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Optimal reciprocal import tariffs under variable elasticity of substitution*

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Abstract

We explore the impact of *reciprocal,* specific or ad valorem, import tariffs on welfare among N symmetric countries (a free-trade agreement)—using the standard Krugman's one-sector trade model, with unspecified variable-elasticity preferences (mostly under decreasing elasticity of utility). Without transport costs, any tariff is harmful, a specific import subsidy (export tariff) can be welfare-improving, whereas ad valorem tariffs or subsidies are always harmful. Under transport costs, a small ad valorem tariff can be beneficial; moreover, under sufficiently high transport costs, both kinds of tariffs can be become beneficial. The reason is mitigated distortion: excessive entry under decreasingly elastic utility.

1 Introduction

International trade liberalization has reduced tariffs substantially through multilateral free-trade agreements and trade organizations, including WTO, NAFTA, APTA, AFTA, CEFTA, GAFTA, etc. However, some tariffs and other reciprocal barriers still remain (see WTO-2003). Their economic rationale remains contradictory. On the one hand, both imports and exports are beneficial to consumers/workers, and in general, trade promotes specialization and efficiency. On the other hand, the protectionists argue that home industries need to avoid competition from imports, unemployment in certain industries, the collapse of "infant industries", and so on. Protectionism has found several theoretical rationales, as has the opposite policy. Our topic is traditional, only the details of the setting are new.

Setting. Most importantly, unlike the vast theoretical literature on "unilaterally optimal market protection," here we limit our attention to *reciprocal* tariffs, and to monopolistic competition with *variable markups.* To motivate this choice, we note that the monopolistic competition assumption is standard in New Trade (see Arkolakis et al. (2012), Demidova et al. (2011)). Unilateral tariffs under constant elasticity of substitution (CES) have already been well studied in Felbermayr et al. (2012), and Demidova (2017) studies them with variable elasticity, i.e., variable markups which are rarely studied. The reciprocity of tariffs is also a rare non-traditional assumption, though, in our view, it looks quite realistic. Countries nowadays typically choose the admissible level of industry protection *cooperatively,* during negotiations. In such cases the traditional assumption of non-cooperative myopic behavior of countries—becomes obsolete. Typically trade agreements postulate *equal* reciprocal tariff caps, because such a common restriction is easily negotiable, and we stick to this realistic assumption.

Specifically, we take the Krugman (1979) model of international trade with monopolistic competition—because it is fairly standard. Its characteristic features are a non-specified additive utility, one differentiated sector, general equilibrium (similar methodology, though without tariffs, is used in Zhelobodko et al. (2012), Mrazova and Neary (2014)). All consumers are identical, each supplies a unit of labor to the market. On the supply side, we consider homogeneous firms and one production factor – labor. There is an endogenous mass of firms; each produces one variety with increasing returns to scale. Two or more identical countries cooperatively (through negotiations) choose a common level of reciprocal import tariffs. We study both, a "specific" tariff of the iceberg type (cost-multiplicative tariff), and an ad valorem tariff, which is a fraction of price. Unlike the usual costs of transportation,

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the tariffs generate proceeds, which are redistributed to consumers in the form of lump-sum transfers. An extension of our model considers ad valorem or specific tariffs in the presence of iceberg transportation costs. The equilibrium under any given tariff coefficient τ is defined through consumers/producer optimization conditions, free entry and labor market clearing.

Our focus is on the comparative statics of equilibria: how country well-being responds to tariffs or subsidies (which are negative tariffs: $\tau < 1$). When the tariff coefficient τ changes from zero to infinity, the welfare peak should be somewhere in between, because both extremes generate too much consumption distortion towards domestic or imported goods. Finding this peak, i.e., a socially optimal level of tariff/subsidy—is a common topic in theoretical debates: does it equal $\tau^0 = 1$ (free trade) or does it lie to the right/left of from this reference point. We tackle this question without the CES assumption, considering instead increasing/decreasing elasticity of substitution, i.e., variable markups. The first reason is that nowadays the CES specification is often recognized as implausible (see Zhelobodko et al. (2012), Mrazova and Neary (2014)). More importantly, the leading idea for our setting is that changing markups can manifest changing *distortion*, which potentially can be a reason to impose tariffs or other regulations.

Our analytical **results** show the importance of increasing or decreasing the elasticity of the utility function. Namely, under CES, any regulation appears harmful, but under realistic (as we explain below) decreasing elasticity of utility (DEU), a *commodity-specific import subsidy (or export tariff) must be welfare-improving* (which contradicts common practice). The reason is that without regulation, under DEU, free entry generates an excessive mass of firms (as shown in Dixit and Stiglitz (1977)). In such a situation, a small subsidy cures this distortion somewhat, for the price of some distortion in the consumption of domestic/imported goods. This unevident conclusion reverses under (less plausible) increasingly elastic utility (IEU): here a small specific tariff is beneficial, whereas subsidies are harmful.

By contrast, *ad valorem tariffs*/subsidies can never be beneficial under any (CES, DEU or IEU) preferences, when physical trade costs are absent. In addition to this characterization of a (free trade) social optimum, we study the monotonicity of such an impact: generally, both output and welfare *decrease* in response to tariffs or subsidies.

Further, we expand this analysis to the realistic case; when specific or ad valorem tariffs operate in the presence of the usual iceberg transportation costs, and the picture changes. Under transport costs, a small ad valorem tariff improves welfare under any preferences (we provide related simulations and an incomplete proof of a related proposition). Similarly, a specific tariff also appears beneficial in some examples under transport costs, but only under sufficiently high transport costs.

Calibrating our model in a simple way, we numerically evaluate the possible extent of the damage from specific tariffs under zero transport costs. Under realistic DEU case, a 4% tariff should reduce imports by about 10%, which reduces welfare (GDP) by about 3%. The mechanism of such an impact of specific tariffs on trade is as follows. Such tariffs shrink firm size, but under DEU, firm size was already too small; the mass of firms in equilibrium exceeds the socially-optimal one Dixit and Stiglitz (1977). Essentially, the introduction of low import tariffs attracts new businesses and exacerbates non-optimality of equilibrium, adding up to the distortion of demand, expressed in the insufficient consumption of imports. By contrast, a small subsidy works in the opposite direction and somewhat cures excessive variety, outweighing the small distortion in import/domestic demand. A Similar interpretation of the effects is found under all assumptions: ad valorem tariffs and presence of transport costs.

In the **literature**, there are variety of conclusions about the benefits from tariffs under perfect or oligopolistic competition (see review in Krugman et al. (1994), Helpman (2011)), considering a differentiated good and monopolistic competition. We consider New Trade. Here, unlike us, the dominant hypothesis is constant elasticity of substitution. In particular, Gros (1987) shows the (negative) effects of a tariff war. Under CES preferences and the second sector of the economy, Venables (1987) considers some benefits from import tariffs, but, unfortunately, the conclusions are based on the hypothesis of unequal utility from domestic and imported goods. Demidova et al. (2011) explores a small open Melitz economy with tariffs; preferences are CES, firms are heterogeneous in costs and only strong firms can export. The number of foreign firms and the foreign demand function are fixed and some import tariffs increase well-being. Another Melitz model (unfortunately, ignoring the effects of general equilibrium) is Cole and Davies (2009), they derive the optimal tariff with quasi-linear preferences. A more elaborated Melitz economy with CES preferences is considered in two articles Felbermayry et al. (2013) and Felbermayr et al. (2012). Like ours, it includes mainly symmetric countries. There is an estimation method for gains/losses from tariff reductions, considering reciprocal tariffs between the two countries. This approach (except for CES) is similar to ours,as is Ossa (2011). The latter article considers a model like ours, but with CES preferences and with the second sector. This feature creates a cross-subsidization between two industries (see also Pflüger and Suedekum (2012)). As a result, a one-way tariff may be beneficial, but reciprocal tariffs (a tariff war) lead to a welfare loss, which is consistent with our findings under CES. An earlier article with CES preferences and similar negative conclusions for bilateral tariff - is Jorgensen et al. (2007). The most closely related is Demidova (2017), where a unilateral tariff is studied in Melitz-Ottavino setting with heterogeneous firms and non-additive quadratic preferences (without an outside good). A small tariff can be beneficial, but the effect is different from ours, being driven by the unilateral nature of the tariff and complicated selection effects among firms.

Summarizing, several articles have considered the optimal tariff problem under various hypotheses about sectors, preferences and heterogeneous firms. However, our tariff effects associated with variable elasticity of substitution and tariffs curing excessive variety – have been little studied. We fill this lacuna in sufficient generality: under any additive utilities.

Section 2 introduces a model with both specific and ad valorem tariffs (or subsidies) and formulates the equilibrium conditions. Section 3 presents the analytic comparative statics: changes in outputs, mass of firms and welfare in response to tariffs, ad valorem or specific , and gives simulation for several utility functions. Section 4 presents the extension of the results for tariffs combined with transport costs – through simulations and proofs. The conclusion summarizes and the appendices contain the mathematical proofs.

2 Model

This section introduces the universal trade model, which includes all cases: specific or ad valorem tariffs/subsidies, with or without transport costs. It investigates the welfare impact of bilateral or multilateral trade tariffs.

We consider a model of trade between $K+1$ countries, comprising the set $\mathcal{K} \equiv \{1, ..., K+1\}$. There is one differentiated good and one aggregate factor of production traditionally called labor. The mass of similar workers – consumers in any country k is L_k (the most conclusive results below relate to the symmetric case $L_k = L_j$, but there are some results for the asymmetric case also). Each firm i from country k produces its own variety indexed ik.

Consumers. The preferences of consumers in all countries are the same and given by the following additively-separable utility function

$$U = \sum_{k \in \mathcal{K}} \int_0^{N_k} u(x_{ikj}) \mathrm{d}i,$$

where $ikj \equiv (i)kj$ is the index of a specific variety produced by firm ik in country k and sold in country j, where i takes values from a continuum. N_k is the total mass of (continuous) varieties produced in country k, related consumption is $x_{ik} \equiv x_{(ik)}$ is a function of i (index $_{ik}$ here after is used interchangeably as the function argument $x_{(ik)}$, to economize brackets), the whole infinite-dimensional consumption vector is $\mathbf{x}_j \equiv \left\{x_{(i)kj}\right\}_{i \in \{0,N_k\}, k \in \mathcal{K}})$.

Function $u(\cdot)$ is the "elementary utility", it depends on consumption x_{ikj} of a single variety.

Assumptions, traditionally for the theory of monopolistic competition, function u is strictly increasing, strictly concave, thrice differentiable, and satisfies the boundary conditions

$$u(0) = 0, \;\; u'(0) > 0, \;\; u'(\infty) \leq 0.$$

The price of a variety is denoted by p_{ikj}. With these notations, utility maximization of every consumer in any country j can be written as:

$$\begin{gathered} \sum_{k \in \mathcal{K}} \int_0^{N_k} u(x_{ikj}) \mathrm{d}i \to \max_{\mathbf{x}^j}, \\ \sum_{k \in \mathcal{K}} \int_0^{N_k} p_{ikj} x_{ikj} \mathrm{d}i = w_j + T_j. \end{gathered} \tag{1}$$

Here T means some transfer from the government paid from its tax revenue, while each consumer's labor endowment is 1denoted as wage w_j, income becomes $w_j + T_j$. However, when maximizing her utility, each "small" consumer ignores the dependence of this transfer on her choice. The the consumer's first-order optimality condition defines the *inverse demand function* for ik-variety as

$$p_{ikj} = \frac{u'(x_{ikj})}{\lambda^j}, \quad k, j \in \mathcal{K}. \tag{2}$$

Here the Lagrange multiplier λ^j is the marginal utility of income. This market statistic, analogous to a price index, expresses the degree of competition in country j.

Demand properties. Generally, we denote elasticity of any function g in several forms

$$\mathcal{E}_g(x) \equiv \overset{\varepsilon}{g}(x) \equiv \frac{xg'(x)}{g(x)}$$

and sometimes write here the straight symbol E instead of calligraphic $\mathcal{E}$ when we need to express *total* elasticity instead of *a partial* one, like the total derivative d differs from the partial derivative ∂. As in Krugman (1979) and Zhelobodko et al. (2012), we use the Arrow-Pratt measure of concavity of elementary utility ("love for variety") and concavity of u':

$$r_u(x) \equiv -\frac{xu''(x)}{u'(x)} > 0, \;\; r_{u'}(x) \equiv -\frac{xu'''(x)}{u''(x)}. \tag{3}$$

Love for variety is the (absolute value of) the inverse demand elasticity:

$$r_u[x(p)] \equiv -\frac{1}{\varepsilon_x(p)} \equiv -\mathcal{E}_{u'}(x). \tag{4}$$

When $r_u(\cdot)$ increases (decreases) with consumption, the inverse demand becomes less (more) sensitive to the consumption of this variety, which yields important market effects. The popular CES utility function ($u(x) = x^\rho : 0 < \rho \leq 1$) entails a constant love for variety $r_u(x) \equiv 1 - \rho$. Another popular utility called $CARA$ in Behrens and Murata (2012) is $u(x) = 1 - \exp(-\alpha x)$ which entails a linear love for variety $r_u(x) = \alpha x$. More generally, we distinguish two cases: increasing elasticity of (inverse) demand (IED): $R'_u(x) > 0$ and decreasing elasticity (DED): $R'_u(x) < 0$. These two cases respond differently to changes in market parameters, the former looking more realistic economically.

Another characteristic, important for comparing equilibrium and social optimum according to Dixit and Stiglitz (1977) and Dhingra and Morrow (2012), is the elasticity of the utility:

$$\mathcal{E}_u(x) \equiv \frac{xu'(x)}{u(x)} > 0.$$

For elasticity of utility, we distinguish two cases: increasing elasticity of utility (IEU): $\mathcal{E}'_u(x) > 0$, and decreasing elasticity of utility (DEU): $\mathcal{E}'_u(x) < 0$, the latter looking more realistic.

Additional assumptions, imposed further on utility u include:

$$r_u(z) < 1, \;\; r_{u'}(z) < 2, \;\; \forall z \geq 0 \tag{5}$$

$$\lim_{z\to 0}[u'(z)] > 0 \qquad \lim_{z\to\infty}[u'(z) + zu''(z)] \leq 0,$$

they are traditional, see Zhelobodko et al. (2012) and other papers on monopolistic competition. The boundary conditions indicate positive marginal utility at zero and saturable demand at infinity, for equilibria existence. The condition on r_u means elasticity of demand greater than 1 is needed for monopolistic pricing. The condition on $r_{u'}$ means the revenue and profit is concave, i.e., second-order condition $2u''(z) + zu'''(z) \leq 0$. In further analysis we assume these conditions are satisfied.

Producers. Each firm produces a single variety, there is a one-to-one correspondence between firms and varieties. Each firm has the same labor requirement $c > 0$ to produce one unit of product and same fixed cost $F > 0$. Respectively, the cost function $C(q) \equiv (F + cL \cdot q)\, w$ for per-consumer output q under wage w — shows economies of scale (gross output is $L \cdot q$).

Wages and all prices are expressed in some world currency, immaterial for equilibria. Assuming some trade agreements, similar to the WTO, we consider *a uniform* trade tariff τ across all countries.

In practice, trade tariffication may consist of two components: t^{sp} denotes a specific tariff per unit of an imported good (per kilogram), whereas $t^{ad} < 1$ denotes an ad valorem tariff, which is the share of the revenue $p_{ikj}x_{ikj}$ which the state receives from importing a quantity x_{ikj}. When positive, $t^{sp} \geq 0$, $t^{ad} \geq 0$ mean tariffs, we also consider import subsidies , i.e., $t^{sp} < 0$, $t^{ad} < 0$. Related per capita governmental transfer T_j, consists of payments from both components of tariffs:

$$T_j = \sum_{k\in\mathcal{K}\setminus\{j\}} \int_0^{N_k} \left(p_{ikj}t^{ad} + t^{sp}\right) x_{ikj} \mathrm{d}i.$$

Thus, similar consumers get back the tariffs they have paid and generally do not gain or lose money, but the taxation changes their behavior.

We reformulate such tariffs in the form of *a uniform* iceberg-type trade coefficient

$$\tau \equiv \tau^{ad} \cdot \tau^{sp} = \frac{1}{1-t^{ad}} \cdot (1 + \frac{t^{sp}}{c})$$

In other words, the revenue of country k from the sales of goods of firm i in country j decreases $\tau^{ad} = \frac{1}{1-t^{ad}}$ times because of the ad valorem tariff, and somehow decreases due to the specific tariff, because the unit cost c increases to $c(1 + \frac{t^{sp}}{c}) = c \cdot \tau^{sp}$. (In principle, we could immediately consider transportation costs combined with customs tariffs, but we postpone analyzing these costs, to simplify the analysis now).

In any model of monopolistic competition, each manufacturer has some degree of monopoly power. She is able to price-discriminate across sub-markets, taking the demand functions and the degree of competition λ_j everywhere as given. Maximizing profit π_{ij} of i -th firm in country j can equivalently be performed in prices or quantities:

$$\pi_{ij} \equiv [p_{ijj} - cw_j] L_j x_{ijj} + \sum_{k \in \mathcal{K} \setminus \{j\}} \left[\frac{p_{ijk}}{\tau^{ad}} - \tau^{sp} c w_j \right] L x_{ijk} - F w_j = \tag{6}$$

$$= \left[\frac{u'(x_{ijj})}{\lambda_j} - cw_j \right] L_j x_{ijj} + \sum_{k \in \mathcal{K} \setminus \{j\}} \left[\frac{u'(x_{ijk})}{\lambda_k \tau^{ad}} - \tau^{sp} c w_j \right] L x_{ijk} - w_j F \to \max_{\mathbf{x}_{ijk}} .$$

Since the profit function is strictly concave under our assumptions (5), each producer i in a country behaves similarly, so, we drop index i . We also introduce the "normalized revenue function"

$$R(z) \equiv z u'(z), \tag{7}$$

extensively exploited further as an alternative "primitive" of the model, instead of $u(\cdot)$.

In these terms, the first-order conditions of profit maximization says that marginal revenue equals marginal costs:

$$\frac{R'(x_{jj})}{\lambda_j} L_j = w_j c L_j, \quad \frac{R'(x_{jk})}{\lambda_k} L_k = \tau \cdot w_j c L_k \equiv \tau^{sp} \cdot \tau^{ad} w_j c L_k. \tag{8}$$

Then, taking into account the identity $R'(x_{jk}) \equiv u'(x_{jk}) \cdot (1 - r_u(x_{jk}))$ and prices $p_{jk} = \frac{u'(x_{jk})}{\lambda_k} = \frac{\tau c w_j}{1-r_u(x_{jk})}$, $p_{jj} = \frac{u'(x_{jj})}{\lambda_j} = \frac{c w_j}{1-r_u(x_{jj})}$, $p_{kj} = \frac{u'(x_{kj})}{\lambda_j} = \frac{\tau c w_k}{1-r_u(x_{kj})}$, the conditions on marginal revenue in all countries can be expressed either in relative λ_j or in relative wages:

$$\frac{R'(x_{jj})}{R'(x_{jk})} = \frac{\lambda_j}{\lambda_k} \cdot \frac{1}{\tau}, \tag{9}$$

$$\frac{R'(x_{jj})}{R'(x_{kj})} = \frac{w_j}{w_k} \cdot \frac{1}{\tau}. \tag{10}$$

These equations imply that the ratio $\frac{\lambda_j}{\lambda_k}$ of the marginal utilities of income equals the ratio of domestic/export prices. We proceed so far without using country symmetry, while afterwards we use $\frac{w_j}{w_k} \equiv 1$.

Equilibrium. Firms enter the market while profit remains positive, while at equilibrium it must vanish, so, the free entry condition for each country j is

$$\pi_j \equiv \left[\frac{u'(x_{jj})}{\lambda_j} - cw_j \right] L_j x_{jj} + \sum_{k \in \mathcal{K} \setminus \{j\}} \left[\frac{u'(x_{jk})}{\lambda_k \tau^{ad}} - \tau^{sp} c w_j \right] L_k x_{jk} - w_j F = 0.$$

Using $\frac{x_{jj} u'(x_{jj})}{\lambda_j} = \frac{x_{jj} c w_j}{1-r_u(x_{jj})} = \frac{R(x_{jj}) c w_j}{R'(x_{jj})}$, the free entry condition can be rearranged as

$$L_j \frac{R(x_{jj})}{R'(x_{jj})} + \tau^{sp} \cdot \sum_{k \in \mathcal{K} \setminus \{j\}} L_k \frac{R(x_{jk}) w_k}{R'(x_{jk}) w_j} = \frac{F}{c} + L_j x_{jj} + \tau^{sp} \cdot \sum_{k \in \mathcal{K} \setminus \{j\}} L_k x_{jk}, \tag{11}$$

Further, labor market clearing means that the total costs of firms should equal the total labor supply in each country:

$$N_j \cdot \left(F + cL_j x_{jj} + \sum_{k\in\mathcal{K}\setminus\{j\}} cL_k x_{jk} \right) = L_j \tag{12}$$

As usual in such models, at equilibrium the labor market clearing entails another constraint (see Appendix 1), which is the consumer's budget for each country,

including the tax revenue $T_j = \sum_{k\in\mathcal{K}\setminus\{j\}} N_k \left(p_{kj}t^{ad} + t^{sp}\right) x_{kj}$:

$$\sum_{k\in\mathcal{K}} N_k p_{kj} x_{kj} = w_j + \sum_{k\in\mathcal{K}\setminus\{j\}} N_k \left(p_{kj}t^{ad} + t^{sp}\right) x_{kj}. \tag{13}$$

Further, we mainly consider symmetric countries: $L_j = L_k = L$ and *the same* reciprocal tariff τ (more common than unilateral tariffs). To summarize the model, for symmetric countries the export-import balance is guaranteed automatically, and the symmetric wage of each country can be used as a numeraire:

$$w_k \equiv 1, \quad \forall k \in \mathcal{K}.$$

Symmetric equilibrium. Under reciprocal tariffs τ^{sp}, τ^{ad} on imports, the *symmetric free-entry equilibrium* consumptions $\mathbf{x} \in \mathbf{R}^{(K+1)^2}$, and number of firms $N \in \mathbf{R}^{K+1}$ are those that satisfy labor balances (12), budgets (13) and the following $(K+1)^2 + K$ firm equations:

$$\frac{R'(x_{jj})}{R'(x_{kj})} = \frac{1}{\tau^{sp}\tau^{ad}}, \forall j,k \in \mathcal{K} \tag{14}$$

$$L\frac{R(x_{jj})}{R'(x_{jj})} + \tau^{sp} \cdot \sum_{k\in\mathcal{K}\setminus\{j\}} L\frac{R(x_{jk})}{R'(x_{jk})} = \frac{F}{c} + Lx_{jj} + \tau^{sp} \sum_{k\in\mathcal{K}\setminus\{j\}} Lx_{jk}, \tag{15}$$

whereas equilibrium prices are determined from x_{jk} as

$$p_{jj} = \frac{c}{1 - r_u(x^{jj})}, \; p_{kj} = \frac{c\tau^{sp}\tau^{ad}}{1 - r_u(x^{kj})}. \tag{16}$$

The Welfare of a citizen in country j is her (symmetric across varieties) utility

$$W_j = \sum_{k\in\mathcal{K}} N_k u(x_{kj}).$$

To simplify our equations using symmetry, all (the same) imported consumptions is denoted $x_{kj} = x_{il} = y$, while the consumptions of domestic goods is denoted $x_{kk} = x_{jj} = x$. The number of firms is also symmetric: $N^k = N^j = N$ and wages are normalized to $w_k \equiv 1$. Then the equilibrium equations (9), (10), (11) for finding consumption are simplified as follows:

$$\frac{R'(x)}{R'(y)} = \frac{1}{\tau^{sp}\tau^{ad}}. \tag{17}$$

$$\frac{R(x)}{R'(x)} - x + K\tau^{sp}\left(\frac{R(y)}{R'(y)} - y\right) = \frac{F}{cL} \equiv \frac{f}{c}, \tag{18}$$

where f is the notation for per-consumer investment used further. The budget or labor balance (13) for finding the mass of firms N in each country takes the form:

$$\frac{f}{c} + x + Ky = \frac{1}{cN}. \tag{19}$$

In these terms, the *symmetric free-entry equilibrium* consumption (x, y) is determined only by two equations (35)–(36), while N is determined subsequently. Using the labor balance, the per-consumer welfare function can be simplified as a function of the consumption variables only (and parameters f, c, K):

$$W(x,y) = \frac{u(x) + Ku(y)}{f + cx + cKy}. \tag{20}$$

3 The impact tariffs when transport costs are absent

Now our task is to study the equilibrium response to reciprocal tariffs, especialy the welfare consequences. To better display ideas, we start with the simple case of K symmetric countries without trade costs, postponing the extensions to Section 4. Unlike Gros (1987), Demidova et al. (2011) and Demidova (2017), we do not focus completely on optimal tariffs, revealing instead the impact of *any* level of tariff, ranging from zero to infinity. Our goal is to see whether any reciprocal tariff/subsidy can be welfare-enhancing for *all* countries involved in a trade agreement.

