possible if the bank does not file for bankruptcy. If depositors feel that the bank they deposited in is not safe enough and that they might not be able to withdraw their whole deposit, they can decide to withdraw their deposits early and deposit them in a safer bank. Depositors are actors of the corporate governance of a bank. Through the decision depositors take, they impose direct or indirect pressure on the bank's decision makers.

Depositors have three ways to exercise governance. First disciplining by price, where depositors require higher interest rates from riskier banks. Higher interest rates contain the risk premium. Second, depositors can discipline by quantity; if bank fundamentals demonstrate greater risks, depositors tend to withdraw their fund from this bank, so it becomes more difficult for the bank to fulfil an investment portfolio. Third, they can discipline by maturity shifts: depositors may switch from riskier long term deposits to short-term or even on-call ones if they face additional risk-taking by banks.

Diamond and Dybvig (1983) have probably contributed the most to the literature on bank runs and therefore contributed to the different extensions of the original Diamond and Dybvig model. A large share of the depositor behaviour literature focuses on the quantity disciplining effect and therefore on the risk of potential bank runs formerly known as the risk of informational contagion. Diamond and Dybvig (1983) have argued that uninsured on-call deposit contracts provide liquidity but leave banks vulnerable to runs (Postlewaite and Vives, 1987). The risk of one individual withdrawing a deposit is, of course, not of significant concern. Nonetheless, the combination of such risk with an implicit rule of first-come, first-served (presented by Chen, 1999) significantly modifies the behaviour of agents as expressed by Chen and Hasan (2008). The authors propose a theory as to why bank runs look like panic. They provide evidence that a depositor with excessive incentives to withdraw (Chen & Hasan, 2006) may force other depositors to respond to mildly adverse information even if depositors would be better off not withdrawing. Therefore, we can wonder if depositors are fully rational.

The risk of a run is taken seriously by regulators because of the interconnectedness of the banking sector. Observing a run on one bank is likely to make uninformed depositors from other banks, run (Aghion et al. 2000) and Acharya and Thakor, 2014). Banks are linked through cross-deposits: negative information about one bank starts a run and this starts runs in the ones linked to it. The contagion of the run is related to a liquidity shock. Cespa and Foucault (2014) show that market illiquidity is contagious: lower liquidity for one bank can trigger a similar drop in other interconnected banks. The consequences of runs can lead to a liquidity driven crisis, market freezes and coordination failures (Benoit et al., 2016). The systemic nature of such risk confirms the necessity of contract design, capital and liquidity regulation to avoid global financial crises. It also emphasizes the need to identify the nature of depositors to detect irrational and ambiguous behaviour. This article fits with this literature.

3. Ambiguity and the decision to withdraw

A bank run arises when depositors decide to withdraw their deposits which leads others to follow[7]. When they are incompletely informed regarding the quality of the bank they deposited in, a bank run can occur. Gorton (1985) presents a model showing the relationship between banks, depositors and governance. The author analyses the impact factor of the suspension of convertibility on potential runs. Governance actions are based on the non-observance by depositors of potential capital losses by the bank during its portfolio investment strategy.

[7] First come, first serve rule applies to deposit withdrawals

3.1. Gorton's (1985) general framework presentation and full information scheme

The framework we use in this article is based on the one proposed by Gorton (1985). The author uses a three-periods model, in which depositors maximize their utility in the first two periods, and then the game ends during the third period. During the last period, depositors live off the gains and savings realized in the first two periods. At the beginning of the game, depositors receive an initial endowment M_0 and are considered risk averse towards the lotteries of the first two periods but remain risk neutral for the retirement period. With the endowment, depositors consume X_i[8] and choose between holding currencies C_i remunerated at a rate λ_i or to deposit (*d*) the endowment in the bank remunerated at a rate r_{di}. The indices *d* and *i* represent respectively the deposit contract and the interest rate paid to depositors for the period *i*. The banking sector is a competitive sector with a two-period investment where debt (deposits) Δ are collected and equity Q are raised. Banks, then choose an investment portfolio with a level of risk θ_i where $\underline{\theta_i}$ is the minimum level of risk of the portfolio. Debt is subject to capital losses based on the result of the portfolio investment choice $\pi_i(\theta_i)$, but cannot incur capital gains. The deposit contract proposed by the banks allows depositors to withdraw at the beginning of the second period.

