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VOLATILITY SPILLOVERS WITH SPATIAL EFFECTS ON THE OIL AND GAS MARKET

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Abstract

The article is devoted to the estimation of volatility spillovers occurred on the oil and gas market taking into account cross-sectional dependence. The latter is implemented via spatial specifications of the BEKK multivariate volatility model. We also use DCC, GO-GARCH and ADCC models as a benchmark.

JEL Classification: C32, C58, G15, G17.

Keywords: multivariate volatility models, spillover effects, spatial specifications, oil and gas market.

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Introduction

In the finance literature the volatility spillovers effects are an important part of understanding market behavior, price changes and risk measurement. They exist widely through all of financial markets and are characterized by shocks that occur at one market and cause changes in asset prices in other markets. It leads to the situation when current value of a variable depends on past and/or current values of other variables, not only on its past values (Schmidt, 2005).

Volatility spillover effects are associated with the spread of market disturbances between countries and sectors as a result of prices, shares, exchange rates or capital flow co-movements and other financial linkages among market economies (Dornbusch et al., 2000). For example, volatilities of exchange rates can significantly influence the prices of commodities and markets behavior in general (Akman and Bozkurt, 2016).

In this case investigation of the cross-market linkages, price co-movements has become a crucial issue for modelling volatility in applications to the tasks set by investors, financial organizations and governments in trading, hedging and financial regulation.

Systematic modelling of financial volatility has started with the auto-regressive conditional heteroscedasticity (ARCH) model in the seminal paper of Engle (1982). To explain correlation transmission and spillover effects ARCH model was generalized to multivariate case in Engle and Kroner (1995). Multivariate GARCH models focus on time-varying conditional variances as well as co-variances, what allows to make better representation of the volatility process.

With a development of econometrics a number of parametric models have been derived to describe the asymmetry of financial volatilities. These models include copulas (Jaworski and Pitera, 2014), DECO–FIEGARCH (Mensi et al., 2017), DCC (Kocaarslan et al., 2017), wavelets (Liu, An, Huang, et al., 2017), complex networks (Liu, An, Li, et al., 2017). Using such specifications some papers document significant volatility spillover between oil and stock markets. Ewing et al. (2002) found the evidence of volatility persistence in both oil and gas markets. They show that volatility in the natural gas sector is directly affected by events in this sector and indirectly by events originating in the oil sector.

El Hedi Arouri et al. (2011), Olson et al. (2014), Serletis and Xu (2016) use BEKK–GARCH model to estimate cross-market volatility spillovers. Their results point to the existence of widespread direct spillover of volatility between oil and stock sector returns whatever the region considered. Moreover, the volatility spillovers across markets are increasing when "the zero lower bound" (situation when the short-run nominal interest rate is at or near zero; liquidity trap) occurs than when it is not, suggesting that unconventional monetary policy at "the zero lower bound" has strengthened the linkages between the crude oil and financial markets.

Aiming to investigating the evolution of mean and volatility spillovers between oil and stock markets Liu, An, Huang, et al. (2017) employ WTI crude oil prices, the S&P 500 (USA) index and the MICEX index (Russia) with a wavelet-based BEKK–GARCH model to examine the spillover features in frequency dimension. They find the evidence of information transmission between the crude oil market and US stock market that is gradually weakened and mainly maintained in short-term scale. The contacts between two markets gradually disappear at the long-term scales.

However, despite the fact that such models are quite clear and convenient in interpreting the spillover effects, the problem of non-linear growth of the number of estimated parameters (so called "curse of dimensionality") arises. To overcome this problem Caporin and Paruolo (2015) propose an intermediate form for multivariate GARCH models with restrictions based on spatial dependencies among the assets to build conditional covariances.

Spatial BEKK–GARCH is used by Chen and Tian (2017) to explore unidirectional and bi–directional spatial volatility spillover effects among the stock markets based on symbolic transfer entropy. Anatolyev and Khrapov (2016) investigate forecast performance of different types of further restrictions on coefficient matrices in spatial BEKK and compare spatial and standard specification.

The spatial specification of the multivariate model of generalized autoregressive conditional heteroscedasticity (spatial BEKK) allows to take into account both temporal and spatial effects in the dynamics of volatility (Caporin and Paruolo, 2015). Such effects are modeled using weight matrix that is given exogenously and can be defined either as a binary matrix or as a function of the economic distances (Borovkova (2016)). There are some evidence that spatial models are good in optimal hedging ratio computation, forecasting performance, modeling the effects of contamination and volatility spillovers in comparison with other multivariate models (Gu et al., 2017, Anatolyev and Khrapov, 2016, Jaworski and Pitera, 2014, Chen and Tian, 2017).

A number of authors have considered the effects of volatility spillovers between oil Brent price and stock markets (return on market indexes) of different countries: Caporale and Spagnolo (2011) examine linkages between the stock markets of the Czech Republic, Hungary, Poland, the UK and Russia; Arouri et al. (2012) — of stock markets in Europe; Lin et al. (2014) — of Ghanaian stock market; Liu, An, Li, et al. (2017) — G20 countries.

The main purpose of this work is an empirical analysis of the volatility spillovers effects presented on the oil and gas market based on the class of the multivariate GARCH models. The research question is whether a spatial BEKK model, based on computation of spatial dependence between assets, has a greater explanatory (or predictive) power than other M–GARCH models, according to statistical tests. As a benchmark we estimate such multivariate volatility models as the generalized orthogonal GARCH (GO-GARCH), dynamic (DCC) and asymmetric dynamic (ADCC) conditional correlations (for more details on volatility models, see Bauwens et al., 2006) that are widely-used to perform spillover effect analysis (Kocaarslan et al., 2017, Gu et al., 2017, for example).

1 Multivariate GARCH models

Let x_t, $x_t = (x_{1t}, x_{2t}, \ldots, x_{nt})'$ be a portfolio consisted of n assets at time moment t, $t \in [1, \ldots, T]$. x_t is represented as a sum of its mathematical expectation $\mathrm{E}(x_t|\mathscr{F}_t)$, conditional on all available at $t-1$ information, and innovations y_t, (1).

$$x_t = \mathrm{E}(x_t|\mathscr{F}_{t-1}) + y_t,\ t = 1, \ldots, T,\ x_t - (n \times 1)\text{-vector}, \tag{1}$$

Innovations y_t are represented by the product of conditional variance-covariance matrix or volatility matrix H_t and idiosyncratic noise term ε_t, which is distributed according to some distribution f with zero mean and additional parameters θ (2).

$$y_t = H_t^{1/2}\varepsilon_t, \varepsilon_t \sim f(0,\theta), \tag{2}$$

Volatility models differ in the specification of dynamic variance-covariance matrix H_t.