The intuition about the impact of import tariffs on equilibria is as follows. Higher import prices redirect spending from foreign goods to domestic ones. This alters the composition of demand (while total demand cannot fall too much, because the tariffs revenue compensates for high prices). Such an active trade policy artificially distorts consumer choice. On the other hand, it can work towards a more efficient number of firms in the world economy, which is known to be non-optimal in a non-CES world. Which effect dominates, harm or benefit?

In practical trade relations, for any specific good, usually either ad valorem or specific tariffs are applied. Therefore, we first consider only ad valorem tariffs τ^{ad} in Subsection 3.1, then study specific ones: τ^{sp} in Subsection 3.2. In addition, in Section 4, we study several extensions of our model: the impact of tariffs for non-zero transport costs. Moreover, in addition to tariffs, we study the trade licensing in foreign markets (the license is considered an unchanged entry cost).

3.1 The impact of ad valorem tariffs

3.1.1 The impact of ad valorem tariffs on consumption and variety

This subsection studies impact of import tariffs on trade, denoted here as $\tau \equiv \tau^{ad}$, $\tau^{sp} \equiv 1$. To highlight the dependence of domestic and foreign consumption upon tariffs, consumptions is denoted as $(x_\tau, y_\tau) \equiv (x(\tau), y(\tau))$, respectively, under free trade (tariff-coefficient $\tau = 1$) these consumption is denoted as (x_1, y_1) and compared with (x_τ, y_τ). Similarly we compare a regulated and non-regulated mass of firms ($N_1 \gtreqless N_\tau$), regulated and non-regulated per-consumer firm output $q_\tau \equiv x_\tau + K y_\tau \gtreqless q_1 \equiv x_1 + K y_1$ (magnitude q_τ is variety sales per consumer, while gross output per firm amounts to $Q_\tau = L x_\tau + L K y_\tau$). Using these notations, we now express the total derivatives of consumption in τ through the elementary revenue function $R(\cdot)$ (7) and points (x, y) of differentiation.

Differentiating our equations (17)-(18) in tariff τ, we come to the following linear equations w.r.t. total derivatives $x'_\tau \equiv dx_\tau/d\tau$, $y'_\tau \equiv dy_\tau/d\tau$:

$$R'(x) + \tau R''(x)\, x'_\tau = R''(y)\, y'_\tau$$

$$\left[1 - \frac{R''(x)}{R'(x)}\right] x'_\tau + K\left[1 - \frac{R''(y)}{R'(y)}\right] y'_\tau = x'_\tau + K y'_\tau.$$

Solving these equations (we recall $R'(x) > 0$, $R''(x) < 0$, see Appendix 2) we express and estimate the impact of a tariff on consumption as follows:

$$x'_\tau = -\frac{KR(y)}{\tau^2 R''(x)\left(x + Ky + \frac{f}{c}\right)} \geq 0\,, \tag{21}$$

$$y'_\tau = \frac{R(x)}{R''(y)\left(x + Ky + \frac{f}{c}\right)} \leq 0, \tag{22}$$

which means *increasing* domestic consumption and *decreasing* imports (naturally).

Combining these expressions (see Appendix 2), we express firm per-consumer output in (x, y):

$$q'_\tau = x'_\tau + K y'_\tau = K \cdot \frac{\tau^2 R(x) R''(x) - R(y) R''(y)}{\tau^2 R''(y) R''(x)\left(x + Ky + \frac{f}{cL}\right)}. \tag{23}$$

In particular, $R''_x = R''_y = -\beta = const < 0$ for linear inverse demand function $\alpha - \beta z$. We conclude that *for linear demand the firm's output q must globally decrease* over the whole interval $(1, \tau_a)$, connecting free trade with autarky.

To find a similar property in a more general situation, observe that $(2u''_x + xu'''_x)$ must be negative because of SOC, but the negativity of the whole sum does not follow from any equilibrium condition. Also, this "uniform demand flatness" does not follow from another kind of "demand flatness." The latter assumption, which is popular in the literature, is increasingly elastic demand (IED), which means

$$r'_u(x) \equiv \frac{\partial}{\partial x}\frac{xu''_{(x)}}{u'_{(x)}} = \frac{\left(xu'''_{(x)} + u''_{(x)}\right)u'_{(x)} - x\left(u''_{(x)}\right)^2}{\left(u'_{(x)}\right)^2} > 0\ \forall x \quad (IED),$$

while the opposite assumption will be called DED: $r'_u(x) < 0\ \forall x$.[1]

We define one more characteristic of the utility function — increasing elasticity (IEU) or decreasing elasticity (DEU) and by the related behavior of the revenue function R:

$$\mathcal{E}_{u(x)} \equiv \frac{u'(x)\,x}{u(x)}, \qquad R(x) \equiv u'(x)\,x = u(x)\cdot\mathcal{E}_{u(x)}.$$

To explain the importance of these characteristics, note that revenue R differs from utility u by multiplier $\mathcal{E}_{u(x)}$. Generally, market forces maximize revenue, whereas a social planner pursues maximum utility; these goals are completely "aligned" only under CES-CEU preferences Dhingra and Morrow (2012). That is why the difference between IEU and DEU cases can generate socially excessive or insufficient consumption x. This brings insufficient or excessive variety N, inversely related to the volume of consumption . Since Dixit and Stiglitz Dixit and Stiglitz (1977), we know that a closed economy with DEU (IEU) preferences generates excessive (insufficient) variety, whereas the only case of a socially-optimal variety N is CES: $\mathcal{E}_{u(x)} = const$. In our trade model, exactly the same discrepancy between DEU and IEU distortion takes place under free trade which is an integrated economy (zero tariffs).

The sign of the numerator in (23) determines the sign of the derivative q'_τ. Using FOC, we can write (23) as

$$q'_\tau = \frac{K\left(R'(y)\right)^2}{\tau^2 R''(y)R''(x)\left(x + Ky + \frac{f}{c}\right)}\cdot\left(\frac{R(x)R''(x)}{\left(R'(x)\right)^2} - \frac{R(y)R''(y)}{\left(R'(y)\right)^2}\right). \tag{24}$$

Let us denote function $\phi(z) \equiv \frac{R(z)R''(z)}{(R'(z))^2}$. Let us note that function $\phi(\cdot)$ is decreasing under IED for the well-known and widely-used functions $CARA$, $AHARA+$, which means $u = (a+z)^\rho - lz - z^\rho$ $(a \geq 0,\ l \geq 0)$ and $LOG+$ $(u = \log(a+z)$ $(a \geq 0)$, see Appendix 2). Therefore, the derivative of the total output q'_τ is negative under a positive tariff $\tau > 1$. In the case of subsidies (i.e., when $\tau < 1$ and $x < y$), we get $q'_\tau > 0$.

We also use the following complicated condition on concavity r_u of utility u, explained later on:

$$\left[\left(\mathcal{E}_{r'_u}(z) + 2\right)\cdot\left(1 - r_u(z)\right) + 2\cdot r'_u(z)\cdot z\right]\cdot r'_u(z) > 0. \tag{25}$$

Using these restrictions, the following proposition describes the general impact of an ad valorem tariff on output, consumption, prices and variety. It discusses the local and global changes over the interval $(1, \tau_a)$ between the free trade point $\tau = 1$ and some finite or infinite autarky-point called $\tau_a:\ y_{\tau_a} = 0,\ (\tau_a \leq \infty)$.[2]

Proposition 1. *Under K symmetric countries, an increase in the reciprocal ad valorem import tariff coefficient τ modifies the trade equilibrium as follows:*

(i) Domestic individual consumption increases: $dx_\tau/d\tau > 0\ \forall\tau \in (0, \tau_a)$, whereas imports decrease: $dy_\tau/d\tau < 0\ \forall\tau \in (0, \tau_a)$, displaying derivatives (21)–(22); domestic consumption always remains bigger than import ($x > y$) under positive tariffs $\tau \in (1, \tau_a)$.[3]

(ii) The firm's output $q_\tau \equiv x_\tau + Ky_\tau$ reacts to the derivative (23), which has zero impact at the free trade situation $\tau = 1$, but negative impact at the autarky point τ_a (if τ_a is finite). In between, at any $\tau \in (1, \tau_a)$ the firm's output monotonically decreases in τ iff the condition (25) on preferences holds. In particular, monotonicity holds under IED preferences satisfying condition $\mathcal{E}_{r'_u}(z) + 2 \geq 0$.[4] *By*

[1] In their comprehensive study of the demand properties (Mrazova and Neary (2017)), call IED "subconvex" whereas DED is "superconvex." Condition (29) involves the fourth derivative of u and therefore cannot be derived from FOC and SOC of the equilibrium, involving only the third derivative and smaller ones.

[2] Our tariff impact on consumptions turns out rather similar to impact of *trade cost* on consumptions, studied in Mrazova and Neary (2014). However, in Mrazova and Neary (2014) monotonicity of outputs/variety is characterized only in the neighborhood of free trade. Specifically, the impact of trade costs/tariffs on per-variety consumption is the *same* in their and our (rather similar) models, but outputs differ.

[3] As a corollary, the finite differences are $\left(x^1 < x^\tau\ \forall\tau < \tau_a\right)$, $\left(y^1 > y^\tau\ \forall\tau < \tau_a\right)$. As to import subsidies $s:\ \tau \equiv 1 - s \in (0,1)$, their impact simply mirror the tariff impact on consumptions: $(dx^s/ds < 0)$, $(dy^s/ds > 0)$.

[4] Examples, satisfying both these conditions and thus yielding a monotone impact of tariffs on output – are linear demands and other "flat" demands, e.g., those generated by utility functions $CARA$, $AHARA+$, $LOG+$. These conditions are known to govern some other demand properties (Mrazova and Neary (2017)).

contrast, under some DED functions output $q_\tau \equiv x_\tau + Ky_\tau$ *may increase with any tariff* $\tau \in (1,\infty)$.[5]

(iii) The equilibrium mass of firms N *always responds to any tariff* τ *inversely to the firm output, i.e.,* $dq_\tau/d\tau \cdot dN_\tau/d\tau < 0$ *and* $dq_\tau/d\tau < 0 \Leftrightarrow dN_\tau/d\tau > 0$.

(iv) Under IED preferences, prices always change opposite to consumption: domestic price $p_x = \frac{c}{1-r_u(x)}$ *decreases with* τ *, while the import price* $p_y = \frac{c\tau}{1-r_u(y)}$ *increases.*

Proof: Behavior of consumption/output (21)–(22), (23) has already been proved above. For proof of items (ii), (iii) see Appendix 2, while (iv) is obvious. ⊠

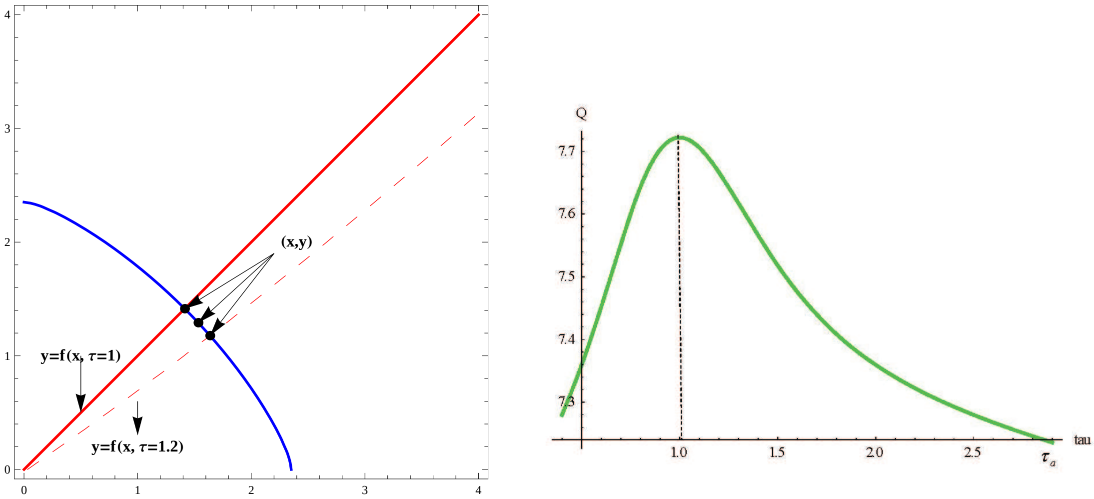

Figure 1: *Ad valorem tariffs under IED: positive impact on domestic consumption x, negative on import y and* output *q*.

For Proposition 1, the geometric intuition behind the changes in consumption, can be explained with Fig. 1 (constructed under specific parameters $u(z) = (0.1+z)^{5.1/6} - 0.1^{5.1/6} - 0.2 \cdot z$, $K = 1$, using the geometry suggested in Mrazova and Neary (2014)). This figure shows that individual domestic consumption x and export consumption y are positively related to the (producer's) FOC equation (17): $y = \psi(x,\tau) = \acute{R}^{-1}(\tau \acute{R}(x))$, the increasing red line. However, these magnitudes are negatively related through the (blue) zero-profit equation (18) under $\tau^{sp} \equiv 1$ and function $\varphi(z) \equiv R(z)/R'(z) - z$, which can be written as $\varphi(x) + K\varphi(y) = f$ (because the function $\varphi(z)$ is increasing). The unique intersection of these two curves is the free-entry equilibrium (x_τ, y_τ) among symmetric countries. The FOC curve should be shifted downwards by tariff τ in such a way (shown by the dashed line), that solution x_τ increases, y_τ declines, and their sum $q_\tau = x_\tau + Ky_\tau$ decreases monotonically whenever $\varphi(\cdot)$ is concave, which is given by condition (25).

Economically, these natural quantity effects mean that import consumption y (becoming expensive) responds to tariffs by shrinking, being replaced by domestic consumption x. Total consumption $q = x + Ky$ must shrink whenever the demand characteristic $R(x)/R'(x) - x$ is concave, which we argue for *IED* preferences, also satisfying the modest technical assumption mentioned. Further, output q always changes opposite to variety N, because the labor balance (19) contains their product Nq.

Next subsection addresses our main question of interest: Are these *opposite changes of quantity and variety* beneficial for our consumer or not?

3.1.2 The impact of ad valorem tariffs on welfare

Now we turn to the welfare consequences of the changes in consumption and variety revealed in Proposition 1. We see that the distinction between the positive and negative results of using ad valorem tariffs — is governed not only by *IED/DED* properties of demand, but also by increasing elasticity (*IEU*) or decreasing elasticity (*DEU*) of the utility function. Since Dixit and Stiglitz (1977), these characteristics are known to govern the distortions in consumption, i.e., deviations from the social

[5] As to import subsidies $s: \ \tau \equiv 1 - s \in (0,1)$, they mirror the tariff effect on outputs: $(dq^s/ds < 0)$ under natural condition (25), i.e., point $\tau = 1$ shows the maximal output among $(0, \tau_a)$. When the autarky point τ_a is finite, *IED* property at τ_a is guaranteed.

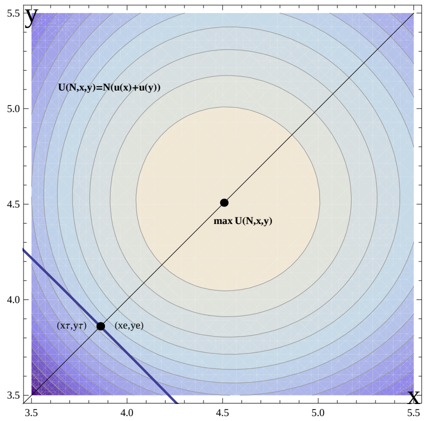

Figure 2: The indifference curves of welfare $W(x, y)$, the (decreasing) loci of all equilibrium consumptions (x_τ, y_τ) $\forall\tau$, and the (free-trade) socially optimal equilibrium (x_1, y_1), under DEU and ad valorem tariffs.

optimum.[6] The size of the firm in the free-trade equilibrium under $DEU(IEU)$ is greater than (less than) the optimal one. Tariffs somehow change this distortion. Our goal is to show, that positive ad valorem tariffs or subsidies *aggravate* such distortions under a natural combination of properties $IED - DEU$, and also under the combination $DED - IEU$ (below, we discuss these assumptions and why other combinations are excluded).

We now formulate such a proposition, using each consumer's welfare function $W(x, y)$ (20), illustrated in Fig. 2. In the formulation, we consider two intervals of changing τ: from some point τ_{x0}, where the domestic consumption disappears ($x_{\tau_{x0}} = 0$) until free trade, and further from the free trade point $\tau = 1$ to some autarky point $\tau_a \leq \infty : \ y_{\tau_a} = 0$.

Proposition 2. *Symmetric ad valorem tariffs in K symmetric countries have the following impact on welfare:*

(i) Locally, at free trade ($\tau = 1$) welfare has a zero first derivative with a tariff τ in any case.

(ii) Under the non-strict $IED - DEU$ assumption ($\mathcal{E}'_u(x) \leq 0$, $r'_u(x) \geq 0$), or the strict $DED - IEU$ assumption ($\mathcal{E}'_u(x) > 0$, $r'_u(x) < 0$), welfare W has a strict argmaximum in a free trade situation $\tau = 1$ and decreases in τ at any positive ad valorem tariff $\tau \in (1, \tau_a]$, it also decreases in s at any subsidy level: ($\tau = 1 - s \in [\tau_{x0}, 1)$).

Proof: see Appendix 2. ⊠

Discussing this proposition, let us explain the main idea of welfare maximum at free trade – with the help of Fig. 2. Recall that consumer welfare is expressed through consumption as

$$W_\tau(x, y) = \frac{u(x) + Ku(y)}{f + c(x + Ky)}.$$

Since welfare is a concave function of (x, y) divided by a positive linear function, $W_\tau(\cdot)$ must be quasi-concave (see Theorem 52 in Martos, 1976).

This implies a convex upper Lebesgue set $L^{++}(x, y) \equiv \{(\tilde{x}, \tilde{y}) | \, W_\tau(\tilde{x}, \tilde{y}) > W_\tau(x, y)\}$, "set of better points", relative to any point (x, y). Then, at the free trade point $(x_1, y_1) : \ x_1 = y_1$, this convex set L^{++} of better points can be *separated* by a constant-output line $x + Ky = const = x_1 + Ky_1$ from all equlibria points. By Proposition 1, all equlibria points lie weakly below the constant-output line ($x + Ky \leq x_1 + Ky_1$), as we see in Fig.2.[7]

Fig.2 explains the geometric intuition for the effects described in Proposition 2 under parameters ($K = 1, L = 1$, $u = (0.1 + x)^{5.1/6} - 0.1^{5.1/6} - 0.2 \cdot x$). We display various levels of welfare $W_\tau(\cdot)$ as a function of consumption x, y (the lighter the higher) and observe quasi-concave welfare. An important DEU case is presented, this welfare function has its argmaximum (x^o, y^o) (social optimum) *above* all equilibria, which means a socially insufficient equilibrium output $Q = L \cdot (x_\tau + y_\tau)$ under any τ. Here all equilibria ($(x_\tau, y_\tau)\,\forall\tau$) are displayed with the black downward-slope curve (x_τ, y_τ) of equilibria

[6] See also Dhingra and Morrow (2014).

[7] The complete proof is more involved, because the statement of Proposition 2 provides more than just welfare maximum at $\tau = 1$: not only welfare is everywhere lower than one at free trade, but also W_τ *strictly decreases* with governmental intervention everywhere outside $\tau = 1$.

responses to τ. At free trade $((x_1, y_1), \tau = 1)$ the 45-degree line $x + Ky = const = x_1 + Ky_1$ is tangent to the curve of all equilibria $((x_\tau, y_\tau) \forall \tau)$ and also tangent to some indifference curve of welfare $W_\tau(x, y) = W_\tau(x_1, y_1)$, thereby separating all lower outputs (all equilibria) from all situations with higher welfare. Hence, any introduction of tariffs (subsidies) deteriorates welfare. Similarly the right panel displays the $DED - IEU$ case, only the zone of better welfare is now separated from *higher* outputs.

The $IED - DEU$ assumption used essentially means that our demand is "not too convex," which holds true for many reasonable utility functions: CES, $CARA$, $HARA$ and for many others (see our Appendix and Mrazova, Neary (Mrazova and Neary (2014)) study of many demand types. Mrazova and Neary explain why flat demand is realistic in terms of their market effects: e.g., prices and markups decrease under increasing competition.

