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PRICE DISPERSION IN INTERNET SALES: DATA FROM AN ONLINE MARKETPLACE CONTRADICT LAB EXPERIMENTS

BASIC RESEARCH PROGRAM WORKING PAPERS

SERIES: ECONOMICS WP BRP 219/EC/2019

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Price dispersion in Internet sales: data from an online marketplace contradict lab experiments

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Abstract

This paper considers three hypotheses about the strategic origin of price dispersion in homogeneous product online sales. The first two are the $\varepsilon$-equilibrium and the quantal-response equilibrium (QRE) in a pure Bertrand setting involving the boundedly rational behavior of sellers. The third introduces the share of loyal consumers into the model of competition. These hypotheses were supported by estimations on experimental lab data. We test the hypotheses on a set of real prices for 30 models of household appliances collected from the largest Russian online marketplace Market.Yandex.ru. In contrast to the previously reported experimental data, we found very limited support for any of these explanations. QRE showed the best performance on the data. For most of the products it accurately predicts central tendency, i.e. the mean and the median. However, the shape of the observed price distributions is not explained well by any of the models. These results pose new challenges for theoretical explanations of observed Internet prices.

JEL Classification: C52, C72, D22, D43, L81.

Keywords: price dispersion, Internet markets, household appliances, $\varepsilon$-equilibrium, quantal-response equilibrium, loyal consumers, e-commerce.

\textsuperscript{*}The article was prepared within the framework of the HSE University Basic Research Program and funded by the Russian Academic Excellence Project ’5-100’. Nikolai Bazenkov acknowledges the financial support by Russian Foundation for Basic Research, project 17-07-01550. The authors gratefully thank the Yandex.Market Customer Support API of commercial services for the data access. The authors are indebted to Jacques-Francois Thisse, Sergei Izmalkov, Alexander Tarasov, Fuad Aleskerov, and Alexis Belianin for fruitful discussions and useful comments.

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1 Introduction

The large price dispersion in Internet sales of homogeneous goods is a steady but still unexplained phenomena. While theoretical analysts originally expected perfect competition in these markets, experimental studies reported enormous price variations which remain over time [Baye and Morgan, 2004], [Gorodnichenko et al., 2018].

Most research in this field focus on understanding heterogeneous consumer shopping behavior. This line is inspired by the theory of consumer’s search behavior ([Stigler, 1961]) and variation in strategies caused by varying costs and access to information about offers ([Salop and Stiglitz, 1977] and [Varian, 1980]). However, the strategic competitive behavior of e-shops seems to be one more source of observed price diversity. The common belief [Baye et al., 2006], [Baylis and Perloff, 2002], [Pan et al., 2004] is that the large price dispersion must be an equilibrium phenomena, but the mechanism is unclear.

In this paper we take three hypotheses about the principles underlying the behavior of online competitors theoretically developed in [Baye and Morgan, 2004], and [Morgan et al., 2006], and test them on price data from price-comparison site Market.Yandex.ru. Our dataset includes complete price lists for 30 popular models of household appliances available from July to October, 2015. The prices at every moment show large dispersion according to three measures, which are the coefficient of variation, the relative price range, and the gap between the two minimal prices.

The Bertrand framework is common for modeling online competition, this is even more reasonable in the presence of price-comparison platforms. Under the classical Bertrand model, the explanation of price dispersion based on the bounded rationality of firms looks promising (see [Baye and Morgan, 2004], and [Ellison, 2005]). This stems from the observation that similar patterns of price dispersion arise both in data from Internet sales and lab experiments, in which the “unobserved heterogeneities” are excluded by design. The authors use $\varepsilon$-equilibrium and the quantal-response equilibrium (QRE) as the operative concepts for generating the dispersion in prices. However, the testing with experimental data does not allow them to make an undoubted conclusion about the quality of these bounded rationality models.

The alternative theoretical model modifies the classical Bertrand setting by introducing the fraction of loyal consumers [Morgan et al., 2006]. In this case the required level of price dispersion is achieved under standard Nash behavior. Though persuasive experimental support was obtained, the question of how this prediction approximates real market behavior remained open.