1.1 DCC–GARCH

The DCC model proposed by Engle (2002) is defined as follows:

$$H_t = D_t R_t D_t, D_t = \text{diag}\left(\sqrt{h_t}\right) \tag{3}$$

$$h_{it} = \omega + A_i y_{i,t-1} \odot y'_{i,t-1} + B_i h_{i,t-1} \tag{4}$$

$$R_t = (\text{diag}(Q_t))^{-\frac{1}{2}} Q_t (\text{diag}(Q_t))^{-\frac{1}{2}} \tag{5}$$

$$Q_t = (1-\alpha-\beta)\bar{Q} + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1} \tag{6}$$

this model meets the requirement of stationarity as long as $\alpha+\beta<1$ *and* $\alpha,\beta>0$
where $\odot$ is the Hadamard product
A,B : $n\times n$ diagonal matrices of parameters
H_t : $n\times n$ matrix of conditional covariances
$H_t^{\frac{1}{2}}$: $n\times n$ matrix of a Cholesky factorization of H_t
D_t : $n\times n$ diagonal matrix of conditional standard deviations of y_t
$\bar{Q}_t$: $n\times n$ unconditional covariance matrix
R_t : $n\times n$ conditional correlation matrix with unities on the main diagonal

1.2 ADCC–GARCH

DCC model was extended by Cappiello et al. (2006) to the case where asymmetries in the conditional correlations between series can be estimated.

To achieve that matrix Q_t is defined as follows:

$$Q_t = (1-\alpha-\beta)\bar{Q} - \gamma\bar{N} + \alpha y_{t-1}y'_{t-1} + \gamma\eta_{t-1}\eta'_{t-1} + \beta Q_{t-1} \tag{7}$$

where $\eta_t = I[y_t<0]\odot y_t$;
$I(\cdot)$ is an indicator function which takes on value 1 if the argument is true and 0 otherwise;
$\bar{N} = [\eta_{t-1}\eta'_{t-1}]$.

Positive definiteness of Q_t is ensured by condition: $\alpha+\beta+\lambda\gamma<1$, where λ is maximum eigenvalue of $[\bar{Q}_t^{-\frac{1}{2}}\bar{N}\bar{Q}_t^{-\frac{1}{2}}]$.

In literature DCC/ADCC models are usually used for small-scale applications (4 or 5), since it becomes cumbrous to analyze hundreds of assets, though that can be important for financial applications (Jaworski and Pitera, 2014; Guidolin, 2016).

Apart from the mentioned models, multivariate BEKK-GARCH model, that have been proposed by Engle and Kroner (1995), gained high popularity, but also is subject to the dimension curse.

The solution to the dimensionality problem of multivariate GARCH models was proposed in Caporin and Paruolo (2015). The authors apply spatial matrix for the parameter matrices in BEKK volatility equation (8), decreasing the number of parameters to $O(n)$. A detailed description of the properties of spatial matrices can be find in Caporin and Paruolo (2015).

1.3 Spatial BEKK–GARCH

The spatial BEKK model has following structure:

$$H_t = C'C + A'y_{t-1}y'_{t-1}A + B'H_{t-1}B \tag{8}$$

where $n \times n$ coefficient matrices A, B, C, D are defined as (9) and are equal to AR(1) component of spatial autoregressive model (LeSage and Pace, 2009).

$$A = \text{diag}(a_0) + \text{diag}(a_1)W, \tag{9}$$

$$B = \text{diag}(b_0) + \text{diag}(b_1)W, \tag{10}$$

where a_0, b_0, a_1, b_1 are $n \times 1$ vectors of parameters, W — weight matrix.

The constant $C'C$ in (8) is borrowed from the spatial error model (see LeSage and Pace, 2009).

$$C'C = D^{-1}\text{diag}(d_0)(D')^{-1}, \tag{11}$$

where D is also a spatial matrix with d_0 equal to identity matrix. The main idea of such parametrization is to define volatility spillovers across assets that belong to the same group (see below).

Assuming various restrictions on parameter matrices, three types of model specifications can be evaluated:

- scalar homogeneous specification is calculated for the case when a_0, a_1, b_0, b_1, d_1 are limited - i.e. are set as constants;
- in the homogeneous specification restrictions are imposed on the vectors a_1, b_1;
- for a heterogeneous specification parameter vectors are unlimited and consist of n different elements.

In general $C'C$ represents the impact of unrelated to the model dynamic factors on volatility matrix. A is associated with ARCH parameters and shows news impact or innovations effects in a volatility matrix H_t. At the same time, matrix B contains covariance feedback effects (Anatolyev and Khrapov, 2016) or GARCH parameters.

The elements of volatility equation in (8) can be decomposed on directs, indirect and mixed effects (Billio et al., 2016).

To begin with the first part, it is decomposed as:

$$A'y_{t-1}y'_{t-1}A = a'_0y_{t-1}y'_{t-1}a_0 + a'_1y_{t-1}y'_{t-1}a_0 + a'_0y_{t-1}y'_{t-1}a_1 + a'_1y_{t-1}y'_{t-1}a_1 \tag{12}$$

where the first component $a'_0y_{t-1}y'_{t-1}a_0$ contains variance (or covariance) direct effects or shocks, the second and third terms show mixed effects originating both direct and indirect elements and the last one $a'_1y_{t-1}y'_{t-1}a_1$ presents indirect effect due to the spillover exposures.

We use scalar homogeneous specification, where matrices of parameters are modeled as follows:

$$\begin{aligned} a_0 &= \alpha_0 1_n, a_1 = \alpha_1 1_n; \\ b_0 &= \beta_0 1_n, b_1 = \beta_1 1_n; \\ d_0 &= \text{free}, d_1 = \delta_1 1_n. \end{aligned}$$

Such modification brings the S–BEKK closer to the diagonal BEKK model defining equal impact of spillovers for each grouping across all assets, but reduce the number of parameters.

The elements of the weight matrix reflect the force of potential interactions between the assets. When determining the elements of a matrix it is natural to use principle that nearby neighbors exert the greatest influence. Fernandez (2007) aims at deepening researchers understanding of economic distance using spatial methodology with high level of efficiency. Arnold et al. (2013) introduce the approach to model three different types of spatial dependence in stock returns: a general dependence, dependence within industrial branches and based on geographic locations. The most commonly used methods for weight calculation are administrative-territorial specification, the method of moving windows, fixed and adaptive kernels (Chasco Yrigoyen et al., 2007; Munnix et al., 2014).