The sources of uncertainty are dual in this model: the return rate of currency holding λ_i is random, and the return rate on banks' investments is partially random as well. It is assumed that currency consists of gold coins, and the rate of return is the rate of appreciation or depreciation against goods and is therefore correlated with the future rate of the currency. At the beginning of the game, the rate of return on currency for period 1 is known, but not for the second period. Since banks invest the collected deposits on real industrial projects, the link between banks and the real sphere is clear. The observation, by depositors, of the bank investment portfolio risk profile is approximated by the observation of the currency rate of return λ_2 (ex post proxy) at the beginning of the second period and by the losses during the first period $\pi_1(\theta_1)$. Therefore, if a negative shock occurs on output, it will be transmitted to banks through the investment channel and will decrease not just bank return but also currency return rates. Depositors are risk-averse with respect to lotteries on consumption during periods 1 and 2, but are risk-neutral with respect to retirement. This assumption causes depositors to choose portfolios which are corner solutions with holding either currency or deposits, but not both.

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[8] For simplification purpose, consumption is normalized to 0

The participation constraint is ensured when:

$$(1+\lambda_2)C_2 < (1+r_{d1})(1+r_{d2})\left[1-E_1\left(\pi_2(\theta_2)\right)\right]D_2 \qquad (1)$$

Table 2 sums up the different parameters and timing of the game with full information between depositors and banks. In order to implement the ambiguity aversion component, we must use a model with asymmetric information components. This is based on the non-observation of the bank state and capital losses, but on its expected value: the investment portfolio is known only by the bank in both periods 1 and 2.

For the purpose of this paper, the introduction of the ambiguity parameter relies on the implementation of multiple distributions of probabilities regarding expected capital losses, which will be presented in the following section. In contrast, in the Gorton (1985) framework in presence of ambiguity, depositors face a continuum of expected capital losses, based on a maxmin expected utility (MEU) model. In order to ensure the implementation of ambiguity aversion, we implemented an asymmetric information scheme between the bank and the depositor based on the author's framework. The original framework remains while we only modify the definition of the expected capital loss.

Gorton's (1985) expected capital loss is a classic equation expected utility function: $E(\pi_i(\theta_i))$ while the implemented expected capital loss ambiguity function is:

$$\alpha.min\int_{\underline{\theta_i}}^{\overline{\theta_i}}\hat{\pi}_i(\hat{\theta}_i)d\hat{\theta}_i+(1-\alpha).max\int_{\underline{\theta_i}}^{\overline{\theta_i}}\hat{\pi}_i(\hat{\theta}_i)d\hat{\theta}_i$$

Table 2 Full information framework, as of Gorton (1985)

	Beginning of period 1	Beginning of period 2	End of period 2
Common Knowledge	Rate of return on currency λ_1 Return on deposit r_{d1} and r_{d2}	λ_2 and state of bank investment θ_i and capital losses $\pi_1(\theta_i)$	$\lambda_2, \pi_2(\theta_2)$
Deposit's action	Endowment M choice either currency C_1 or deposit D_1	Decision to withdraw or not (C_1 and D_1)	Receive end of game wealth
Bank's action	Equity choice Q given r_{d1} and r_{d2}, given bank's investment $= Q + \Delta$	If $\underline{\theta_1} \leq \theta_1 \leq \theta_1^*$ then set $\pi_1(\theta_1) > 0$	If $\underline{\theta_2} \leq \theta_2 \leq \theta_2^*$ then set $\pi_2(\theta_2) > 0$

In our case, the asymmetry of information is the state of the bank investment θ_i and the capital losses π_i: the investment portfolio is known only by the bank in both period 1 and 2. In the meantime, the minimum value of the state of bank is materialized by $\underline{\theta_i}$ for each period *i*.

3.2. Optimal withdrawing decision under ambiguity

The core value of this article concerns the implementation of ambiguity aversion in the framework suggested by Gorton (1985). The original paper shows evidence that the suspension of convertibility within a deposit contract eliminates the risk of an inefficient bank run in an asymmetric information scheme. In this paper, we do not focus on the suspension of convertibility, but on the effect of ambiguity on depositor discipline. We use the independent state of banks from the first period to the second. We introduce MEU model, presented in appendix *b*. The MEU model is one possible ambiguity model. As revealed by Machina and Siniscalchi (2014) in a survey involving ambiguity models, this model presents advantages among which are mathematical tractability, readability and accessibility to depositors. The choice for this specification is also justified by its applicability to the real world. It allows depositors to compute a value of the bank's investment state, which is expected to reduce the size of withdrawal mistakes (Gorton, 1985). The decision to implement ambiguity aversion is justified by the natural aversion of depositors toward risk and by their high level of withdrawing incentives (Chen and Hasan, 2006, 2008). Depositors' decisions to withdraw are based on the amount of information they have: depositors are sensitive to not only trustworthy information coming from banks or regulators, and also to hearsay and rumours.

