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DO PORTFOLIO INVESTORS NEED TO CONSIDER THE ASYMMETRY OF RETURNS ON THE RUSSIAN STOCK MARKET?

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DO PORTFOLIO INVESTORS NEED TO CONSIDER THE ASYMMETRY OF RETURNS ON THE RUSSIAN STOCK MARKET?[2]

Abstract

This paper uses the parsimonious method of embedding skewness in asset allocation based on the Taylor expansion of the investor utility function up to the third term and maximizing it by portfolio weights. This approach also enables us to consider investor risk aversion. Time-dependent multivariate asset moments are obtained via the GO-GARCH volatility model with a normal-inverse Gaussian distribution for the error term. We explore the performance of the usual 2 moment utility and its 3 moment counterpart for a portfolio consisted of twenty assets traded on the Russian stock market. The results demonstrate that the 3 moment utility significantly outperforms the 2 moment utility by SD, MAD and CVaR for low levels of absolute risk aversion and by portfolio returns and investor utility level during the whole forecast period.

Keywords: portfolio optimization, asymmetry of returns, risk aversion, GO-GARCH, normal-inverse Gaussian distribution, utility approach.

JEL Classification: C13, C22, C58, G11, G17.

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1 Introduction

The traditional approach to portfolio optimization was developed in the seminal paper of Harry Markowitz (Markowitz, 1952), where it is implied that the investor utility function has an infinite risk aversion coefficient. Hence the maximization of his/her utility is equivalent to the minimization of the portfolio variance subject to some fixed return. It is natural to suggest, however, that the investor risk aversion coefficient is less than infinity and investor utility increases if the distribution of portfolio returns is positively skewed (Arrow, 1971; Eisdorfer, 2010).

Recent literature provides a great variety of methods to introduce skewness in asset allocation: minimizing portfolio variance subject to a fixed mean and skewness (Athayde and Flôres, 2004), solving the multi-objective problem using fuzzy variables for asset returns (Li et al., 2010), a mean-absolute deviation-skewness model with linearized second and third terms (Konno et al., 1993), using shortage functions (Briec et al., 2007; Kerstens et al., 2011), and neural networks (Yu et al., 2008). Kim et al. (2014) deals with higher moments without their direct implementation into the goal function. The proper specification of the model facilitates an automatic increase of skewness and a decrease of kurtosis. Another example of such an approach is (Zuluaga and Cox, 2010). It's also possible to find the optimal portfolio by maximizing the positive skewness coefficient. This method is realized in (Mencia and Sentana, 2009), where asset returns are assumed to be distributed as a location scale mixture of normals. Remarkably, this method gives the optimal weights in a closed form.

In this article we use the utility-based approach to account for skewness in distribution of returns. The utility function is approximated by a Taylor series near the average investor's wealth. Varying the number of terms in the Taylor expansion one can include any number of return distribution higher order moments in the utility (see (Jondeau and Rockinger, 2006; Harvey et al., 2010)). We chose an exponential utility, which has constant absolute risk aversion, and represents an investor who dislikes risk. Exponential utility is used frequently in the portfolio optimization literature. For instance, in (Birge and Chavez-Bedoya, 2016) this utility together with a generalized hyperbolic distribution is used to obtain a closed-form solution for the asset allocation problem. Other examples of closed-form solutions with exponential utility include (Çanakoğlu and Özekici, 2009), where the asset returns are independent, and (Bodnar et al., 2015), where the asset returns follow a vector autoregressive process of order one. Using exponential utility Palczewski et al. (2015) study the influence of transaction costs on the optimal portfolio choice.

We compare two asset allocation procedures — with and without asymmetry. The former corresponds to the case when the utility function Taylor expansion consists of three terms (3 moment utility); the latter — two terms (2 moment utility)[3].

The utility-based approach also enables us to take into account the degree of investor risk

[3]Portfolio optimization considering moments of higher than the third order are discussed in (Beardsley et al., 2012; Akbar and Nguyen, 2016).

aversion. These methods of implementing skewness in the portfolio optimization problem do not usually consider investor risk aversion. The related literature, where risk aversion is discussed, usually discards higher order dynamics. For instance, in (Cui et al., 2015) the portfolio optimization problem with risk aversion is solved only within the framework of mean-variance analysis.

We optimize a portfolio consisting of twenty stocks traded on the Moscow Exchange (MOEX) with different levels of risk aversion and calculate three risk measures for each case. The risk measures include Conditional Value-at-Risk (CVaR), standard deviation (SD) and mean absolute deviation (MAD). We also consider investor utility level and portfolio returns. The comparison of two portfolios reveals that the 3 moment utility portfolio allows greater utility, lower risk and substantially higher returns.

The rest of the article is organized as follows. Section 2 is devoted to the portfolio optimization with asymmetry and risk aversion; it describes the objective utility function and moment estimation methods. Section 3 discusses the empirical results and Section 4 concludes.

2 Methodology

2.1 Portfolio optimization with asymmetry and risk aversion

We use the elementary utility function with constant absolute risk aversion, (1).

$$U(W) = -\exp(-\lambda W), \tag{1}$$

where $U(\cdot)$ is the utility function, W is the investor's wealth, and λ is the absolute risk aversion coefficient.

