COMMODITY CYCLES AND FINANCIAL INSTABILITY IN EMERGING ECONOMIES

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Commodity exporting economies display procyclicality with the price of commodity exports. Although financial frictions may amplify commodity price shocks, how they do so for net exporters is unclear. Using Russian data from 2001–2018 we estimate a small open economy New Keynesian model with a banking system and leveraged domestic firms who issue secured debt and may default on their unsecured domestic debt. The collateral constraint and default generate financial intermediation wedges that vary endogenously over the business cycle, amplify the estimated contribution of commodity price shocks, and reduce the importance of investment and discount factor shocks. With financial frictions, optimal policy is characterized by monetary policy with a lower inflation and GDP target, but has a significant role for targeting the credit-to-GDP ratio through a combination of macroprudential tools.

Keywords: Business cycles, Small open economy, Emerging markets, Commodity prices, Financial Stability, Macroprudential policy

JEL classification: E3 F34 G15 G18

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1 Introduction

The large aggregate fluctuations of emerging economies have motivated competing explanations for their underlying cause. These include terms of trade (and price) shocks (Mendoza (1995) and Kose (2002)), commodity price shocks (Fernández et al. (2017), Bergholt et al. (2017), and Fernández et al. (2018)), productivity shocks (Aguiar and Gopinath (2007) and García-Cicco et al. (2010)), and financial frictions and interest-rate premia (Neumeyer and Perri (2005), Uribe and Yue (2006), Chang and Fernández (2013), and Fernández and Gulan (2015)). Recently, an emergent literature has attempted to quantify the role of commodity prices through financial conditions. For example, Drechsel and Tenreyro (2018) show that commodity price shocks account for 38% of the variation in output when the external risk premium fluctuates with the price of oil, while the feedback from Sovereign and fiscal concerns to private debt and default is further emphasised in Kaas et al. (2020). For emerging economies with a positive net external asset position, such a mechanism is difficult to rationalize as a source of financial instability and aggregate volatility. On the other hand, the importance of default on domestic corporate credit markets for aggregate fluctuations is well documented, for the US (Gilchrist and Zakrašek (2012)) and for closed economies (Cui and Kaas (2020)). The puzzle is whether domestic credit markets alone can serve as an amplification mechanism for commodity price shocks for economies with a current account surplus.

In this paper we ask three questions: 1) to what extent are emerging market business cycle dynamics driven by commodity prices? 2) how does the inclusion of financial frictions affect the estimation of the contribution of the shocks that drive the business cycle? 3) how should monetary-macroprudential policy be conducted in an economy susceptible to commodity price shocks? We show, with Russian data from 2001-2018, that the extent to which commodity price shocks account for the variation in output increases significantly when financial frictions on the domestic credit market are included in the endogenous structure of the model being estimated. When they are included, commodity price shocks (total factor produc-
activity shocks) explain 33% (64%) of the variation in output while when they are not, the number falls to 6% (44%). This fall is largely compensated by an increase in the importance of investment shocks which increase from 0% to 31%. Furthermore, the importance of investment shocks in Loans and Deposits increases from 19% and 7% to 36% and 60% when financial frictions are excluded while the importance of discount factor shocks in explaining deposits increases from 8% to 30%. With financial frictions we also find strong evidence of a “Dutch Disease” type effect in the Russian economy following a commodity price shock. Our normative results show that macroprudential policies that respond to the growth in domestic credit substitutes for monetary policy that strongly targets inflation and GDP growth. We argue that consideration of the optimal selection of macroprudential policy tools depends crucially on the inclusion of the cyclical wedges, especially arising from default on unsecured loans, in the endogenous structure of the estimated model.

We study the Russian economy, the benefits of which are two-fold. First, the Russian economy was subject to several episodes of severe economic fluctuations over the last 20 years. Second these fluctuations correspond to large declines in the primary export: oil/gas commodities. In contrast to emerging economies in Latin America, Russia runs a current account surplus, has low external debt in the sample period and has a diverse number of trading partners in spite of exports being concentrated in commodities. This implies two things. First, that variations in the external interest rate results in a muted effect on the banking system. Second, that shocks will be amplified in the economy only through a mechanism that amplifies domestic interest rates. For this, we examine the role of financial frictions between the domestic banking and production sectors.

To understand the interaction between commodity shocks and financial frictions we estimate a small open commodity exporting New Keynesian DSGE model augmented with a banking sector and a leveraged firm sector that defaults on its debt. There are two frictions in the financial intermediation process that generate pecuniary externalities and give macroprudential policy a role. These are due to a collateral constraint and a (deadweight) cost of default. Finally, a meaningful in-
Interaction between monetary and macroprudential policies requires the inclusion of nominal rigidities in the form of price and wage stickiness.