Ambiguity is only compatible with asymmetric information where hearsay can affect depositor decisions. In the asymmetric framework, the state of bank investment θ_i and the capital loss $\pi_i(\theta_i)$ are only known by the bank. Depositors will be able to observe the potential losses of the bank at the end of the game based on the payment of the deposit contract. The maximization model, including ambiguity aversion and interest rate *r* paid on deposits d at time *1* (r_{d1}), is here modelled by the different probability distributions of α. The α parameter is a component of the level of depositor confidence. It is exogenous and randomly distributed among the depositors. This parameter is the realization of depositor confidence in the state of the economy and in the ability of the bank to pay back deposits prior to the decision to deposit. α is observed by depositors before the game and is not available to the bank even at the end of the game. It is also constant throughout the game. It is defined as $\alpha \, \epsilon (0, 1)$. In other words, when they are more optimistic than pessimistic regarding microeconomic and macroeconomic environment, α tends to be close to 1, inversely when depositors are pessimistic α tends to 0. It is only observable by the individual depositor at the beginning of the period and possibly by the bank at the end of the game, if a run occurs. The use of the original endowment M_0 remains either to consume X_i, to hold currencies C_i or to deposit in a bank D_i.

$$max\, V = E[\beta U(X_1) + \beta_1 U(X_2|\lambda_2)] + \beta_2[\Lambda(W)|\lambda_2] \tag{2}$$

under the following constraints:

$$
\begin{gathered}
X_1 + \; C_1 + D_1 \leq M_0 \\
X_2 + C_2 \leq (1+\lambda_1)C_1 + (1+r_{d1})\left[1 - E\left(\alpha.min\int_{\underline{\theta_1}}^{\overline{\theta_1}} \hat{\pi}_1(\hat{\theta}_1)d\hat{\theta}_1 + (1-\alpha).max\int_{\underline{\theta_i}}^{\overline{\theta_i}} \hat{\pi}_1(\hat{\theta}_1)d\hat{\theta}_1\right)\right](D_1 - D_2) \\
W = (1+\lambda_2)C_2 + (1+r_{d1})(1+r_{d2})\left[1 - E\left(\alpha.min\int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2)d\hat{\theta}_2 + (1-\alpha).max\int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2)d\hat{\theta}_2\right)\right]
\end{gathered}
\tag{3}
$$

The difference with the framework of Gorton (1985) and our model are in the constraints. To capture ambiguity aversion, we implemented the MEU function instead of the marginal classical von Neumann and Morgenstern utility function. The change of the expectation function materializes the ambiguity with the parameter α. The framework relies on observing λ_2. The ambiguity aversion problem $E\left(\alpha.min\int_{\underline{\theta_i}}^{\overline{\theta_i}} \hat{\pi}_i(\hat{\theta}_i)d\hat{\theta}_i + (1-\alpha).max\int_{\underline{\theta_i}}^{\overline{\theta_i}} \hat{\pi}_i(\hat{\theta}_i)d\hat{\theta}_i\right)$ represents the value depositors are expecting for potential capital loss in period i. Where $\underline{\theta_i}$ and $\overline{\theta_i}$ represent respectively the minimum and maximum value of the state of banks $\hat{\theta}_i$ investment anticipated by depositors for period i.

We replaced the original capital losses with a maxmin ambiguity function in the second period as well. With these functions, we endogenize the value of θ_1 and θ_2 in the decision model of depositors. In this setup, depositors have a variety of expectations concerning the state of the bank, contrary to the original case where the state of bank has a unique distribution. The multiplicity of these distributions is permitted with the "sentiment" of depositors, throughout the α parameter.

The resolution of the depositors' optimal decision, to withdraw, is based conditionally on observing the return rate of currency λ_2. For the purpose of this article and in line with the literature devoted to bank runs, we define a "correct withdrawal" as a withdrawal threshold above which a depositor refuses to accept the risk of the bank, and therefore decides to withdraw. In other words, such withdrawals would reflect the riskiness of a bank. Under a certain threshold, depositors accept the riskiness of the bank, compensated by payment of interest. Above it, depositors withdraw their deposit prior to the end of the contract. The withdrawal is then a signal prior to bankruptcy, based on the observance of the low quality of bank fundamentals. On the other hand, a withdrawal by mistake does not reflect bank fundamentals, but the confidence of the depositors. At the beginning of the second period, if depositors are able to observe the value of λ_2, they are able to compute their concern regarding the state of the bank, since λ_2 reflects partially the state of the real sphere after the first period. λ_2 and θ_1 are negatively correlate and serve as an indicator of the value of a bank's portfolio. Therefore, when the financial health of the bank is compromised, the price of holding currency rises. By observing λ_2, depositors are able to visualize indirectly the state of the bank of the previous period.