Following (Ghalanos, 2012) we use the Taylor series expansion for expected utility to represent it in form of (2).

$$E\left[U(W)\right] = \sum_{k=0}^{\infty} \frac{U^{(k)}}{k!} E\left[\left(W - \overline{W}\right)^k\right], \tag{2}$$

where $U^{(k)}$ denotes the k-th derivative of U. Since we need the first three moments to capture the asymmetry of returns, we retain the first three terms from (2) to get the expansion of utility function, (3).

$$\begin{aligned} E\left[U(W)\right] &= U(\overline{W}) + U^{(1)}(\overline{W})E[W - \overline{W}] + \frac{1}{2}U^{(2)}(\overline{W})E[W - \overline{W}]^2 \\ &\quad + \frac{1}{3!}U^{(3)}(\overline{W})E[W - \overline{W}]^3 + O(W^4) \\ &\approx U(\overline{W}) + \frac{1}{2}U^{(2)}(\overline{W})E[W - \overline{W}]^2 + \frac{1}{3!}U^{(3)}(\overline{W})E[W - \overline{W}]^3. \end{aligned} \tag{3}$$

Applying the Taylor expansion directly to exponential utility (1), we get (4).

$$E[U(W)] \approx -\exp(-\lambda m_p)\left[1+\frac{\lambda^2}{2}\sigma_p^2-\frac{\lambda^3}{3!}s_p^3\right], \tag{4}$$

where m_p — portfolio return, σ_p^2 — portfolio variance and s_p^3 — unnormalized portfolio skewness.

Later, in Section 2.2, we introduce portfolio moments, which depend on time. Consequently, we obtain (5).

$$E[U_t(W)] \approx -\exp(-\lambda m_{p,t})\left[1+\frac{\lambda^2}{2}\sigma_{p,t}^2-\frac{\lambda^3}{3!}s_{p,t}^3\right], \tag{5}$$

with

$$m_p = w'x_t, \tag{6a}$$

$$\sigma_p^2 = w'\Sigma_t w, \tag{6b}$$

$$s_p^3 = w'S_t(w \otimes w), \tag{6c}$$

where w are the portfolio weights, obtained from the maximization of (5), x_t are the portfolio asset returns at time t, (7), Σ_t is the estimated conditional second moment of return distribution, i.e. volatility, (10), S_t is the estimated conditional third moment of return distribution (see Section 2.2 for more details about moments estimation), $\otimes$ is the Kronecker product. The 2 moment utility function differs from (5) by the absence of the third term in brackets.

For the exponential utility (1) the convergence conditions of the Taylor series to expected utility do not put any substantial restrictions on wealth W (Lhabitant, 1998).

2.2 Moment estimation

To estimate portfolio moments m_p, σ_p^2 and s_p^3 we use generalized orthogonal autoregressive conditional heteroskedasticity model, or GO-GARCH (Van der Weide, 2002) with normal-inverse Gaussian (NIG) distribution (O. E. Barndorff-Nielsen, 1997) for the error term. The model is described below.

Let x_t be an $n \times 1$ vector of n assets' logarithmic returns at time t, (7), where t varies from 1 to T.

$$x_t = (x_{1t}, ..., x_{nt})'. \tag{7}$$

The observed returns x_t can be represented as the sum of their mathematical expectation $E(x_t|\mathcal{F}_{t-1})$ conditional on all available at moment $t-1$ information $\mathcal{F}_{t-1}$ and n-dimensional random process y_t, (8).

$$x_t = E(x_t|\mathcal{F}_{t-1}) + y_t. \tag{8}$$

The random process y_t has a multivariate normal (MN) or NIG conditional distribution for the 2 moment or the 3 moment utility respectively, (9).

$$y_t|\mathcal{F}_{t-1} \sim MN(0, \Sigma_t), \tag{9a}$$

$$y_t|\mathcal{F}_{t-1} \sim NIG(0, \Sigma_t, \beta, \tau). \tag{9b}$$

Matrix $\Sigma_t = E(y_t y_t'|\mathcal{F}_{t-1})$ is the conditional covariance matrix of y_t, or volatility matrix. We model innovations y_t as a NIG random process. Parameters β and τ correspond to asymmetry and tail dependence respectively. In the GO-GARCH model volatility matrix is parametrized as in (10).

$$\Sigma_t = XV_tX', \tag{10}$$

where X is a matrix, based on the singular value decomposition of unconditional variance returns (for more details see (Van der Weide, 2002)), V_t is the diagonal matrix with non-zero elements v_t equal to the univariate volatilities of the portfolio assets. Univariate volatility can be defined by any GARCH-type process, for example standard GARCH(p,q) of Bollerslev (1986). In our paper we use the following notation, (11).

$$v_t = c_0 + \sum_{i=1}^{p} \kappa_i y_{t-i}^2 + \sum_{j=1}^{q} \mu_j v_{t-j}^2. \tag{11}$$

The choice of the model is justified by the feasibility of the two-step estimation procedure. At the first step the X matrix from (10) is obtained from the data, at the second step the univariate volatilities together with the other distribution parameters are estimated via the likelihood function, see details in (Hyvärinen and Oja, 2000).

Optimal weights are calculated by maximizing the utility function (4), using the algorithm from *NLopt* library (for more details refer to (Ypma, 2014)) and *parma* package (Ghalanos and Pfaff, 2016) in the R software platform. The analytical gradient of (4) is provided to improve the accuracy and computing speed.

3 Empirical results

We estimate portfolio moments using the GO-GARCH(1,1) model with NIG errors and obtain the optimal portfolios by maximizing utility (4). The results are compared with the no-asymmetry 2 moment portfolio consisting of the same assets. The portfolio moments for the latter portfolio are assessed by means of GO-GARCH(2,2). The number of lags for the GO-GARCH model is chosen according to the Hannan–Quinn information criterion (Hannan and Quinn, 1979).

We use data from *Yahoo Finance* (2019). The sample includes twenty randomly chosen stocks from MOEX and lasts from March 2010 until March 2019, including nine years of weekly data or 560 observations after the elimination of missing values. The list of stocks considered and the descriptive statistics are presented in Appendix, Tables A1–A2. The out-of-sample period is 100 observations and is approximately 18% of the sample.

The parameter estimates of the V_t matrix from (10)–(11) in the GO-GARCH model are in Appendix, Table A3. The first three parameters are intercept, ARCH effect and GARCH effect respectively. The last two parameters, β and τ, refer to the skewness and shape parameters of the NIG distribution (see (9b)).