Our paper contributes to three strands of the literature. First, to the understanding the role of commodity shocks in explaining business cycle fluctuations in emerging markets; Second, to the literature on the identification of the mechanisms that propagate and amplify structural shocks; Third, to the literature on financial stability and macroprudential analysis in estimated dynamic models.

Our results relate to those of Drechsel and Tenreyro (2018) and Fernández et al. (2018) in as much as the oil price shock in our economy dampens domestic demand, raises expectations of corporate default and interest rates. In contrast to Drechsel and Tenreyro (2018), the external interest-rate premium is constant in our framework\(^1\), and so the effect of the negative shock to the foreign price of oil works through the effect of the shock on domestic income. In that sense our results are closer to Fernández et al. (2018), but there they are amplified via domestic interest rates without ascribing a role for financial frictions. We extend both papers by using a New Keynesian framework that allows a role for monetary policy and an optimizing banking system that allows us to study macroprudential tools. Although corporate unsecured lending has not been emphasized till recently as a source risk in emerging markets, papers such as Fernández and Gulen (2015), Chang et al. (2017) and Caballero et al. (2018) have shown the importance of explaining the countercyclicality of interest rates and leverage. However in those papers the focus is default on external credit. In contrast, we emphasize the role of default by domestic firms to the domestic banking sector in domestic currency.

Chari et al. (2007) argue that the business cycle can be described as wedges in the endogenous structure of the prototype Real Business Cycle model. We show that wedges, specifically inefficiencies arising from financial intermediation, are essential to identify the importance of structural shocks. When these wedges are held

\(^1\)We estimate the adjustment costs on external debt for the sake of obtaining stationarity along the lines of Schmitt-Grohe et al. (2003) and find them to be extremely small and orders of magnitude smaller than necessary to be effective endogenous interest-rate premia.
constant, or, equivalently, when financial frictions are absent, the transmission of foreign exogenous shocks (specifically, the foreign price of oil), are relatively undervalued in the estimation. Furthermore, when the wedges from financial frictions are held constant over the business cycle, the relative importance of investment shocks increases greatly. In this sense our results are closely related to Justiniano et al. (2010) who show that investment shocks help to explain a large proportion of GDP fluctuations in the US and supports the intuition of Justiniano et al. (2011) that investment shocks may be related to financial frictions. In our paper, the superior fit of a model with endogenous financial frictions wedges is driven by the wedge arising from the dead-weight cost of default as it affects how loans depend on expected default (non-performing loans) rates. Following a positive shock to the foreign price of oil, the exchange rate appreciates, decreasing inflation and stimulating demand. Higher demand leads to a sharp decline in expected default rates and borrowing costs, and a rise in firm investment. Unsecured loans increase sharply while secured loans increase gradually due to the gradual rise in the value of collateral. Thus financial frictions wedges affect the composition of debt in addition to the level.

The normative analysis in our paper finds the optimal combination of simple monetary and macroprudential rules that maximize household welfare. We contribute to the literature on the interaction and potential complementarity of multiple prudential tools such as Goodhart et al. (2013), Goodhart et al. (2012) Walther (2016), Kara and Ozsoy (2019), and Boissay and Collard (2016). Kashyap et al. (2017) who show that the quantity of the optimal policy instruments should not equal the number of arising externalities but rather the number of distortions in intermediation margins. In our set up there are two wedges or inefficiencies arising from intermediation - from the collateral constraint and from the deadweight cost of default on unsecured debt. These wedges fluctuate with the business cycle and the “financial cycle”, or the cycles that characterize the financial system (see Claessens et al. (2011) and Drehmann et al. (2012) among others). We focus our normative analysis on using financial instruments to target these two wedges. In
particular, we study how these wedges are affected by augmenting the Taylor rule (Lean Against the Wind monetary policy)\(^2\), deposit requirement (Liquidity Coverage Ratio), Countercyclical Capital Buffer, and Loan-To-Value ratio. We show that the introduction of wedges requires optimal monetary policy to have a smaller emphasis on inflation and GDP but a large response to the credit-to-GDP ratio. The countercyclical capital buffer and liquidity coverage ratio were also found to be important and compliment each other. The capital requirement increases the amount of equity in the banking system while the liquidity coverage ratio penalizes expansions in the balance sheet when credit growth is high. It is important to note that the primary purpose of monetary and prudential policies is demand management whilst the statement we are making is primarily, though not only, with respect to supply side shocks such as TFP or commodity prices. However even in response to such supply side shocks the intermediation of funds by the financial system affects aggregate demand through interest rates and the supply of loanable funds. Hence a clear role exists for policy to manage demand through the financial system.