In this paper spatial weight matrices are modeled in two different ways. In the first type of matrices each element w_{ij} is computed as follows:

$$w_{ij} = \begin{cases} 1, & \text{if j is neighbour for i;} \\ 0, & \text{else.} \end{cases} \tag{13}$$

Second type of matrices is based on computation of weights with bi-square kernel (see details in Balash et al., 2011).

$$w_{ij} = \left(1 - \left(\frac{d_{ij}}{b}\right)^2\right)^2, \text{ if j is one of m neighbours for i.} \tag{14}$$

In this matrix element d_{ij} — is the distance between assets that can be calculated as:

$$d_{ij} = \left(\sum_1^k \left(p_i^{(k)} - p_j^{(k)}\right)^2\right)^{\frac{1}{k}} \tag{15}$$

where k — number of indicators that used for distance defining.

As the distance increases, the Interaction power between assets is declining. The influence of neighbors on the ith-element defines with a respect of the b — distance to the farthest neighbor.

Each element in i-row of matrix $W := (w_{ij})$ should show the share of influence of j-neighbor on i. To achieve this effect, the rows of matrix W should be normalized to unit:

$$w_{ij} = \begin{cases} w_{ij}/\sum_{j=1}^{n} w_{ij}, & \text{if } \sum_{j=1}^{n} w_{ij} > 0; \\ 0, & \text{else.} \end{cases} \tag{16}$$

1.4 GO–GARCH

In GO–GARCH model the volatility matrix is parametrized as follows(Weide, 2002):

$$H_t = XV_tX', \tag{17}$$

where V_t — diagonal $n \times n$ matrix with $v_t = c + A(y \odot y) + Bv_{t-1}$ on the main diagonal, X — $n \times n$ matrix based on singular value decomposition into the de–whitening matrix, invertible, not time depended (see Weide, 2002 for detail), A, B — $n \times n$ diagonal matrices of parameters with a restriction $a_{ii,t} + b_{ii,t} < 1$.

2 Data description

Our sample covers the data of the 67 companies from the oil and gas sector in 13 countries. The countries under consideration are classified as upper-middle income level and higher income according to the World bank classification. The dates range is from April 27, 2015 until January 18, 2018 apart from the public holidays, therefore the full sample contains 634 close price data for each asset. Financial indicators, namely company market capitalization, gross profit and total assets, are taken on 2016 year. Countries of Headquarters include Argentina, China, Colombia, Gabon, Kuwait, Nigeria, Qatar, Russia, Saudi Arabia, South Africa, Thailand, Turkey and Peru.

List of TRBC Activity Codes is presented in table A1.

All the data are obtained from the Thomson Reuters Eikon[3]. Histograms of descriptive statistics of returns can be found in Appendix figure A1.

We construct two weight matrices. The first one is based on the criteria of belonging to the country and activity (by TRBC); the second one — on the economic distance calculated as a difference in market capitalization, total assets and profit.

The calculations were carried out on Amazon Elastic Compute Cloud service with RStudio Amazon Machine Image (AMI) installed.[4] The AMI contains R version 3.3.1 running on Ubuntu 16.04 LTS and R packages including “rmgarch” for GARCH estimation and “optimx” to log-likelihood maximization (Ghalanos, 2015; Nash and Varadhan, 2011; Nash, 2014).

[3]https://eikon.thomsonreuters.com/
[4]https://aws.amazon.com/

3 Empirical results

3.1 Parameters estimation

We estimate parameters for multivariate GARCH models and compare them to see what model demonstrates the highest explaining power in modeling volatility spillovers.

The models under consideration are estimated by means of maximum likelihood method. The log-likelihood function LL defined in (18).

$$\mathrm{LL} = -\frac{1}{2}\sum_{t=1}^{T}(\ln(\det H_t)) + y_t' H_t^{-1} y_t \tag{18}$$

This method fits well on time series for both linear and non-linear models (Brooks, 2008). Estimations obtained by this method are consistent and asymptotically normal but require a modified calculation of standard errors (Greene, 2003).

3.2 Information criteria

For general fitting information following criteria are used (m — number of parameters in model):
Akaike information criterion:

$$\mathrm{AIC} = \frac{-LL}{N} + \frac{2m}{N} \tag{19}$$

Bayesian information criterion:

$$\mathrm{BIC} = \frac{-LL}{N} + \frac{2m\ln N}{N} \tag{20}$$

Tables 1, 2, present the results of comparison of volatility models based on mentioned criteria. Apparently, the spatial structure in the S–BEKK volatility equation improve explaining power of the model according to the chosen criteria. In our sample for all of two criteria scalar S–BEKK has the highest values for both types of weight matrices. Due to the statistical criteria the spatial dependencies that estimated as an economic distance between assets can increase the explanation power of the model on in–sample dataset.

Table 1: Information criteria (in-sample)

Criterion/model	AIC	BIC
S-BEKK(activity and country)	199.46	201.18
DCC/ADCC	294.36	318.75
GO-GARCH	297.85	409.77

Table 2: Information criteria (full sample)

Criterion/model	AIC	BIC
S-BEKK(economic distance)	192.91	194.15
DCC/ADCC	285.71	303.61
GO-GARCH	289.54	370.20

Results point to the existence of volatility spillover among assets grouping by TRBC activity (see in Appendix fig.A2). Large outliers are concentrating near the period of high oil Brent price volatility (for example, in the Q3 of 2015 and beginning of 2016) in all groups. Thus, the interdependence between volatility of companies from different activities increases during the period of oil price turbulence. The dynamics of volatility covariances between one asset and the rest of the various groups is similar, the outliers also occur at the same period of time through all of groups (see fig.1).