The hypotheses of our model are the following:

> **Hypothesis 1:** *Depositors are fully rational before the introduction of ambiguity. They base their optimal withdrawal decision on the rate of return of currency λ_2, the contracted interest rate payment r_{d2}, and the expected state of a bank's investment* (Gorton, 1985). *Depositors are risk-averse for the investment period and neutral to risk for retirement wealth.*

> **Hypothesis 2:** *Depositors under ambiguity are not fully rational anymore. The level of confidence α drives their incentives to withdraw.*

The certainty independence axiom and uncertainty aversion axiom mean depositors are not fully rational. Not only do depositors base their withdrawal decision on common criteria such as interest rates, currency rates or the state of the bank. They also base their decision on their aversion to ambiguity, degree of optimism, their perceived reliability of the bank, and by extension to the macroeconomic environment they are in. We use a backward induction method at the end of period 1, where depositors are in a situation of incomplete information. Therefore, they maximize their utility in the second period conditionally on having observed λ_2. The rest of the constraints remains.

$$\max V = E[\beta U(X_2|\lambda_2)] + \beta_1 E[\Lambda(W)|\lambda_2]$$

$$X_1 + C_1 + D_1 \leq M_0$$

$$X_2 + C_2 \leq (1+\lambda_1)C_1 + (1 + r_{d1})\left[1 - E\left(\alpha.min \int_{\underline{\theta_1}}^{\overline{\theta_1}} \hat{\pi}_1(\hat{\theta}_1)d\hat{\theta}_1 + (1-\alpha).max \int_{\underline{\theta_i}}^{\overline{\theta_i}} \hat{\pi}_1(\hat{\theta}_1)d\hat{\theta}_1\right)\right](D_1 - D_2) \tag{4}$$

$$W = (1+\lambda_2)C_2 + (1+r_{d1})(1+r_{d2})\left[1 - E\left(\alpha.min \int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2)d\hat{\theta}_2 + (1-\alpha).max \int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2)d\hat{\theta}_2\right)\right] D_2$$

X_i is the consumption level at period *i*. To simplify the calculation, depositors decide to use the totality of their endowment M_0 as either currency or deposits but not both at the same time. The withdrawal decision happens when $\lambda_2 > \lambda_2^{***}$, where λ_2 is the level of currency remuneration observed and λ_2^{***} is the optimal level of currency remuneration during the second period under the ambiguity hypothesis.

The resolution of the maximization of equation 3 presented by the following critical value of the currency return rate at period 2 λ_2 is:

<u>Result 1:</u> Depositors will be withdrawing if $\lambda_2 > \lambda_2^{***}$. *The critical value is given by the following expression:*

$$(1+\lambda_2^{***}) = \frac{(1+r_{d2})\left[1-\left(E\left(\alpha.min\int_{\underline{\theta_2}}^{\hat{\theta}_2}\hat{\pi}_2(\hat{\theta}_2)d\hat{\theta}_2+(1-\alpha).max\int_{\underline{\theta_2}}^{\hat{\theta}_2}\hat{\pi}_2(\hat{\theta}_2)d\hat{\theta}_2]\right)\right)\right]}{E[\alpha.min\int_{\underline{\theta_1}}^{\hat{\theta}_1}\hat{\pi}_1(\hat{\theta}_1)d\hat{\theta}_1+(1-\alpha).max\int_{\underline{\theta_1}}^{\hat{\theta}_1}\hat{\pi}_1(\hat{\theta}_1)d\hat{\theta}_1]} \quad (5)$$

The mathematical proof is available in appendix *c*. Since depositors are unable to know the exact value of θ_1 and θ_2, the optimal decision to withdraw is composed of the expected marginal utility based on expected value of $\hat{\pi}_1$, $\hat{\theta}_1$ and α, the degree of confidence.

The withdrawal decision impacts the bank's choice for its equity level Q^F and therefore the stability of the overall sector. Deposits and equity are the only source of bank financing in our model, a change in the deposit level will impact directly the financial health of the bank. The ambiguity actively impacts the level of equity of the bank. Similar to Gorton (1985), the level of equity chosen is the solution of the following problem:

Result 2: *The bank level of equity is a function of the degree of ambiguity of depositors.*

$$\frac{Q^F}{\Delta} = \frac{E_0[(1+r)|\ NW] - E_0[(1+r_d)^2|NW]}{1 + r_q - E[(1+r \mid NW)]} \tag{6}$$

Where:

$$E_0[(1+r)|\ NW] = G(1+r) \int_{\mu^*}^{\overline{\mu}} [\hat{\theta}_2 + \mu] Z(\mu) d\mu$$

$$E_0[(1+r_d)^2|NW] = G(1+r_d)^2 \hat{\theta}_1 \int_{\mu^*}^{\overline{\mu}} Z(\mu) d\mu \tag{7}$$

$$G = \int_{\underline{\theta}_1}^{\overline{\theta}_1} \int_{\mu^*}^{\overline{\mu}} \alpha . g(\lambda_2) . f(\theta_1) d\lambda_2 d\theta_1$$

Where E_0 is the expectation at the beginning of the period and Δ is the amount of debt collected by the bank in the form of deposits. NW is the condition on not withdrawing, μ is a noise indication of the quality of θ following a distribution $Z(\mu)$ and r is the remuneration of the investment portfolio of the bank.