In Section 2 we show the strong countercyclicality of non-performing loans (NPLs) which motivates the model we present in Section 3. Section 4 presents the measurement equations, Section 5 the parameterization, and Section 6 presents the estimation results and compares the fit of our model when financial frictions wedges vary endogenously over the business cycle with the case when we hold it fixed. Section 7 presents the quantitative results of the estimation including the historical decomposition and the forecast error variance decomposition. In Section

\(^2\)The optimal interaction between macroprudential and monetary policies still remains an important challenge for policy. (Nachane et al. (2006); Ghosh (2008); Gavalas (2015); Gambacorta and Shin (2016) ) show that the more restrictive the rules (in particular, capital requirements), the more contractionary effect the monetary policy may have. Gale (2010) suggests that too restrictive capital requirements may encourage banks to take higher risks in order to earn higher expected profits. In this case when monetary authorities increase interest rates this may not have a contractionary effect on credit market and the banks will form highly risky loan portfolios as costs of funding increase. As a result, defaults of the risky firms may create the threat to financial stability. It is also worth noting that not only macroprudential regulation has an impact on the monetary transmission mechanism. According to (Borio and Zhu (2012); de Moraes et al. (2016)), the stance of monetary policy may affect the optimal level of macroprudential regulation.
8 we study candidate simple macroprudential policy tools and find the optimal set of parameters that maximize welfare.

2 Commodity Cycles and Empirical Regularities

2.1 Data

For the estimation of the model parameters and historical decomposition we used eight data series: GDP, household consumption, CPI, interest rate, total loans, ratio of non-performing loans to total loans, deposits and international oil price. The data on GDP, consumption and inflation were taken from Rosstat sources. For the international oil price series we took Urals oil price in dollars per barrel. Data for all other series were taken from the Bank of Russia website. For the total loans series we took loans issued by Russian banks to domestic enterprises. The amount of non-performing loans based on the non-performing loans from all the loans given by Russian banks to domestic enterprises. The series for the ratio of non-performing loans to total loans was constructed by dividing the corresponding amount of non-performing loans by loans issued by Russian banks to domestic enterprises. For the deposit data we used the stock of household deposits nominated in domestic currency. As an interest rate series we took one-day interbank rate in Russia.\footnote{The nominal exchange rate in Russia was fixed for most of the period we consider as the country switched from exchange rate targeting only in the second half of 2014. Given that nominal exchange rate in the model is determined endogenously, we do not include these data series in our estimation.}

The data covers period of 70 quarters: from Q2 2001 to Q2 2018. We took the first quarter of 2001 as the starting point for our series because by that time the influence of 1998 crisis on the economy had vanished and Russian economy started to experience positive effect of the rising international oil prices.

We eliminate the seasonal component from the data. GDP, household consumption, total loans, deposits and international oil price are represented as seasonally adjusted real data with Q4 2013 being the base period. The interest rate, CPI and the NPL to loans ratio are represented as seasonally adjusted values.
Figure 1 below shows the evolution of these variables over the sample period. Interest Rates and CPI are annualized rates at the quarterly frequency. The major crisis events in Russia in 2008 and 2014 coincide with declining oil prices and rising inflation and NPL rates. The fixed or managed nominal exchange rate system that Russia had until 2013 is not reflected in any structural change in the variables, and so we consider the real exchange rate to be the relevant variable for making decisions with the external economy.

![Graph](image1.png)

(a) GDP, consumption, loans, oil price, deposits. Seasonally adjusted real values (Q4 2013 = 100%).

(b) Interest rate (quarterly), CPI (quarterly), NPL to Loans. Seasonally adjusted.

Figure 1

### 2.2 Business Cycle Moments

Table 1 represents the key business cycle moments of Russian economy. It summarizes statistics on mean, standard deviation and cross-correlation of GDP growth, consumption growth, oil price growth, real loans growth, real deposits growth, ratio of NPL to Loans, annual CPI and annual interest rate. The results indicate that there is a high correlation between consumption and GDP, which corresponds to the correlation of these variables in advanced economies.

However, standard deviation of consumption growth is more volatile as compared to the standard deviation of GDP growth, which is a feature of emerging economies.
The important feature of the Russian business cycle is high correlation between GDP growth and oil price growth as well as between consumption growth and oil price growth. We also observe that there is high correlation between the growth of GDP and real loans as well as real deposits, while annual interest rate and GDP growth are negatively correlated. Another striking feature of the business cycle statistics of Russian economy is strong negative correlation between the growth of GDP and ratio of NPL to Loans. Among others we see that there is negative correlation between oil price and ratio of NPL to Loans, while oil price growth is positively correlated with the growth of real deposits.