Economically, we explain such a negative response to a small ad valorem tariff under DEU as follows. When at free trade our countries ($\tau = 1$) start increasing their tariff τ, the equilibrium mass of firms increases until a certain value $N^{\overline{\tau}} > N^1$ by Proposition 1. Does this shift enhance or deteriorate well-being? We know that under DEU ($\mathcal{E}_u'(x) < 0$) variety N^1, even in free trade, exceeds the socially optimal one: $N^1 > N^o$, that mirrors the insufficient output $q^1 < q^o$. However, variety further rises with tariffs — up to the level $N^\tau > N^1 > N^o$. As a result, even if we were able to keep symmetrical consumption $x = y$ (which we cannot), welfare would further decline. Beyond this harm, consumption asymmetry $x > y$ brings additional distortion, therefore the comparison $U^o > U^1 > U^\tau$ remains true for $x > y$ as well. Under subsidies, all the effects are reversed: the growth of subsidies (i.e., the decreasing of τ) the number of firms in the economy increases, which deteriorates welfare.

Under CES ($\mathcal{E}_u'(x) = 0$), the same two-sided mechanism works against welfare. At free trade equilibrium the number of firms is optimal, whereas imposing a tariff or a subsidy may distort this optimal variety, while the distortion of consumption further deteriorates welfare. We conclude that the negative impact of a positive tariff on welfare in DEU and CES cases *is driven by distorting variety and bringing asymmetry into consumption.* This fact can serve as an argument against protectionism under these assumptions.

Under IEU (i.e., when $\mathcal{E}_u'(x) > 0$), tariffs and subsidies reduce the already small number of firms, thereby harming welfare.

3.2 The impact of specific tariffs

3.2.1 The impact of specific tariffs on consumption and variety

Now we turn to another, more involved form of protectionism: specific tariffs. When studying the impact of specific tariffs on trade, we denote tariffs, unlike in the previous subsection, as $\tau \equiv \tau^{sp}$, $\tau^{ad} \equiv 1$, using the model (17)-(18).

To derive the impact of tariffs on consumption, we totally differentiate our equations (17)-(18) for the specific tariff τ. We come to the following linear equations w.r.t. total derivatives $x_\tau' \equiv dx_\tau/d\tau$, $y_\tau' \equiv dy_\tau/d\tau$:

$$R'(x) + \tau R''(x) x_\tau' = R''(y) y_\tau'$$

$$\left[1 - \frac{R''(x)}{R'(x)}\right] x_\tau' + K\frac{R(y)}{R'(y)} + K\tau \left[1 - \frac{R''(y)}{R'(y)}\right] y_\tau' = x_\tau' + K\tau y_\tau' + Ky.$$

Solving these equations (and recalling $R'(x) > 0$, $R''(x) < 0$) we express and roughly estimate the impact of a tariff on consumption:

$$x_\tau' = -\frac{Ky \cdot R'(x)}{R''(x) \cdot \left(\frac{f}{c} + x + K\tau y\right)} > 0 \text{ (when } y > 0), \tag{26}$$

$$y_\tau' = \frac{R'(x) \cdot \left(\frac{f}{c} + x\right)}{R''(y) \cdot \left(\frac{f}{c} + x + K\tau y\right)} < 0, \tag{27}$$

(naturally) observing increasing domestication and decreasing imports.

Decreasing output. Combining these expressions (see Appendix 3), we find the local *necessary-and-sufficient condition for a negative impact of tariffs* on the firm per-consumer output at any point $\tau > 1$:

$$q_\tau' = x_\tau' + Ky_\tau' = \frac{KR'(x)\left(\left(x + \frac{f}{c}\right) R''(x) - yR''(y)\right)}{R''(y) R''(x) \left(\frac{f}{c} + x + K\tau y\right)} < 0. \tag{28}$$

By adding the multiplier L here, one can easily conclude, that the negative tariff impact on firm *gross output* $Q = Lq$, also obeys the same condition (28). Can we reduce this general condition to some specific forms which are more easy to check?

Alternative conditions for decreasing output. By dropping summ and $\frac{f}{c}$, we can formulate a **stronger** restriction (28) on our general-form revenue function R (and, thereby, on demand), making the condition only a sufficient one, but simple to check for any demand or revenue function. Reformulating (28), using $x > y$ and $R'_x > 0$, we conclude that *locally* (at any point $z \in [y, x]$) output *decreases* under any demand which satisfies at this point the sufficient condition on revenue/utility

$$\frac{\partial}{\partial z}\left(zR''(z)\right) = \left(2u''_z + zu'''_z\right) - \left(3u'''_z + zu''''_z\right) \leq 0 \ \forall z \in [y_\tau, x_\tau]. \tag{29}$$

This condition, decreasing function $zR''(z)$, suffices for decreasing output under $x > y$, which yields $\left(x + \frac{f}{c}\right) R''(x) - yR''(y) < 0$ in inequality (28), which makes output *globally decreasing* over the whole interval $(1, \tau_a)$. This assumption (29) is called "uniformly-flat demand" and it looks realistic. In particular, it holds for *linear inverse demand* $\alpha - \beta z$, where $R''(x) = R''(y) = -\beta = const < 0$. Similarly, for CES demand $x^{\rho-1}$, the related function $xR''(x)$ also decreases in x and therefore output decreases with tariffs. For $HARA$ and $CARA$ utilities, the condition (29) may be locally violated in some zone, here the output can locally decrease, as in Fig. 3, and revealing that the harm from tariffs become more involved.

Bearing this in mind, we formulate a **weaker** possible restriction on our functions: requiring everywhere only a smaller output $q_{\tau>1}$ than the initial q_1, not monotonicity in τ everywhere. Such a condition is formulated through the following function $\phi_{x_1}(\cdot)$, defined at initial point (x_1) and "eventually decreasing" in ξ in the sense:

$$\phi_{x_1}(\xi) \equiv \frac{R(x_1+\xi)}{R'(x_1+\xi)} + \frac{R(x_1-\xi)}{R'(x_1-\xi)} = \frac{x_1+\xi}{1-r_u(x_1+\xi)} + \frac{x_1-\xi}{1-r_u(x_1-\xi)} \leq \phi_{x_1}(0) \ \forall \xi \in (0, x_a]. \tag{30}$$

This sufficient condition is not easy to reduce to other conditions on u, so, we use it "as is" in our propositions. It does not guarantee monotone behavior of output, but at least yields smaller output under a tariff, than at the free-trade one. Therefore definite conclusions about negative gains from tariffs become possible.

To get a **third condition** of this kind, instead of directly varying the tariff τ, we can indirectly study the evolution of output and welfare through varying consumption x. Indeed, we can substitute FOC $\tau = \frac{R'(y)}{R'(x)}$ into the free-entry condition (18), obtain the equation $-x^2u''(x) - Ky^2u''(y) - \frac{f}{c}R'(x) = 0$, use an auxiliary function $\psi(z) \equiv z^2u''(z)$, and express imports through domestic consumption as

$$y^*(x) = \psi^{-1}\left(\frac{-x^2u''(x) - \frac{f}{c}R'(x)}{K}\right). \tag{31}$$

Then, the inequality

$$q_{(\tau)} = x + \psi^{-1}\left(\frac{-x^2u''(x) - \frac{f}{c}R'(x)}{K}\right) < q_1 \ \forall x \in (x_1, x_a] \tag{32}$$

entails the output, at least *globally* decreasing on $(1, \tau_a]$, equivalent to condition (30). By contrast, to make $q_{(x)}$ also *locally* decreasing in x everywhere, the second term in (31) must have a negative derivative smaller than -1, which means the inequality

$$-K < \left(y^2u''(y)\right)' = yu''_{(y)}\left(2 + \frac{yu'''_{(y)}}{u''_{(y)}}\right) \ \forall y < y_1, \tag{33}$$

serving as an alternative to the necessary and sufficient condition for local decrease in output of (28). One can notice the coincidence of the parentheses in this inequality with SOC ($2 + yu'''_{(y)}/u''_{(y)} > 0$, which means a concave revenue function). We can combine this inequality with SOC (using $u''_{(\cdot)} < 0$) as $-K/yu''_{(y)} > \left(2 + yu'''_{(y)}/u''_{(y)}\right) > 0$. We conclude, that *for locally decreasing output, both utility and revenue should not be too concave*, and that high number of trading countries *increases* the probability of decreasing output. We summarize.

Proposition 3. *Under K symmetric countries, their reciprocal specific import tariff coefficient $\tau^{sp} \equiv \tau = 1 + t$ modifies the equilibrium as follows:*

(i) *Domestic individual consumption increases ($dx^{\tau}/d\tau > 0\ \forall\tau \in (0, \tau_a)$, whereas imports decrease ($dy^{\tau}/d\tau < 0\ \forall\tau \in (0, \tau_a)$ with derivatives (26)–(27); domestic consumption exceeds imports ($x > y$) under positive tariffs $\tau \in (1, \tau_a)$.*[8]

(ii) *Firm output $q_{\tau} \equiv x_{\tau} + Ky_{\tau}$ decreases at the free trade $\tau = 1$ and at the autarky point τ_a (when τ_a is finite).*[9] *At any $\tau > 1$, output locally decreases iff inequality (28) holds, which is equivalent to the condition (33), whereas a sufficient condition for a decrease is (29). Under weaker conditions (30) or (32), output remains lower over interval $(1, \tau_a)$ than at free trade ($q_{\tau\in(1,\tau_a)} < q_1$), though it may show down-up-down evolution.*

(iii) *The equilibrium mass of firms N always responds to a tariff τ inversely to the firm output, i.e., $dq_{\tau}/d\tau > 0 \Leftrightarrow dN_{\tau}/d\tau < 0$ and $dq_{\tau}/d\tau < 0 \Leftrightarrow dN_{\tau}/d\tau > 0$.*[10]

Proof: The behavior of consumption/output, summarized in (26)–(27), (28) is already proven above. For the proof of items (ii), (iii) see the above derivation of the conditions and Appendix 3. ⊠

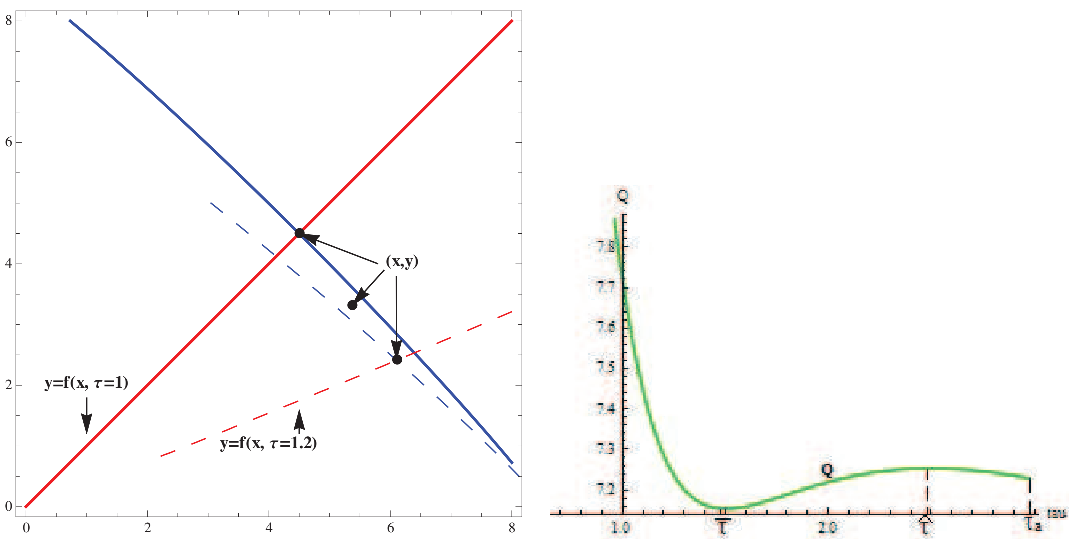

Figure 3: *A specific tariff: possibly non-monotone impact on output under DEU and IED.*

To discuss Proposition 3, Fig.3 illustrates part (ii) with an example using the utility from the previous section: $u(x) = (0.1 + x)^{5.1/6} - 0.1^{5.1/6} - 0.2 \cdot x$. It shows the change in firm output Q under a specific tariff: it can behave non-monotonically but remains lower than at free trade. In Fig.4, expanding the same illustration, the green 45-degree line shows constant output $x+y = const = x_1+y_1$. Namely, it shows the change in firm size with respect to tariffs under *DEU* (left) or *IEU* (right) preferences of type $u(x) = (a + x)^{\rho} + bx$. The thick black thick line shows the evolution of equilibria from the free-trade diagonal ($x = y$) towards autarky ($y = 0$), and we see a non-monotone change of the firm size under growing a specific tariff.

Economic intuition says that any import tariff induces a redistribution of spending towards domestic goods; domestic production rises, exports fall. However, these changes do not cancel each other out within firm output. In particular, for $IED - DEU$ case the firm size *decreases in tariff* due to discouraging higher costs, under a sufficiently small tariff $\tau < \bar{\tau}$. This general decrease leads to lower economies of scale and a higher domestic price $p_x = c/(1 - r_u(x))$. The mass of firms is linked inversely to output through the labor balance (12), therefore variety increases. Transfers (tariff proceeds) partially compensate to consumers the hike in prices, so, the aggregate demand does not fall. The smaller per-variety consumption is compensated for creation of new firms. However, this tendency may be reversed under a sufficiently high tariff $\tau > \bar{\tau}$, which is difficult to explain intuitively.

[8] As a corollary, finite differences are ($x^1 < x^{\tau}\ \forall\tau < \tau_a$), ($y^1 > y^{\tau}\ \forall\tau < \tau_a$), and import subsidies $s: \ \tau \equiv 1-s \in (0, 1)$ yield the inverse effect: ($dx^s/ds < 0$), ($dy^s/ds > 0$).

[9] Thereby, an import subsidy $s: \ \tau \equiv 1 - s \in (0, 1]$, which is a negative tariff, pulls output *up* near free trade: $dq^s/ds > 0\ \forall s \approx 0$.

[10] Additionally, one can prove that under increasingly-elastic demand ($r'_u(z) > 0\,\forall z$), domestic and import prices increase: $dp^{jj}/d\tau > 0$, $dp^{kj}/d\tau > 0\ \forall\tau \in (0, \tau_a)$. Under decreasingly-elastic demand, domestic prices decrease *Zhelobodko et al. (2012)*.

The right panel shows that in the unnatural case of decreasingly-elastic demand (DED: $r'_u(\cdot) < 0$, not included in the previous proposition), there can be a controversial impact on output. This yields non-monotone prices, either growing because of additional tariffs, or reducing because of increased competition. In both cases, total physical consumption Nq always changes oppositely to variety N because of the labor balance and economies of scale.

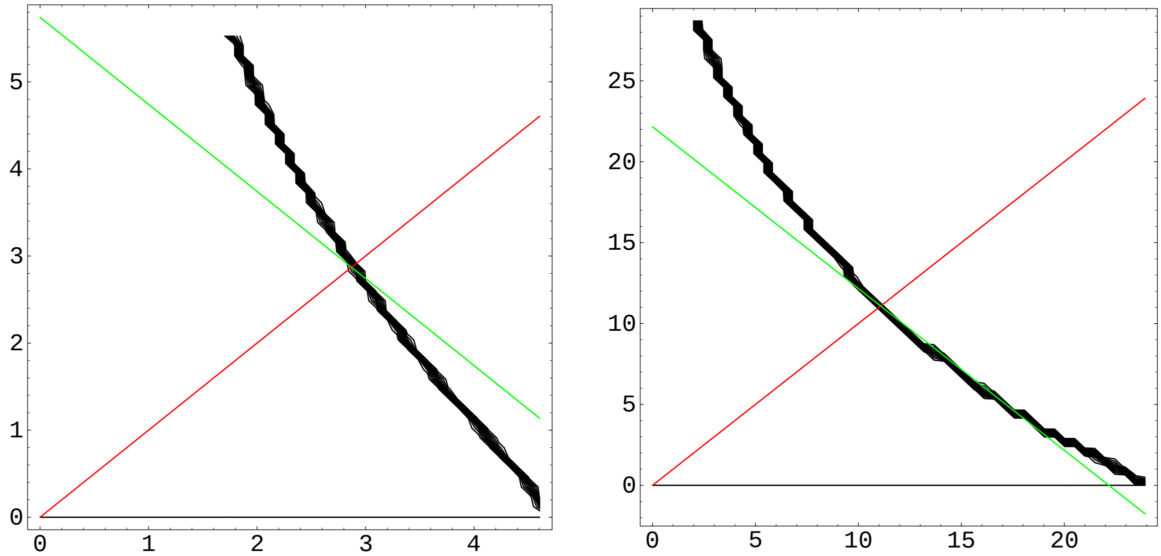

Figure 4: *Negative (DEU) or locally positive (IEU) impact of specific tariff on output.*

Next section shows whether or not the revealed changes in quantity and variety beneficial for consumer.

3.2.2 The impact of specific tariffs on welfare

Now we turn to the welfare consequences of changes in consumption and variety, revealed in Proposition 3. Generally, market forces maximize revenue, whereas a social planner pursues maximum utility. Following Dhingra and Morrow (2012), these goals are completely "aligned" only under CES-CEU preferences. Since Dixit and Stiglitz Dixit and Stiglitz (1977), we know that a closed economy with DEU (IEU) preferences generates excessive (insufficient) variety (whereas the only case of socially-optimal variety N is CES: $\mathcal{E}_{u(x)} = const$). This quantity distortion brings insufficient or excessive variety N, inversely related to the amount of consumption volume. We study a trade model but exactly the same discrepancy between DEU and IEU distortion takes place under free trade, because it is essentially an integrated economy (zero tariffs). Our task now is to show whether positive specific tariffs further distort or cure quantity/variety distortions (see Fig. 2 for the geometric intuition).

In the formulation below, we mention the autarky point $\tau_a \leq \infty$ and the switching point $\overline{\tau} \leq \tau_a$ (guaranteed by Proposition 3) of output, restricting the zone where output changes monotonically: $\overline{\tau}: \; dq_\tau/d\tau < 0 \; \forall \tau \in (1, \overline{\tau})$. This helps to specify some intervals where welfare decreases/increases with tariffs, and the location of the optimal tariff/subsidy.

Proposition 4. *Assume a symmetric specific tariff in K symmetric countries. Then: (i) Under marginal utility restricted as $\lim_{x\to\infty} u'(x) \leq c$, there exists a socially optimal reciprocal import tariff $\tau^* = \arg\max_\tau W(\tau)$ with positive consumption $x_\tau > 0, y_\tau > 0$.*[11]

(ii) Under $DEU-IED$ preferences ($\mathcal{E}'_u(x) < 0$, $r'_u(x) > 0$), all positive specific tariffs $\tau \in (1, \tau_a]$ bring lower welfare than free trade, in the sense $W(\tau) < W(1): \; \tau \in (1, \tau_a]$; more specifically, welfare W reaches its local argmaximum $\tau^* < 1$ at a negative tariff and strictly decreases in tariff over on interval $[1, \overline{\tau}]$ of monotone outputs, and at the autarky point τ_a.[12]

(iii) Under IEU preferences ($\mathcal{E}'_u(x) > 0$), any negative specific tariff brings lower welfare than free trade, in the sense $W(\tau) < W(1): \; \tau \in (0, 1]$; welfare W reaches its local argmaximum $\tau^* > 1$ at a positive tariff $t^* = \tau^* - 1$, and welfare increases on interval $[1, \tau^*)$.

[11] Without condition $\lim_{x\to\infty} u'(x) \leq c$, existence of social optimum becomes really problematic, unless the mass of firms is not positively restricted from below with some $N_{min} > 0$. Otherwise under $u'(x) > c$ there can be infinitely increasing x and decreasing N to enhance welfare without violating the labor balance.

[12] Actually, strict decrease of welfare *everywhere* – is observed in *all* examples that we have studied, but we are unable to prove under general-form utility u.

(iv) Under CES utility ($\mathcal{E}'_u(x) = 0$), welfare W reaches its argmaximum at free trade ($\tau^* = 1$) and declines w.r.t. any non-zero tariff or subsidy.

Proof: see Appendix 3. ⊠

For this proposition, the $DEU - IED$ assumption used here, holds true for many reasonable utility functions: quadratic, CES, $CARA$, $HARA$ and others (see Appendix 3 and Mrazova and Neary (2014)).

We illustrate these (and other) findings in Fig. 5 and in the following Remark, which mentions four threshold values of tariff:

$$\tau = 1 : [x = y],\ \overline{\tau} : \left[\frac{dq(\overline{\tau})}{d\tau} = 0\& \frac{d^2q(\overline{\tau})}{d^2\tau} > 0\right], \underline{\tau} : \left[\frac{dq(\underline{\tau})}{d\tau} = 0,\ \frac{d^2q(\underline{\tau})}{d^2\tau} < 0\right], \tau_a : [y(\tau_a) = 0].$$

Remark. *In symmetric countries, under three types of utilities, the impact of a specific tariff on welfare (W'_τ) can be positive (+), zero or negative (−) over certain tariff intervals, as follows*:

Case\ Tariff size:	$\tau \in (\underline{\tau}, 1) : q'_\tau < 0$	1	$\tau \in (1, \overline{\tau}) : q'_\tau < 0$	$\overline{\tau} : q'_\tau = 0$	τ_a
IEU: $\mathcal{E}'_u(z) > 0$	$+$	$+$	$\pm$	$-$	$-?$
CES: $\mathcal{E}'_u(z) = 0$	$+$	0	$-$	$-$	$-$
DEU:$\mathcal{E}'_u(z) < 0$	$\pm$	$-$	$-$	$-$	$-$

where symbol "±" means that both positive and negative examples have been found, while "-?" means that simulations always show decreasing utility but this decrease is not proven analytically for all additive utilities.

Proof: see Appendix 3. ⊠

Comparing the new Remark to Proposition 4, there is additional information in the Remark about DEU case, and several indecisive cases $\pm, -?$ studied through simulations (described in the next subsection).

The geometric intuition for the effects described in our propositions is shown in Fig. 5. All equilibria $(x_\tau, y_\tau)\,\forall\tau$ belong to the black downward-sloped curve, containing the market responses to any τ. Welfare decreasing in $\tau \in (1, \overline{\tau})$ means, that this response-curve intersects all indifference curves of welfare function W *downwards*: $dW^\tau/d\tau < 0$. Moreover, even any 45-degree line $x + y = const$ is intersected by this curve *downwards* in this zone, because here $dq/d\tau < 0$ under DEU. This fact entails a downward intersection with the indifference curves of $W(x, y)$, tangent to lines $x+y = const$. (However, IEU shows a different tendency.)

Economically, we explain such response to a small tariff under DEU as follows. When at free trade ($\tau = 1$) our countries start increasing their tariff τ, the equilibrium mass of firms increases until a certain value $N^{\overline{\tau}} > N^1$, by Proposition 3. We know that under DEU ($\mathcal{E}'_u(x) < 0$) variety N^1 under free trade exceeds the socially optimal one: $N^1 > N^o$, which mirrors the insufficient output $q^1 < q^o$. However, variety further increases with tariff—up to the level $N^\tau > N^1 > N^o$ which yields $U^o > U^1 > U^\tau$ for $x > y$. Further, under higher tariffs, the consumption distortion works as previously and even more strongly, though N may slightly decrease. Reversing this argument, under DEU a small import subsidy $s \approx 0$ (or export tariff) enhances welfare because it cures the variety distortion, though causing a small consumption distortion.