3.3. The effect of ambiguity on withdrawal decisions and depositor expectation distribution

The ambiguity parameter, α, represents the degree of optimism or pessimism coming from the depositors. The level of optimism concerns the state of the bank investment portfolio and indirectly the macroeconomic environment. In a non-ambiguous situation, depositors only have one distribution of $\hat{\theta}$ and therefore one distribution of $\hat{\pi}(\hat{\theta})$, but in the ambiguity situation, they have multiple distribution of both $\hat{\pi}(\hat{\theta})$ and $\hat{\pi}$. In order to verify the properties of α, we make a hypothesis on the distribution of $\hat{\pi}(\hat{\theta})$, which is the expected capital loss based on the expected state of the bank. We expect α to have a positive influence on the decision not to withdraw. The most optimistic depositors are expected to be less likely to mistakenly withdraw.

We use the envelope theorem. As we modify parameter α of the maximization of the depositors' utility function, it shows that changes in the optimizer of the objective function do not contribute to the change in the objective function. At the threshold level, the envelope theorem indicates how the optimum fluctuates regarding the variation of a parameter (here α). For small variations of α, we can observe the variation of $1 + \lambda_2^{***}$ by using $\frac{\partial V}{\partial \alpha}$.

<u>Result 3:</u> *The more optimistic the depositors, the less likely they are to mistakenly withdraw*

$$\frac{\partial V}{\partial \alpha} = (1+\lambda_2)(1+r_{d1})\left[E\left(max\int_{\underline{\theta}_1}^{\overline{\theta}_1}\hat{\pi}_1(\hat{\theta}_1)d\hat{\theta}_1\right) - E\left(min\int_{\underline{\theta}_1}^{\overline{\theta}_1}\hat{\pi}_1(\hat{\theta}_1)d\hat{\theta}_1\right)\right](D_1 - D_2)$$
$$+ D_2(1+r_{d1})(1+r_{d2})\left[E\left(max\int_{\underline{\theta}_2}^{\overline{\theta}_2}\hat{\pi}_2(\hat{\theta}_2)d\hat{\theta}_2\right)\right. \tag{8}$$
$$\left. - E\left(min\int_{\underline{\theta}_2}^{\overline{\theta}_2}\hat{\pi}_2(\hat{\theta}_2)d\hat{\theta}_2\right)\right] > 0$$

Depositors never deposit more in period 2 than they did at the end of period 1, implying $D_1 > D_2$. Based on the distribution function of $\hat{\pi}_i(\hat{\theta}_i)$, we have a positive effect of α on the withdrawal threshold. In other words, the higher the confidence of depositors, the higher the withdrawal threshold. This situation is possible when the distribution of $\hat{\pi}_i(\hat{\theta}_i)$ is growing with the value of $\hat{\theta}_i$. As a reminder, to be considered optimistic, depositors have $\alpha > \frac{1}{2}$, to be considered pessimistic $\alpha < \frac{1}{2}$, when $\alpha = \frac{1}{2}$, they are considered indifferent which is identical to the case of asymmetric information exhibited in Gorton (1985).

3.4. The effect of ambiguity on withdrawal decisions: a visual summary

Given depositors' decisions to withdraw and the bank's decision regarding the level of equity, the situation can be summarized in Figure 1, which shows a change in the optimal decision to withdraw. The blue line indicates the optimal currency rate threshold over a decision to withdraw in a full information scheme.

Gorton (1985) explained that due to the structure of asymmetric information some depositors can mistakenly choose to withdraw when unnecessary or not to withdraw when necessary. This is materialized by the black line in Figure 1. Within the ambiguity framework, the same result can be observed, but only when depositors are indifferent, ie ($\alpha = \frac{1}{2}$).