We accurately reformulate these three hypotheses in a form suitable for fitting to our dataset. Applying a similar strategy to [Baye and Morgan, 2004], we estimate the parameters of each model. Then we check how good the estimated values are in three
formal tests: on equity of means, medians, and empirical and estimated theoretical distributions, for each hypothesis.

Our study demonstrates that only the QRE hypothesis deserves further attention since its predictions about the mean and the median are valid, and the prediction about the shape of price distribution works much better than the default uniform distribution. Two other hypotheses provide poor-quality explanations of real Internet prices.

The paper is organized as follows. Section 2 reproduces the theoretical concepts for further estimations and presents the analytical formulae for price distributions. Section 3 provides a brief description of the dataset from Market.Yandex.ru. Section 4 contains the results of the structural estimations of the parameters for each hypothesis. In the last section we discuss the quality of theoretical models under consideration and highlight their limitations.

2 Theory

2.1 Price competition with homogeneous product

Here we reproduce briefly the competition models and solution concepts from [Baye and Morgan, 2004] and [Morgan et al., 2006] with some modifications required for fitting them to the data.

Let us start with the pure homogeneous Bertrand setting. Assume \( n \) agents (firms) sell a unique type of product with costs \( c \). Firm \( i \in \{1, \ldots, n\} \) proposes price \( p_i \) simultaneously and independently of all other firms. For simplicity, let \( p_i \in [c, p^m] \), \( p^m \) be the monopoly price, and \( \pi(p^m) \) be the monopoly payoff. The expected utility of firm \( i \) under the price vector \((p_1, \ldots, p_i, \ldots, p_n)\) is given by

\[
\pi_i(p_1, \ldots, p_i, \ldots, p_n) = \begin{cases} 
\frac{\pi(p_i)}{k}, & p_i = \min\{p_1, \ldots, p_n\}, \\
0, & \text{otherwise.}
\end{cases}
\]

(1)

For estimation purposes we use the monopoly profit function with non-elastic demand normalized to 1:

\[
\pi(p) = p - c.
\]

(2)

We consider the mixed strategy expansion with a price cumulative distribution vector \( \{F_1, \ldots, F_n\} \). The expected payoff of firm \( i \) is \( E\pi_i(F) = \int \int_{p_1, \ldots, p_n} \pi_i(p) dF \).

The two equilibrium concepts that will be tested for the model above are the \( \varepsilon \)-equilibrium and QRE.
2.1.1 $\varepsilon$-equilibrium

The $\varepsilon$-equilibrium implicates the minimal level of extra payoff which is expected to be reached after a deviation. This also can be interpreted as the decision maker being insensitive to small price differences or having limited cognitive abilities. Another logic beyond this concept involves the possible costly price changes and as a result of the lack of motivation for small extra gains from frequent price adjustments.

**Definition 1.** Fix $\varepsilon > 0$. A strategy vector $F^\varepsilon$ is an $\varepsilon$-equilibrium if for any unilateral deviation $F'_i$ of player $i$

$$E\pi_i(F'_i, F^\varepsilon_{-i}) - E\pi_i(F^\varepsilon) \leq \varepsilon, \quad i = 1, \ldots, n.$$  

In [Baye and Morgan, 2004], the solution for the Bertrand model with arbitrary function $\pi$ was obtained. For the profit function (2), the $\varepsilon$-equilibrium price distribution is given by

$$F^\varepsilon_i(p) = \begin{cases} 0, & p < c + \theta; \\ 1 - \left(\frac{\theta}{p^m - c}\right)^{1/n}, & p \in [c + \theta, p^m); \\ 1, & p \geq p^m, \end{cases}$$  

where

$$\theta = \left(\frac{1}{\varepsilon} \left(\frac{n}{n-1}\right)^{n-1} (p^m - c)^{1/n}\right)^{1/n}$$

$\varepsilon$-hypothesis: The firms selling identical products on the Internet play their $\varepsilon$-equilibrium price strategies in the Bertrand model with homogeneous products.

2.1.2 Quantal-response equilibrium

In QRE the probability of setting a particular price depends on the expectations of how this price will influence the profit level under some fixed price distributions of competitors. The motivation for this concept stems from preference shocks [McFadden, 1984], or decision errors [Luce, 1959].

Formally, let $T_i$ be a map from $E\pi_i(p_i, F_{-i})$ into the probability of setting the price less than or equal to $p_i$.