Overall, we observe that the dynamics of the variables that represent financial cycle (loans, deposits, NPL to Loans) are strongly correlated with the dynamics of GDP, while the later is positively correlated with the oil price.

2.3 Unsecured Credit and Loans

The importance of unsecured credit in Russia is reflected in the importance of credit lines as a source of liquidity to firms and loans to early-stage firms who have limited collateral. Table 8 in the Appendix displays point estimates for different types of loans. According to this partial data\(^4\) only 17-18% of corporate loans have real estate as collateral. 56-75% of loans are uncollateralized or have financial collateral. The importance of “risky” borrowers in evaluating financial stability was central to the policy debate in the US following the crisis of 2007-08. Aikman et al. (2019) describe how the aggregate loan-to-value ratio on mortgages remained stable in the years leading up to the US crisis, but there was an increase in the concentration of debt among riskier borrowers. The build-up in debt concentrated at riskier, heavily indebted borrowers was not being adequately picked up (see Eichner and Palumbo (2013)). Unfortunately aggregate statistics on secured vs unsecured credit is not available but we can infer the role that it plays through proxies.

In Figure 2a we decompose credit growth across types of borrowers. We posit

\(^4\)We were able to obtain information on this for only 2 of the 12 largest Russian banks.
that ‘Big firms’ have the ability to pledge physical capital while other types of commercial borrowers cannot. In the crisis period following December 2014 there are sharp declines in all categories except the loans to large firms.

In Figure 2b we decompose loans by maturity with the proxy that shorter maturity loans are more likely to be unsecured. We can see both in the 2009 crisis period as well as the end of 2014 crisis period that shorter maturity loans fell the first and by the largest amount.

In Figure 2c we look at the ratio of non-performing loans across borrower

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5Mortgages being the only exception though the collateral posted there is newly purchased rather than already existing.
types. The large reduction in loans to small and medium firms, and individual entrepreneurs is coincident with a sharp rise in non-performing loan rates. This tells us that these loan types are more similar than others in both the sensitivity to the business cycle and delinquency rates.

We use these stylized facts to motivate the construction of our model where we emphasize the role of unsecured firm credit.
2.4 Summary of Stylized Facts

The stylized facts that motivate our model and analysis can be summarized as:

1. Positive correlation of output and consumption .66.
2. Positive correlation of output with oil price .52.
3. Excess volatility of consumption over output 2.1/1.47.
4. Positive correlation between GDP growth and loans .61.
5. Negative correlation of GDP growth and interest rates -.53.
6. Negative correlation between Loan growth and NPLs -.69.

Our stylized facts 1 and 2 are similar to those documented for Argentina in Drechsel and Tenreyro (2018). While their focus is on the external interest rate spread, ours is on the non-performing loans rates but here again our results are comparable. The commodity price and output growth are found to be negatively correlated with the spread there, and here it is with NPLs. As we use domestic bank loans, we find a strong correlation between their growth rates and NPLs.

3 A NK Small Open Economy Model with a Banking Sector

We now present our small open economy NK model, developed along the lines of Galí and Monacelli (2005) and Gertler et al. (2007) among others. While otherwise standard, our model has two distinguishing features: an explicit optimizing banking sector and the way in which we model corporate default. Our closest methodological precursors for modeling default and banks are Peiris and Tsomocos (2015), De Walque et al. (2010), Cui and Kaas (2020), Goodhart et al. (2018), and Walsh (2015). In Cui and Kaas (2020) debtors face a non-pecuniary cost if they default while in the last two papers the marginal cost of default depends on
debt to capital ratio and the level of wealth respectively, so the propensity to default depends on business cycle fluctuations. We follow this notion here by introducing a macro-variable that governs the marginal cost of renegotiating debt (default), termed ‘credit conditions’. This reflects changing motivations and incentives of debtors to make the necessary sacrifices to repay their obligations, as emphasized by Roch et al. (2016). We introduce optimizing banks subject to regulatory requirements along the lines of Tsomocos (2003) and Martinez and Tsomocos (2018).

The inclusion of the explicit banking sector allows us to model loan and regulatory requirement decisions formally which then allows us to better match the financial data we use.

3.1 Circular Flow of Funds

Firms require funding to invest in physical capital in order to produce non-tradable goods. They use capital and labor to produce intermediate non-tradable goods. Unsecured loans are repaid next period, but are defaultable. Secured borrowing is subject to a collateral constraint. Capital producers use imported intermediate goods as an input in production of capital together with undepreciated capital and domestic final goods. Oil reserves belong to the Government and it gets all the oil revenue. Banks combine households’ deposits with their equity and lend to Wholesale producers. Loan origination requires banks to satisfy capital adequacy requirements imposed by the Monetary Authority.