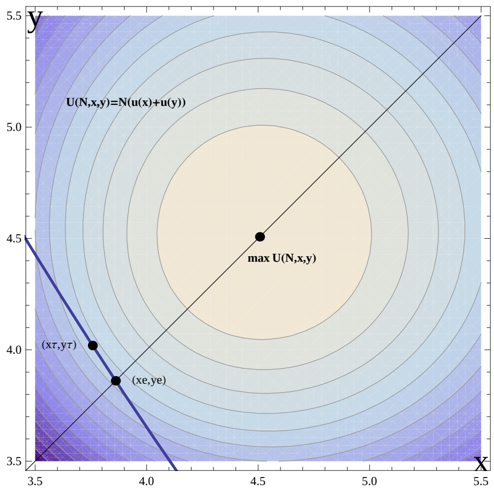

Figure 5: The indifference curves of welfare $W(x, y)$, all equilibria $(x_\tau, y_\tau)\,\forall\tau$ and optimal subsidy under specific tariff in DEU case.

Under the IEU case $\mathcal{E}'_u(x) > 0$, a small tariff brings changes that mirror what was said about DEU. At free-trade equilibrium, the mass of firms (competition) is insufficient. So, imposing a small tariff somewhat corrects this non-optimality. This benefit could be offset by asymmetric consumption but with a small tariff asymmetry does not outweigh the positive effect (see Appendix). It dominates only under sufficiently high tariffs, where welfare start decreasing with τ.

Subsidies have opposit effect everywhere because they are negative tariffs. They are beneficial in the realistic DEU case. The excessive number of firms at free-trade equilibrium is somewhat corrected by small import subsidies, thereby enhancing country welfare. However, this positive effect is quickly exhausted under higher subsidies that yield a stronger asymmetry effect. Politically, import subsidies look infeasible, looking "anti-domestic" policy. But in a symmetric world, such subsidies are the equivalent to some export tariffs, bringing the same transfers and same consumption asymmetry. This beneficial trade policy looks more feasible, though being a sort of anti-protectionist governmental intervention. However, a *direct* correction of excessive competition/variety (such as a costly sales license) applied equally to foreign and domestic firms, would look even more beneficial. It brings no consumption distortion, and is politically feasible. It may perform a better job than export tariffs/subsidies.

3.2.3 Quantification and simulations

This section quantifies our model under several utility specifications, to roughly estimate the size of welfare losses from several tariff rates: 4%, 8%, 12%. Our quantification partially relies on the methodology from Arkolakis et al. (2012), using equivalent variation for the welfare measure and similar trade elasticity. Arkolakis et al. (2012) study the impact of trade costs and CES preferences on a single country, however, we assume: (1) tariffs; (2) VES preferences and; (3) global tariff impact in a K-country world.

Our study exploits the parameters of utility consistent with the trade elasticity reported by Arkolakis et al. (2012) and Anderson and Wincoop (2004), where the elasticity coefficient is found empirically using gravity equations (elasticity of imports from data on world trade). Specifically, following Anderson and Wincoop (2004), the utility function must generate the elasticity of substitution $\sigma \in [-5, -10]$. The coefficient $\sigma = -6$ is said to be the most plausible value. A corresponding price margin of about 20-25% is quite typical. To be consistent with $\sigma = -6$, the CES utility function $u(x) = x^{\rho}$ must have a power of $\rho = 5/6 = 1 - 1/\sigma$.

Below we compare simulations for three cases: (1) CES ; (2) DEU and (3) IEU .

To investigate the utility functions with a variable elasticity of substitution a rather wide class of functions was selected, covering events such as rising and falling elasticity. More specifically, the function used is $AHARA$ (augmented hyperbolic absolute risk aversion) of the following form:

$$u(x) = \frac{(d+x)^{\rho} - d^{\rho}}{h} + l \cdot x, \ \rho \in (0,1), \ d \geq 0, \ h > 0. \tag{34}$$

Various properties of the utility function can be set by varying factors. In this example, we will highlight the situation (conditional on $d, \ l$):

- CES is the case under $d = 0$, $l = 0$, $u(x) = x^{5/6}$, $\mathcal{E}'_{u(x)} = 0$
- DEU — specific demand when the economy appears to have more than the optimum number of firms: $\mathcal{E}'_u(x) < 0$ the parameters $d \geq 0$ and $l < 0$. The simulation function $u(x) = (0.1 + x)^{5.1/6} - 0.1^{5.1/6} - 0.2 \cdot x$.
- IEU — the derivative elasticity utility is positive $\mathcal{E}'_{u(x)} > 0$ for $x > 0$ when the parameters $d = 0$ and $l > 0$. The simulation function $u(x) = x^{4.4/6} + 0.2 \cdot x$.

Thus the elasticity of substitution for all cases at point $\tau = 1$ is as close to $\sigma = -6$, as for the functionCES.

The table shows the results of calculations: there is a decline in imports due to the introduction of different levels of tariffs, as well as a change in welfare (the result is equivalent to a variation of income counted by the price index). The rate is expressed as a percentage of the price. The effects of welfare are calculated by compensating for income as a percentage. For CES you can see that in spite of the fact that the economy loses from the reduction of foreign trade, the size of the loss is small (middle pane). For example, with the introduction of a 4% of import tariff there is a fall in foreign trade of about 10% relative to the initial value, and the total welfare decreases by only 3.4%. The fact that the domestic product effectively replaces the goods imported from abroad, due to the elasticity of substitution ($\sigma = 6$), as we originally anticipated.

For DEU (first pane) as a result of changing the value of the tariff $\tau \in [0, 1, +\infty]$ in increments of 1%, we get a fall in welfare (counted as the equivalent to the variation in income and compensating

Table 1: *Tariff impact on welfare for: DEU, CES and IEU .*

t,%	DEU E_u	DEU %,import change	DEU ΔW,%	CES E_u	CES %,import change	CES ΔW,%	IEU E_u	IEU %,import change	IEU ΔW,%
-12	-6.018	-36	-3.8		-36	-12.4	-6.03	34	-6.8
-8	-6.011	-20	-2.1		-26.2	-7.7	-6.025	26	-4.2
-4	-6.004	-12	0.9		-12.2	-3.4	-6.023	10	-2.6
-0.9	-6.001	-2.6	1.2		-2.9	-0.8	-6.00	-3	-0.6
0	-6	0	0	-6	0	0	-6	0	0
1.4	-6.00	-2.1	-0.9		-3.1	-0.9	-6.00	-2.9	1.6
4	-6.02	-10	-3.9		-10.9	-2.9	-5.97	-10	-5.4
8	-6.05	-20	-7.5		-28.1	-7.9	-5.93	-22	-12
12	-6.07	-28	-11.2		-32.3	-13.4	-5.89	-32	-17

for it) in both countries on the entire range at a positive rate of growth and well-being with the introduction of small subsidies for the imports (the maximum increase is equal to subsidies (export duties) of approximately 1% (0.9)). Here the results are similar to the first example with the introduction of an import tariff of 4%: the fall of imports by 10% with the introduction of the tariff leads to a reduction in welfare of 3.9%. This reduction in welfare is somewhat less than in the model with CES preferences (3.4%). This difference is consistent with the observation that the elasticity of import interchange with domestic goods increases with the rate (column 2), i.e., consumers easily switch from imports to domestic consumption. In this case, the welfare falls for any positive value of the tariff, and increases by only a small value for subsidies.

The third pane of table shows the maximally increasing well-being ($\approx 1,5\%$) for IEU: 1.4%. The rest of the effects of reducing welfare repeated the values given above: the average value of the rate of 4% reduces imports approximately 10% and reduces well-being from 5 to 10%. In all the other examples the usefulness of IEU preferences to the consumer increases only for small values of the tariff, and a further increase of the tariff leads to a decrease of utility in both countries. An increase in the number of firms leads to the movement of the point of balance to the optimum point, but when you enter large values the equilibrium rate does not improved well-being.

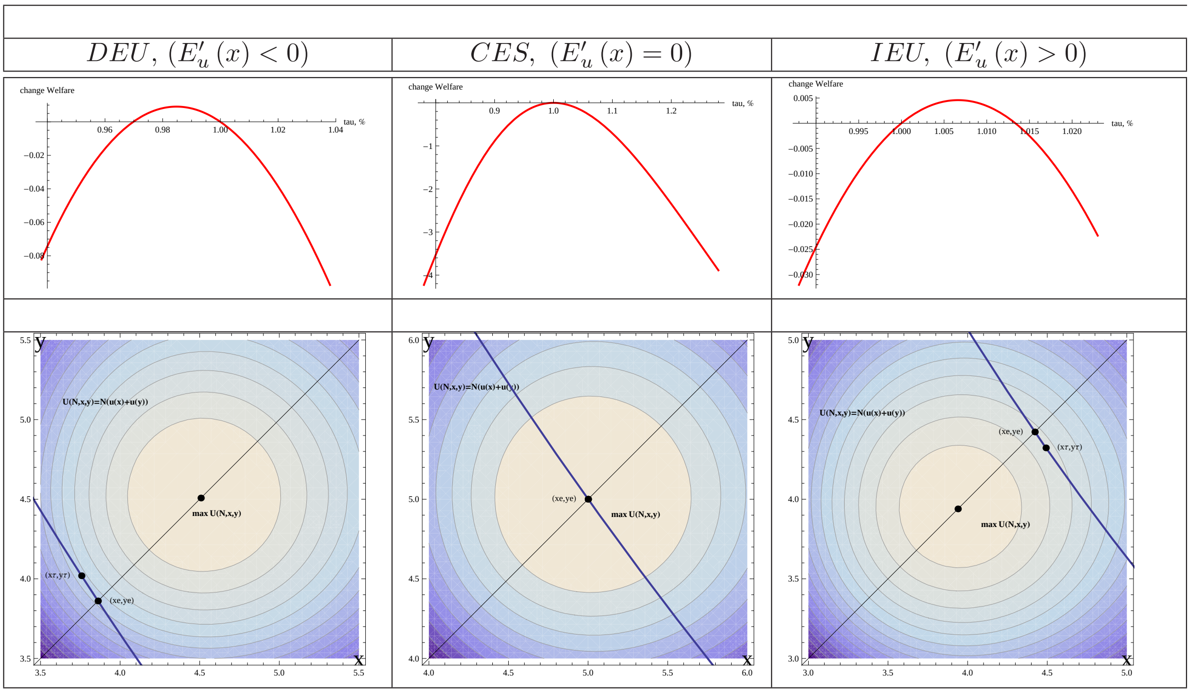

Figure 6: *Welfare function under DEU, CES, IEU preferences: costless trade points and optimal specific tariff: negative, zero or positive.*

Fig. 6 shows the evolution of well-being under changing tariffs/subsidies and related shifts in domestic/foreign consumption (x, y). The abscissa in the upper three graphs show the specific mutual tariff coefficient τ, the ordinate is welfare W. One can see that welfare falls under any regulation in the CES case (middle pane). The indifference curves of consumer welfare $W(x, y)$ presented in the middle lower panel explain this effect. The free-trade equilibrium (x_τ, y_τ) under $\tau = 1$ *coincides* with the symmetric socially optimal consumption (x^e, y^e) in this case. Any shifts of attainable consumption (x_τ, y_τ) along the (thick, decreasing) admissible curve of equilibria—would be harmful, i.e., any $\tau \neq 1$ is not beneficial.

By contrast, according to the first (third) panel of Fig. 6, subsidies (tariffs) can be beneficial in the case DEU (IEU). It shows the overall picture of changes generated by the model of balance, in particular the effect of the tariff on a company in the economy. For small tariff increases, the same behavior is seen in the near prohibitive tariff for trade (autarky). But a change in the overall usefulness of the approach to self-sufficiency (for this case at a rate equal to 270%) did not have a significant impact. At the point of near-autarky welfare falls, which is consistent with our theoretical result.

In a sense, a more understandable trade policy is the introduction of import tariffs . The beneficial effect of a mutually positive rate was found during the simulation for the utility functions with IEU. Fig. 4 (third pane) shows that when the value of the tariff $\tau \in [1, +\infty]$ increases in increments of 1%, the utility increases for both countries, but only for a small ($\tau \in [1,\ 1.07)$) positive rate. In this zone, keeping the tariff increases the number of active firms in the economy, which was originally less than optimal (the IEU condition). So the economy is close to the optimum and social welfare increases. For a large tariff, distortion between domestic consumption and imports of goods is outweighed by this correction input effect.

We add that a variety of simulations were carried out not only to show examples, but to explore a wide range of function parameters (34) at different intervals x   the existence of equilibrium (17)-(19). All experiments confirmed the general findings.

3.3 Why “flat” demand curves look natural?

To draw the meaningful conclusion that in the real economy, the benefits of tax are hardly possible, we discuss why the DEU $(\mathcal{E}'_u(x) < 0)$ case seems natural. Let us try to justify the idea of the "explicit" decreasing marginal utility of such functions, that is, saturable demand. For a natural elementary utility function $u(x)$ (increasing, concave, coming from zero) elasticity $\frac{xu'(x)}{u(x)}$ is less than unity at every point because it is the quotient of the average value of the derivative function. Power CES functions have this constant fraction and the increase in this fraction (IEU , $\mathcal{E}'_u(x) > 0$) indicates that the derivative approaches the average utility consumption grows. That is, when IEU "everywhere" utility to infinity approaches straight, linear function, i.e., the consumer would be willing to consume infinitely , and be willing to pay about the same price for each additional unit of product . From a consumer perspective, this is a strange effect of decreasing marginal utility. We (and many economists, including Dixit and Stiglitz (1977), discussing the effect) believe it is unnatural, and the saturation of demand $(\exists \bar{x} :\ u'(x) \leq 0\, \forall x \geq \bar{x})$ It is difficult to imagine a product that the consumer would be willing to consume infinitely many. When saturable, property IEU is not possible "everywhere" , because $\mathcal{E}_u$ eventually goes to zero.

In terms of demand and firm behavior, a typical IEU function is also "unnatural." For example, the function $u = x^\rho + ax$ belongs to IEU and its marginal utility (the inverse demand function) not only crosses the horizontal axis of the graph supply and demand graph, but even separated from zero by a constant $a > 0$ (See. Fig. 1). This means that for a sufficiently small cost some firms would be able to get an infinite gain , which is strange. It looks more natural to have an upper bound on the demand (the horizontal axis), which means "strong" saturation utility and contradicts IEU . A more natural class DEU belong to as a quadratic utility function– $u(x) = d \cdot x - h \cdot x^2$, $CARA$ – $u(x) = 1 - \exp(-ax)$, $HARA - u(x) = h \cdot \left(\sqrt{x+d} - \sqrt{d}\right)$.

4 Tariff impacts under positive transport costs

In this section we consider an extension of our analysis— the joint influence of tariffs and transport costs on trade, which is realistic and is important for welfare conclusions.

4.1 The impact of a specific tariff under positive transport costs

Let us modify the basic model outlined above, adding non-zero transport costs denoted $\theta \geq 1$. This iceberg type coefficient works as in Krugman’s model together with a tariff in the multiplicative form $\theta \cdot \tau$. The difference between them is that the tariffs are redistributed to consumers whereas the costs are lost. This model explains the general patterns of reactions of equilibrium and welfare to tariffs and also estimates the relative value θ/τ leading to opposite outcomes for welfare, especially for DEU.

The basic model (17)–(19) described for K symmetric countries, is now modified by including parameter θ of trade cost, as follows:

$$\frac{R'(x)}{R'(y)} = \frac{1}{\theta \cdot \tau}, \tag{35}$$

$$\frac{R(x)}{R'(x)} - x + K \cdot \theta \cdot \tau \left(\frac{R(y)}{R'(y)} - y \right) = \frac{f}{c}. \tag{36}$$

The budget or labor balance (13) for finding the mass of firms N in each country takes the form

$$\frac{f}{c} + x + K \cdot \theta \cdot y = \frac{1}{cN}. \tag{37}$$

One can see that the new parameter θ comes together with τ in two equations (35)–(36) defining the reaction of firms to costs/tariffs and firms do not distinguish between these two, therefor the comparative statics of the reaction (x, y, q) remains as above, as if $\tilde{\tau} = \theta \cdot \tau$. However, the mass of firms and welfare react differently because cost θ changes the labor balance, unlike tariff τ (37). The impact of tariffs on consumptions and outputs (x, y, q), i.e, the comparative statics with respect to the input tariff/subsidy is somewhat similar to Proposition 2 of the basic model. It starts with consumption differentiation (27)-(26):

$$x_\tau^{\theta\prime} = \frac{-Ky \cdot R'(x)}{R''(x) \cdot ((f + cx)/\theta + cK \cdot \tau y)} > 0\,, \qquad y_\tau^{\theta\prime} = \frac{\left(\frac{f}{c} + x\right) \cdot R'(x)}{R''(y) \cdot ((f + cx)/\theta + cK \cdot \tau y)} < 0\,. \tag{38}$$

After this, we study output, considering not only autarky $\tau_a = \tau_a(\theta)$ and free trade $\tau = 1$ but also two other important points of comparative statics: the point $\tau_1 = \frac{1}{\theta}$ where a small tariff (subsidy) exactly compensates for the transport cost, which now plays the role played by free trade: $x = y$.

Proposition 5. *Assume positive transport costs (losses) $\theta > 1$ under K symmetric countries. The symmetric import tariff coefficient τ modifies the equilibrium as follows: (*i*) Domestic individual consumption increases ($dx_\tau^\theta/d\tau > 0\ \forall \tau \in (0, \tau_a)$, whereas imports decrease: ($dy_\tau^\theta/d\tau < 0\ \ \forall \tau \in (0, \tau_a)$. The higher the transport cost θ, the higher the rate of change in import and domestic consumption, as in (38), and the smaller the autarky point $\tau_a(\theta)$.*

*(*ii*) Firm output $q_\tau^\theta = x_\tau^\theta + \theta \cdot K y_\tau^\theta$ in any case is decreasing at the compensating point $\tau_1 = \frac{1}{\theta}$ and at the autarky point $\tau_a(\theta)$; there are examples where it responds to τ in down-up-down manner, and examples of the monotone decrease.*

*(*iii*) The equilibrium mass of firms N always responds to the tariff τ inversely to the firm's output: $dq_\tau^\theta/d\tau > 0 \Rightarrow dN_\tau^\theta/d\tau < 0$, $dq_\tau^\theta/d\tau < 0 \Rightarrow dN_\tau^\theta/d\tau > 0$.*

Proof. The equilibrium equations (35)-(37) differ from the equations studied in Proposition 3 only by composite parameter $\theta \cdot \tau$ replacing the previous parameter τ. Thereby, any changes in this composition have the same impact as changes in τ and Proposition 5 is a direct corollary of Proposition 3. The only the compensation point $\tau_1 = \frac{1}{\theta}$ replaces the previous free trade point. ⊠

Discussing the difference between Proposition 5 and its basic version (Proposition 3), we note that the stronger impact of tariff τ under trade cost θ is predictable, because both variables come into the equations determining x, y in a multiplicative way $\theta\tau$, they reinforce each other. The higher the transport cost, the more disproportion x/y in the consumption of domestic and imported goods. The value of the impact on output q and the mass of firms N slightly changes but the sign remains the same. Another difference is that without the tariff ($\tau = 1$) we do not really have free trade, because $\theta\tau > 1$. The compensating point, where $\theta\tau = 1$ cannot be called free trade, plays an important role in studying welfare that we address now.

Now we turn to changes in social welfare $W_\tau^{\theta\prime}(x, y)$. These depend on both types of trade impediments: costs and tariffs. Keeping costs unchanged in our comparative statics, we have in mind short periods when losses from transportation do not change and establish only the impact of tariffs, that may change with new trade agreements. Such an impact is illustrated in Fig. 4 (remaining valid for a small trade cost θ), Fig.5, and is classified as follows.

Proposition 6. *Assume a symmetric tariff in K symmetric countries, and positive transport costs $\theta > 1$. Then:*

(i) Under CES ($\mathcal{E}_u'(x) = 0$), welfare $W^\theta(\tau)$ first increases with a tariff over the on interval $\frac{1}{\theta} \le \tau < 1$ (interval of subsidization), then decreases on interval $1 < \tau < \infty$, reaching its maximum at unregulated trade $\tau^* = 1$.

(ii) Under IEU ($\mathcal{E}_u'(x) > 0$), welfare $W^\theta(\tau)$ also increases with tariffs over some initial interval, at least at point $\tau_1 = \frac{1}{\theta}$ of the compensating subsidy ($y = x \Rightarrow W_\tau^{\theta\prime}(\tau_1) > 0$) but $W^\theta(\tau)$ decreases at the (finite or infinite) autarky point ($y \approx 0 \Rightarrow W_\tau^{\theta\prime}(\tau_a) < 0$).

(iii) Under DEU ($\mathcal{E}_u'(x) < 0$), decreasing welfare $W_\tau^{\theta\prime}(\tau_1) < 0$ at the compensating point $\tau_1 = \frac{1}{\theta}$ is equivalent to a restriction from above on trade costs

$$\theta < \theta_1 \equiv \frac{1+\zeta}{1-K\zeta},$$

where

$$\zeta \equiv -\frac{\mathcal{E}_u'(\bar{x})\, r_u(\bar{x})}{\mathcal{E}_u(\bar{x})\left(1 - r_u(\bar{x}) + K\mathcal{E}_u(\bar{x})\, r_u^2(\bar{x})\right)} > 0,$$

and $\bar{x} = \bar{y}$ is the solution to equations (35)–(36) with $\tau_1 = \frac{1}{\theta}$, while for large $\theta > \theta_1$ welfare increases: $W_\tau^{\theta\prime}(\frac{1}{\theta}) > 0$.[13]

(iv) Assume finite derivatives $u'(0)$, $u''(0)$, then there is an autarky point $\tau_a : y = 0$, local DEU, IED properties hold there ($\mathcal{E}_u'(0) < 0$, $r_u'(0) > 0$), and the welfare impact of the tariff obeys the rule

$$W_\tau^{\theta\prime}(\tau_a) < 0 \Leftrightarrow 1/\mathcal{E}_u(x_a) < \tau_a, \quad W_\tau^{\theta\prime}(\tau_a) > 0 \Leftrightarrow 1/\mathcal{E}_u(x_a) > \tau_a.$$

In particular, welfare $W^\theta(\tau)$ decreases within any tariff under a sufficiently small trade cost $\theta < u'(0)/u'(x_a)$, whereas under a higher cost $\theta > u'(0)/u'(x_a)$ welfare $W^\theta(\tau)$ increases with a small tariff $\tau_a < 1/\mathcal{E}_u(x_a)$.