**Definition 2.** A strategy vector $F^Q$ is a quantal-response equilibrium if for any $p_i$

$$F^Q_i(p_i) = T_i(E\pi_i(p_i, F^Q_{-i})), \quad i = 1, \ldots, n.$$  

A standard form of the decision rule function $T$ is the power function with parameter $\lambda \in \left[0, \frac{1}{n-1}\right]$

$$T_i(E\pi_i(p, F_{-i})) = \int_c^p \frac{E\pi_i(q, F_{-i})^\lambda}{\int_c^p E\pi_i(t, F_{-i})^\lambda} dq.$$  

5
For $T_i$ defined by (5), the symmetric QRE is provided by the following price distribution:

$$F^Q_i(p) = 1 - \left( \frac{g(p^m) - g(p)}{g(p^m) - g(c)} \right)^{1/(1+\lambda-n\lambda)}, \quad i = 1, \ldots, n, \ p \in [c, p^m],$$

with $g(p) \equiv \int \pi(p) dp + K$.

The parameter $\lambda$ expresses the degree of irrationality in a specific sense. The two limit cases correspond to Nash equilibrium ($\lambda \to \frac{1}{n-1}$) and random behavior ($\lambda \to 0$). The intermediate values of $\lambda$ generate the set of price distribution functions with a larger mass of firms setting low prices in comparison to $\varepsilon$-equilibrium.

Accounting for (2), the final form of the QRE price distribution is

$$F^Q_i(p) = \begin{cases} 
0, & p < c; \\
1 - \left( 1 - \left( \frac{p-c}{p^m-c} \right)^{(1+\lambda)\frac{1}{1+\lambda-n\lambda}} \right)^{1/(1+\lambda-n\lambda)}, & p \in [c, p^m); \\
1, & p \geq p^m.
\end{cases}$$

(6)

**Q-hypothesis:** The firms selling identical products on the Internet set their prices according to the QRE price distribution in the Bertrand model with homogeneous products.

### 2.2 Price competition with a share of loyal consumers

An alternative model for explaining the behavior of online competitors is based on consumer heterogeneity in access to the complete list of offers. In particular, this line relates to modelling multi-channel competition via online-sites and bricks-and-mortar stores.

[Morgan et al., 2006] presents the clearinghouse model which modifies Bertrand price competition by introducing a share of captive consumers. Assume that the fraction $\alpha$ of all consumers (the total mass is normalized to 1) use a price aggregator and choose the firm with the minimal price. The rest of the consumers are not informed about the whole range of firms: they are loyal to a certain firm from the list and buy from it with the reservation price normalized to 1. As above, costs are equal to $c$ for all firms. The utility of firm $i$ is combined from two sources: non-competitive revenue coming from loyal consumers and a possible gain for the winner of the Bertrand competition

$$\pi_i(p_1, \ldots, p_n) = \begin{cases} 
\alpha(p_i-c) + \frac{(1-\alpha)(p_i-c)}{n}, & p_i = \min\{p_1, \ldots, p^n\}, \\
\frac{(1-\alpha)(p_i-c)}{n}, & k \text{ is the number of firms charging } p_i; \\
\end{cases}$$

(7)

$k$ is the number of firms charging $p_i$. 

6
By analogy with the logic in [Morgan et al., 2006], it is easy to derive the (Nash) equilibrium mixed strategy of a firm. The price distribution is given by

$$F_i^\alpha(p) = \begin{cases} 
0, & p < p_i \\
1 - \left[\frac{1-\alpha}{\alpha n} \frac{p^m - p}{p-c}\right]^{\frac{1}{n-1}}, & p \in [p, p^m); \\
1, & p \geq p^m,
\end{cases}$$

where $p = \frac{p^m(1-\alpha)+\alpha c}{1-\alpha+\alpha n}$ is the minimal price for an active firm in the market. Thus, the lower bound for the fraction $\alpha$ is determined by

$$\alpha = \frac{p^m - p}{p^m - nc + p(n - 1)}. \quad (9)$$

$\alpha$-hypothesis: The firms selling identical products on the Internet compete in price for the share of informed consumers within the Bertrand model with homogeneous product and simultaneously get extra revenue from the consumers loyal to them.