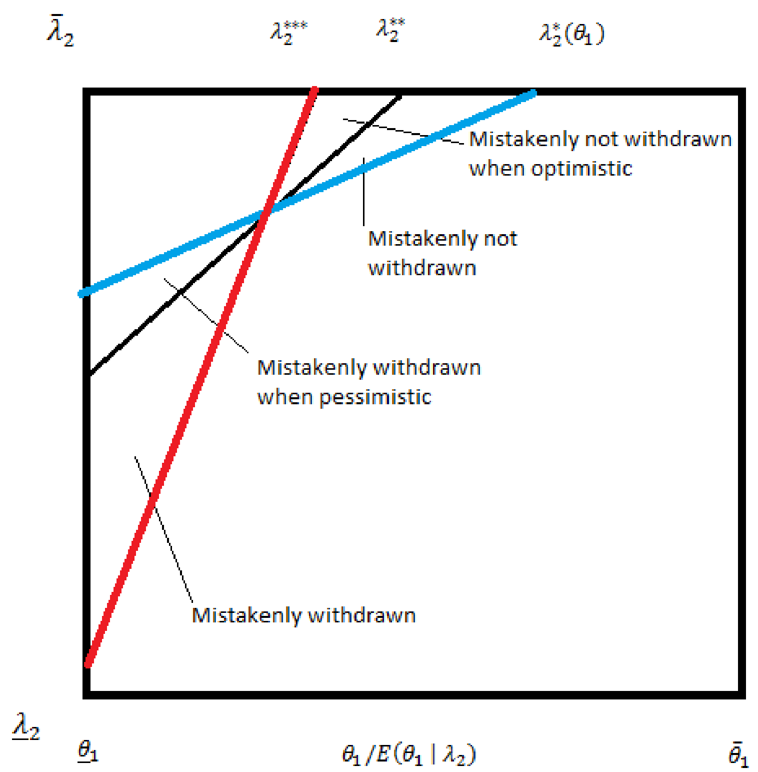

Figure 1 Withdrawing decision under ambiguity, λ_2^{**} the optimal currency return rate threshold in case of asymmetric information as of Gorton (1985) and λ_2^{***} is the optimal currency return rate threshold in case ambiguity

Ambiguity, and more especially ambiguity aversion, modifies the incentives of depositors and is represented by the red line. The preference for a known distribution of probability over an unknown distribution of probability triggers an incentive to withdraw. There are two possible cases: either a depositor is more optimistic or more pessimistic. We can interpret this as the degree of confidence based on characteristics of the depositor's economic environment, at an individual and a macro level. The degree of information the depositors have about bank fundamentals will affect their confidence. The global economic and financial outlook will modify the incentives as well. In case of optimism ($\alpha > \frac{1}{2}$) depositors are in a sort of euphoria which reduces their stimulus to withdraw early, when they should. In this situation, the ambiguity aversion reduces the ability of the depositor to efficiently discipline the bank they deposited in. The situation is represented in Figure 1. The size of the area when depositors mistakenly do not withdraw increases with and is a function of the degree of optimism. When pessimistic, the opposite effect occurs and reinforces the decision to withdraw even if it is completely irrational. The results obtained are important and confirm that ambiguity aversion creates a bias in the decision-making process. Higher levels of ambiguity do not play an active role during a bank run, but an inefficient run can occur because of ambiguity.

4. Conclusion and Policy Implications

This article shows interesting results concerning the decision of depositors to withdraw in the presence of ambiguity aversion. As expected, the nature of the relationship between these two is negative. When depositors are pessimistic about the state of the investment portfolio of a bank, the decision to withdraw is anticipated and leads to a situation which jeopardizes banking activity. Pessimistic depositors mistakenly withdraw their deposits and generate an inefficient bank run as in Chen and Hasan (2008). Those bank runs do not trigger any governance implications and emphasize banking instability.

This article is the first address the impact of ambiguity on the withdrawal decision-making process. A change in the expected probability distribution, increases significantly the possibility of depositors withdrawing earlier by mistake: depositors decide to withdraw no matter the threshold of a bank's risk. Nonetheless, the degree of optimism relies on the observance, direct or not, of macroeconomic shocks while the decision to withdraw relies on both the anticipated ability of the bank to pay back deposits and the macro shocks. The combination of the two can then reinforce the degree of pessimism at both macro- and micro-level. In the meantime, the results obtained above also show the presence of ambiguity reduces equity anticipation of the bank, emphasizing the overall financial instability.

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Appendix

a. Ambiguity and ambiguity aversion axioms

Ambiguity-averse individuals would rather choose an alternative act where the occurrence distribution of probability is known over unknown according to Epstein (1999). To model ambiguity preference schemes, Machina and Siniscalchi (2014) proposed a horse-roulette gamble where $f = \left(\ldots; P_j\ if\ E_j\ ; \ldots\right) = \left(\ldots; \left(\ldots x_{ij}, p_{ij}; \ldots\right), E_{ij}; \ldots\right)$, P_j is the roulette lottery, from a state space S and X the set of payoff. The independence property over this act is identical to the independence axiom of objective expected utility, except for the more general notion of probability mixing it entails. The probability mixtures of the horse-roulette acts are defined state wise: given act $f = (\ldots; P_j\ if\ E_j; \ldots)$ and $g = (\ldots; Q_j\ if\ E_j\ ; \ldots)$ over a common partition $\{E_1; \ldots; E_n\}$ of the state S, and probability $\alpha\ \epsilon\ (0,1)$, the mixture $\alpha. f + (1-\alpha). g$ is defined as the act:

$$\alpha. f + (1-\alpha). g = \left(\ldots; \alpha. P_j + (1-\alpha)Q_j; \ldots\right) \tag{9}$$

The axioms that characterize subjective expected utility in this framework are accordingly to Fishburn (1970):