The first five columns in Table A3 shows GO-GARCH parameter estimates for the 2 moment utility and the last five for the 3 moment one. For each asset either the ARCH effect κ or GARCH effect μ is significant at 5%. In the 3 moment utility case for GAZA, IRAO and TANL both effects are significant. The shape parameter τ is significant at the same level for each asset except GAZA. This asset is distributed according to the variance-gamma distribution as a special case of NIG distribution (O. Barndorff-Nielsen and Blaesild, 1981). The skewness parameter β is significant at 5% in half of the cases. IRKT, AMEZ, IRAO and ODVA reveal negative skewness coefficient (for NVTK β is insignificant), the other assets have a positive one.

Next, we pass the obtained moment estimates to the investor utility function (5) and carry out the portfolio optimization procedure. We allow short positions and employ full investment constraint during the optimization procedure. We also investigate the influence of risk aversion λ by varying its values from 0.1 to 10. The choice of λ is based on similar research (see references in Section 1) and covers the range investigated in those articles. The smallest λ corresponds to the riskiest behavior.

Hereinafter we focus on the out-of-sample period, which seems more relevant to practical applications. Average optimal weights for twenty assets are presented in Appendix, Fig. A1. Most weights demonstrate monotonic dynamics. There are nine stocks, whose weights are increasing with the growth of risk aversion, and eleven stocks, whose weights are decreasing under the same conditions. Stocks whose weights decrease demonstrate higher skewness and kurtosis, than the other assets in the sample. The former include MGNZ, ZVEZ and IRKT (see Table A2 for skewness and kurtosis coefficients). Investor preferences differ between the 2 moment and the 3 moment cases primarily when λ is low, i.e. if investor loves risk.

3.1 Out-of-sample risk measure comparison

For each portfolio we calculate three risk measures: SD, MAD, and CVaR with a 5% probability.

Fig. 1 presents weekly SD, calculated for the 2 and 3 moment utilities. The SD obtained from the 3 moment utility, outperforms its 2 moment counterpart for low values of risk aversion. With the growth of risk aversion, the SD for the 3 moment utility also grows, outpacing the SD from the 2 moment utility. The other two risk measures demonstrate similar behavior. The conditional boxplots for all risk measures under investigation can be found in Appendix, Fig. A2–A4. Each boxplot displays the distribution of weekly risk measures, evaluated in the out-of-sample period. Figures give evidence that the medians

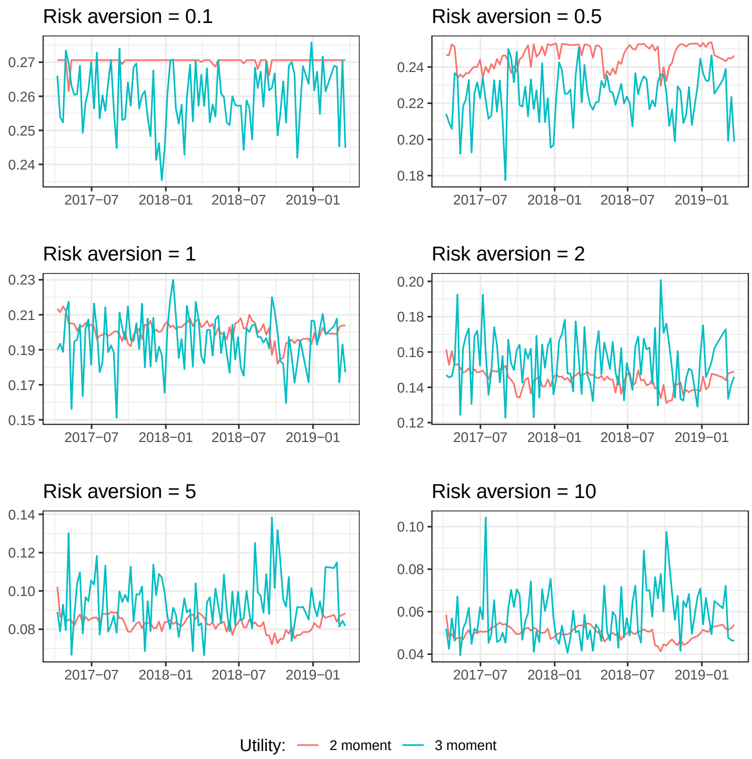

Figure 1: SD, obtained from maximization of 2 and 3 moment utility

λ	0.1	0.5	1	2	5	10
2 moment	6.80	6.57	5.77	4.49	2.65	1.47
3 moment	8.16	7.56	6.93	5.88	3.74	2.24

Table 1: Cumulative portfolio returns obtained from maximization of 2 and 3 moment utility

of SD and MAD are lower for the 3 moment utility for $\lambda \leqslant 1$ and CVaR for $\lambda \leqslant 0.5$. If risk aversion increases, the distribution of risk measures for the 3 moment utility goes up, demonstrating higher risk, than the 2 moment utility. The risk measures obtained from the 3 moment utility are more volatile, compared to their 2 moment counterparts and the pattern described above is apparent mainly for the central tendency of risk measures distributions.

We also present some performance measures for the portfolios: portfolio return and investor utility at every moment of time. Weekly returns of two portfolios based on the 2 and 3 moment utilities are plotted in Fig. 2. The returns from the 3 moment utility portfolio clearly outperform the 2 moment ones, despite being more volatile. In order to compare the general performance during the whole out-of-sample period we calculate the cumulative portfolio returns, Table 1. According to Table 1, the 3 moment portfolio provides higher returns during the period under consideration and its performance remains stable with the growth of risk aversion, λ.

Investor utility is slightly higher for the 3 moment case, and the difference increases if investor prefers to avoid risk, Table 2.