### 3 Data

For the dataset, we collected prices from the online marketplace Market.Yandex.ru during several months from July 24 to October 20, 2015. Market.Yandex.ru is the most popular platform for online shopping in Russia. More than 40% of population and more than 50% of Internet users in Moscow make online purchases\(^1\).

We fixed a set of 30 the most popular goods in 5 subcategories of household appliances, i.e. fridges, cooker hoods, warm ovens, dishwashers, cooktops, and washing-machines, and downloaded the whole set of actual prices 4 times per day for e-shops in the Moscow region. Household appliances are in the top-5 categories with the largest share of online sales. For this category the share of consumers who prefers to purchase on Market.Yandex.ru exceeds 30%.

Not every e-shop specializing in household appliances sells every product from our sample, and moreover, even if it sells the specific product, the e-shop may not sell it at every moment. This gives unbalanced panel data with 30 goods, 258 sellers and 324 moments containing 463502 unique price offers.

We argue that our dataset is representative in several aspects. First, one may be sure that all the offers presented on Yandex.Market.ru are available for the announced price because the platform checks regularly and sanctions the violators. Second, the metropolitan online market is highly localized: 90% of respondents prefer to buy locally, in Moscow, [Yandex report, 2014]. Third, geo-location inside the city is not important

\(^1\)Here and further the survey characteristics of industry are based on [E-commerce report, 2014], [Yandex report, 2014]
since for the purchase of major household appliances most consumers use delivery services.

The prices in our dataset demonstrate a huge dispersion both in the coefficient of variation and in the relative price range at each moment. The aggregative description of our data is presented in Table 1. The following measures of price dispersion are used. The coefficient of variation is the standard deviation divided by the average. The relative price range is the difference between the maximal and minimal prices divided by the minimal price. The gap is the difference between the two minimal prices divided by the minimal price.

In our sample, at each moment at least two firms sold every product such that we never observe a monopoly. The influence of the number of active sellers is uncertain: the gap depends positively on the number of sellers for 17 of 30 products, the range depends positively on the number of sellers for 20 of 30 products, the variance depends negatively for 15 of 30 products (with the significance level 5%), see Table 2.

4 Results

For all estimations we apply techniques similar to [Baye and Morgan, 2004]. For the three hypotheses about the price distribution function, we tested how well they predict the mean, the median, and the histogram of the observed price distributions. The important difference is that in the previous work the monopoly price $p^m$ and the cost $c$ were fixed by the experimental design, while we estimate them from the data.

Data for the given product is a set of price vectors $p^1, \ldots, p^T$ where each vector $p^t$ contains prices observed at moment $t$. In the analyzed behavioral models the number of sellers $n$ greatly influences the price distribution, so we estimate the parameters separately for the different number of sellers. We grouped the data such that each group consists only of price vectors of the given length $n$ and the total number of prices in each group was at least $N = 25$, which is required for a reliable chi-square test. Then the parameters were estimated separately for each group.

The monopoly price $p^m$ and the cost $c$ were estimated as the sample maximum and minimum, respectively. Our data were cleaned of outliers, so the estimation is justified. The third parameter, specific for each behavioral hypothesis ($\lambda$, $\varepsilon$ or $\alpha$) was obtained by minimizing the sum of squared errors between the empirical and the theoretical cumulative distribution functions, following the same approach as [Baye and Morgan, 2004].

After the parameters had been estimated for the given price group, three tests were performed. The T-test was used to compare the observed and the predicted mean. The sign test was used to compare the medians. The goodness of fit was evaluated with the chi-square test. We used 5 bins, chosen such that the expected frequencies were equal for each bin. The significance level was chosen as $p = 0.05$ for each test. For each
### Table 1: Average characteristics of products and price dispersion for the whole period