Weak order: $\succcurlyeq$ is complete and transitive

Non-Degeneracy: It exists acts f and g for which $f \succ g$

Continuity: For all acts f, g, h if $f \succ g$ and $g \succ h$, it exists $\alpha, \beta\ \epsilon\ (0,1)$ such that $\alpha. f + (1-\alpha). h \succ g$ and $\beta. f + (1-\beta). h$

Independence: For all acts f, g, h and $all\ \alpha\ \epsilon(0,1)$, $f \succcurlyeq g$ if and only if $\alpha. f + (1-\alpha). h \succcurlyeq \alpha. g + (1-\alpha). h$

Monotonicity: For all acts f, g if the roulette lottery $f(s)$ is weakly preferred to the roulette lottery $g(s)$ for every state s, then $f \succcurlyeq g$

The five axioms taken into account, the subjective expected utility representation of the agent's preferences over the horse roulette gamble is:

$$W(f) = \int_S U[f(s)]d\mu(s) = \sum_{j=1}^{n} U(P_j).\mu(E_j) = \sum_{j=1}^{n}\left[\sum_{j=1}^{n} U(x_{ij}).p_{ij}\right]\mu(E_j) \tag{10}$$

$U(.)$ is a classic von Neumann-Morgenstern objective expected utility function. μ is a finitely additive probability measure ex-ante as in Savage (1954) axiomatization. The independence axiom implies the sure-thing principle mentioned before. The definition proposed above implies that any ambiguity model in a horse roulette act framework must relax the independence axiom.

b. Relaxing independence axiom: maxmin expected utility model

The global presentation of ambiguity was a necessary step to understand the variety of models which are used in the literature[9]. The model which applies the most to our research questions was proposed by Gilboa and Schmeidler (1989). It suggests that agents facing ambiguity aversion "are taking into account the minimal expected utility while evaluating a bet", while under an asymmetric information scheme. The Maxmin Expected Utility (MEU) is a direct extension of Ellsberg's paradox and the model is common reference for applied literature around ambiguity aversion scheme.

While using Ellsberg urn presented in section 2.2, individuals evaluate the bet on the appearance of a black ball as if absolutely none of the unknown balls in the urn were black. The associated utility function is then as follow:

$$W(f(.)) = \rho.\int U(f.))d\mu + (1-\rho).\min_{\mu \epsilon D}\int U(f(.))d\mu \tag{11}$$

Where $\rho\,\epsilon(0,1)$ represents the individual's "degree of confidence" in the estimate of the act μ_0. Gilboa and Schmeidler (1989) axiomatizes the MEU decision criteria based on horse-roulette axioms presented above. In addition Ellsberg proposed, by careful deliberation, an agent facing ambiguous situation may never "arrive at a composite estimated distribution of μ_0 that represents all his available information on relative likelihoods". Therefore, in presence of ambiguity, agents are facing more than one distribution of probability. In addition to Ellsberg (1961), Gilboa and Schmeidler (1989) weaken the independence axiom and replace it with the following one:

> ***Certainty Independence:*** *For all acts f, g all constant acts x, and all $\alpha\,\epsilon(0,1)$: $f \succcurlyeq g$ if and only if $\alpha.f + (1-\alpha).x \succcurlyeq \alpha.g + (1-\alpha).x$*

[9] The handbook chapter Machina and Siniscalchi (2014) is a perfect survey to illustrate the variety of models present in the literature which used a relaxed independence axiom

Uncertainty Aversion: *For all acts f, g and all* $\alpha \, \epsilon (0,1)$*:* $f \succcurlyeq g$ *implies* $\alpha . f + (1-\alpha) . g \succcurlyeq g$

The uncertainty aversion axiom reflects the preference for hedging. The quasi-concavity of the axiom preference's representation offers a convenient analytic property. The authors show axioms are both necessary and sufficient for the existence of the MEU representation. The model can be then generalized with α-maxmin or α-MEU model:

$$W\big(f(.)\big) = \alpha . \min_{\mu \epsilon C} \int U(f.))d\mu + (1-\alpha) . \max_{\mu \epsilon D} \int U\big(f(.)\big)d\mu \qquad (12)$$

Based upon the attitudes toward ambiguity, the generalized MEU model can be reduced to the original MEU model when $\alpha = 0$. Unfortunately, axiomatization has not been possible unless for maximum and minimum value of α.

c. Critical value computation: technical appendix.