To summarize, we employ the utility approach to build two portfolios, consisting of twenty assets, which are traded on MOEX. The multivariate distribution of returns is estimated by

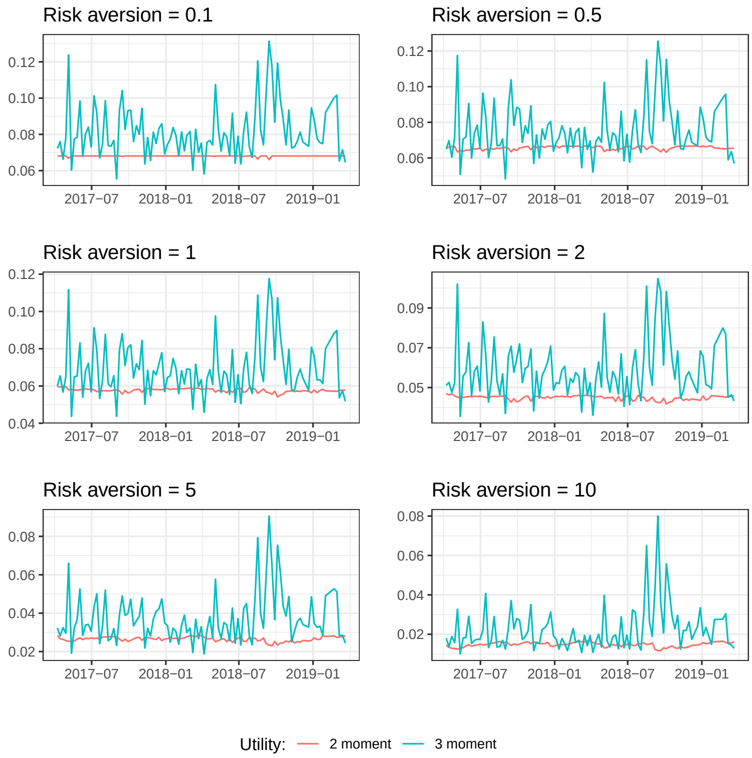

Figure 2: Portfolio returns obtained from maximization of 2 and 3 moment utility

	λ	0.1	0.5	1	2	5	10
Mean	2 moment	-0.9936	-0.9747	-0.9613	-0.9481	-0.9387	-0.9473
	3 moment	-0.9923	-0.9692	-0.9502	-0.9288	-0.9102	-0.9146
1st quartile	2 moment	-0.9936	-0.9749	-0.9618	-0.9492	-0.9413	-0.9515
	3 moment	-0.9931	-0.9735	-0.9579	-0.9404	-0.9331	-0.9457
Median	2 moment	-0.9936	-0.9747	-0.9610	-0.9478	-0.9381	-0.9469
	3 moment	-0.9926	-0.9703	-0.9524	-0.9337	-0.9194	-0.9309
3rd quartile	2 moment	-0.9936	-0.9744	-0.9608	-0.9470	-0.9356	-0.9415
	3 moment	-0.9916	-0.9657	-0.9447	-0.9187	-0.8978	-0.9039

Table 2: Statistical characteristics of investor's utility for 2 and 3 moment cases

means of the GO-GARCH model with MN and NIG distributions for errors. The risk levels of the portfolios are compared by SD, MAD and CVaR with a 5% probability. The results demonstrate the substantial outperformance of the 3 moment utility, according to the risk measures mentioned, for low levels of risk aversion, providing evidence for the fact that taking into account the asymmetry of returns allows risk to be reduced in situations when the investor does not avoid risk-taking. Such a strategy provides higher portfolio returns and investor utility.

4 Conclusion

The necessity of taking into account the skewness of returns has been investigated in much research and has a long history, see (Fogler et al., 1977; Kane, 1982). In our work we use the parsimonious method of embedding skewness in asset allocation based on the Taylor expansion of the investor utility function up to the third term and maximizing it by portfolio weights. This approach also enables us to consider investor risk aversion. Time-dependent multivariate asset moments are obtained via the GO-GARCH volatility model with NIG distribution for the error term.

We explore the performance of the usual 2 moment utility and its 3 moment counterpart for a portfolio consisting of twenty assets traded on MOEX. We compare the portfolio risk estimated by CVaR and SD, as well as mean portfolio returns and investor's utility for an out-of-sample period varying the risk aversion parameter from 0.1 to 10.

The results demonstrate that the 3 moment utility significantly outperforms the 2 moment utility by SD, MAD and CVaR for low levels of absolute risk aversion and by portfolio return and investor utility during the whole forecast period.

The work can be continued by considering other types of multivariate models for returns, including BEKK (Engle and Kroner, 1995), dynamic conditional correlations model (Engle, 2002), copula GARCH (Jaworski and Pitera, 2014) and others.

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Appendix

Table A1: Tickers and companies' names

Ticker	Company name
ZVEZ	PJSC Zvezda
GAZA	PJSC GAZ
CNTL	Central Telegraph PJSC
LNTA	Lenta Ltd
IRKT	Irkut Corporation
LSNG	PJSC Lenenergo
MGNZ	PJSC Solikamsk magnesium works
NMTP	PJSC Novorossiysk Commercial Sea Port
AMEZ	PJSC Ashinskiy metallurgical works
IRAO	PJSC Inter RAO UES
RUSP	PJSC Ruspolymet
AQUA	PJSC Russian Aquaculture
MVID	PJSC M.Video
NKSH	PJSC Nizhnekamskshina
ODVA	PJSC Mediaholding
TANL	Tantal PJSC
BSPB	Bank Saint-Petersburg PJSC
NVTK	PJSC NovatekK
URKA	PJSC Uralkali
MFGS	Slavneft-Megionneftegaz PJSC