Proof is in Appendix 4. ⊠

Commenting on this comparative static, we should say that (naturally) the impact of tariffs under *small* transport costs behaves like in our basic model shown in Fig.4. The welfare curve is single-peaked in all our examples (though we have not managed to prove this analytically). In the natural DEU case and small trade cost, the welfare peak is situated to the left of the non-regulated trade point $\tau = 1$, thereby, any positive tariff decreases welfare, while small subsidy increases it.[14]

Thus, if the transport costs are low, the increase in subsidies to a level equivalent to the transport costs, increases welfare, whereas welfare reduces in a positive tariff. However an increase in subsidies is not useful with high transport costs. Moreover, there exists a threshold θ which does not give any positive or negative effect, i.e., there exist the subsidies $\tau = 1/\theta$ that fully compensated the transport losses and such that $\left(W^\theta\right)'_{(at\,1/\theta)} = 0$.

Under an infinitesimal tariff, the welfare change depends on the magnitude of the transport losses, as well as on the characteristics of the sub-utility function. For DEU, the welfare reduces under (64). For high transport costs, introducing a positive tariff has a positive effect.

Let us consider now the situation of trade opening (a move from autarky). If there are transport costs θ, welfare increases at low θ (more precisely, when θ is less than the ratio of the imported and domestic prices). Under high θ, the behavior of welfare is completely determined by the size of the tariff: welfare falls, if the tariff is greater than the inverse elasticity of the sub-utility, and vice versa. Thus, if the elasticity of utility and transport costs is high, autarky occurs at sufficiently low import tariffs (it is well known that tariffs accelerate the process of sliding into autarky), in these conditions, the trade opening reduces welfare. With a low elasticity of the sub-utility, whatever the transportation costs, autarky occurs under higher import tariffs, therefore the trade opening can increase welfare, even under high transport costs.

Fig. 7 shows an example of the impact of tariffs/subsidies on welfare change for the sub-utility functions with variable elasticity (a natural type of sub-utility functions is DEU). The calculation is

[13] Essentially, the value $\bar{x}$ is the value of consumption in autarky of a country with population $(K+1)L$.

[14] For small transport costs, which are less than the ratio of the prices of imported and domestic goods at the point of autarky $1 \le \theta \le \frac{u'(y)}{u'(x)}$; and for large transport costs which are more than the ratio of the prices of imported and domestic goods at the point of autarky $\theta > \frac{u'(y)}{u'(x)}$ and at the same time the tariff less, than inverse elasticity $\tau < \frac{1}{E(x)}$; welfare W^θ increases only for large transport costs, which are more than the ratio of the prices of imported and domestic goods at the point of autarky $\theta > \frac{u'(y)}{u'(x)}$ and at the same time the tariff more, than inverse elasticity $\tau > \frac{1}{E(x)}$.

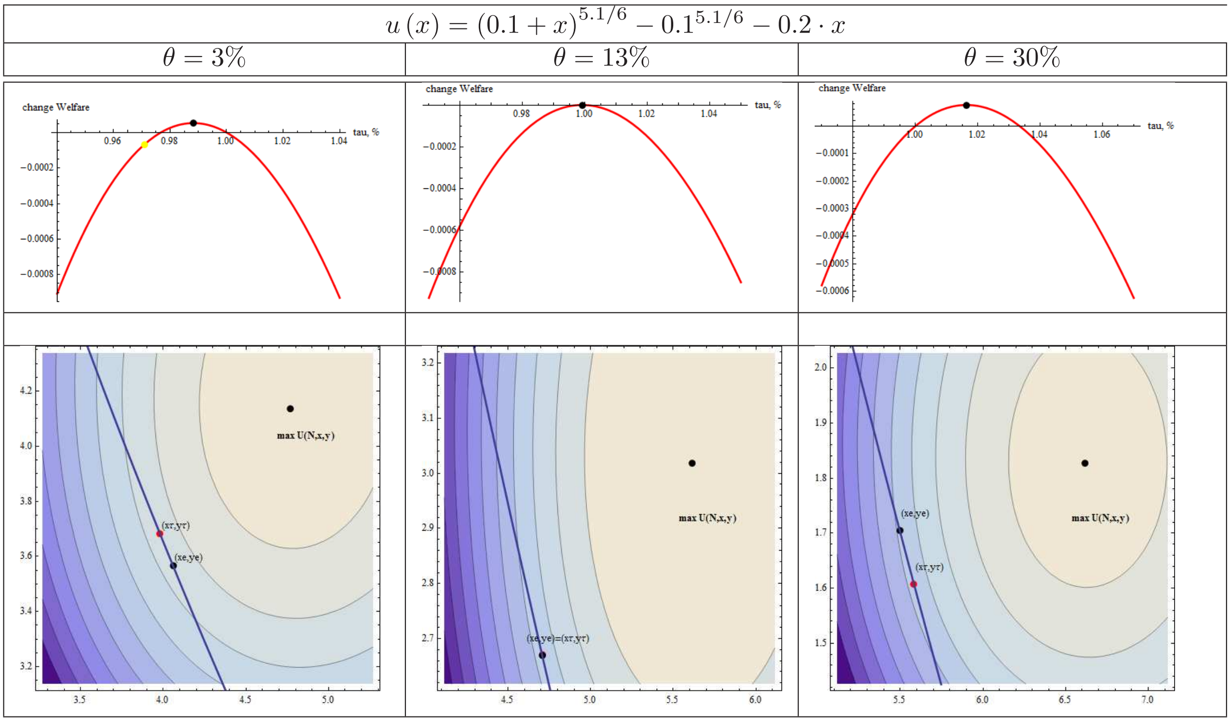

Figure 7: *Welfare function and optimal specific tariffs (red points) in DEU case: negative, zero or positive, depending on small/moderate/big transport costs θ.*

carried out at the free trade point. The optimal tariff / subsidy calculated for three cases: low small (3%), medium (13%) and high (30%) transport costs. It turned out that

1) For low costs (3%), subsidies should be introduced, the concentration of firms in the economy is excessive (this case is similar to the characteristics of equilibrium under the tariff/subsidy introduction in the basic model).

2) For the medium costs (13%), the introduction of any regulation leads to a negative effect.

3) High costs (30%) make the introduction a positive tariff profitable, which reduces incentives for trade. These high transport costs reduce the number of firms due to economies of scale, the equilibrium number of firms is smaller than socially optimal. Tariffs increase the number of firms shifting welfare closer to an optimum.

4.2 The impact of an ad valorem tariff under positive transport costs

The basic model with an ad valorem tariff $\tau^{ad} = \tau$, $\tau^{sp} = 1$ (17)–(19) described for K symmetric countries, is now modified by including parameter θ for trade costs, as follows:

$$\frac{R'(x)}{R'(y)} = \frac{1}{\theta \cdot \tau}, \tag{39}$$

$$\frac{R(x)}{R'(x)} - x + K \cdot \theta \left(\frac{R(y)}{R'(y)} - y \right) = \frac{f}{c}. \tag{40}$$

The budget or labor balance for finding mass N in each country takes the form

$$\frac{f}{c} + x + K \cdot \theta \cdot y = \frac{1}{cN}. \tag{41}$$

The impact of the tariff on consumption and output (x, y, q), i.e, comparative statics with respect to input tariff/subsidy is similar to Proposition 2 for the basic model. It starts with consumption differentiation (39-40):

$$x_\tau^{\theta\prime} = -\frac{KR(y)}{\tau^2 R''(x)\left(x + K\theta \cdot y + \frac{f}{c}\right)} > 0\,, \qquad y_\tau^{\theta\prime} = \frac{\theta \cdot R(x)}{R''(y)\left(x + K\theta \cdot y + \frac{f}{c}\right)} < 0\,. \tag{42}$$

$$q_\tau^{\theta\prime} = \frac{LK}{\left(x + K\theta \cdot y + \frac{f}{c}\right)} \left(\frac{\theta^2 \cdot R(x)}{R''(y)} - \frac{KR(y)}{\tau^2 R''(x)} \right). \tag{43}$$

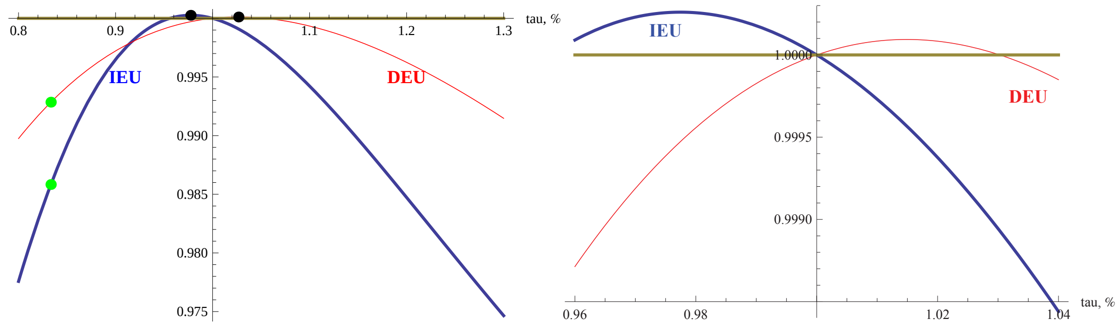

Figure 8: Welfare functions and optimal ad-valorem tariffs (black points) under transport cost $\theta = 1.2$: positive in DEU case ($u(x) = (0.1 + x)^{5.1/6} - 0.1^{5.1/6} - 0.2 \cdot x$), negative in IEU case ($u(x) = (0.1 + x)^{4.3/6} + 0.2 \cdot x$) case (right panel - scaled).

After this, we study outputs, considering not only autarky $\tau_a = \tau_a(\theta)$ and free trade $\tau = 1$ but also two other important points of comparative statics: the point $\tau_1 = \frac{1}{\theta}$ where small tariff (subsidy) exactly compensates for transport costs, which was plays the role which was played by free trade: $x = y$, and the point τ_{x0}, where domestic consumption disappears $(x_{\tau_{x0}} = 0)$.

Proposition 7. *Assume positive transport cost (losses) $\theta > 1$ for K symmetric countries. A symmetric import tariff coefficient τ modifies the equilibrium as follows: (i) Domestic individual consumption increases $(dx_\tau^\theta/d\tau > 0\ \forall\tau \in (0, \tau_a))$, whereas imports decrease: $(dy_\tau^\theta/d\tau < 0\ \ \forall\tau \in (0, \tau_a))$. The higher the transport cost θ, the higher the rate of change in import and domestic consumption, as in (42), and the smaller the autarky point $\tau_a(\theta)$.*

(ii) Firm output $q_\tau^\theta \equiv Lx_\tau^\theta + \theta \cdot LKy_\tau^\theta$ has a derivative expressed as (43), which equals 0 at the compensating point $\tau_1 = \frac{1}{\theta}$, but becomes negative at the autarky point τ_a (if any) and becomes positive at point $\tau_{x=0}$, where domestic individual consumption is $x = 0$. At any $\tau \in (\tau_1, \tau_a)$ the firm's output monotonically decreases in τ iff condition (25) on preferences holds. In particular, it holds under IED preferences supplemented by condition $\mathcal{E}_{r'_u}(\cdot) + 2 \geq 0$. Examples are linear demands and other "flat" demands, including functions CARA, AHARA+, LOG+. By contrast, for some DED functions output $q_\tau^\theta \equiv Lx_\tau^\theta + \theta \cdot LKy_\tau^\theta$ may increase with in tariff at any $\tau \in (\tau_1, \tau_a)$.

(iii) The equilibrium mass of firms N always responds to tariff τ inversely to firm output: $dq_\tau^\theta/d\tau > 0 \Rightarrow dN_\tau^\theta/d\tau < 0$, $dq_\tau^\theta/d\tau < 0 \Rightarrow dN_\tau^\theta/d\tau > 0$, $dN_\tau^\theta/d\tau = 0 \Rightarrow dN_\tau^\theta/d\tau = 0$.

Proof. The proof repeats one of Proposition 1. (see Appendix 5) ⊠

Positive transport costs θ make import smaller than domestic consumption and a tariff further aggravates this disproportion. However, under the $IED + DEU$ assumption, output q decreases while the number of firms grows.

Proposition 8. *Assume a symmetric tariff in K symmetric countries, and positive transport cost $\theta > 1$. Then:*

(i) Under DEU $(\mathcal{E}'_u(x) < 0)$, IEU $(\mathcal{E}'_u(x) > 0)$, CES $(\mathcal{E}'_u(x) = 0)$ increases welfare $W_\tau^{\theta\prime}(\tau) > 0$ at the compensating point $\tau = \frac{1}{\theta}$.

(ii) Assuming finite derivatives $u'(0)$, $u''(0)$, then there is an autarky point $\tau_a : y = 0$, local DEU, IED properties hold there $(\mathcal{E}'_u(0) < 0,\ r'_u(0) > 0)$, and the welfare impact of tariff obeys the rule

$$1/\mathcal{E}_u(x_a) < \tau_a \Rightarrow W_\tau^{\theta\prime}(\tau_a) < 0, \quad 1/\mathcal{E}_u(x_a) > \tau_a \Rightarrow W_\tau^{\theta\prime}(\tau_a) > 0.$$

In particular, welfare $W^\theta(\tau)$ decreases with DEU in any tariff under DEU and $\theta < u'(0)/u'(x_a)$ i.e. trade costs are sufficiently small $\theta < u'(0)/u'(x_a)$, whereas under higher cost $\theta > u'(0)/u'(x_a)$ welfare $W^\theta(\tau)$ increases with a small tariff $\tau_a < 1/\mathcal{E}_u(x_a)$.

Proof is in Appendix *5*. ⊠

Summarizing, under DEU a small positive tariff is always welfare improving (optimal tariff is positive) under positive transport costs.

We have a conjecture that the optimal tariff is negative (subsidy) in the IEU case, whereas it is positive in the DEU case, as Fig. 8 shows, under any trade costs (i.e., a small tariff is always beneficial under DEU). However, our proof of this important fact is incomplete so far.

5 Conclusion

We study *reciprocal* ad valorem or specific import tariffs in the general-form Krugman model of international trade among several symmetric countries (complimenting the theory of *unilateral* tariffs under monopolistic competition).

It is shown that under preferences with constant elasticity of substitution – CES – any tariffs or subsidies are harmful, because they induce product distortion, whereas variety is socially optimal without regulation. Under "flatter" demands, satisfying the realistic assumptions of increasingly elastic demand (IED) and decreasingly-elastic utility (DEU), any specific or ad valorem tariff deteriorates welfare, when transportation costs are zero. Under the same conditions, a specific small import subsidy (or export tariff) can improve welfare. In the realistic situation, when tariffs are combined with some transport cost, *a small ad valorem tariff can improve welfare* Similarly, a specific tariff is also beneficial in some cases, at least under transport costs, exceeding a certain critical level.

In other words, in some realistic cases, a small mutual export tariff can be socially optimal for both trading countries. The reason is that it cures the market distortion connected with excessive entry (an inefficiently high mass of firms), though somewhat distorts the demand for imports. This argument for slight protectionism is not found in the literature. Its mechanism does not stem from the well-known "infant industry" arguments or oligopoly arguments, only from love for variety and inefficiently high free entry under variable elasticity, known since Dixit and Stiglitz (1977).

Further research in this direction can consider heterogeneous firms and empirical calibration of the effect shown.

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Appendices

Appendix 1. Derivation of export-import balance

Let us derive the balance of revenues and expenditures for each country from the budget constraints. The left side is the proceeds from sales of all goods, the right side is the country's income from wages and transfers.

The free entry condition for each country j for firms:

$$\pi_j \equiv [p_{jj} - cw_j] L_j x_{jj} + \sum_{k \in \mathcal{K} \setminus \{j\}} [p_{jk} - cw_j - t] L_k x_{jk} - w_j f = 0.$$

Using (12) we get

$$p_{jj} N_j L_j x_{jj} + \sum_{k \in \mathcal{K} \setminus \{j\}} [p_{jk} - t] N_j L_k x_{jk} - w_j L_j = 0.$$

The budget constraint (13) for country j:

$$p_{jj} N_j L_j x_{jj} + \sum_{k \in \mathcal{K} \setminus \{j\}} [p_{kj} - t] N_k L_j x_{kj} - w_j L_j = 0.$$

From these two equations we get the balance of trade, i.e., export equals import:

$$\sum_{k \in \mathcal{K} \setminus \{j\}} [p_{kj} - t] N_k L_j x_{kj} = \sum_{k \in \mathcal{K} \setminus \{j\}} [p_{jk} - t] N_j L_k x_{jk}.$$

Appendix 2. How equilibrium characteristics of symmetric countries respond to ad-valorem import tariff

Proof of Proposition 1. (i) Reaction of **consumption**. To find how consumptions in $K+1$ symmetric countries respond to tariff τ, we denote total derivatives as $x'_\tau \equiv \frac{dx}{d\tau}$, $y'_\tau \equiv \frac{dy}{d\tau}$ and $R'_z \equiv R'(z)$. Our symmetric-equilibrium equations are:

Free entry:

$$\pi(x, y) \cdot \lambda \equiv R(x) + K \frac{R(y)}{\tau} - c \cdot (x + Ky) \lambda - f\lambda = 0,$$

FOC:

$$R'(x) = c\lambda, \;\; R'(y) = c\tau\lambda.$$

So

$$R'(x)\,\tau = R'(y),$$

i.e.,

$$\tau = \frac{R'(y)}{R'(x)}.$$

Thus,

$$\frac{R(x)}{R'(x)} + K \frac{R(y)}{R'(y)} = x + Ky + \frac{f}{c}.$$

Totally differentiating the latter equations in τ (and applying $\frac{d\pi}{dx} = 0$, $\frac{d\pi}{dy} = 0$ or Envelope Theorem to the third equation) we get

$$R''(x) x'_\tau = c\lambda'_\tau, \;\; R''(y) y'_\tau = c\lambda + c\tau\lambda'_\tau$$

and

$$\lambda'_\tau = -\frac{KR(y)}{\tau^2 \left(c\,(x + Ky) + f\right)},$$

i.e.,

$$\lambda'_\tau = -\frac{KR(y)\left(R'(x)\right)^2}{\left(R'(y)\right)^2\left(c\left(x+Ky\right)+f\right)}.$$

It follows that the total derivatives of consumptions are

$$x'_\tau = -\frac{KR(y)}{\tau^2 R''(x)\left(x+Ky+\frac{f}{c}\right)} > 0,$$

i.e.,

$$x'_\tau = -\frac{KR(y)\left(R'(x)\right)^2}{\left(R'(y)\right)^2 R''(x)\left(x+Ky+\frac{f}{c}\right)} > 0;$$

$$y'_\tau = \frac{R'(y)\left(x+Ky+\frac{f}{c}\right)-KR(y)}{\tau^2 R''(y)\left(x+Ky+\frac{f}{c}\right)},$$

i.e.,

$$y'_\tau = \frac{\left(x+Ky+\frac{f}{c}\right)-\left(x+Ky+\frac{f}{c}\right)+\frac{R(x)}{R'(x)}}{\tau R''(y)\left(x+Ky+\frac{f}{c}\right)},$$

i.e.,

$$y'_\tau = \frac{R'(y)\frac{R(x)}{R'(x)}}{\tau R''(y)\left(x+Ky+\frac{f}{c}\right)},$$

i.e.,

$$y'_\tau = \frac{R(x)}{R''(y)\left(x+Ky+\frac{f}{c}\right)} < 0.$$

(Here we used that $K\frac{R(y)}{R'(y)} = \left(x+Ky+\frac{f}{c}\right)-\frac{R(x)}{R'(x)}$.)

(ii) **Reaction of output**. Under *general* tariff, to find its impact on sales (output), we combine the changes in x and y:

$$q'_\tau = x'_\tau + Ky'_\tau = \frac{KR(x)}{R''(y)\left(x+Ky+\frac{f}{c}\right)} - \frac{KR(y)}{\tau^2 R''(x)\left(x+Ky+\frac{f}{c}\right)} =$$

$$= \frac{K\left(\tau^2 R(x)R''(x)-R(y)R''(y)\right)}{\tau^2 R''(y)R''(x)\left(x+Ky+\frac{f}{c}\right)}. \quad (44)$$

We would like to know the sign of this derivative. For linear demand $R''(x)=R''(y)=$ *constant*, so, the sign is clear: output decreases in τ on the whole interval.

Let us find out the derivative of output at **free trade,** where $\tau=1,\ x=y$,

$$x'_\tau = -\frac{KR(x)}{1\cdot R''(x)\left(\left(1+K\right)x+\frac{f}{c}\right)} > 0, \quad (45)$$

$$y'_\tau = \frac{R(x)}{R''(y)\left(\left(1+K\right)x+\frac{f}{c}\right)} < 0. \quad (46)$$

To find the derivative of output $q'_\tau = x'_\tau + Ky'_\tau$ we substitute (45), (46) into $q'_\tau = x'_\tau + Ky'_\tau$ and

$$q'_\tau\mid_{\tau=1} = 0.$$

To find the derivative of sales at **autarky** $(\tau_a :\ y(\tau_a)=0)$ we just plug $y(\tau_a)=0$ into our formulate and obtain

$$x'_{\tau_a} = -\frac{K \cdot R(0)}{\tau^2 \cdot R''(x)\left(x + \frac{f}{c}\right)} = 0,$$

$$y'_{\tau_a} = \frac{K \cdot R(x)}{R''(0)\left(x + \frac{f}{c}\right)} < 0,$$

$$q'_{\tau} < 0.$$

To find the derivative of sales at **global point** ($\tau_{x0}: \ x(\tau_{x0}) = 0$) we just plug $x(\tau_{x0}) = 0$ into our formula and obtain

$$x'_{\tau_{x0}} = -\frac{KR(y)}{\tau^2 \cdot R''(0)\left(y + \frac{f}{c}\right)} > 0,$$

$$y'_{\tau_{x0}} = \frac{K\theta \cdot R(0)}{R''(y)\left(y + \frac{f}{c}\right)} = 0,$$

$$q'_{\tau_{x0}} > 0.$$

So it decreases when subsidy grows.

Global impact q'_τ of τ. We use that $\tau = \frac{R'(y)}{R'(x)}$. Substitute τ into (44):

$$q'_{\tau} = \frac{K\left(\left(\frac{R'(y)}{R'(x)}\right)^2 R(x)R''(x) - R(y)R''(y)\right)}{\tau^2 R''(y)R''(x)\left(x + Ky + \frac{f}{c}\right)} =$$

$$= \frac{K\left(R'\left(y\right)\right)^2}{\tau^2 R''(y)R''(x)\left(x + Ky + \frac{f}{c}\right)} \cdot \left(\frac{R(x)R''(x)}{\left(R'\left(x\right)\right)^2} - \frac{R(y)R''(y)}{\left(R'\left(y\right)\right)^2}\right). \tag{47}$$

Sign of the bracket determines the sign of the derivative. Let us introduce the function $\phi(z) \equiv \frac{R(z)R''(z)}{(R'(z))^2}$. If function $\phi(\cdot)$ is decreasing then the derivative q'_τ of the total output is negative under a positive tariff $\tau > 1$.