Households who are infinitely lived own capital producers, non-tradable goods producers, banks and other firms except for oil producers. They save through deposits at banks and domestic and foreign bonds. Monetary authority sets the nominal interest rate on domestic bonds. Fiscal Authority spends its revenues on non-tradable and imported goods.

The circular flow of funds is summarized in figure 3.

We use a first order Taylor approximation to estimate and simulate the model. We use the Bayesian estimation procedure in the Dynare package with 1,000,000 replications for the Metropolis-Hastings algorithm with two MCMC chains and a
3.2 Households

There is a continuum of households who are infinitely lived. Each of them consumes both domestically produced \( (c_{N,t}) \) and imported goods \( (c_{T,t}) \) and gets utility from consuming their consumption bundle \( (c_t) \). The domestic price of imported goods is \( p^{\text{imp}}_t \). Households get disutility from labor \( (l^h_t) \) and receive wage \( (w_t) \) that they choose. Households own all the firms (wholesale and intermediate producers, retailers and capital producers) and banks in the economy except for the oil producer (owned by the government) and receive profits from them. Households capitalize banks and wholesales producers with equity \( (e_t^{\text{bank}} \text{ and } e_t^{w,\text{total}}) \). Equity to the wholesale producers is composed of the net equity \( (e_t^{w}) \) and undepreciated capital that households receive from the firm that shuts down in the current period.

\[ \text{We computed the mode by using the } \text{mode}_{\text{compute}} = 4 \text{ option. Estimation was deemed successful as the two chains attained multivariate convergence in the first three moments.} \]
\((1 - \tau)p^K_t k^w_t\). The second component arises due to the OLG structure of firms that we use. Households can also make savings through the deposits \((d^h_{t+1})\), foreign bonds \((B^f_{t+1})\) and domestic government bonds \((B^{g,h}_{t+1})\).

The consumption bundle is:

\[
c^h_t = A^c[(\phi^h)\frac{1}{\sigma}c^{\nu}_{N,t} + (1 - \phi^h)\frac{1}{\sigma}c^{\nu}_{P,t}]\frac{1}{\nu},
\]  

(1)

where \(\nu_c\) is the elasticity of substitution between domestic and foreign goods. Budget Constraint of a Household:

\[
d^h_{t+1} + p^imp_t c_{T,t} + c_{N,t} + e^{w,total}_t + e^{bank}_t + Q_t B^f_{t+1} + B^{g,h}_{t+1} \\
\leq (1 + r^d_t)d^h_t + Q_t B^f_t (1 + r^f_t) + B^{g,h}_t (1 + r^b_t) + w^l_t + (1 - \theta^w)\Pi^w_t + \theta^w\Pi^w_t + \Pi^{bank}_t + \Pi^{cap}_t + \Pi^{ret}_t + \Pi^{exp}_t - adj^h_t
\]

(2)

where \(Q_t\) is an exchange rate, \(e^{w,total}_t = (e^w_t + (1 - \tau)p^K_t k^w_t)\), adjustment costs of household, \(adj^h_t = 0.5a^{h,b,e}_t (e^{bank}_t - e^{ss}_t)^2 + 0.5a^{h,w,e}_t (e^{w,total}_t - e^{ss}_t)^2 + 0.5a^{h,b,f}_t (Q^f_t B^f_{t+1} - Q^{ss}_t B^{f ss}_{t+1})^2 + 0.5a^{h,b,g}_t (B^{g,h}_{t+1} - B^{g,h ss}_{t+1})^2\).

Households maximize their discounted utility s.t. their BC:

\[
\max_{c_{T,t},c_{N,t},e^{w,total}_t,e^{bank}_t,d^h_{t+1},w_t,B^f_t,B^{g,h}_{t+1}} \sum_{t=1}^{\infty} (\beta^h_{t+1})^{1-\sigma} \left[ \frac{(c^h_t)^{1-\sigma}}{1-\sigma} - \theta^h (l^h_t)^{1+\gamma_h} \right] +
\]

(3)

Households supply their labor in a monopolistically competitive market where their optimally chosen wage may be revised in the future with probability \(1 - \theta^pw\).