Table A2: Descriptive statistics for log returns

	Min.	1stQ.	Mean	Median	3rdQ.	Max.	St.dev.	Skewn.	Kurt.
ZVEZ	−0.431	−0.020	0.010	0.000	0.020	0.701	0.129	1.901	10.247
GAZA	−0.336	−0.029	−0.003	−0.002	0.026	0.192	0.060	−0.577	5.083
CNTL	−0.298	−0.035	−0.001	0.000	0.026	0.387	0.067	0.385	4.406
LNTA	−0.114	−0.026	−0.006	−0.008	0.014	0.096	0.037	−0.257	0.876
IRKT	−0.219	−0.017	0.004	0.002	0.026	0.619	0.059	2.864	27.597
LSNG	−0.204	−0.028	−0.003	−0.003	0.021	0.395	0.054	1.047	8.247
MGNZ	−0.292	−0.027	0.000	−0.000	0.023	0.586	0.076	1.822	15.928
NMTP	−0.214	−0.022	0.004	−0.000	0.024	0.322	0.050	0.718	5.137
AMEZ	−0.208	−0.029	−0.003	−0.005	0.020	0.375	0.048	1.235	10.229
IRAO	−0.239	−0.027	0.003	0.000	0.029	0.250	0.055	0.437	3.110
RUSP	−0.206	−0.031	−0.004	−0.002	0.019	0.312	0.056	0.550	4.976
AQUA	−0.331	−0.045	−0.001	−0.004	0.027	0.524	0.085	1.298	6.776
MVID	−0.215	−0.016	0.005	0.002	0.028	0.167	0.046	−0.429	2.917
NKSH	−0.433	−0.032	0.002	0.000	0.028	0.442	0.075	0.959	10.100
ODVA	−0.354	−0.038	−0.007	−0.009	0.016	0.422	0.072	1.031	7.869
TANL	−0.385	−0.044	−0.003	−0.010	0.027	0.666	0.107	1.282	6.910
BSPB	−0.216	−0.027	−0.000	−0.003	0.028	0.293	0.055	0.350	5.289
NVTK	−0.149	−0.017	0.005	0.003	0.026	0.134	0.035	0.091	1.197
URKA	−0.296	−0.019	−0.001	−0.001	0.020	0.157	0.043	−0.904	5.859
MFGS	−0.143	−0.025	−0.000	0.000	0.019	0.303	0.044	1.243	6.429

Table A3: The estimates of GO-GARCH parameters
(standard errors are in parenthesis, * denotes significance on 5% level)

	c_0	κ_1	κ_2	μ_1	μ_2	c_0	κ_1	μ_1	β	τ
	2 moment utility					3 moment utility				
ZVEZ	0.002	0.000	0.000	0.021*	0.978*	0.029	0.026	0.942*	0.098	0.629*
	(0.009)	(0.006)	(0.009)	(0.006)	(0.000)	(0.023)	(0.016)	(0.031)	(0.078)	(0.177)
GAZA	0.001	0.000	0.000	0.028*	0.971*	0.109*	0.130*	0.745*	0.267*	3.768
	(0.010)	(0.011)	(0.011)	(0.001)	(0.000)	(0.050)	(0.046)	(0.083)	(0.118)	(2.353)
CNTL	0.002	0.000	0.000	0.027*	0.972*	0.001	0.000	0.999*	0.018	0.574*
	(0.006)	(0.008)	(0.006)	(0.003)	(0.000)	(0.005)	(0.005)	(0.000)	(0.076)	(0.135)
LNTA	0.030*	0.071*	0.000	0.000	0.900*	0.000	0.000	0.999*	0.156	2.148*
	(0.015)	(0.010)	(0.011)	(0.090)	(0.090)	(0.001)	(0.001)	(0.000)	(0.091)	(0.761)
IRKT	0.141	0.218*	0.035	0.000	0.641*	0.001	0.000	0.999*	−0.187*	1.637*
	(0.088)	(0.074)	(0.051)	(0.115)	(0.129)	(0.002)	(0.003)	(0.000)	(0.088)	(0.561)
LSNG	0.198*	0.000	0.123*	0.000	0.692*	0.001	0.000	0.999*	0.274*	0.339*
	(0.081)	(0.019)	(0.048)	(0.022)	(0.088)	(0.004)	(0.004)	(0.000)	(0.070)	(0.073)
MGNZ	0.001	0.000	0.000	0.177*	0.822*	0.023	0.024	0.949*	0.102	1.337*
	(0.006)	(0.019)	(0.017)	(0.001)	(0.000)	(0.016)	(0.013)	(0.024)	(0.093)	(0.445)
NMTP	0.661*	0.378*	0.000	0.000	0.000	0.012	0.000	0.989*	0.104	2.022*
	(0.277)	(0.070)	(0.195)	(0.474)	(0.053)	(0.013)	(0.013)	(0.002)	(0.094)	(0.977)
AMEZ	0.002	0.000	0.000	0.000	0.999*	0.435	0.115	0.415	−0.179*	0.964*
	(0.004)	(0.002)	(0.003)	(0.003)	(0.000)	(0.251)	(0.079)	(0.310)	(0.084)	(0.301)
IRAO	0.016	0.043	0.000	0.404*	0.535*	0.008*	0.020*	0.969*	−0.167*	0.241*
	(0.017)	(0.059)	(0.055)	(0.061)	(0.057)	(0.004)	(0.002)	(0.013)	(0.069)	(0.065)