For *IED:*

$$\frac{R(z)R''(z)}{\left(R'\left(z\right)\right)^2} = \frac{u'\left(z\right) \cdot x \cdot u''\left(z\right)\left(2 - r_{u'}\left(z\right)\right)}{\left(u'\left(z\right)\left(1 - r_u\left(z\right)\right)\right)^2} =$$

$$= -\frac{r_u\left(z\right)\left(2 - r_{u'}\left(z\right)\right)}{\left(1 - r_u\left(z\right)\right)^2} \equiv \frac{r'_u\left(z\right) \cdot z + r_u\left(z\right) - \left(r_u\left(z\right)\right)^2}{\left(1 - r_u\left(z\right)\right)^2}. \tag{48}$$

Find derivative:

$$-\left(\frac{r'_u\left(z\right) \cdot z + r_u\left(z\right) - \left(r_u\left(z\right)\right)^2}{\left(1 - r_u\left(z\right)\right)^2}\right)' =$$

$$= -\frac{r''_u\left(z\right) \cdot z \cdot \left(1 - r_u\left(z\right)\right) + 2 \cdot \left(r'_u\left(z\right)\right)^2 \cdot z + 2 \cdot \left(1 - r_u\left(z\right)\right) \cdot r'_u\left(z\right)}{\left(1 - r_u\left(z\right)\right)^3} =$$

$$= -\frac{\frac{r''_u\left(z\right) \cdot z}{r'_u\left(z\right)} \cdot \left(1 - r_u\left(z\right)\right) + 2 \cdot r'_u\left(z\right) \cdot z + 2 \cdot \left(1 - r_u\left(z\right)\right)}{\left(1 - r_u\left(z\right)\right)^3} \cdot r'_u\left(z\right) =$$

$$= -\frac{\left(\mathcal{E}_{r'_u}\left(z\right) + 2\right) \cdot \left(1 - r_u\left(z\right)\right) + 2 \cdot r'_u\left(z\right) \cdot z}{\left(1 - r_u\left(z\right)\right)^3} \cdot r'_u\left(z\right).$$

For function *AHARA*, $u(z) = z^\rho + l \cdot z$, where $l \in (-\infty, +\infty)$:

$$\mathcal{E}_u\left(z\right) = \frac{\rho z^\rho + l \cdot z}{z^\rho + l \cdot z} < 1,\ r_u\left(z\right) = \frac{\rho\left(1 - \rho\right) z^{\rho - 1}}{\rho z^{\rho - 1} + l},\ r_{u'}\left(z\right) = 2 - \rho,\ r_{u''}\left(z\right) = 3 - \rho,$$

$$r_u\left(z\right) = \frac{\left(\rho z^{\rho - 1} + l\right) \cdot \left(1 - \rho\right) - l \cdot \left(1 - \rho\right)}{\rho z^{\rho - 1} + l} = \left(1 - \rho\right) - \frac{l \cdot \left(1 - \rho\right)}{\rho z^{\rho - 1} + l},$$

$$r_u'(z) = -l\cdot(1-\rho)^2\rho\cdot\frac{z^{\rho-2}}{(\rho z^{\rho-1}+l)^2} > 0,$$

$$r_u''(z) = -l\cdot(1-\rho)^2\rho\cdot\frac{-\rho^2\cdot z^{\rho-1}+(\rho-2)\cdot l}{(\rho z^{\rho-1}+l)^3}\cdot z^{\rho-3},$$

$$r_u''(z)\cdot z + 2\cdot r_u'(z) = -l\cdot(1-\rho)^2\rho\cdot\frac{-\rho^2\cdot z^{\rho-1}+(\rho-2)\cdot l}{(\rho z^{\rho-1}+l)^3}\cdot z^{\rho-2} - 2\cdot l\cdot(1-\rho)^2\rho\cdot\frac{z^{\rho-2}}{(\rho z^{\rho-1}+l)^2} =$$

$$= -l\cdot(1-\rho)^2\rho^2\cdot\frac{(2-\rho)\cdot z^{\rho-1}+l}{(\rho z^{\rho-1}+l)^3}\cdot z^{\rho-2}.$$

For IED:

$$\left(\rho z^{\rho-1}+l>0 \qquad and \qquad l<0\right)$$

$$-l\cdot(1-\rho)^2\rho^2\cdot\frac{(2-\rho)\cdot z^{\rho-1}-\rho z^{\rho-1}}{(\rho z^{\rho-1}+l)^3}\cdot z^{\rho-2} =$$

$$= -2\cdot l\cdot(1-\rho)^3\rho^2\cdot\frac{z^{2\rho-3}}{(\rho z^{\rho-1}+l)^3} > 0.$$

Hence, for IED (48) is a decreasing function.

For DED:

$$\left(\rho z^{\rho-1}+l>0 \qquad and \qquad l>0\right)$$

$$-l\cdot(1-\rho)^2\rho^2\cdot\frac{(2-\rho)\cdot z^{\rho-1}-\rho z^{\rho-1}}{(\rho z^{\rho-1}+l)^3}\cdot z^{\rho-2} =$$

$$= -2\cdot l\cdot(1-\rho)^3\rho^2\cdot\frac{z^{2\rho-3}}{(\rho z^{\rho-1}+l)^3} < 0.$$

Hence, for DED (48) is increasing function.

For $u(z) = log(z+d) - log(d)$ $(DEU - IED)$:

$$\left(-\frac{r_u(z)(2-r_{u'}(z))}{(1-r_u(z))^2}\right)' = -\frac{2(d+z)}{d}.$$

In this case (48) is decreasing function on $\tau > 1$.

For $CARA$ $u(z) = 1 - \exp(-z)$ $(DEU - IED)$:

$$\mathcal{E}_u(z) = \frac{z\cdot\exp(-z)}{1-\exp(-z)},\ r_u(z) = z,\ r_{u'}(z) = z,\ r_{u''}(z) = z,$$

$$(1 - r_u(z)) = 1 - z,\ \ (2 - r_{u'}(z)) = 1 - z,\ \ (3 - r_{u''}(z)) = 1 - z,$$

$$r_u'(z) = 1,\ \ r_{u'}'(z) = 1,$$

$$\left(-\frac{r_u(z)(2-r_{u'}(z))}{(1-r_u(z))^2}\right)' = -\frac{1}{(1-z)^2} < 0.$$

Hence, for the case of $CARA$, (48) is decreasing function on $\tau > 1$.

Proof of Proposition 2 (about welfare)

Recall that any consumer's welfare to be studied is expressed through consumptions as

$$W_\tau(x,y) = \frac{u(x)+Ku(y)}{f+c(x+Ky)}.$$

Using notations $C_q \equiv C(x+Ky) \equiv f + c(x+Ky)$, we estimate the welfare total derivative W_τ' w.r.t. tariff τ:

$$W_\tau'(x,y) = \frac{u'(x)\,x_\tau' + Ku'(y)\,y_\tau'}{(f+c(x+Ky))} - \frac{u(x)+Ku(y)}{(f+c(x+Ky))}\cdot\frac{c(x_\tau'+Ky_\tau')}{(f+c(x+Ky))}.$$

Multiplying everything by $\frac{C_q}{U}$ we come to

$$\frac{C_q}{U}\cdot W'_\tau(x,y)=\frac{u'(x)\,x'_\tau+Ku'(y)\,y'_\tau}{u(x)+Ku(y)}-\frac{c\,(x'_\tau+Ky'_\tau)}{(f+c(x+Ky))}=$$

$$=x'_\tau\left[\frac{u'(x)}{u(x)+Ku(y)}-\frac{c}{(f+c(x+Ky))}\right]+y'_\tau K\left[\frac{u'(y)}{u(x)+Ku(y)}-\frac{c}{(f+c(x+Ky))}\right], \tag{49}$$

which can be expressed in elasticities as

$$\frac{C_q}{U}\cdot W'_\tau=\frac{x'_\tau}{x}\cdot\left[\mathcal{E}_{U|x}-\mathcal{E}_{C|x}\right]+K\frac{y'_\tau}{y}\cdot\left[\mathcal{E}_{U|y}-\mathcal{E}_{C|y}\right], \tag{50}$$

*(i)***Unchanging welfare at the point of free trade.** Using elasticities, at free trade we plug $\tau=1$, $x=y$ into expression (50), substitute $\mathcal{E}_{C|y}=\mathcal{E}_{C|x}$ (which is true everywhere, not only at $x=y$) and obtain

$$\frac{C_q}{U}\cdot W'_\tau=\left[\mathcal{E}_{U|x}-\mathcal{E}_{C|x}\right]\left(\frac{x'_\tau}{x}+K\frac{y'_\tau}{x}\right)=0$$

because of zero change $q'_\tau=x'_\tau+Ky'_\tau=0$ at $\tau=1$, by Proposition 1.

The global welfare change under *IED+DEU*

Rearranging (49) we get

$$\frac{C_q}{U}\cdot W'_\tau=x'_\tau\left[\frac{u'(x)}{u(x)+Ku(y)}-\frac{c}{(f+c(x+Ky))}\right]+y'_\tau K\left[\frac{u'(y)}{u(x)+Ku(y)}-\frac{c}{(f+c(x+Ky))}\right]. \tag{51}$$

This expression, denoted further by $x'_\tau\cdot[A]+Ky'_\tau\cdot[B]\equiv\frac{C_q}{U}\cdot W'_\tau$ we would like to prove being negative everywhere, except the point of free trade. At free trade, this $\frac{C_q}{U}\cdot W'_\tau$ is zero, because the first bracket is equal to the second one ($A=B$), while $q'_\tau=x'_\tau+Ky'_\tau=0$ at $\tau=1$. Both brackets at free trade ($\tau=1,\ x=y=z$) are positive under *DEU* ($\mathcal{E}'_u(z)<0$) because

$$[B]_{\tau=1}=\frac{u'(z)}{2u(z)}-\frac{c}{f+c(1+K)z}=-\frac{1}{2z}\left(\mathcal{E}_u(z)-1+r_u(z)\right)=-\frac{\mathcal{E}'_u(z)}{2\mathcal{E}_u(z)}>0,$$

using (53) and identity $z\cdot\mathcal{E}'_u(z)\equiv\mathcal{E}_u(z)\cdot(1-\mathcal{E}_u(z)-r_u(z))$ that can be easily derived for any function u.

Further, under positive tariff, the second bracket $[B]$ should *increase* (and remain positive) when τ increases and thereby y decreases. Indeed, differentiating $[B]$ we get

$$[B]'_\tau=\frac{u''(y)\,y'_\tau}{u(x)+Ku(y)}-\frac{u'(y)\,(u'(x)\,x'_\tau+Ku'(y)\,y'_\tau)}{(u(x)+Ku(y))^2}+\frac{c^2\,(x'_\tau+Ky'_\tau)}{(f+c(x+Ky))^2}>0$$

under *DEU*. First summands here is positive due to $u''(y)\,y'_\tau>0$, see part (i) of the proposition. Two of the remaining amount are

$$y'_\tau\nu_y+x'_\tau\nu_x\equiv y'_\tau\left[-\frac{(u'(y))^2}{(u(x)+Ku(y))^2}+\frac{1}{\left(\frac{f}{c}+(x+Ky)\right)^2}\right]+x'_\tau\left[-\frac{u'(y)\,u'(x)}{(u(x)+Ku(y))^2}+\frac{1}{\left(\frac{f}{c}+(x+Ky)\right)^2}\right].$$

Let's compare two of the remaining amount ν_y and ν_x. If $-\frac{(u'(y))^2}{(u(x)+Ku(y))^2}+\frac{1}{\left(\frac{f}{c}+(x+Ky)\right)^2}<0$ then we have $y'_\tau\nu_y>-x'_\tau\nu_x$ (using $\frac{(u'(y))^2}{(u(x)+Ku(y))^2}>\frac{(u'(x))^2}{(u(x)+Ku(y))^2}$, i.e., $u'(z)$is decreasing function). Then we get that positive summand x'_τ weighted with small positive or negative multiplier ν_x, whereas the negative summand y'_τ is weighted with bigger negative multiplier ν_{y}, while without multipliers $x'_\tau+Ky'_\tau<0$ under *IED* by Proposition 1. Then $y'_\tau\nu_y+x'_\tau\nu_x>0$.

We prove that $-\frac{(u'(y))^2}{(u(x)+Ku(y))^2}+\frac{1}{\left(\frac{f}{c}+(x+Ky)\right)^2}<0$. So, bracket $[B]$ increases in τ, remaining positive, whereas its multiplier Ky'_τ is negative. At the same time, for all positive tariffs $[A]<[B]$, because $u'(x)<u'(y)$, other parts of these expressions being similar. Further, consider the sum $x'_\tau\cdot[A]+Ky'_\tau\cdot[B]$, where the first positive summand is weighted with a smaller multiplier $[A]$ (positive or negative), than the negative summand Ky'_τ. So, the sum remains negative (provided it was negative: $x'_\tau+Ky'_\tau<0$ without any multipliers).

Thus, under $IED - DEU$ we have proven strict decrease of welfare everywhere except free trade.

One can extend exactly the same reasoning to the case of subsidies, where $\tau = 1 - s < 1$, here welfare decreases in subsidy.

The global welfare change under *DED+IEU* - just exactly mirror the proof for previous $IED - DEU$ case. Only both signs change in the derivation: $x'_\tau + Ky'_\tau > 0$ for $\tau \in (1, \tau_a)$ under IED and $\mathcal{E}'_u(z) > 0$ under DEU in assessing the sign of term $[A]$, instead of term $[B]$, both of them starting from a negative value, and $[A]$ decreasing. ⊠

Appendix 3. How equilibrium variables of trading symmetric countries respond to specific import tariff

Proof of Proposition *3*: *comparative statics of quantities.*

(i) **Reaction of consumption**. To find how consumptions in $K+1$ symmetric countries respond to tariff τ, we denote total derivatives as $x'_\tau \equiv \frac{dx}{d\tau}$, $y'_\tau \equiv \frac{dy}{d\tau}$ and $R'_x \equiv R'(x)$, $\varphi(x) \equiv \frac{R(x)}{R'(x)}$, $\varphi'(x) \equiv 1 - \frac{R(x)R''(x)}{(R'(x))^2}$. Our symmetric-equilibrium equations (17-19)

$$R'(x)\tau = R'(y). \tag{52}$$

$$\frac{R(x)}{R'(x)} - x + K\tau\left(\frac{R(y)}{R'(y)} - y\right) = \frac{f}{c}, \tag{53}$$

can be reformulated as

$$R'(x) = c\lambda, \;\; R'(y) = c\tau\lambda,$$

$$\pi(x, y)\cdot\lambda \equiv R(x) + KR(y) - \left(\frac{f}{c} + x + K\tau y\right)\lambda = 0.$$

Totally differentiating the latter equations in τ (and applying $\frac{d\pi}{dx} = 0, \frac{d\pi}{dy} = 0$ or Envelope Theorem to the third equation) we get

$$R''(x)x'_\tau = c\lambda'_\tau, \;\; R''(y)y'_\tau = c\lambda + c\tau\lambda'_\tau,$$

$$\lambda'_\tau = -\frac{\lambda K y}{\left(\frac{f}{c} + x + K\tau y\right)}.$$

It follows that the total derivatives of consumptions are

$$x'_\tau = \lambda c\cdot\frac{-cKy}{R''(x)\left(\frac{f}{c} + x + K\tau y\right)}, \qquad y'_\tau = \lambda c\cdot\frac{1 - \dfrac{\tau K y}{\left(\frac{f}{c} + x + K\tau y\right)}}{R''(y)}.$$

After excluding λ, the expressions show the reaction of equilibria consumptions to tariffs (or to trade costs, interchangeably):

$$x'_\tau = -\frac{Ky\cdot R'(x)}{R''(x)\left(\frac{f}{c} + x + K\tau y\right)} > 0, \tag{54}$$

$$y'_\tau = \frac{R'_x\left(\frac{f}{c} + x\right)}{R''(y)\left(\frac{f}{c} + x + K\tau y\right)} < 0. \tag{55}$$

Thus, the elasticity of consumption of imported goods is negative because consumers shift their purchases to cheaper domestic goods.

(ii) **Reaction of output**.

Under globally changing specific tariff, to find its impact on sales (output), we combine the changes in x and y:

$$q'_\tau = x'_\tau + Ky'_\tau = \frac{K(x + \frac{F}{cL})\cdot R'(x)}{R''(y)C(x + K\tau y)} - \frac{Ky\cdot R'(x)}{R''(x)C(x + K\tau y)} =$$

$$= \frac{KR'(x)\left(xR''(x) + \frac{f}{c}R''(x) - yR''(y)\right)}{R''(y)R''(x)C(x+K\tau y)}. \tag{56}$$

For non-linear demands, output may react as down-up-down even under IED: see Example in Fig. 1. However, for linear demand $R''(x) = R''(y) = constant < 0$, so, the sign is clear: output is decreasing in τ on the whole domain. Somewhat more general restriction - "uniform flatness" (29), states decreasing function xR''_x. It also provides decreasing output because, under $x > y$, it yields negative term $\left(\left(x + \frac{f}{c}\right)R''(x) - yR''(y)\right) < 0$ in (56), other terms being positive. So, output shrinks everywhere under concavity condition (29).

Now we use the weaker condition to show a weaker statement. In other words, decreasing xR''_x guarantees a convex "lower Lebesgue set" of gross profit under $\tau = 1$:

$$\frac{R(x)}{R'(x)} - x + \frac{R(y)}{R'(y)} - y \le \frac{f}{c} \quad \forall (x,y) : x + y = q_1$$

which means that maximal output is attained at point $x = y$, under $\tau = 1$. Using Fig. 1 we explain general change in output under specific tariff.

In equation $\frac{R(x)}{R'(x)} - x + \tau\left(\frac{R(y)}{R'(y)} - y\right) = \frac{f}{c}$, parameter $\tau > 1$ makes the admissible points *lower* in coordinate y under any x (shrinking), whereas admissible point remains the same. In another equation, $R'(x)\tau = R'(y)$, parameter τ similarly suppresses y under any x, so, the intersection (equilibrium under $\tau > 1$) must lie below initial curve $\frac{R(x)}{R'(x)} - x + \frac{R(y)}{R'(y)} - y = \frac{f}{c}$ and below initial curve $R'(x) = R'(y)$. This means that it is below line $x + y = x_1 + y_1$, and output shrinks under concavity condition (29) imposed only on the initial curve $y(x)$ determined by $\frac{R(x)}{R'(x)} - x + \frac{R(y)}{R'(y)} - y = f/c$.

Arguments for other conditions of shrinking output are studied before formulating Proposition 3.

What remains for general behavior of output is showing and example of down-up-down evolution. It is our Example in Fig. 3.

Now we turn to point-wise characterization of output.

To find the derivative of output at **free trade** ($\tau = 1$), we substitute the consumptions (55), (54) into $q'_\tau = x'_\tau + Ky'_\tau$ and express q'_τ as (28). Because of $R''(x) < 0$, at the particular point $\tau = 1,\ x = y$, the numerator in this formula is less than zero, that yields the sign of tariff impact

$$q'_\tau \mid_{\tau=1} < 0.$$

Taking the opposite direction (introducing subsidies on imports at the point $\tau = 1$), the result is an increase in the size of the firm q_τ with the subsidy.

To find the derivative of sales at **autarky** ($\tau_a :\ y(\tau_a) = 0$) we just plug $y(\tau_a) = 0$ into our formula and obtain

$$\frac{dx}{d\tau} = -\frac{y \cdot R'(x)}{R''(x)\tilde{C}_q} = 0,$$

$$\frac{dy}{d\tau} = \frac{R'(x)\left(\frac{f}{c} + x\right)}{R''(y)\tilde{C}_q} = \frac{R'(x)}{R''(y)} < 0,$$

$$\frac{dN}{d\tau} = -N \cdot \frac{\frac{dy}{d\tau}}{C_q} > 0.$$

(iii) **The mass of firms in the economy**.

Mass of firms has the opposite sign with respect to the output of one firm and it follows from the equation

$$N'_\tau\left(\frac{F}{cL} + q\right) \cdot c + N \cdot (x'_\tau + Ky'_\tau) \cdot c = 0.$$

Therefore

$$N'_\tau = -N \cdot \frac{x'_\tau + Ky'_\tau}{c \cdot C(x + Ky)}.$$

The consequence is an increase in the mass of firms in the economy with the introduction of subsidies on imports at the point $\tau = 1$.

Proof of Proposition 4: *Comparative statics of welfare.*

(i) To ensure existence of our maximum, we would like that the domain of optimization be compact. We exploit Proposition 1, especially increasing x_τ and decreasing y_τ at equilibria. Such increase of x is bounded from above by x_a, which we define as an autarky point achieved by a closed economy (absent trade). For the case like CES preferences, when rising tariff does not achieve complete autarky, this point x_a arise as a limit $x_a = \lim_{\tau\to\infty} x_\tau$. Anyway, admissible $x \le x_a$. To restrict another variable, y, we express our per-citizen **welfare** $W(x,y)$ maximized as:

$$\max_{\{x,y=y^*(x)\}} W = W(x,y) \equiv \frac{U(x,y)}{C(x+Ky)} = \frac{u(x)+Ku(y)}{f+c(x+Ky)},$$

where continuous decreasing function $y^*(x)$ is defined in (31). Variables (x,y) indirectly depend on τ, thereby tariff is optimized indirectly, by choosing (x,y) (which determines τ using monotone x_τ). To restrict admissible y, let us denote one of the admissible points, the free-trade consumption as (x_1,y_1). We artificially restrict the domain for optimization, to consider only such (x,y) that bring welfare not lower than welfare at the free trade point:

$$Z \equiv \{(x,y)|\, W(x,y) \ge W(x_1,y_1),\, y = y^*(x)\}.$$

Is Z bounded? Relying on our assumption of non-negative limit $\lim_{x\to x_a} u'(x) \ge 0$, we conclude that at any fixed x, the denominator of function $W(x,y)$ increases linearly in y whereas the numerator essentially cease to increase sooner or later. When we tend x to zero, related $y^*(x)$ may increase up to some finite limit $\breve{y}$ (this case suffices for compactification) or to infinity. In the latter case, welfare $\frac{u(0)+Ku(y)}{f+c(0+Ky)}$ sooner or later (under some big $\breve{y}$) start to decrease and becomes lower than the starting point, i.e., $W(x_1,y_1) > W(0,y)$ for $y > \breve{y}$. Excluding such big y from consideration, we conclude that set $Z \subset [0,x_a] \times [0,\breve{y}]$ where we should seek for optimal tariffs, can be considered *bounded*. Instead of proving that it is closed, we take its *closure* Z_c, where continuous function $W(x,y)$ must have an artificial maximum

$$(x^{op}, y^{op}) \equiv \arg \max_{\{(x,y)\in Z_c|\, y=y^*(x)\}} W(x,y).$$

What remains is to ensure that this argmaximum is a real one, belonging to initial Z and not to artificial zone $Z_c \setminus Z$, and ensure positivity of $(x^{op}, y^{op}) \gg 0$. The latter fact would prove the former, because our decreasing curve $y^*(x)$ is continuous everywhere, except, maybe, its left end ($x \to 0$) and its right end ($y \to 0$).