The solution of the maximization problem is conditional on observing the currency rate λ_2. The resolution is then as follow:

$$\max V = E[\beta U(X_1) + \beta_1 U(X_2|\lambda_2)] + \beta_2[\Lambda(W)|\lambda_2] \qquad (13)$$

The maximization occurs on the gain at the end of the game using:

$$\begin{aligned} E_1(W) &= (1+\lambda_2)C_2 \\ &\quad + (1+r_{d1})(1+r_{d2})\Bigg[1 \\ &\quad - E\left(\alpha . min \int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2)d\hat{\theta}_2 + (1-\alpha) . max \int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2)d\hat{\theta}_2\right)\Bigg] \end{aligned} \qquad (14)$$

Where

$$C_2 = (1 + r_{d1}) \left[1 - E \left(\alpha . min \int_{\underline{\theta_1}}^{\overline{\theta_1}} \hat{\pi}_1(\hat{\theta}_1) d\hat{\theta}_1 + (1 - \alpha) . max \int_{\underline{\theta_i}}^{\overline{\theta_i}} \hat{\pi}_1(\hat{\theta}_1) d\hat{\theta}_1 \right) \right] \tag{15}$$

The solution of the critical value is then given by:

$$\Leftrightarrow (1 + \lambda_2)(1 + r_{d1}) \left[1 - E \left(\alpha . min \int_{\underline{\theta_1}}^{\overline{\theta_1}} \hat{\pi}_1(\hat{\theta}_1) d\hat{\theta}_1 + (1 - \alpha) . max \int_{\underline{\theta_i}}^{\overline{\theta_i}} \hat{\pi}_1(\hat{\theta}_1) d\hat{\theta}_1 \right) \right]$$

$$> (1 + r_{d1})(1 + r_{d2}) \left[1 \right.$$

$$\left. - E \left(\alpha . min \int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2) d\hat{\theta}_2 + (1 - \alpha) . max \int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2) d\hat{\theta}_2 \right) \right] \tag{16}$$

$$\Leftrightarrow (1 + \lambda_2) > \frac{(1 + r_{d2}) \left[1 - E \left(\alpha . min \int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2) d\hat{\theta}_2 + (1 - \alpha) . max \int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2) d\hat{\theta}_2 \right) \right]}{\left[1 - E \left(\alpha . min \int_{\underline{\theta_1}}^{\overline{\theta_1}} \hat{\pi}_1(\hat{\theta}_1) d\hat{\theta}_1 + (1 - \alpha) . max \int_{\underline{\theta_i}}^{\overline{\theta_i}} \hat{\pi}_1(\hat{\theta}_1) d\hat{\theta}_1 \right) \right]}$$

The withdrawing threshold appears when $\lambda_2 > \lambda_2^{***}$.

d. Result 3: envelop theorem technical appendix

The effect of α on the decision to early withdraw is determine by the following expression:

$$\frac{\partial V}{\partial \alpha} \tag{17}$$

Where V is:

$$V =$$

$$(1 + \lambda_2)(1 + r_{d1}) \left[1 - E \left(\alpha . min \int_{\underline{\theta_1}}^{\overline{\theta_1}} \hat{\pi}_1(\hat{\theta}_1) d\hat{\theta}_1 + (1 - \alpha) . max \int_{\underline{\theta_i}}^{\overline{\theta_i}} \hat{\pi}_1(\hat{\theta}_1) d\hat{\theta}_1 \right) \right] (D_1 - \tag{18}$$

$$D_2) + (1 + r_{d1})(1 + r_{d2}) \left[1 - E\left(\alpha. min \int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2) d\hat{\theta}_2 + (1 - \alpha). max \int_{\underline{\theta_2}}^{\overline{\theta_2}} \hat{\pi}_2(\hat{\theta}_2) d\hat{\theta}_2\right)\right]$$

$$\frac{\partial V}{\partial \alpha} = (1 + \lambda_2)(1 + r_{d1}) \left[E\left(max \int_{\underline{\theta}_1}^{\overline{\theta}_1} \hat{\pi}_1(\hat{\theta}_1) d\hat{\theta}_1\right) - E\left(min \int_{\underline{\theta}_1}^{\overline{\theta}_1} \hat{\pi}_1(\hat{\theta}_1) d\hat{\theta}_1\right)\right] (D_1 - D_2)$$
$$+ D_2(1 + r_{d1})(1 + r_{d2}) \left[E\left(max \int_{\underline{\theta}_2}^{\overline{\theta}_2} \hat{\pi}_2(\hat{\theta}_2) d\hat{\theta}_2\right) - E\left(min \int_{\underline{\theta}_2}^{\overline{\theta}_2} \hat{\pi}_2(\hat{\theta}_2) d\hat{\theta}_2\right)\right] > 0 \tag{19}$$

The endowment M_0 is used either to hold currency C_i or to deposit D_i. To facilitate the tractability, depositors only choose to either deposit or to hold currency, but not both at the time. M_0 is use to deposit the amount D_1 minus potential consumption X_1. D_1 is never larger than D_2: the bank cannot incur capital gain (to be redistributed) and neither receive another endowment, therefore $D_1 - D_2 \geq 0$.

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