RUSP	0.001	0.000	0.000	0.003∗	0.996∗	0.015∗	0.000	0.985∗	0.084	1.034∗
	(0.011)	(0.007)	(0.007)	(0.001)	(0.000)	(0.002)	(0.007)	(0.006)	(0.084)	(0.323)
AQUA	0.175	0.188∗	0.000	0.313	0.342	0.001	0.001	0.998∗	0.205∗	0.601∗
	(0.095)	(0.093)	(0.123)	(0.370)	(0.257)	(0.005)	(0.005)	(0.003)	(0.071)	(0.108)
MVID	0.002	0.000	0.000	0.000	0.999∗	0.052	0.038	0.903∗	−0.080	0.962∗
	(0.010)	(0.006)	(0.007)	(0.004)	(0.001)	(0.029)	(0.022)	(0.046)	(0.088)	(0.285)
NKSH	0.007	0.000	0.000	0.185∗	0.809∗	0.002	0.000	0.999∗	0.126	0.391∗
	(0.017)	(0.035)	(0.034)	(0.002)	(0.000)	(0.005)	(0.005)	(0.000)	(0.075)	(0.099)
ODVA	0.084∗	0.168∗	0.000	0.000	0.761∗	0.001	0.000	0.999∗	−0.251∗	0.457∗
	(0.033)	(0.033)	(0.013)	(0.085)	(0.018)	(0.004)	(0.005)	(0.000)	(0.075)	(0.112)
TANL	0.000	0.000	0.000	0.000	0.999∗	0.071∗	0.095∗	0.839∗	0.340∗	0.844∗
	(0.005)	(0.003)	(0.003)	(0.001)	(0.000)	(0.036)	(0.039)	(0.055)	(0.072)	(0.259)
BSPB	0.001	0.000	0.000	0.034∗	0.965∗	0.022	0.025	0.950∗	0.179∗	0.752∗
	(0.010)	(0.007)	(0.007)	(0.006)	(0.000)	(0.017)	(0.014)	(0.023)	(0.076)	(0.215)
NVTK	0.201	0.219∗	0.000	0.270	0.286	0.001	0.000	0.999∗	−0.031	1.487∗
	(0.186)	(0.035)	(0.350)	(0.892)	(0.380)	(0.007)	(0.007)	(0.000)	(0.084)	(0.535)
URKA	0.000	0.055	0.000	0.441∗	0.503∗	0.001	0.000	0.999∗	0.178∗	1.092∗
	(0.000)	(0.034)	(0.035)	(0.086)	(0.083)	(0.007)	(0.007)	(0.000)	(0.080)	(0.271)
MFGS	0.002	0.000	0.000	0.011	0.988∗	0.017	0.027	0.954∗	0.009	1.563∗
	(0.025)	(0.011)	(0.008)	(0.037)	(0.003)	(0.013)	(0.015)	(0.023)	(0.089)	(0.544)
	$LL = 13253.47$					$LL = 14098.87$				

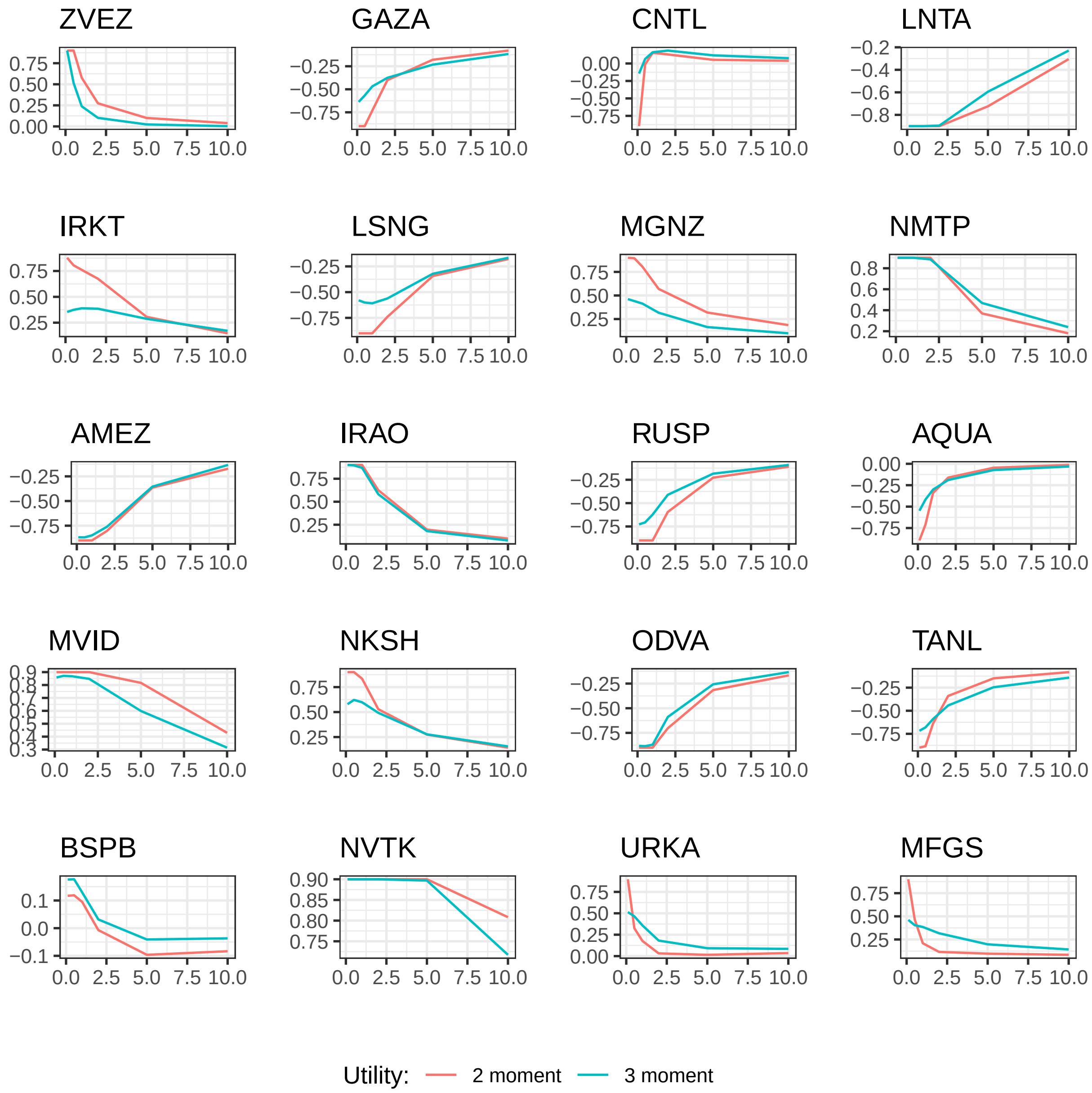

Figure A1: Average optimal weights obtained from maximization of 2 and 3 moment utility