<table>
<thead>
<tr>
<th>Product</th>
<th>Coeff of var</th>
<th>Relative price gap</th>
<th>Relative price range</th>
<th>Average price</th>
<th>Lowest price</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cata Ceres 600 Negra</td>
<td>22.6</td>
<td>143.6</td>
<td>4.0</td>
<td>19443</td>
<td>12431</td>
<td>36</td>
</tr>
<tr>
<td>Electrolux EHH 6340 FOK</td>
<td>15.1</td>
<td>73</td>
<td>2.7</td>
<td>24524</td>
<td>19949</td>
<td>50</td>
</tr>
<tr>
<td>Gorenje BO 73 CLI</td>
<td>13.7</td>
<td>70.1</td>
<td>1.4</td>
<td>27435</td>
<td>22947</td>
<td>47</td>
</tr>
<tr>
<td>Gorenje gw 65 cl i</td>
<td>13.5</td>
<td>69.3</td>
<td>2.1</td>
<td>17900</td>
<td>14836</td>
<td>28</td>
</tr>
<tr>
<td>Electrolux EZB 53400 AX</td>
<td>13.5</td>
<td>51.2</td>
<td>5.2</td>
<td>19944</td>
<td>15750</td>
<td>18</td>
</tr>
<tr>
<td>Hansa OSC 511 WH</td>
<td>13.4</td>
<td>79.4</td>
<td>8.4</td>
<td>2813</td>
<td>2124</td>
<td>47</td>
</tr>
<tr>
<td>Hotpoint-Ariston 7HPC 640</td>
<td>13</td>
<td>80.5</td>
<td>2.2</td>
<td>11876</td>
<td>9833</td>
<td>67</td>
</tr>
<tr>
<td>Krona Kamilla 600</td>
<td>12.7</td>
<td>76.7</td>
<td>13.2</td>
<td>7546</td>
<td>5348</td>
<td>30</td>
</tr>
<tr>
<td>Atlant XM 4008-022</td>
<td>12.5</td>
<td>101.1</td>
<td>0.7</td>
<td>14281</td>
<td>12698</td>
<td>59</td>
</tr>
<tr>
<td>Hansa ZIM 436 EH</td>
<td>12.2</td>
<td>52.7</td>
<td>1.3</td>
<td>20252</td>
<td>17542</td>
<td>31</td>
</tr>
<tr>
<td>Hansa BOEI62000015</td>
<td>11.7</td>
<td>85.7</td>
<td>0.6</td>
<td>16744</td>
<td>14442</td>
<td>68</td>
</tr>
<tr>
<td>Electrolux EWS 1052 NDU</td>
<td>11.6</td>
<td>57.5</td>
<td>2.5</td>
<td>19539</td>
<td>16906</td>
<td>31</td>
</tr>
<tr>
<td>Hansa BH68300</td>
<td>11.6</td>
<td>55.1</td>
<td>1.4</td>
<td>16494</td>
<td>14035</td>
<td>45</td>
</tr>
<tr>
<td>Bosch DHL 545 S</td>
<td>11.3</td>
<td>65.7</td>
<td>3.3</td>
<td>9220</td>
<td>7706</td>
<td>36</td>
</tr>
<tr>
<td>Elikor integra 60h-400</td>
<td>11.1</td>
<td>64.6</td>
<td>2.1</td>
<td>3778</td>
<td>3125</td>
<td>46</td>
</tr>
<tr>
<td>Bosch PIC645F17E</td>
<td>10.8</td>
<td>58.4</td>
<td>0.3</td>
<td>28000</td>
<td>23686</td>
<td>58</td>
</tr>
<tr>
<td>Indesit BIA 18</td>
<td>10.3</td>
<td>60.4</td>
<td>1.4</td>
<td>21264</td>
<td>17741</td>
<td>71</td>
</tr>
<tr>
<td>Bosch HBG43T450</td>
<td>10</td>
<td>60.3</td>
<td>2.7</td>
<td>30429</td>
<td>25717</td>
<td>51</td>
</tr>
<tr>
<td>Samsung WF8590NMW9</td>
<td>9.1</td>
<td>78.2</td>
<td>0.7</td>
<td>19111</td>
<td>17217</td>
<td>75</td>
</tr>
<tr>
<td>Bosch ActiveWater SPV40E10RU</td>
<td>9.1</td>
<td>47.6</td>
<td>0.8</td>
<td>24489</td>
<td>21671</td>
<td>56</td>
</tr>
<tr>
<td>Siemens SR 66T090</td>
<td>8.6</td>
<td>58.3</td>
<td>3.2</td>
<td>43908</td>
<td>37238</td>
<td>68</td>
</tr>
<tr>
<td>Hotpoint-Ariston FTR 850</td>
<td>8.5</td>
<td>52.2</td>
<td>1.4</td>
<td>21085</td>
<td>18539</td>
<td>65</td>
</tr>
<tr>
<td>Bosch WLG 20061 OE</td>
<td>8.1</td>
<td>54.2</td>
<td>3.0</td>
<td>20417</td>
<td>17506</td>
<td>66</td>
</tr>
<tr>
<td>Ariston LSTB 4B00 RU</td>
<td>7.9</td>
<td>43.2</td>
<td>2.2</td>
<td>17519</td>
<td>15710</td>
<td>34</td>
</tr>
<tr>
<td>Bosch KGS 30XW20 R</td>
<td>7.8</td>
<td>44.8</td>
<td>1.4</td>
<td>34584</td>
<td>30170</td>
<td>52</td>
</tr>
<tr>
<td>Indesit wiuw 81 (csi) F053525</td>
<td>7.6</td>
<td>26.7</td>
<td>4.4</td>
<td>13340</td>
<td>12102</td>
<td>9</td>
</tr>
<tr>
<td>Candy CDCF 6-07</td>
<td>7</td>
<td>36.4</td>
<td>2.6</td>
<td>13619</td>
<td>12284</td>
<td>25</td>
</tr>
<tr>
<td>LG F-1096SD3</td>
<td>6.2</td>
<td>39.6</td>
<td>0.5</td>
<td>21768</td>
<td>19605</td>
<td>67</td>
</tr>
<tr>
<td>LG GAB409SVQA</td>
<td>5</td>
<td>28.8</td>
<td>1.6</td>
<td>32571</td>
<td>29610</td>
<td>44</td>
</tr>
<tr>
<td>Liebherr SBSesf 7212</td>
<td>4.3</td>
<td>25.1</td>
<td>4.8</td>
<td>124142</td>
<td>111186</td>
<td>29</td>
</tr>
</tbody>
</table>