This nominal wage rigidity construction results in labor supply accommodating demand in a similar manner that firm output responds to demand when there is stickiness in nominal prices. The demand for individual labor becomes a function of
the total demand for labor, aggregate wage and wage of the individual. In particular, it takes the form:

$$l^h_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{\epsilon_w} l^h_t$$  \hspace{1cm} (4)

Individual real wage can be expressed as:

$$w_t(j) = \frac{W_t(j)}{P_t}$$  \hspace{1cm} (5)

Aggregate real wage can be expressed as:

$$w_t = \frac{W_t}{P_t}$$  \hspace{1cm} (6)

Given that an individual can reset their nominal wage next period with probability $1 - \theta^{pw}$, real wage that individual gets at period $t + s$ if they are stuck with the wage they chose at time $t$ can be represented as:

$$w_{t+s}(j) = \frac{W_t(j)}{P_{t+s}} = \frac{W_t(j)}{P_t} \frac{P_t}{P_{t+s}} = w_t(j)\Pi_{t,t+s}^{-1},$$  \hspace{1cm} (7)

where $\Pi_{t,t+s} = \prod_{m=1}^{s} \Pi_{t+m} = \frac{P_{t+1}}{P_t} \frac{P_{t+2}}{P_{t+1}} \cdots \frac{P_{t+s}}{P_{t+s-1}}$.

By denoting the optimal choice of $w_t(j)$ at time $t$ by $w^*_t$ we get the following expression:

$$w^*,1+\epsilon_w^{h} = \frac{\epsilon_w}{\epsilon_w - 1} H_1,$$  \hspace{1cm} (8)

where $\epsilon_w$ - elasticity of labor substitution.

$$H_{1,t} = \theta^h w_t^{\epsilon_w(1+\gamma^h)} l_t^{h,1+\gamma^h} + \beta^h \theta^{pw} \Pi_{t+1}^{\epsilon_w(1+\gamma^h)} H_{1,t+1},$$  \hspace{1cm} (9)

where $\theta^{pw}$ - probability of saver household not to be able to adjust their wage rate next period.

$$H_{2,t} = \lambda^h w_t^{\epsilon_w} l^h + \beta^h \theta^{pw} \Pi_{t+1}^{w-1} H_{2,t+1}$$  \hspace{1cm} (10)
And labor wage rate dynamics follows equation (similar to the dynamics of inflation in case of price stickiness):

$$w_t^{1-\epsilon_w} = (1 - \theta p_{w \cdot u}^t)w_{t-1}^{1-\epsilon_w} + \theta p_{w \cdot u}^t \Pi_t w_{t-1}^{1-\epsilon_w}$$ (11)

### 3.3 Firms

#### 3.3.1 Wholesale producer

Wholesale producers in the economy have an Overlapping Generations (OLG) structure. All newly-born firms are identical. In its first period each firm receives equity from households (HH) and issues secured ($\mu_{w \cdot s}^t + 1$) and unsecured ($\mu_{w \cdot u}^t + 1$) debt to the banking system to finance the purchase of capital ($k_{t+1}^w$) at price $p_K^t$.

In the next period each firm realizes its level of productivity ($A_t$), which can be either high ($\bar{A}_t$) or low ($A_t$). Given its level of productivity each firm decides how much labor ($l_t^w$) it wants to hire. We assume that a fraction of firms ($1 - \theta_{w}$) are “lucky” and experience high level of productivity while the fraction ($\theta_{w}$) are “unlucky” and experience low level of productivity. So, firms are identical ex-ante but different ex-post. When firms borrow secured, they are subject to the collateral constraint under which the amount due to repayment can’t be higher than the expected value of undepreciated capital in the next period. This expected value of the undeprecated capital is accounted with the collateral discount ($coll$). Each “unlucky” firm can default on a fraction of their unsecured debt with the default rate ($\delta_t^w$), which we call the ‘loss given default’.

The total production is given by a constant returns to scale production function:

$$y_t^j = A_t^j (k_t^j)^{\alpha} (l_t^j)^{1-\alpha}.$$ (12)

The objective function that firms solve is:

$$k_{t+1}^{w,s}, \mu_{t+1}^{w,s}, \mu_{t+1}^{w,u}, \delta_{t+1}, \delta_{t+1}^w \max \mathbb{E}_{t^1} \beta^h \lambda^h \left[ \Pi_{t+1}^{w} \right]$$ (13)
subject to 14, 15, and 16. \( \lambda_{t+1}^{h} \) is the marginal utility of households (the owner).