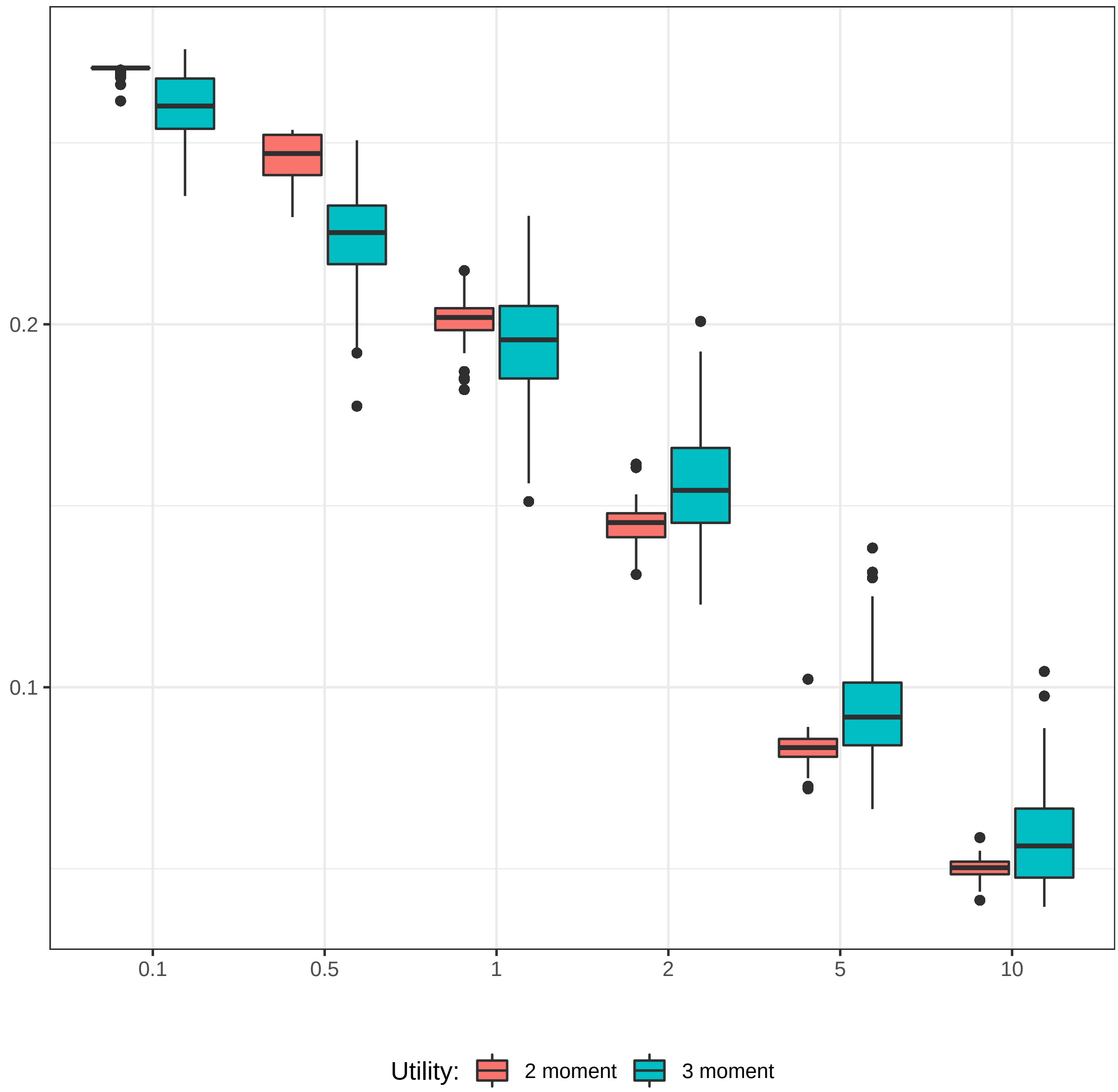

Figure A2: SD, obtained from maximization of 2 and 3 moment utility

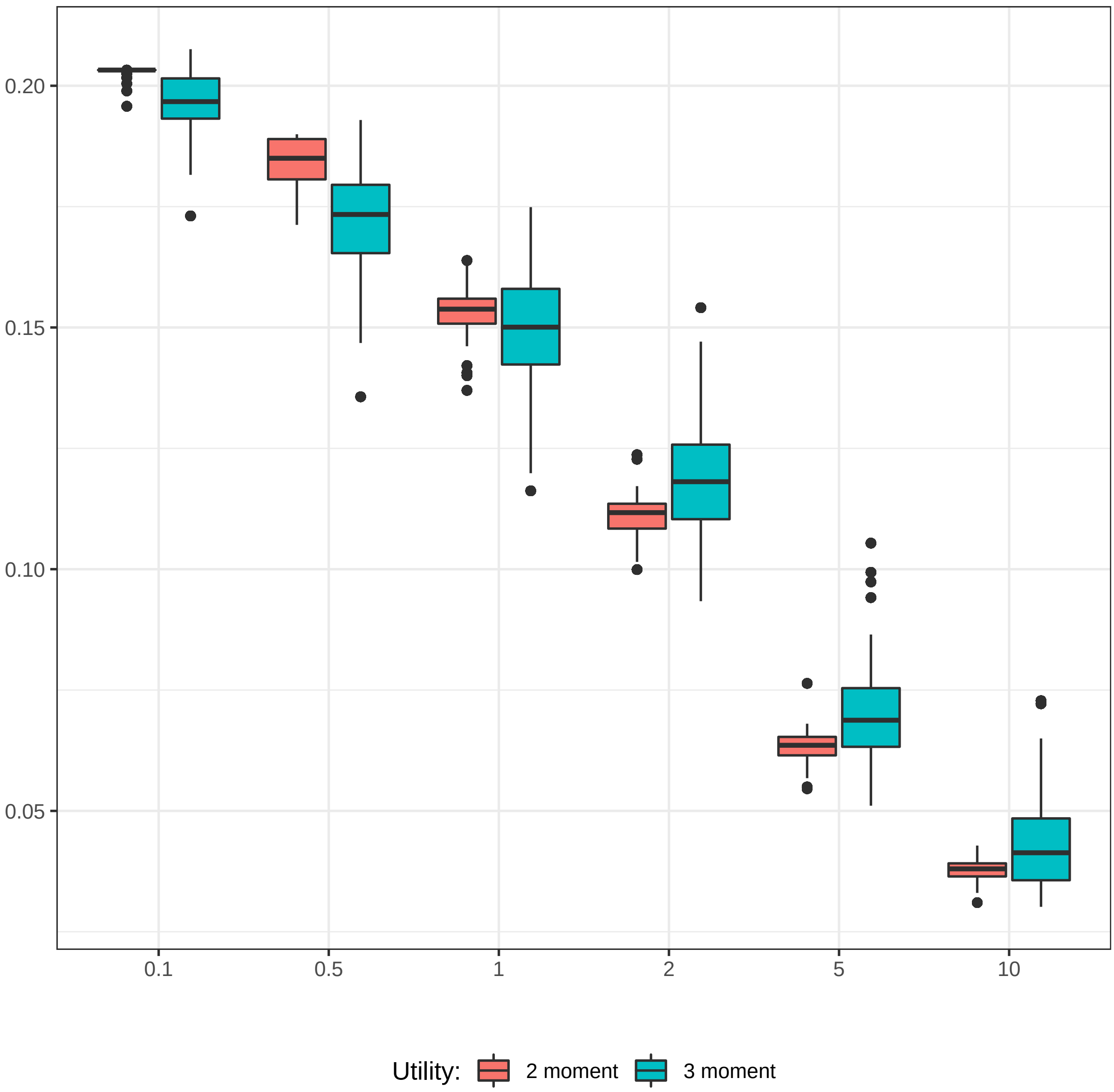

Figure A3: MAD, obtained from maximization of 2 and 3 moment utility