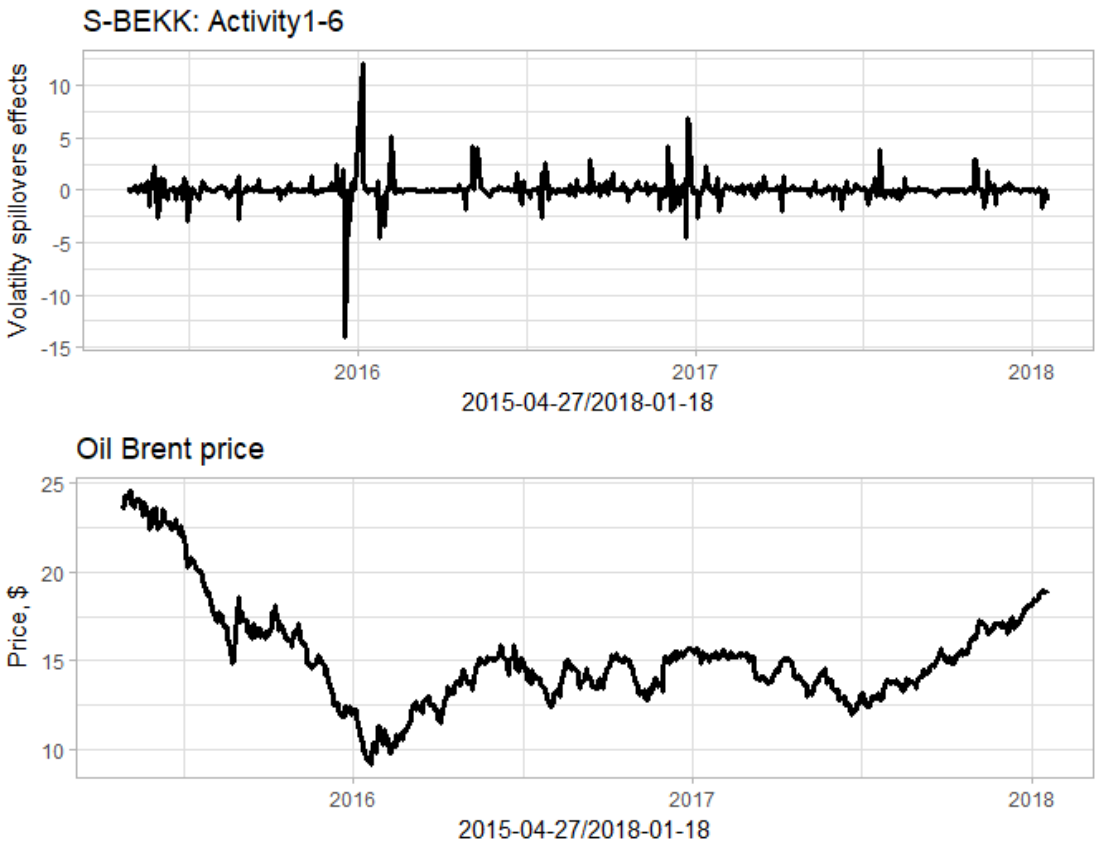

Figure 1: Selected volatility spillovers effects and oil Brent price

4 Forecasting

4.1 One-step forecast

Volatility forecasting was performed using a rolling window of fixed size $L < T$. For subsample L we estimate parameters of S–BEKK model as described above in Subsection 1.3 and compute one-step forecast from recursion (21):

$$\tilde{H}^{f}_{L+1} = C'C + A'y_L y'_L A + B'H_L B \tag{21}$$

4.2 Multistep forecast

One of the most common techniques to make a robust multistep forecast is the use of bootstrapping procedure. The main idea of bootstrap method is to obtain the distributions and calculate a bootstrap test statistic generating a large number of simulated samples. The circular block bootstrap proposed by Politis and Romano (1992) and Hall et al. (1995) allows to keep the time structure of the time-series data when simulating samples. Following the idea of Billio et al. (2016) a short description of the S-BEKK model forecast is given bellow:

1 step: Obtain S-BEKK parameters matrices $\hat{A}, \hat{B}, \hat{C}$ and variance matrix $\hat{H}_t$ for the estimation window t, $t \in [1, \ldots, L]$;

2 step: Compute the $n \times 1$ vector of filtered innovations (standardized residuals) for period L as follows:

$$\varepsilon_t = H_t^{-1/2} y_t \tag{22}$$

3 step: Bootstrap N_b samples of length h from ε_t using block method with automatic block length selection k (see details in Politis and White (2004));

4 step: Compute bootstrap innovations y_{T+l} for each b, $b \in [1, \ldots, N_b]$, and l, $l \in [1, \ldots, h]$:

$$\tilde{y}_{L+l}^{[b]} = \tilde{H}_{L+l}^{-\frac{1}{2}} \tilde{\varepsilon}_{L+l}^{[b]} \tag{23}$$

5 step: Compute the covariance matrix H_{L+l}:

$$\tilde{H}_{L+l}^{[b]} = \hat{C}'\hat{C} + \hat{A}'\tilde{y}_{L+l}^{[b]}\tilde{y}'^{[b]}_{L+l}\hat{A} + \hat{B}'\tilde{H}_{L+l-1}^{[b]}\hat{B} \tag{24}$$

6 step: Set the forecast value of covariance matrix equal to the mean of bootstrap variances $\tilde{H}_{L+l}^{[b]}$:

$$\tilde{H}_{L+l}^{f} = \frac{1}{L_b} \sum_{1}^{L_b} \tilde{H}_{L+l}^{[b]} \tag{25}$$

Repeat previous steps to obtain the path of forecast volatility matrix from $L+1$ to T.

In this paper the bootstrap block length is maximum value for all time–series and equal 19 observation. The number of bootstrap samples is 10000.

The confidence interval on 5% and 95% of bootstrapped volatility between two selected assets are presented in fig. 2.

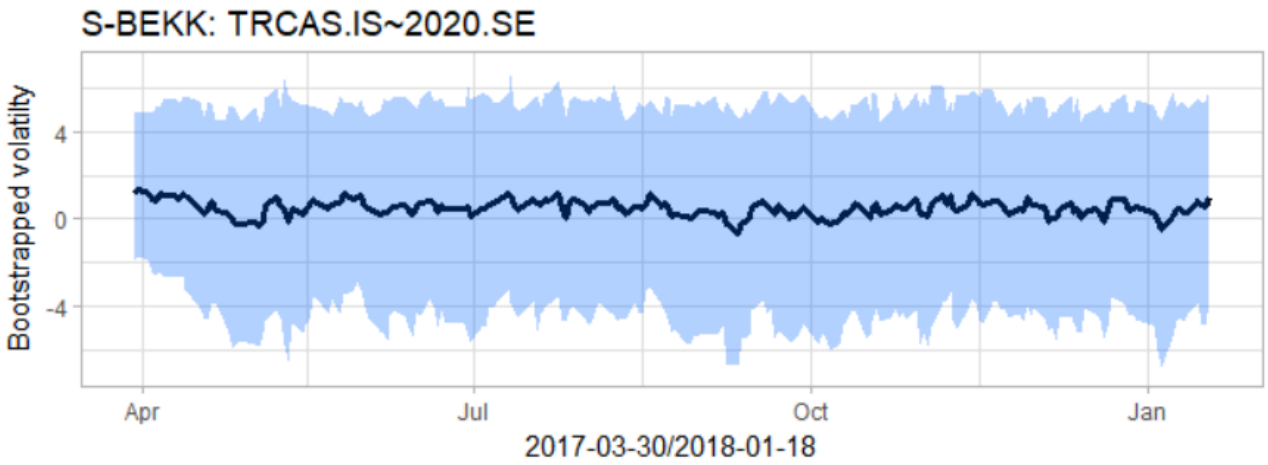

Figure 2: Bootstrapped volatility confidence interval

4.3 Forecast evaluation

4.3.1 Diebold–Mariano test

There are a lot of new testing procedures that has been developed in past years to answer the question what model provides the "best fitting results" (see Giacomini and White, 2006, for example). The statistics are evaluated for an arbitrary loss function, which essentially means that it is possible to test models on various aspects depending on the chosen loss function. To compare the forecasting ability of the spatial BEKK model with benchmark Diebold–Mariano test was accomplished (Diebold and Mariano, 2002). The main idea of the test to estimate the difference in forecast accuracy of competing models. To achieve this the first step is to compute time–t loss of forecast and arbitrary function of realized volatility $g(H_t, \tilde{H}_t^f)$ as described in Laurent et al. (2012) , Caporin and McAleer (2014):