(ii) Studying welfare under DEU, IED case to the right from free trade, let us first show that for all positive tariffs $\tau \in (1,\infty)$ related consumptions $(x(\tau), y(\tau))$ lie *below* the 45-degree line $q = x+y = x_1+y_1$ (and to the right from line $y = x$). Indeed, each equilibrium $(x(\tau), y(\tau))$ is the intersection of the curve $\frac{R(x)}{R'(x)} - x + K\tau\left(\frac{R(y)}{R'(y)} - y\right) - \frac{F}{cL} = 0$ defined in (18) with the curve $R'(x)\tau - R'(y) = 0$ defined in (17). The second curve shifts to the right (in x at any y), because of increasing function $R'(x)$. Similarly but more importantly for us, the first curve decreases (in y) everywhere (for all x) when τ grows, because of increasing function $\left(\frac{R(y)}{R'(y)} - y\right)$. The right end of this curve, $(x_{max}, 0)$ shows lower total output than free trade: $x_{max} < x_1 + y_1$, as proved in (Zhelobodko et al. (2012)) under IED. This fact under DEU proves that per-consumer output $q(x(\tau), y(\tau))$ under positive tariffs $\tau \in (1,\infty)$ is *lower* than the free trade one: $q(x(\tau), y(\tau)) < q(x_1,y_1)$ and $x > y$. We would like to see that $q(x(\tau), y(\tau)) < q(x_1,y_1)$ for all $\tau > 1$. Let us study all $(x,y):\ q = x(\tau)+y(\tau) = q(x_1,y_1)$ and differentiate (starting from x_1) in auxiliary variable z new function

$$\chi(z) \equiv \frac{x_1+z}{1/r_u(x_1+z)-1} + \frac{x_1-z}{1/r_u(x_1-z)-1},$$

it increases when $\frac{x_1+z}{1/r_u(x_1+z)-1}$ is convex.

With notations $C_q \equiv C(x+Ky) \equiv f + c(x+Ky)$, we estimate the welfare total derivative W'_τ w.r.t. tariff τ:

$$W'_\tau(x,y) = \frac{u'(x)\,x'_\tau + Ku'(y)\,y'_\tau}{C(x+Ky)} - \frac{u(x)+Ku(y)}{C(x+Ky)} \cdot \frac{c\,(x'_\tau + Ky'_\tau)}{C(x+Ky)},$$

i.e.,

$$C_q \cdot W'_\tau = x'_\tau \left(\frac{u'(x)}{U(x,y)} - \frac{c}{C(x+Ky)} \right) + Ky'_\tau \left(\frac{u'(y)}{U(x,y)} - \frac{c}{C(x+Ky)} \right). \tag{57}$$

Hence

$$C_q \cdot W'_\tau = \frac{x'_\tau}{x} \cdot \left[\mathcal{E}_{U|x} - \mathcal{E}_{C|x} \right] + \frac{y'_\tau}{y} \cdot \left[\mathcal{E}_{U|y} - \mathcal{E}_{C|y} \right], \tag{58}$$

which we would like to prove being negative.

In particular, at free trade ($\tau = 1,\ x = y$) this derivative becomes

$$C_q \cdot W'_\tau|_{\tau=1} = \frac{x'_\tau + Ky'_\tau}{(K+1)\cdot x} \cdot \left(\frac{u'(x)\cdot(K+1)\cdot x}{(K+1)\cdot u(x)} - \frac{c\cdot(K+1)\cdot x}{f+c\cdot(K+1)\cdot x} \right) = \frac{x'_\tau + Ky'_\tau}{(K+1)\cdot x} \cdot \left[\mathcal{E}_{u|x} - \mathcal{E}_{C|q} \right]$$

and output decreases here: $q'_\tau = x'_\tau + Ky'_\tau < 0$. So, positive or *negative* derivative $C_q \cdot W'_\tau$ of welfare at free trade depends on the sign of bracket $\left[\mathcal{E}_{u|x} - \mathcal{E}_{C|q} \right]$.

Three particular cases arise. Under CES one has $\left[\mathcal{E}_{u|x} - \mathcal{E}_{C|q} \right] = 0|_{\tau=1}$ because of social optimality proven through $\mathcal{E}_{R|x} = \mathcal{E}_{u|x} + \mathcal{E}_{\mathcal{E}_u|x} = \mathcal{E}_{u|x}$, therefore

$$W'_\tau|_{\tau=1,CES} = 0$$

(which we have seen in simulations).

Under DEU (IEU) case one has $\left[\mathcal{E}_{u|x} - \mathcal{E}_{C|q} \right]|_{\tau=1} > 0 \quad (< 0)$ because of socially insufficient consumption (proven through $\mathcal{E}_{R|x} = \mathcal{E}_{u|x} + \mathcal{E}_{\mathcal{E}_u|x} < \mathcal{E}_{u|x}$, $\mathcal{E}_{R|x} = \mathcal{E}_{u|x} + \mathcal{E}_{\mathcal{E}_u|x} > \mathcal{E}_{u|x}$, whereas equilibrium elasticities $\left[\mathcal{E}_{R|x} - \mathcal{E}_{C|q} \right]|_{\tau=1} = 0$), therefore

$$W'_\tau|_{\tau=1,DEU} < 0, \qquad W'_\tau|_{\tau=1,IEU} > 0.$$

An alternative proof goes through direct calculation:

$$W'_\tau|_{\tau=1} = \frac{(-x'_\tau - Ky'_\tau)\cdot u(x)}{C(x+Ky)x} \cdot \frac{\mathcal{E}'_u(x)}{\mathcal{E}_u(x)}. \tag{59}$$

It also distinguish DEU, CES, and IEU cases for decreasing, constant or increasing welfare at free trade, since all multipliers are positive, except for $\mathcal{E}'_u(x)$.

It is possible to estimate the utility derived substituting into expression of the derivatives at the threshold point $\tilde{\tau}, \overline{\tau}, \hat{\tau}$, where $q' = 0$. As $u'(x)$ decreasing function, we have

$$C_q \cdot W'_\tau|_{\tau\in\{\tilde{\tau},\overline{\tau},\hat{\tau}\}} = \frac{dx}{d\tau}\cdot(u'(x) - cW(x,y)) - \frac{dy}{d\tau}\cdot(u'(y) - cW(x,y)) = \frac{dx}{d\tau}\cdot(u'(x) - u'(y)) < 0.$$

It is important to evaluate the change of welfare in the intervals between such points.

Global impact of τ. Now, to infer the sign of W'_τ under all τ, we plug the equilibrium values of derivatives $x'_\tau = R'(x)\frac{-cKy}{R''(x)C(x+K\tau y)}$, $y'_\tau = \frac{R'(y)}{\tau} \cdot \frac{1-\frac{c\tau Ky}{C(x+K\tau y)}}{R''(y)}$, introduce new symbol $\tilde{C}_q \equiv C(x + K\tau y) \equiv f + c(x + K\tau y)$, $U(x,y) \equiv u(x) + Ku(y)$ and replacing $\tau = R'(y)/R'(x)$ to everything, and express the welfare derivative as

$$\frac{C_q}{K\tilde{C}_q} \cdot W'_\tau = R'(x)\left(\frac{-cy}{R''(x)} \cdot \left[\frac{u'(x)}{U(x,y)} - \frac{c}{C_q} \right] + \frac{f+cx}{R''(y)} \cdot \left[\frac{u'(y)}{U(x,y)} - \frac{c}{C_q} \right] \right).$$

For such expression, at free trade we have seen positive brackets **under** DEU **case**, to be studied now. Since $\frac{u'(x)}{-xR''(x)}$ remains positive, the welfare derivative can be estimated from above as

$$\widetilde{w} \equiv C_q \cdot W'_\tau = \frac{Ky\cdot R'(x)}{-R''(x)\tilde{C}_q} \cdot \left[u'(x) - \frac{c\cdot U(x,y)}{C_q} \right] - \frac{K\left(\frac{f}{c}+x\right)R'(x)}{-R''(y)\tilde{C}_q} \cdot \left[u'(y) - \frac{c\cdot U(x,y)}{C_q} \right] <$$

$$< \overline{w} \equiv \frac{K\cdot R'(x)}{\tilde{C}_q} \left(\frac{y}{-R''(x)} - \frac{\left(\frac{f}{c}+x\right)}{-R''(y)} \right) \cdot \left[u'(y) - \frac{c\cdot U(x,y)}{C_q} \right]. \tag{60}$$

Indeed, the second bracket remains higher than the first one on $\tau \in (1,\infty)$, because, due to concavity of u,

$$(x'_\tau > 0,\ y'_\tau < 0) \Longrightarrow x > y \Longrightarrow u'(x) < u'(y),$$

so, when we replace the first bracket by $\left[u'(y) - \frac{c \cdot U(x,y)}{C_q}\right]$, the sum increases. To ensure positive brackets everywhere, we use induction for case DEU. Whenever U/C_q decreases on some interval $\tau \in [1, 1+\varepsilon)$, it must strictly decrease on its upper bound also, as we have proven. Any point τ where U/C_q stop decreasing would contradict previous decrease. This entails increasing (and thus positive) bracket everywhere (because $y(\tau) \downarrow$, $u'(y(\tau)) \uparrow$, $U/C_q \downarrow$). In other words, since we have started from decreasing welfare $U(x,y)/C_q$ at the free trade point (from positive brackets), this decrease will continue for all higher τ.

The case of tariff. Further, under DEU, the path of comparative statics in $\tau \in [1, \infty)$ starts from inequality $\widetilde{w} = \overline{w} < 0|_{\tau=1,DEU}$ and equal positive brackets, with $x = y|_{\tau=1}$.

Let us consider all the intervals of monotony change of output where the size of firm decreases$q'_\tau < 0$ or $x'_\tau < -y'_\tau$. There the first bracket is negative. Then $\overline{w}$<0 and $W \downarrow$ for the case DEU (59) and $\tau \geq 1$. The global welfare decreases in all interval $\tau \in [1,\, \overline{\tau}]$: $q'_\tau \leq 0$ or $N'_\tau \geq 0$.

Similarly, for the case IEU.

At free trade we have seen positive brackets under IEU case. The welfare derivative can be estimated for the case of subsidy $\tau \in [\underline{\tau}, 1]$ and $x < y$ from above as

$$\widetilde{w} \equiv C_q \cdot W'_\tau = \frac{Ky \cdot R'(x)}{-R''(x)\tilde{C}_q} \cdot \left[u'(x) - \frac{c \cdot U(x,y)}{C_q}\right] - \frac{K\left(\frac{f}{c} + x\right) R'(x)}{-R''(y)\tilde{C}_q} \cdot \left[u'(y) - \frac{c \cdot U(x,y)}{C_q}\right] <$$

$$< \overline{w} \equiv \frac{K \cdot R'(x)}{\tilde{C}_q} \left(\frac{y}{-R''(x)} - \frac{\left(\frac{f}{c} + x\right)}{-R''(y)}\right) \cdot \left[u'(x) - \frac{c \cdot U(x,y)}{C_q}\right].$$

Similarly DEU, we prove that bracket $\left[u'(x) - \frac{c \cdot U(x,y)}{C_q}\right] < 0$ in interval $\tau \in [\underline{\tau}, 1]$. Let us consider all the intervals of monotony change of output where the size of firm increases $q'_\tau > 0$ or $x'_\tau > -y'_\tau$. There the first bracket is positive. Then $\overline{w}$<0 and $W \downarrow$ for the case IEU and (59) $\tau \leq 1$. The global welfare decreases in all interval $\tau \in [\underline{\tau},\, 1]$: $q'_\tau > 0$ or $N'_\tau \leq 0$.

Similarly, we can show that welfare decreases in interval $\tau \in [\hat{\tau},\, \tau^a]$ for case IEU suggesting that in this interval the second bracket (60) is positive.

Autarky. From

$$C_q \cdot W'_\tau = \frac{c \cdot K \cdot R'(x)}{-R''(y) \cdot C_q} u(x) + u'(y) \frac{K \cdot R'(x)}{-R''(y)} =$$

$$= -u(x) \cdot K \cdot \frac{R'(x)}{R''(y)} \cdot \frac{1 - r_u(x)}{x} (1 - \mathcal{E}_u(x) \cdot \tau)$$

using

$$\frac{R'(x)}{R'(y)} = \frac{1}{\tau}$$

we have $u'(y) = \tau \cdot u'(x)(1 - r_u(x))$. Moreover by (18) we have $\frac{1}{C_p} = \frac{1 - r_u(x)}{x}$.

Knowing that $\frac{u'(x)}{u'(y)} = \frac{1}{\tau(1 - r_u(x))} < 1$ or $\tau > \frac{1}{(1 - r_u(x))}$ we get

$$C_q \cdot W'_\tau = -u(x) \cdot K \cdot \frac{R'(x)}{R''(y)} \cdot \frac{1}{x} \cdot (1 - r_u(x) - \mathcal{E}_u(x) \cdot \tau(1 - r_u(x))) <$$

$$< -u(x) \cdot K \cdot \frac{R'(x)}{R''(y)} \cdot \frac{1}{x} \cdot (1 - r_u(x) - \mathcal{E}_u(x)) = -u(x) \cdot K \cdot \frac{R'(x)}{R''(y)} \cdot \frac{1}{x} \cdot \frac{\mathcal{E}'(x)}{\mathcal{E}_u(x)}.$$

Then the welfare decreases in autarky point in the case $\mathcal{E}'(z) \leq 0$, i.e., DEU and CES. In the case IEU, the sign of change of welfare is uncertain, but in simulation it is negative.

The case when $x = 0$.

If $x = 0$ and f is near 0, that $y'_\tau = 0$, then $f + y = \tau \cdot \frac{y}{(1 - r_u(y))}$, $u'(0) = \frac{u'(y)(1 - r_u(y))}{\tau}$

$$C_q \cdot W'_\tau = x' \cdot \left(u'(0) - \frac{u(y)}{f + y}\right) =$$

$$= x' \cdot \left(\frac{u'(y)(1 - r_u(y))(f + y) - \tau \cdot u(y)}{\tau(f + y)}\right) = \frac{x'}{(f + y)} \cdot \left(u'(y)(1 - r_u(y)) \frac{y}{(1 - r_u(y))} - u(y)\right) =$$

$$= \frac{x'}{(f+y)} \cdot (u'(y)\,y - u(y)) = \frac{u(y) \cdot x'}{(f+y)} \cdot (\mathcal{E}(y) - 1) < 0.$$

Appendix 4

Proof of Proposition 6.

For situations with positive transportation costs θ, the labor balance enables us to express welfare without mass of firms N as:

$$W_\tau^\theta(x,y) = \frac{u(x) + Ku(y)}{C(x+\theta\cdot Ky)}.$$

Totally differentiating W in tariff, we find its impact on welfare as follows:

$$W_\tau^{\theta\prime}(x,y) = \frac{x_\tau^{\theta\prime}\cdot u'(x) + Ky_\tau^{\theta\prime}\cdot u'(y)}{C(x+\theta\cdot Ky)} - \frac{\left(x_\tau^{\theta\prime} + \theta\cdot Ky_\tau^{\theta\prime}\right)c}{C^2(x+\theta\cdot Ky)} \cdot (u(x) + Ku(y)). \tag{61}$$

(i) We should prove that under CES ($\mathcal{E}_u'(x) = 0$), welfare $W^\theta(\tau)$ first increases in tariff on interval $\frac{1}{\theta} \le \tau < 1$ (interval of subsidization), then decreases on interval $1 < \tau < \infty$, reaching its maximum at unregulated trade $\tau^* = 1$.

(ii)–(iii) **at the point** $\breve{\tau} = \frac{1}{\theta}$.

Here the subsidy to firms equals the transport cost, $x = y$ and the welfare change can be reformulated as

$$w' \equiv W_\tau^{\theta\prime}(x,x)\left[\frac{C(x+\theta\cdot Kx)}{(u(x)+Ku(x))}\right] = \left[\frac{x_\tau^{\theta\prime} + Ky_\tau^{\theta\prime}}{(u(x)+Ku(x))}\cdot u'(x) - \frac{\left(x_\tau^{\theta\prime} + \theta\cdot Ky_\tau^{\theta\prime}\right)c}{C(x+\theta\cdot Kx)}\right].$$

We use the equilibrium conditions (38) which can be reformulated as $x_\tau^{\theta\prime} = \frac{-\theta Ky\cdot R'(x)c}{R''(x)\cdot C(x+Kx)} > 0\,,\qquad y_\tau^{\theta\prime} = \frac{\theta\left(\frac{f}{c}+x\right)\cdot R'(x)c}{R''(y)\cdot C(x+Kx)} = -x_\tau^{\theta\prime}\frac{\left(\frac{f}{c}+x\right)R''(x)}{KyR''(y)} < 0.$

Substituting these $x_\tau^{\theta\prime}$, $y_\tau^{\theta\prime}$, under $x = y$ we obtain

$$w' = \left[\frac{x_\tau^{\theta\prime} - x_\tau^{\theta\prime}\frac{\left(\frac{f}{c}+x\right)}{x}}{(u(x)+Ku(x))}\cdot u'(x) - \frac{\left(x_\tau^{\theta\prime} - x_\tau^{\theta\prime}\theta\cdot\frac{\left(\frac{f}{c}+x\right)}{x}\right)c}{C(x+\theta\cdot Kx)}\right] =$$

$$= x_\tau^{\theta\prime}\left[\frac{1 - \frac{\left(\frac{f}{c}+x\right)}{x}}{(u(x)+Ku(x))}\cdot u'(x) - \frac{\left(1-\theta\cdot\frac{\left(\frac{f}{c}+x\right)}{x}\right)c}{C(x+\theta\cdot Kx)}\right] =$$

$$= \frac{x_\tau^{\theta\prime}}{x}\left[\frac{-\frac{f}{c}\cdot u'(x)}{u(x)(1+K)} - \frac{\left(x - \theta\cdot\left(\frac{f}{c}+x\right)\right)c}{f + cLx(1+\theta\cdot K)}\right].$$

Further, taking out $\frac{1}{(1+K)x}$ and reformulating the terms we get

$$w' = \frac{x_\tau^{\theta\prime}}{x^2}\frac{1}{(1+K)}\left[-\frac{f}{c}\cdot\frac{xu'(x)}{u(x)} + \frac{\left((\theta-1)x + \theta\cdot\frac{f}{c}\right)}{\frac{f}{c} + x(1+\theta\cdot K)}(1+K)\,x\right].$$

Taking out $\frac{f}{c}$ and replacing elasticity $\mathcal{E}_u(x) \equiv \frac{xu'(x)}{u(x)}$ we obtain

$$w' = \frac{x_\tau^{\theta\prime}}{x^2}\frac{\frac{f}{c}}{(1+K)}\left[\frac{\left(\left(\frac{\theta-1}{\theta}\right)\frac{x}{\frac{f}{c}} + 1\right)}{\left(\frac{f}{c} + x(1+\theta\cdot K)\right)}\theta(1+K)\,x - \mathcal{E}_u(x)\right].$$

Recall that because of pricing rule $p(x) = c/(1 - r_u(x))$, zero-profit condition at point where $x = y$ becomes $px + Kpy = x(1+K)/(1-r_u(x)) = \frac{f}{c} + x(1+K)$, and we replace $(1+K)\,x$ as follows

$$w' = \frac{x_\tau^{\theta\prime}}{x^2}\frac{\frac{f}{c}}{(1+K)}\left[\frac{\left(\left(\frac{\theta-1}{\theta}\right)\frac{x}{\frac{f}{c}} + 1\right)\left(\frac{f}{c} + x(1+K)\right)}{\frac{f}{c} + x(1+\theta\cdot K)}\theta\,(1 - r_u(x)) - \mathcal{E}_u(x)\right].$$

For the simple basic model where $\theta = 1$, we can immediately replace by 1 the coefficient γ denoted

$$\gamma \equiv \frac{\left((\frac{\theta-1}{\theta})\frac{x}{\frac{f}{c}}+1\right)\left(\frac{f}{c}+x\,(1+K)\right)\theta}{\frac{f}{c}+x\,(1+\theta\cdot K)}. \tag{62}$$

Then, (recalling $x_{\tau}^{\theta\prime} > 0$) we come to condition $(1 - r_u(x) - \mathcal{E}_u(x)) > 0 \Rightarrow w' > 0$. Using $1 - r_u(x) - \mathcal{E}_u(x) = \frac{x\mathcal{E}_u'(x)}{\mathcal{E}_u(x)}$ this condition gives the needed classification of cases of welfare behavior into IEU ($w' > 0$), $CES(w' = 0)$, DEU ($w' < 0$) at point $x = y$.

In more complicated model with $\theta > 1$, we drop multiplier $\left((\frac{\theta-1}{\theta})\frac{x}{\frac{f}{c}}+1\right) > 1$ and estimate the size of coefficient γ (see (62)) as

$$\gamma > \frac{\theta\cdot Kx+\theta\cdot x+\theta\cdot\frac{f}{c}}{\theta\cdot Kx+x+\frac{f}{c}} > 1.$$

Introducing some positive multiplier $B > 0$ we can study the sign of welfare change depending on γ as

$$W_{\breve{\tau}}^{\theta\prime}(x,y) = B\cdot(\gamma\,(1-r_u(x))-\mathcal{E}_u)\,. \tag{63}$$

Using again $1 - r_u(x) - \mathcal{E}_u(x) = \frac{\mathcal{E}_u'(x)x}{\mathcal{E}_u(x)}$, the estimate $\gamma > 1$ brings conclusion $W_{\breve{\tau}}^{\theta\prime}(x,y) > 0$ for cases IEU, CES ($\frac{\mathcal{E}_u'(x)}{\mathcal{E}_u(x)} \geq 0$), i.e., welfare increases in tariff τ at point $\breve{\tau}$ where $x = y$.

For more complicated case DEU ($\frac{\mathcal{E}_u'(x)}{\mathcal{E}_u(x)} < 0$) we are going to derive two conditions on economic parameters: one for decreasing welfare $W_{\breve{\tau}}^{\theta}$ and one for increasing $W_{\breve{\tau}}^{\theta}$.

From (63) we see that welfare decreases ($W_{\breve{\tau}}^{\theta\prime} < 0$) under condition $\gamma\cdot(1-r_u(x)) < \mathcal{E}_u(x)$.