### Table 2: Correlation of price dispersion measures with the number of sellers

<table>
<thead>
<tr>
<th># of products</th>
<th>Positive corr.</th>
<th>Negative corr.</th>
<th>Not significant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average price</td>
<td>5</td>
<td>21</td>
<td>4</td>
</tr>
<tr>
<td>Gap</td>
<td>3</td>
<td>17</td>
<td>10</td>
</tr>
<tr>
<td>Range</td>
<td>20</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Coeff of var</td>
<td>12</td>
<td>15</td>
<td>3</td>
</tr>
</tbody>
</table>
of the tests we calculated the fraction of all time moments when the hypothesis was not rejected. This fraction estimates how well the given behavioral model explains the data. The complete results are summarized in Tables 4, 5, 6, provided in Appendix.

The total shares of successful tests are shown in Table 3. The “Random” column corresponds to uniformly distributed prices. A detailed discussion for each hypothesis is presented below.

Table 3: The shares of moments for all products for which $Q$- and $\varepsilon$-hypotheses are not rejected, the significance level is 0.05

<table>
<thead>
<tr>
<th>Test</th>
<th>$Q$-hypothesis</th>
<th>$\varepsilon$-hypothesis</th>
<th>$\alpha$-hypothesis</th>
<th>Random</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>95%</td>
<td>14%</td>
<td>15%</td>
<td>14%</td>
</tr>
<tr>
<td>Median</td>
<td>88%</td>
<td>5%</td>
<td>9%</td>
<td>15%</td>
</tr>
<tr>
<td>Distribution</td>
<td>16%</td>
<td>0.1%</td>
<td>0.4%</td>
<td>0.4%</td>
</tr>
</tbody>
</table>

4.1 Testing of $Q$-hypothesis

The results of estimation of $Q$-hypothesis are the most optimistic. The mean and median tests fail to reject QRE in almost 100% of the tests. This is caused by the fact that most of the observed price distributions are unimodal with positive skewness and this shape is in general similar to that induced by the QRE hypothesis. One of the successful fits is presented in Figure 1.