$$g1 = tr\left[\left(\tilde{H}_{L+l}^f - H_{L+l})\right)'\left(\tilde{H}_{L+l}^f - H_{L+l}\right)\right] \tag{26}$$

$$g2 = log\left|\tilde{H}_{L+l}^f\right| + y'_{L+l}\left(\tilde{H}_{L+l}^f\right)^{-1} y_{L+l} \tag{27}$$

$$g3 = \frac{1}{6}tr\left[\left(\tilde{H}_{L+l}^f\right)^3 - (H_{L+l})^3\right] - \frac{1}{2}tr\left[\left(\tilde{H}_{L+l}^f\right)^2\left(\tilde{H}_{L+l}^f - H_{L+l}\right)\right] \tag{28}$$

where H_{L+l} is defined as $y'_{L+l}y_{L+l}$ and tr is an operator of diagonal elements sum.

According to the objectives of the further model application the loss functions may vary. For the purpose of portfolio investment and risk management we choose following functions: $g1$ is Frobenius norm of volatility matrix square errors, $g2$ – Stein loss fiction which evaluates simply(twice) a negative contribution of loglikelihood, $g3$ – loss function that proposes penalty for over-estimation of forecast volatility.

The sample mean of loss differential is :

$$\bar{d} = \frac{1}{T-L} \sum_{t=1}^{T-L} [g_{it} - g_{jt}] \tag{29}$$

with spectral density at frequency 0:

$$f_d(0) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \gamma_d(\tau) \tag{30}$$

where γ_d is an autocovariance of the loss differential at displacement τ and μ – population mean loss differential:

$$\gamma_d(\tau) = E[(d_t - \mu)(d_{t-\tau} - \mu)] \tag{31}$$

In large samples $\bar{d}_t$ is normally distributed with mean μ and variance $2\pi \hat{f_d}(0)$, statistics of the Diebold–Mariano test is follows:

$$S = \frac{\bar{d}}{\sqrt{\frac{2\pi \hat{f_d}(0)}{T-L}}} \sim N(0,1) \tag{32}$$

where $\hat{f_d}(0)$ is a consistent estimate of $f_d(0)$.

The null hypothesis is that two models have the same forecast accuracy, alternative is that model j is more accurate.

The results of model comparison for one–step forecast procedure are presented in table 3. According to first two loss functions S-BEKK model has worse forecasting performance than benchmark on 1% level of significance, but in the case of 3-rd function models are equal (p–value and test statistics are in Appendix table A2).

Table 3: Diebold–Mariano test (one–step forecast)

Model/Loss function	g1	g2	g3
S–BEKK and GO–GARCH	GO–GARCH	GO–GARCH	equal
S–BEKK and DCC/ADCC	DCC/ADCC	DCC/ADCC	equal

As for multistep forecast Diebold–Mariano test emphasizes that only for stein loss function DCC/ADCC and S–BEKK have equal forecast accuracy, in the case of other functions benchmark models are better (p–value and test statistics are in table 4 and Appendix table A3).

Table 4: Diebold–Mariano test (multistep forecast)

Model/Loss function	g1	g2	g3
S–BEKK and GO–GARCH	GO–GARCH	GO–GARCH	GO–GARCH
S–BEKK and DCC/ADCC	DCC/ADCC	equal	DCC/ADCC

4.3.2 Model confidence set procedure

Another testing procedure that was used in this work is Model confidence set (MCS). The idea of procedure is to create a set of "superior" models (SSM) making a sequence of statistic tests. For such set the null hypothesis is not declined. It can be defined into two different ways:

$$c_{ij} = 0 \, for \, some \, i, j \in [1, \ldots, m] \, or \tag{33}$$

$$c_{i\cdot} = 0 \, for \, all \, i, j \in [1, \ldots, m] \tag{34}$$

where $c_{ij} = \mathbb{E}(d_{ij})$ and $c_{i\cdot} = \mathbb{E}(d_{i\cdot})$,

m – is number o models in the set M,

$d_{i\cdot} = (m-1)^{-1} \sum_{j \in M} d_{ij}$ – loss differential between i-th model and the average in set M and d_{ij} – between i-th and j-th.

For every type of null hypothesis test statistics is constructed as follows:

$$t_{ij} = \frac{\bar{d}_{ij}}{\sqrt{\hat{\sigma}(\bar{d}_{ij})}} \, and \, t_{i\cdot} = \frac{\bar{d}_{i\cdot}}{\sqrt{\hat{\sigma}(\bar{d}_{i\cdot})}} \tag{35}$$

where $\hat{\sigma}(\bar{d}_{i\cdot})$ and $\hat{\sigma}(\bar{d}_{ij})$ are bootstrapped estimates of $\sigma(\bar{d}_{i\cdot})$ and $\sigma(\bar{d}_{ij})$.

Two hypothesis of equal predictive ability from (33) and (34) are combined into following statistics:

$$T_{R,M} = \max |t_{ij}| \, and \, T_{max,M} = \max |t_{i\cdot}| \tag{36}$$

The sequence of testing in MCS procedure eliminates at each step the worst model, until the hypothesis of equal predictive ability (EPA) is accepted for all elements belonging to the set of models. Model that should be eliminated is chosen according to statistics described in (35).

The results of MCS procedure for one–step forecast performance are presented in table 5. It is noted that the best model of the set for $g1$ and $g2$ loss functions is DCC/ADCC. As for $g3$ the set of "superior" models consists of S–BEKK as well as DCC/ADCC.

Table 5: The set of "superior" models according to MCS (one–step forecast)

Loss function	g1	g2	g3
Model	DCC/ADCC	DCC/ADCC	S–BEKK and DCC/ADCC

In the case of bootstrap approach in forecasting the results point out that DCC/ADCC model is in the set of "superior" models in two cases from three for loss functions $g1$ and $g3$, as for $g2$ — GO–GARCH presents the best forecasting performance (see details in table 6). S–BEKK is not inlcuded in SSM in for all of three loss functions.