To find the condition of decreasing social welfare, using (62) in the form $\gamma\cdot(1-r_u(x)) = \frac{\theta(1+K)x\left((\frac{\theta-1}{\theta})\frac{x}{\frac{f}{c}}+1\right)}{(\frac{f}{c}+x(1+\theta\cdot K))}$ we divide it by x, plug in the equality $\frac{\frac{f}{c}}{x} = \frac{(1+K)r_u(x)}{(1-r_u(x))}$ (already used) and get

$$\gamma\,(1-r_u(x)) = \frac{\theta\cdot(K+1)\,x\left(1+\frac{\theta-1}{\frac{f}{c}\cdot\theta/x}\right)}{\frac{f}{c}+x\,(1+\theta\cdot K)} = \frac{(K+1)\left(\theta+(\theta-1)\,\frac{(1-r_u(x))}{(1+K)r_u(x)}\right)}{\left((1+\theta\cdot K)+\frac{(1+K)r_u(x)}{(1-r_u(x))}\right)}.$$

Multiplying the numerator and denominator by $(1-r_u(x))\,/r_u(x)$ we can express the (necessary and sufficient) condition for decreasing welfare as

$$\begin{aligned}\gamma\,(1-r_u(x)) &= \frac{1-r_u(x)}{r_u(x)}\cdot\frac{(1+K)\,\theta r_u(x)+(\theta-1)\,(1-r_u(x))}{(1+\theta K)\,(1-r_u(x))+(1+K)\,r_u(x)} = \\ &= \frac{1-r_u(x)}{r_u(x)}\cdot\frac{\theta r_u(x)+K\theta r_u(x)+\theta-\theta r_u(x)-1+r_u(x)}{1+\theta K-r_u(x)-\theta K r_u(x)+r_u(x)+K r_u(x)} = \\ &= \frac{(1-r_u(x))}{r_u(x)}\cdot\frac{(\theta K+1)\,r_u(x)+\theta-1}{1+\theta K-K r_u(x)\,(\theta-1)} = \\ &= \frac{1-r_u(x)}{r_u(x)}\cdot\frac{r_u(x)+\frac{\theta-1}{\theta K+1}}{1-K r_u(x)\,\frac{\theta-1}{\theta K+1}} < \mathcal{E}_u(x)\,.\end{aligned}$$

To exerts this condition for decreasing welfare as a condition on θ, the required inequality

$$\frac{r_u(x)+\frac{\theta-1}{\theta K+1}}{1-K r_u(x)\cdot\frac{\theta-1}{\theta K+1}} < \frac{\mathcal{E}_u(x)\cdot r_u(x)}{1-r_u(x)}$$

can be rewritten (because under $\gamma > 1$ all terms are positive) as

$$r_u(x)+\frac{\theta-1}{\theta K+1} < \frac{\mathcal{E}_u(x)\cdot r_u(x)}{1-r_u(x)}\left(1-K r_u(x)\cdot\frac{\theta-1}{\theta K+1}\right),$$

i.e.,

$$\frac{\theta-1}{\theta K+1}\left(1+\frac{\mathcal{E}_u(x)\cdot r_u(x)}{1-r_u(x)}K r_u(x)\right) < \frac{\mathcal{E}_u(x)\cdot r_u(x)}{1-r_u(x)}-r_u(x)\,.$$

Multiplying everything by $(1-r_u(x))$ we get

$$\frac{\theta-1}{\theta K+1}\left((1-r_u(x))+\mathcal{E}_u(x)\cdot r_u(x)\,K r_u(x)\right) < r_u(x)\left(\mathcal{E}_u(x)-1+r_u(x)\right),$$

i.e.,

$$\frac{\theta-1}{\theta K+1}<\frac{r_u(x)\left(\mathcal{E}_u(x)-1+r_u(x)\right)}{1-r_u(x)+\mathcal{E}_u(x)\cdot r_u^2(x)K}.$$

Using again $\frac{x\mathcal{E}_u'(x)}{\mathcal{E}_u(x)}=(1-r_u(x)-\mathcal{E}_u(x))$ we get

$$\frac{\theta-1}{K\theta+1}<-\frac{\mathcal{E}_u'(x)\,r_u(x)}{\mathcal{E}_u(x)}\cdot\frac{1}{1-r_u(x)+K\mathcal{E}_u(x)\,r_u^2(x)}.$$

The right side is positive under the case DEU and we solve the linear equation w.r.t. θ to express θ. We denote

$$\zeta\equiv-\frac{\mathcal{E}_u'(x)\,r_u(x)}{\mathcal{E}_u(x)\left(1-r_u(x)+K\mathcal{E}_u(x)\,r_u^2(x)\right)}>0$$

and conclude that $W_{\tau=1}^{\theta\prime}(x,y)<0$ if and only if the transport costs satisfy the inequality[15]

$$1<\theta<\frac{1+\zeta}{1-K\zeta}. \tag{64}$$

Obviously, the right-hand side is greater than 1. Here the value x used is the value of consumption in autarky with double population $2L$.

(ii)-(iii) **at autarky**.

Here, at τ_a export $y=0$ (one can check that properties DEU, IED are guaranteed at any exact autarky point, otherwise export tends to zero at infinite cost or tariff). In the case $y=0$ one can see, that $x_\tau^{\theta\prime}=\frac{-\theta Ky\cdot R'(x)c}{R''(x)\cdot C(x+Kx)}=0\,,\qquad y_\tau^{\theta\prime}=\frac{\theta\left(\frac{F}{cL}+x\right)\cdot R'(x)c}{R''(y)\cdot C(x+Kx)}<0\ q_\tau^{\theta\prime}=\theta Ky_\tau^{\theta\prime}<0,\ N_\tau^{\theta\prime}>0$. Costs at this point equal $C(x+K\cdot 0)=F+cLx$. Plugging this and $y=0$ into (61), we find its impact on welfare as follows:

$$W_\tau^{\theta\prime}(\tau_a)=\frac{Ky_\tau^{\theta\prime}u'(0)}{f+cx}-\frac{c\cdot\theta\cdot Ky_\tau^{\theta\prime}\cdot u(x)}{(f+cx)^2}=$$

$$=\frac{Ky_\tau^{\theta\prime}\frac{u(x)}{x}}{f+cx}\left(\frac{u'(0)\,x}{u(x)}-\frac{c\cdot\theta\cdot x}{(f+cx)}\right)=$$

$$=-\frac{Ky_\tau^{\theta\prime}u(x)}{(f+cx)\,x}\left(\frac{\theta x}{f/c+x}-\frac{xu'(0)}{u(x)}\cdot\frac{u'(x)}{u'(x)}\right).$$

As previously, we use zero-profit condition in the form $x/\left(1-r_u(x)\right)=f/c+x$ to get

$$W_\tau^{\theta\prime}(\tau_a)=-\frac{Ky_\tau^{\theta\prime}u(x)}{(f+cx)\,x}\theta\left(1-r_u(x)-\mathcal{E}_u(x)\frac{u'(0)}{\theta\cdot u'(x)}\right).$$

For DEU we use again $0>\frac{\mathcal{E}_u'(x)x}{\mathcal{E}_u(x)}=(1-r_u(x)-\mathcal{E}_u(x))$ negative term $-\mathcal{E}_u(x)$ outweighs positive $1-r_u(x)$. In the case point $\frac{u'(0)}{\theta\cdot u'(x_a)}\geq 1$ (which means $1\leq\theta\leq\frac{u'(0)}{u'(x_a)}$), this coefficients enforce negativity of the whole expression and we conclude that $W_\tau^{\theta\prime}(\tau_a)<0$ under sufficiently low transport costs. Here x_a means the (domestic) consumption at autarky, found from equation in elasticities $1-r_u(x_a)=\mathcal{E}_C(Lx_a)$. (Under CES function and for IEU case similar effects need to be proven at infinite τ.)

For large transport costs, which are more than the ratio of prices on the imported and domestic products in the considered point $\frac{u'(0)}{\theta\cdot u'(x_a)}<1$ (which means $\frac{u'(0)}{u'(x_a)}<\theta$) for IEU, we could state welfare increase $W_\tau^{\theta\prime}(\tau_a)>0$ but we need to study limit transition.

For DEU under $\frac{u'(0)}{\theta\cdot u'(x_a)}<1$ (that means big $\theta>\frac{u'(0)}{u'(x_a)}$, which must hold under sufficiently small τ and IED) we rewrite the condition $W_\tau^{\theta\prime}(\tau_a)<0$ as

$$(1-r_u(x_a))-\mathcal{E}_u(x_a)\frac{u'(0)}{\theta\cdot u'(x_a)}<0,$$

and use the producer's FOC in the form $u'(x)\left(1-r_u(x)\right)=\frac{u'(y)(1-r_u(y))}{\theta\cdot\tau}$ where $y=0$, and $r_u(0)=0$ (because we assume finite derivatives $u'(0),u''(0)$ at 0) to get

[15] We can somewhat relax the inequality by dropping $KE_u(x)\,r_u^2(x)$ and simplify similar constant as $\tilde{\zeta}=-E_u'(x)\,r_u(x)\,/\left(E_u(x)\left(1-r_u(x)\right)\right)$.

$$W_{\tau}^{\theta\prime}(\tau_a) < 0 \Leftrightarrow (1 - r_u(x_a))(1 - \tau_a \cdot \mathcal{E}_u(x_a)) < 0,$$

under big $\theta > \frac{u'(0)}{u'(x_a)}$. Here welfare increases when tariff is small in the sense $\tau_a < 1/\mathcal{E}_u(x_a)$ (resp. decreases when tariff is big in the sense $\tau_a > 1/\mathcal{E}_u(x_a)$).

Appendix 5

Proof of Proposition 7.

(i) **Reaction of consumption**. To find how consumptions in $K+1$ symmetric countries respond to tariff τ, we denote total derivatives as $x'_\tau \equiv \frac{dx}{d\tau}$, $y'_\tau \equiv \frac{dy}{d\tau}$ and $R'_z \equiv R'(z)$. Our symmetric-equilibrium equations are:

$$\pi(x,y)\cdot\lambda \equiv R(x) + K\frac{R(y)}{\tau} - c\cdot(x + \theta\cdot Ky)\lambda - f\lambda = 0.$$

FOC:

$$R'(x) = c\lambda, \;\; R'(y) = \theta \cdot c\tau\lambda,$$

hence

$$R'(x)\theta\cdot\tau = R'(y)$$

and

$$\tau = \frac{R'(y)}{R'(x)\theta}. \tag{65}$$

Free entry:

$$\frac{R(x)}{\lambda} + K\frac{R(y)}{\tau\lambda} - c\cdot(x + \theta\cdot Ky) - f = 0.$$

Let us substitute (65)

$$\frac{R(x)}{R'(x)} + K\theta\cdot\frac{R(y)}{R'(y)} = x + K\theta\cdot y + \frac{f}{c}. \tag{66}$$

Totally differentiating the latter equations in τ (and applying $\frac{d\pi}{dx} = 0$, $\frac{d\pi}{dy} = 0$ or Envelope Theorem to the third equation) we get

$$R''(x)x'_\tau = c\lambda'_\tau, \;\; R''(y)y'_\tau = c\lambda + c\tau\theta\cdot\lambda'_\tau.$$

Hence

$$\lambda'_\tau = -\frac{KR(y)}{c\tau^2\left(x + K\theta\cdot y + \frac{f}{c}\right)}$$

or

$$\lambda'_\tau = -\frac{KR(y)\left(R'(x)\theta\right)^2}{c\left(R'(y)\right)^2\left(x + K\theta\cdot y + \frac{f}{c}\right)}.$$

It follows that the total derivatives of consumptions are

$$x'_\tau = -\frac{KR(y)}{\tau^2 R''(x)\left(x + K\theta\cdot y + \frac{f}{c}\right)} > 0,$$

$$y'_\tau = \frac{\theta\cdot R(x)}{R''(y)\left(x + K\theta\cdot y + \frac{f}{c}\right)} < 0.$$

(We used that $K\theta\cdot\frac{R(y)}{R'(y)} = \left(x + K\theta\cdot y + \frac{f}{c}\right) - \frac{R(x)}{R'(x)}$.)

(ii) **Reaction of output.**

Under ***general*** tariff, to find its impact on sales (output), we combine the changes in x and y:

$$q'_\tau = x'_\tau + K\theta \cdot y'_\tau = \frac{\theta^2 \cdot KR(x)}{R''(y)\left(x + K\theta \cdot y + \frac{f}{c}\right)} - \frac{KR(y)}{\tau^2 R''(x)\left(x + K\theta \cdot y + \frac{f}{c}\right)} =$$

$$= \frac{K\left(\tau^2\theta^2 \cdot R(x)R''(x) - R(y)R''(y)\right)}{\tau^2 R''(y)R''(x)\left(x + K\theta \cdot y + \frac{f}{c}\right)}. \tag{67}$$

For linear demand $R'(x) = R'(y) = constant$, so, the sign is clear: output is decreasing in τ on the whole interval.

The derivative of output at **free trade *(compensation point),*** where $\tau = \frac{1}{\theta},\ x = y$,

$$K\theta \cdot y'_\tau = -\frac{\theta^2 KR(x)}{R''(x)\left((1 + K\theta)\, x + \frac{f}{c}\right)} < 0, \tag{68}$$

$$x'_\tau = \frac{\theta^2 KR\,(x)}{R''(x)\left((1 + K\theta)\, x + \frac{f}{c}\right)} > 0. \tag{69}$$

To find the derivative of output $q'_\tau = Lx'_\tau + LK\theta \cdot y'_\tau$. We substitute (68), (69) into $q'_\tau = Lx'_\tau + LK\theta \cdot y'_\tau$ and

$$q'_\tau \mid_{\tau=\frac{1}{\theta}} = 0.$$

Taking the opposite direction (introducing subsidies on imports at the point $\tau = \frac{1}{\theta}$), the result is an increase in the size of the firm q_τ with the subsidy.

To find the derivative of sales at **autarky** ($\tau_a :\ y\,(\tau_a) = 0$) we just plug $y\,(\tau_a) = 0$ into our formulate and obtain

$$x'_{\tau_a} = -\frac{KR(0)}{\tau^2 \cdot R''(x)\left(x + \frac{f}{c}\right)} = 0,$$

$$y'_{\tau_a} = \frac{K\theta \cdot R(x)}{R''(0)\left(x + \frac{f}{c}\right)} < 0,$$

hence

$$q'_\tau < 0.$$

To find the derivative of sales at **point, where $x = 0$:** ($\tau_{x0} :\ x\,(\tau_{x0}) = 0$) we just plug $x\,(\tau_{x0}) = 0$ into our formulate and obtain

$$x'_{\tau_{x0}} = -\frac{KR(y)}{\tau^2 \cdot R''(0)\left(y + \frac{f}{c}\right)} > 0,$$

$$y'_{\tau_{x0}} = \frac{K\theta \cdot R(0)}{R''(y)\left(y + \frac{f}{c}\right)} = 0,$$

hence

$$q'_{\tau_{x0}} > 0.$$

Similarly subsidy works.

Global impact q'_τ of τ. We use that $\tau = \frac{R'(y)}{\theta \cdot R'(x)}$. Substitute τ into (67):

$$q'_\tau = \frac{K\left(\left(\frac{R'(y)}{R'(x)}\right)^2 \frac{\theta^2}{\theta^2} R(x)R''(x) - R(y)R''(y)\right)}{\tau^2 R''(y)R''(x)\left(x + K\theta \cdot y + \frac{f}{c}\right)} =$$

$$= \frac{K\left(R'\,(y)\right)^2}{\tau^2 R''(y)R''(x)\left(x + K\theta \cdot y + \frac{f}{c}\right)} \cdot \left(\frac{R(x)R''(x)}{\left(R'\,(x)\right)^2} - \frac{R(y)R''(y)}{\left(R'\,(y)\right)^2}\right). \tag{70}$$

Sign of the bracket determines the sign of the derivative. As in Appendix 2, consider again the function $\phi(z) \equiv \frac{R(z)R''(z)}{(R'(z))^2}$. If the function $\phi(\cdot)$ is decreasing then the derivative of the total output q'_τ is positive under a positive tariff $\tau > 1$. For IED $q'_\tau < 0$ wherever $x > y$, i.e., to the right of the point$\tau > \frac{1}{\theta}$. To prove this, we use arguments from Appendix 2, where it is proved that the function $\phi(\cdot)$ is decreasing on condition $\mathcal{E}_{r'_u}(z) + 2 \geq 0$. And vice versa: for DED $q'_\tau > 0$ wherever $x > y$, i.e., to the right of the point $\tau > \frac{1}{\theta}$ and on condition $\mathcal{E}_{r'_u}(z) + 2 \leq 0$.

Proof of Proposition 8.

(iii) **Welfare.** With notations $C_q \equiv C(x + Ky) \equiv f + c(x + K\theta \cdot y)$, we estimate the welfare total derivative $W_\tau^{\theta\prime}$ w.r.t. tariff τ:

$$W_\tau^{\theta\prime}(x, y) = \frac{u'(x)\, x'_\tau + Ku'(y)\, y'_\tau}{\left(x + K\theta \cdot y + \frac{f}{c}\right)} - \frac{u(x) + Ku(y)}{\left(x + K\theta \cdot y + \frac{f}{c}\right)} \cdot \frac{c\left(x'_\tau + K\theta \cdot y'_\tau\right)}{\left(x + K\theta \cdot y + \frac{f}{c}\right)},$$

hence

$$C_q \cdot W_\tau^{\theta\prime} = \frac{x'_\tau}{x} \cdot \left[\mathcal{E}_{U|x} - \mathcal{E}_{C|x}\right] + \frac{y'_\tau}{y} \cdot \left[\theta \cdot \mathcal{E}_{U|y} - \mathcal{E}_{C|y}\right], \tag{71}$$

which we would like to prove being negative.

Welfare in point of compensation. In particular, at free trade ($\tau = \frac{1}{\theta},\ x = y$) this derivative becomes

$$W_\tau^{\theta\prime}(x, y) = \frac{u'(x)\, x'_\tau + Ku'(y)\, y'_\tau}{\left(x + K\theta \cdot y + \frac{f}{c}\right)} - \frac{u(x) + Ku(y)}{\left(x + K\theta \cdot y + \frac{f}{c}\right)} \cdot \frac{c\left(x'_\tau + K\theta \cdot y'_\tau\right)}{\left(x + K\theta \cdot y + \frac{f}{c}\right)},$$

and output here: $q'_\tau = x'_\tau + K\theta \cdot y'_\tau = 0$. We get

$$W_{\tau=\frac{1}{\theta}}^{\theta\prime}(x, y) = \frac{u'(x)\, x'_\tau + Ku'(x)\, y'_\tau}{\left(x + K\theta \cdot x + \frac{f}{c}\right)} - 0,$$

i.e.,

$$W_{\tau=\frac{1}{\theta}}^{\theta\prime}(x, y) = \frac{u'(x)\left(x'_\tau + Ky'_\tau\right)}{\left(x + K\theta \cdot x + \frac{f}{c}\right)} > 0.$$

Under $\tau = \frac{1}{\theta}$, the sum $x'_\tau + Ky'_\tau = -\frac{\theta^2 KR(x)}{R(x)\left((1+K\theta)x + \frac{f}{c}\right)} + \frac{\theta KR(x)}{R''(x)\left((1+K\theta)x + \frac{f}{c}\right)} = -\frac{\theta KR(x)(\theta - 1)}{R''(x)\left((1+K\theta)x + \frac{f}{c}\right)} > 0$.

The welfare in the autarky economy. We get

$$\begin{aligned} W^{\theta\prime}(x, 0) &= Ky'_\tau \left(u'(0) - \frac{\theta u(x)}{\frac{f}{c} + x}\right) = \\ &= \frac{\theta KR(x)}{R''(0)} \left(\tau\theta R'(x) - \frac{\theta u(x)}{R(x)} R'(x)\right) = \\ &= -\frac{K\theta^2 R(x) R'(x) \mathcal{E}_u(x)}{R''(0)} \left(1 - \mathcal{E}_u(x)\, \tau\right). \end{aligned}$$

(Here we used $u'(0) = \tau\theta \cdot u'(x)\left(1 - r_u(x)\right)$, knowing that $\frac{u'(x)}{u'(0)} = \frac{1}{\tau\theta(1 - r_u(x))} < 1$ or $\tau > \frac{1}{\theta(1 - r_u(x))}$.)

Therefore

$$W_{\tau=\tau_a}^{\theta\prime}(x, 0) = -\frac{K\theta R(x) R'(x) \mathcal{E}_u(x)}{R''(0)\left(1 - r_u(x)\right)} \frac{u'(0)}{u'(x)} \left(\frac{u'(x)}{u'(0)} \theta \left(1 - r_u(x)\right) - \mathcal{E}_u(x)\right).$$

For DEU we use again $0 > \frac{x\mathcal{E}'_u(x)}{\mathcal{E}_u(x)} = \left(1 - r_u(x) - \mathcal{E}_u(x)\right)$, i.e., negative term $-\mathcal{E}_u(x)$ outweighs positive $1 - r_u(x)$. In the case point, the coefficient $\frac{u'(0)}{\theta \cdot u'(x_a)} \geq 1$ enforces negativity of the whole expression, thus $W_\tau^{\theta\prime}(\tau_a) < 0$ for DEU. This transport cost is $1 \leq \theta \leq \frac{u'(0)}{u'(x_a)}$), which means sufficiently *low* transport costs. Here x_a means the (domestic) consumption at autarky, found from equation in elasticities $1 - r_u(x_a) = \mathcal{E}_C(Lx_a)$. (Under CES function and for IEU case similar effect needs to be proven at infinite τ.)

The consequence is an increase in the mass of firms in the economy with the introduction of subsidies on imports at the point $\tau = \frac{1}{\theta}$.

The welfare in free trade point. In particular, at free trade ($\tau = 1$) this derivative becomes

$$W^{\theta\prime}_{\tau}(x, y) = \frac{u'(x)\, x'_{\tau} + K u'(y)\, y'_{\tau}}{\left(x + K\theta \cdot y + \frac{f}{c}\right)} - \frac{u(x) + K u(y)}{\left(x + K\theta \cdot y + \frac{f}{c}\right)} \cdot \frac{c\,(x'_{\tau} + K\theta \cdot y'_{\tau})}{\left(x + K\theta \cdot y + \frac{f}{c}\right)},$$

and output in case DEU: $q'_{\tau} = x'_{\tau} + K\theta \cdot y'_{\tau} < 0$.

$$W^{\theta\prime}_{\tau=1}(x, y) = \frac{u'(x)\, x'_{\tau} + K u'(x)\, y'_{\tau}}{\left(x + K\theta \cdot x + \frac{f}{c}\right)} - 0,$$

i.e.,

$$W^{\theta\prime}_{\tau=1}(x, y) = \frac{u'(x)\,(x'_{\tau} + K y'_{\tau})}{\left(x + K\theta \cdot x + \frac{f}{c}\right)} > 0.$$

Under $\tau = 1$, the sum $x'_{\tau} + K y'_{\tau} = -\frac{\theta^2 K R(x)}{R(x)\left((1+K\theta)x + \frac{f}{c}\right)} + \frac{\theta K R(x)}{R''(x)\left((1+K\theta)x + \frac{f}{c}\right)} = -\frac{\theta K R(x)(\theta - 1)}{R''(x)\left((1+K\theta)x + \frac{f}{c}\right)} > 0$.

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