The first period budget constraint of a firm is:

\[
p_t^K k_{t+1}^w + T^w + \text{adj}_t^w = \mu_{t+1}^w + e_t^{w,\text{total}},
\]

where \( \mu_{t+1}^w = \mu_{t+1}^{w,u} + \mu_{t+1}^{w,s}, \text{adj}_t^w - \text{adjustment costs of firm}, \text{adj}_t^w = 0.5a_{w,u}^w (\mu_{t+1}^{w,u} - \mu_{ss}^{w,u})^2 + 0.5a_{w,s}^w (\mu_{t+1}^{w,s} - \mu_{ss}^{w,s})^2 + 0.5a_{w,k}^w k_{t+1}^w - k_{ss}^w)^2 \). We assume that firms can only issue non-state-contingent nominal bonds to banks, or, equivalently, nominally riskless loans are obtained from banks. Firms that suffer a negative idiosyncratic productivity shock may choose to renege on some of their debt obligations, but then suffer a renegotiation cost proportional to the scale of loss given default.\(^7\) As firms vanish after their second period of life, their ability to liquidate assets and pay dividends to shareholders is predicated on successfully negotiating their existing debt burden. In this sense, the decision on how much of their debt to default on is strategic.

The collateral constraint of a firm takes the form:

\[
\mathbb{E}_t(1 + r_{t+1}^{w,s}) \mu_{t+1}^{w,s} \leq \text{coll}(1 - \tau) k_{t+1}^w \mathbb{E}_t p_t^K
\]

Profits are defined as:

\[
\Pi_{t+1}^w = p_{t+1}^w A_{t+1}^w (k_{t+1}^w)^{1-\alpha} (l_{t+1}^w)^{1-\alpha} - (1 - \delta_{t+1}^w) \mu_{t+1}^{w,u} (1 + r_{t+1}^{w,u}) - \mu_{t+1}^{w,b} (1 + r_{t+1}^{w,b}) - \frac{\Omega_{t+1}^w}{1 + \psi} (\delta_{t+1}^w \mu_{t+1}^{w,u} (1 + r_{t+1}^{w,u}))^{1+\psi} + p_{t+1}^K k_{t+1}^w (1 - \tau)
\]

So, depending on the level of technology firm’s profit can either be high (\( \Pi_t \)) or low (\( \Pi_l \)).

\(^7\)Allowing for default in the high idiosyncratic productivity state would allow us to normalize payoffs and costs resulting in similar results.
$\Omega^w_t$ is a macro variable that represents the aggregate credit conditions. $\Omega^w_t = \Omega^w_{ss} \left( \frac{\mu^w_{u,t} (1 + r^w_{u,t})}{GDP_{ss}} \right) \omega (\delta^w_{ss})^\gamma \left( \frac{GDP_t}{\mu^w_{u,t} (1 + r^w_{u,t})} \right) \omega \left( \frac{1}{(\delta^w_t)} \right)^\psi$. This pecuniary cost for renegotiating debt effectively creates a borrowing constraint and stems from Shubik and Wilson (1977) and Dubey et al. (2005) applied in Tsomocos (2003), Goodhart et al. (2005), Goodhart et al. (2006) and Goodhart et al. (2018).

$\Omega^w_t$ evolves according to:

$$\Omega^w_t = \Omega^w_{ss} \left( \frac{\mu^w_{u,t} (1 + r^w_{u,t})}{GDP_{ss}} \right) \omega (\delta^w_{ss})^\gamma \left( \frac{GDP_t}{\mu^w_{u,t} (1 + r^w_{u,t})} \right) \omega \left( \frac{1}{(\delta^w_t)} \right)^\psi. \tag{17}$$

$\Omega^w_t$ varies with the aggregate debt, but individual firms do not internalize how their borrowing decisions affect the aggregate credit conditions. We follow Goodhart et al. (2018) by introducing this macrovariable that governs the marginal cost of renegotiating debt (default), termed ‘credit conditions’. This reflects changing motivations and incentives of debtors to make the necessary sacrifices to repay their obligations, as emphasized by Roch et al. (2016). The debtor firm takes the credit conditions variable as given since creditors are capable of imposing institutional arrangements that are non-negotiable.

The pecuniary cost of default methodology and credit conditions variable allows us to calibrate the model to realized average loss given default rates (fraction of firms who default times loss given default, or, equivalently, total non-performing loans rates on bank lending). The estimation of $\omega, \gamma,$ and $\psi$ allows us to capture the endogenous relationship between default rates and the rest of the economy over the business cycle. The way we model default is analogous to a reduced form version of the equilibrium default threshold in Bernanke et al. (1999) and a richer version of the credit spread variable in Cúrdia and Woodford (2016). We estimate the relevant parameters and have a nested case that allows us to falsify our approach (full details

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8See appendix for the discussion of this variable.