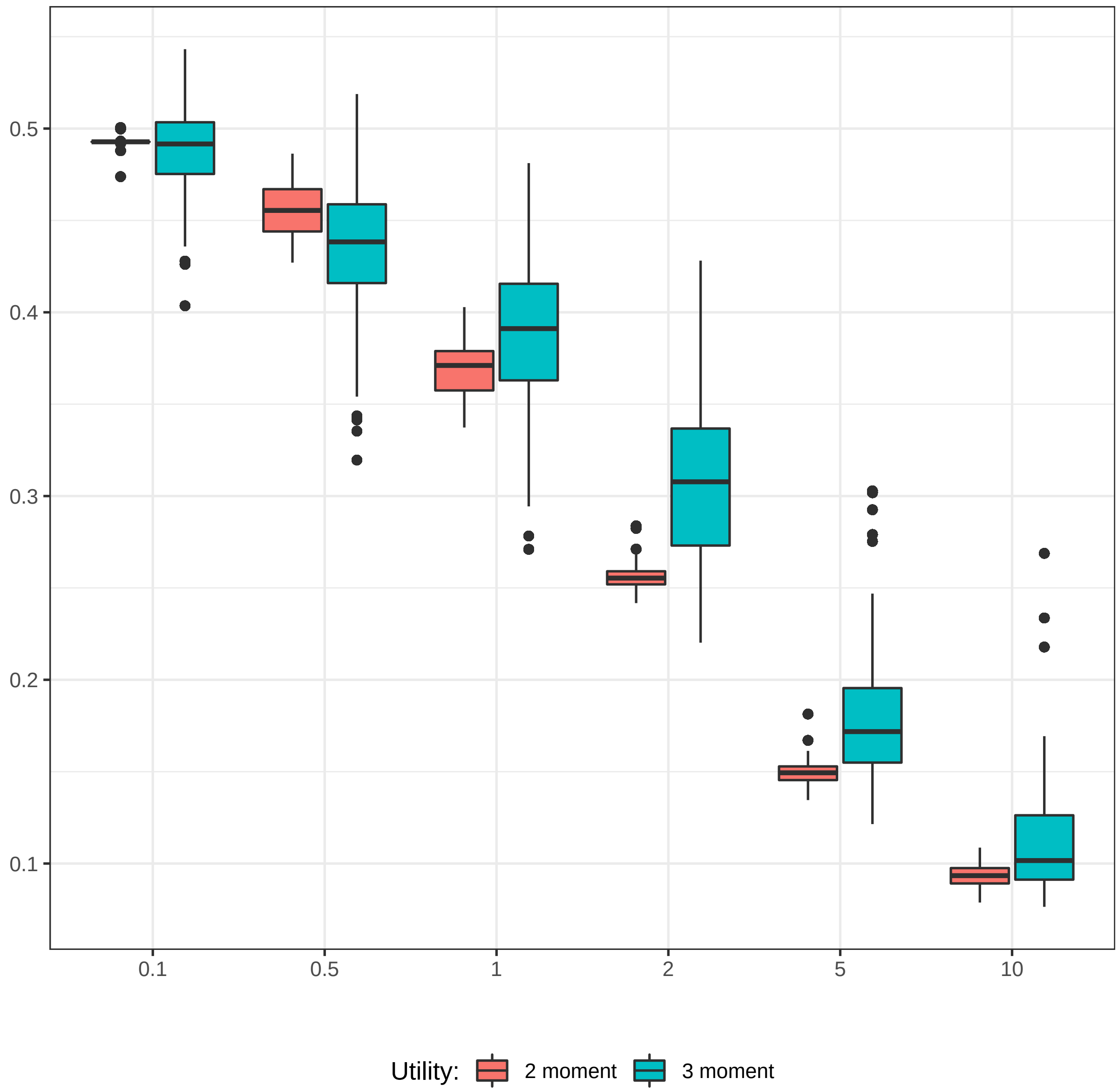

Figure A4: CVaR, obtained from maximization of 2 and 3 moment utility

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