Table 6: The set of "superior" models according to MCS (multistep forecast)

Loss function	g1	g2	g3
Model	DCC/ADCC	GO–GARCH	DCC/ADCC

Summary and concluding remarks

Volatility is one of the most important statistical financial indicator that characterizes the changeability of prices. The volatility spillover effects play significant role in financial risks management and connect with a spatial dependencies between assets. It confirms the relevance of the use of S–BEKK model that has in structure spatial components, where weight matrices express the degree of assets' proximity or economic distance between assets. Moreover such specification overcomes the problem of non-linear growth of number of parameters.

In this paper we explore 67 companies from oil and gas sector of 13 countries considering proximity of assets based on belonging to economic sector, country or economic distance. As benchmark M–GARCH models with dynamic conditional correlation and generalized orthogonal one are used.

Estimation of S–BEKK gives some proofs of companies integration in oil and gas sector that is measured as volatility spillovers effects between assets price that can be numerically expressed through weight matrices. The total number of S–BEKK model parameters is 72 due to the scalar homogeneous structure of volatility equation (see 1.3). All parameters for both types of weight matrices are statistically significant on 1% or 5% level. The main idea of using the obtained estimates of the model is the possibility of identifying the relationship between assets with a purpose of building investment or speculative strategies.

To visualize the dynamic volatility spillover effects the condition variances (on the main diagonal) and covariances between pairs of assets are presented in Appendix fig.A2. The network visualization of volatility spillover effects can be found in Appendix fig. A3, A4. There are two groups of networks for both types of volatility matrices. The edge of graphs is computed as a median of volatility covariance between assets.

Models are compared by Akaike and Bayesian information criteria. Proceeding from lower information criteria S–BEKK is close to the real data generating process as well as DCC/ADCC and GO–GARCH models. Diebold–Mariano test and Modern Confidence Set procedure are performed to estimate accuracy of one-step and multistep forecast made by circular block bootstrap approach. Based on Modern Confidence Set procedure S–BEKK model does not perform very well for most loss function, excepting $g3$ for which "superior" set includes S–BEKK model as well as DCC/ADCC. Diebold–Mariano test does not decline null hypothesis of equal error measurement of S–BEKK and benchmark models forecast for $g3$ loss functions in case of one–step forecast and for $g2$ in case of multistep one. For others GO–GARCH and DCC/ADCC are more accuracy. Therefore, S–BEKK model provides better results on short forecasting horizon. For instance, S–BEKK fits better to hedging strategy building according to the $g3$ loss function that imposes higher penalty for volatility over-estimation leading to negative consequences in hedging.

Such forecasting performance of S–BEKK model can be caused by the static structure of weight matrices due to that some patterns occured on in–sample data are not reflect the assets‘ behavior on out–of–sample period. To overcome this problem future research should be focused on dynamic weight matrices computing.

Also the main result of this work is that concerning the oil and gas market the cross sectional dependencies between assets and spillover effects between them can be estimate through as differences in financial indicators or belonging to the same activity group.

As a future research the companies can be separated into two groups as volatility spillover effects sources and recipients using different specification of spatial matrices and precise analysis of networks is also desirable. Moreover, it is possible to extended scalar model to full version or calculate spatial dependencies between assets based on cluster analysis. As well the results of volatility matrix estimation are useful for building hedging strategy and portfolio management.

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A Appendix

Table A1: TRBC Activity Codes

Code	Activity	Group
5010202010	Oil & Gas Exploration and Production (NEC)	Activity1
5010201010	Integrated Oil & Gas	Activity2
5010202013	Natural Gas Exploration & Production - Onshore	Activity3
5010203010	Oil & Gas Refining and Marketing (NEC)	Activity4
5010203011	Petroleum Refining	Activity5
5010203012	Gasoline Stations	Activity6
5010202011	Oil Exploration & Production - Onshore	Activity7
5010202015	Unconventional Oil & Gas Production	Activity8

Source: Thomson Reuters.

Table A2: The statistics and p–value for Diebold–Mariano test (one–step)

Model/Loss function	g1	g2	g3
S–BEKK and GO GARCH	6.26^{***}	7.04^{***}	-0.18
S–BEKK and DCC/ADCC	10.60^{***}	11.15^{***}	1.00

Note: (-) $p>0.1$; $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table A3: The statistics and p–value for Diebold–Mariano test (multistep)

Model/Loss function	g1	g2	g3
S–BEKK and GO GARCH	130.26^{***}	-7.42^{***}	47.60^{***}
S–BEKK and DCC/ADCC	129.91^{***}	14.97^{***}	47.60^{***}