However, the goodness-of-fit chi-squared test rejects the null hypothesis for most of the observed moments. The best result is in the last row in the table when the QRE hypothesis passed the test in 60% of the time moments. In sum, for 6 products the null hypothesis is not rejected in more than 20% of the tests, for 11 products it is not rejected in 10-20% of the tests, for 9 products it is not rejected in less than 10% of the tests, and for 4 products it is always rejected. The percentage of success does not depend on the number of sellers. This distribution of successful estimations convinces us that the QRE hypothesis is far from an ideal explanation for the highly dispersed prices in Internet sales. Nevertheless, QRE performs much better than the uniform prices and the other two hypothesis. Therefore, our study shows that QRE can serve as a useful baseline for more sophisticated behavioral models.

4.2 Testing of $\varepsilon$- and $\alpha$-hypotheses

The estimations of $\varepsilon$- and $\alpha$-hypotheses are much less successful. Even the mean and the median tests provide weak predictions, while the chi-squared test fails dramatically. In fact, these two models predict the observed prices less accurately than the uniform distribution. The examples of successful fits are shown in Figures 2 and 3.
A possible reason for the poor performance is that both \( \varepsilon \)- and \( \alpha \)-hypotheses predict a high concentration of prices near the monopoly price for \( n \geq 8 \). But this pattern is rarely observed in our data, most of the time the prices lie close to the minimal price, not maximal. Moreover, for most of the products the average price correlates negatively with the number of sellers. This tendency directly contradicts the models’ predictions.

These two hypotheses demonstrate similar poor performance, despite the intuition behind them seeming reasonable and there being some experimental support in their favor.

5 Discussion

The objective of our study was to verify the models from [Baye and Morgan, 2004], and [Morgan et al., 2006] on real price data. The results show that from the three models to explain Internet price dispersion, only one performed better than the hypothesis of random behavior.

The principle difficulty concerns the number of competitors. In the Bertrand model with \( \varepsilon \)-equilibrium and in the model with loyal consumers the higher the number of sellers, the higher the prices must be concentrated at the monopoly price, while our data shows the opposite tendency. In the experiments, reported in [Baye and Morgan, 2004],
Figure 2: Empirical CDF and the theoretical distributions for the washing machine Indesit wiiun 81 (csi) F053525. Number of sellers is \( n = 4 \). Prices are grouped from 7 time points from August 26, 1am to August 27, 1pm. Only \( \varepsilon \)-equilibrium passed \( \chi^2 \) test: \( \varepsilon = 0.41, p_{\chi^2} = 0.1 \).

Figure 3: Kitchen hood Krona Kamilla 600. Number of sellers \( n = 18 \), prices are grouped from September 16, 1am and 7am. Only \( \alpha \)-hypothesis passed \( \chi^2 \) test: \( \alpha = 0.37, p_{\chi^2} = 0.063 \).
and [Morgan et al., 2006] there were not more than four competitors, which is in line with the model. However, in Internet sales, the number of active firms is generally greater, such that these models are not applicable. Even in the case of a relatively small number of sellers, the predictions under $\varepsilon-$ and $\alpha-$hypotheses are not valid. These results were not obvious, because the assumptions underlying both models have a reasonable interpretation in the context of online markets.

QRE performs much better in comparison with the two others. It is shown that the central tendency is predicted very accurately by QRE. However, the fraction of successful distribution fits does not favour this hypothesis unreservedly. The value of the predicted degree of rationality $\lambda$ is closer to 0 than in the experiments. This means that real firms are less rational than the participants of the lab session according to the model.

This observation supports the important role of bounded rationality for the adequate modeling of online competition. The limited success of QRE shows some potential for this approach. The question of what model is suitable still remains open, but QRE may be viewed as a useful starting point for further investigation. One of the possible directions of future research is to extend the homogeneous Bertrand competition in a way which is more suitable for online markets.

**References**


Table 4: Tests of equality of Means, Medians, and Distributions: the shares of moments for every product for which $Q$- and $\varepsilon$-hypotheses are not rejected, the significance level is 0.05

<table>
<thead>
<tr>
<th>Product</th>
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<th>QRE Median</th>
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<th>$\varepsilon$-equilibrium Mean</th>
<th>$\varepsilon$-equilibrium Median</th>
<th>$\chi^2$</th>
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Table 5: Tests of equality of Means, Medians, and Distributions: the shares of moments for every product for which $\alpha$-hypothesis and Random behavior are not rejected, the significance level is 0.05

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Table 6: Parameters of QRE and $\alpha$-hypothesis

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