Note: (-) $p>0.1$; $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

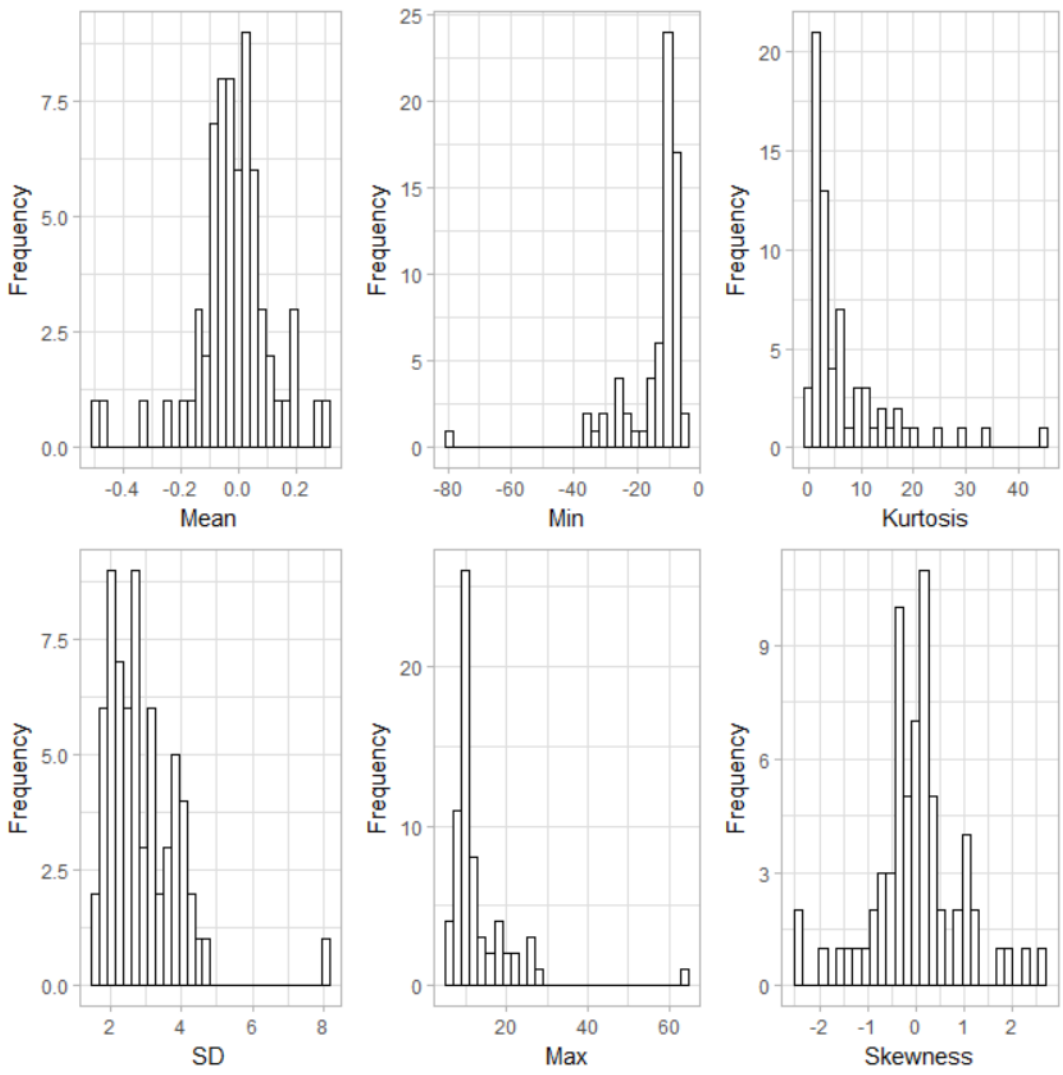

Figure A1: Descriptive statistics of data

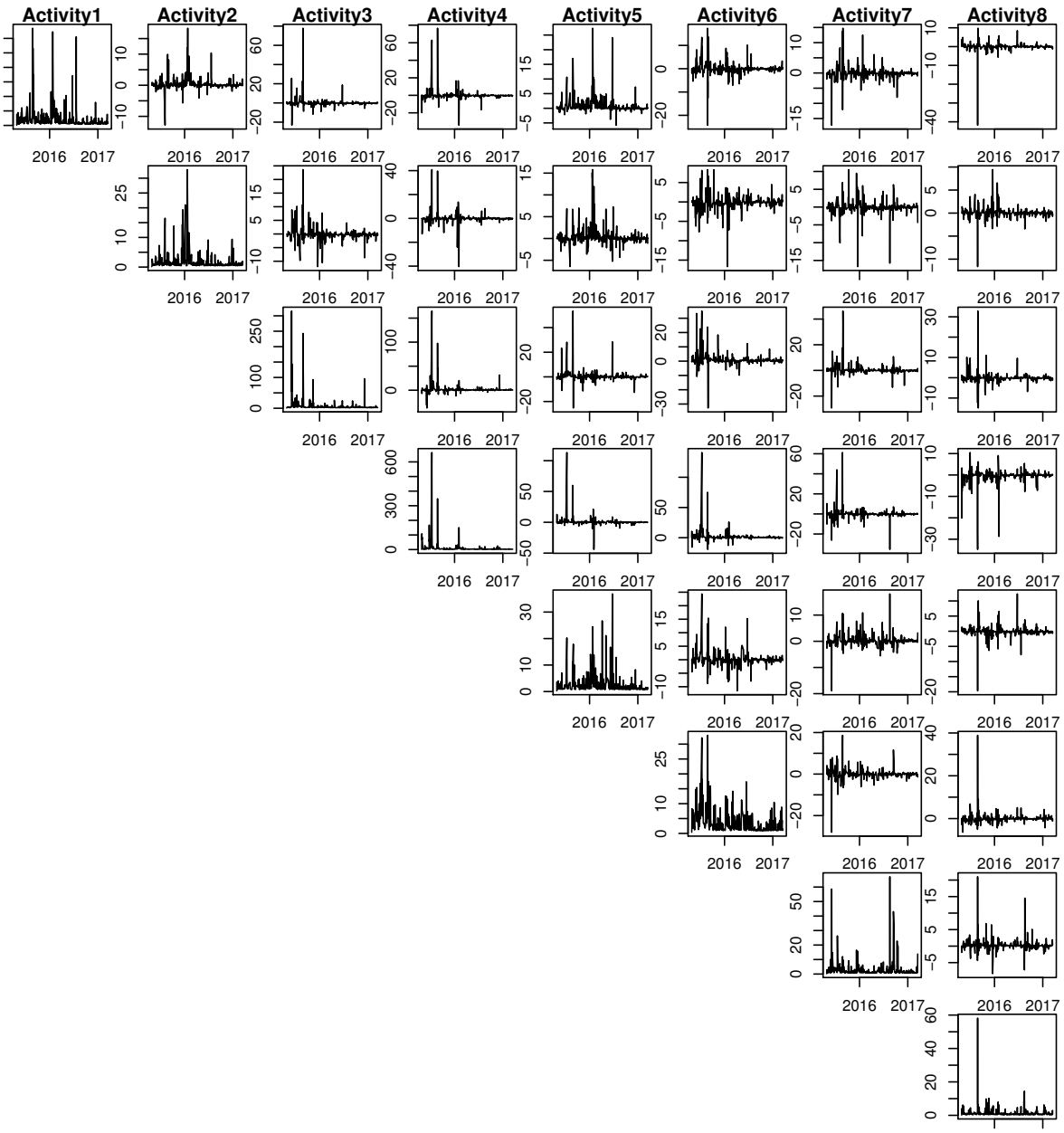

Figure A2: Dynamic volatility covariances

Neighbours in activity, country → weight matrix

Volatility covariances → edge width

In red positive coefficients, in blue negative

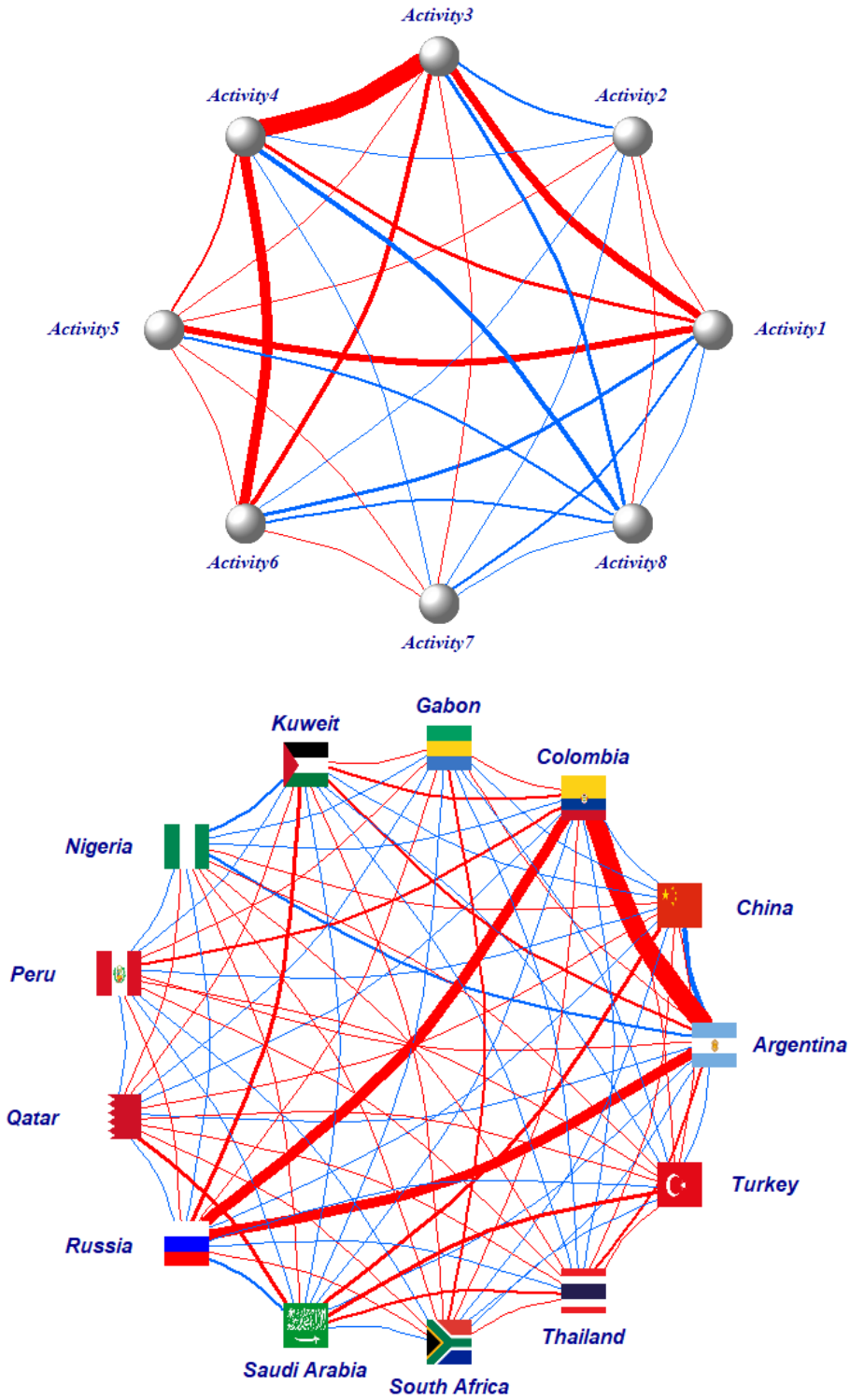

Figure A3: Volatility covariances networks

Neighbours in economics distance → weight matrix

Volatility covariances → edge width

In red positive coefficients, in blue negative

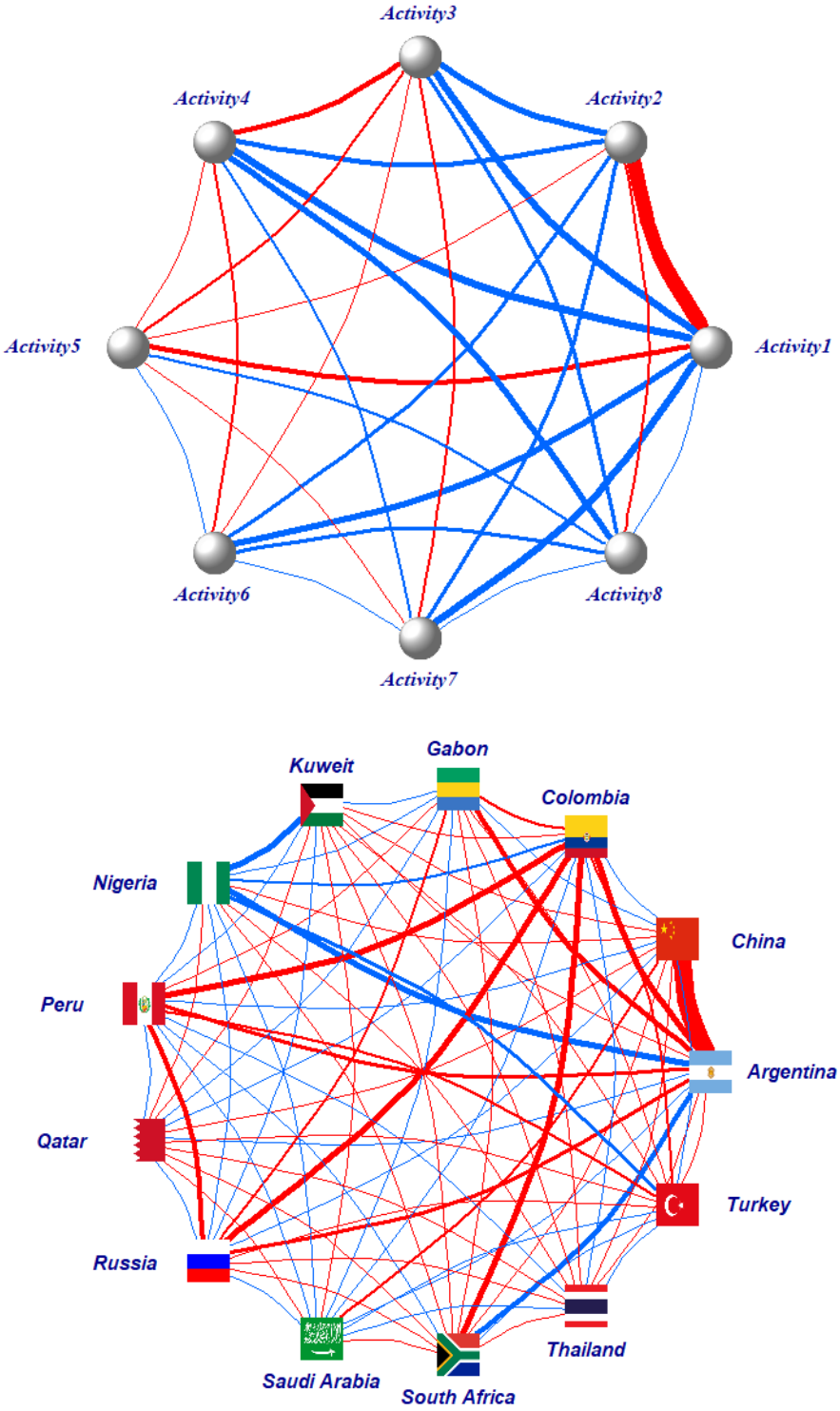

Figure A4: Volatility covariances networks

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