9The optimality condition for the default rate, substituting in the value of the credit conditions variable is

$$\Omega^w_t \left( \frac{\mu^w_{u,t} (1 + r^w_{u,t})}{GDP_{ss}} \right) \omega (\delta^w_{ss})^\gamma \left( \frac{GDP_t}{\mu^w_{u,t} (1 + r^w_{u,t})} \right) \omega \left( \frac{1}{(\delta^w_t)} \right)^\psi = \mu^w_{u,t} (1 + r^w_{u,t}), \tag{18}$$
are in Appendix IV). In our set up, lenders recover an endogenously chosen fraction of the debt due. This is in contrast to Cui and Kaas (2020) where the recovery rate follows an exogenous process though their “credit conditions” variable which is endogenously determined from a surplus based on the value function of the debtor.

3.3.2 Intermediate goods producers

Intermediate Goods Producers are monopolistically competitive and produce a differentiated intermediate good using wholesale goods:

\[ Y^\text{ret}_t(k) = Y^w_t(k) \]  \hspace{1cm} (19)

Hence, they solve:

\[ \min_{Y^\text{ret}_t(k)} \frac{P^w_t}{P_t} Y^\text{ret}_t(k) + \lambda^\text{ret}_t(Y^\text{ret}_t(k) - Y^w_t(k)). \]  \hspace{1cm} (20)

The first order condition gives:

\[ \lambda^\text{ret}_t = \frac{P^w_t}{P_t} = p^w_t. \]  \hspace{1cm} (21)

Intermediate goods producer sets the price \( p_t(k) \) by solving:

\[
\max_{p_t(k)} \lambda^h_t \left[ \frac{P_t(k)}{P_t} c_t(k) - \lambda^\text{ret}_t c_t(k) \right] + \mathbb{E}_t \sum_{i=1}^{\infty} \left( \beta_t \theta_{t+i} \right) \lambda^h_{t+i} \left[ \frac{P_t(k)}{P_{t+i}} c_{t+i}(k) - \lambda^\text{ret}_{t+i} c_{t+i}(k) \right]
\]

\[ \text{s.t. } Y^\text{ret}_t(k) = \left( \frac{P_t(k)}{P_t} \right)^{-\theta_t} Y^\text{ret}_t \].

(22)

The solution to this problem is given by

from which one can see that the default rate depends on the stock of unsecured debt due and GDP (which includes the stock of capital and level of TFP). In Bernanke et al. (1999), the default rates depend on the stock of debt, the production function (via the expected return on capital) and the expected consumption of the owners. For a thorough derivation see Dmitriev and Hoddenbagh (2017). In Cúrdia and Woodford (2016) the credit spread depends on the stock of debt only.
\[ \lambda_t^b \left[ (1 - \theta_c) \frac{P_t^r}{P_t^*} + \lambda_t^{ret} \theta_c \right] \left( \frac{P_t^r}{P_t^*} \right)^{-\theta_c} \left( \frac{1}{P_t^*} \right) Y_t^{ret} + \\
+ E_t \sum_{i=1}^{\infty} (\beta_{t+i-1}^{h} \theta_{ps})^i \lambda_{t+i}^h \left[ (1 - \theta_c) \frac{P_{t+i}^r}{P_{t+i}^*} + \lambda_{t+i}^{ret} \theta_c \right] \left( \frac{P_{t+i}^r}{P_{t+i}^*} \right)^{-\theta_c} \left( \frac{1}{P_{t+i}^*} \right) Y_{t+i}^{ret} = 0 \]

(23)

It can be shown that

\[(1 + \pi_t)^{1-\theta_c} = (1 - \theta_{ps})(1 + \pi_t^*)^{1-\theta_c} + \theta_{ps} \quad \text{(24)}\]

where \( \pi_t \) is the inflation rate and

\[ Y_t^{ret} = Y_t^w/v_t^p \quad \text{(25)}\]

Price persistence \( v_t^p \) is defined as:

\[ v_t^p = (1 - \theta_{ps}) \left( \frac{1}{1 + \pi_t^*} \right)^{\theta_c} + \theta_{ps} (1 + \pi_t)^{\theta_c} v_{t-1}^p \quad \text{(26)}\]

### 3.3.3 Domestically-Priced Final Goods Producers (Retailers)

Domestically-Priced Final Goods Producers create a composite final good using as inputs goods purchased from intermediate goods producers that is then demanded by Households, the Government and Capital Producers, and is given by:

\[ Y_t^{ret} = \left( \int_0^1 Y_t^{ret}(k) \theta_c (-1)/\theta_c dk \right) \frac{d\theta_c}{(\theta_c-1)} \quad \text{(27)}\]

It can be shown that the demand for the individual good \( k \) is given by:

\[ Y_t^{ret}(k) = \left( \frac{P_t(k)}{P_t} \right)^{-\theta_c} Y_t^{ret} \quad \text{(28)}\]

Where \( Y_t^{ret} \) is the bundle of domestically-priced final goods consumed by each of the agents.
3.3.4 Capital producers

Capital producers purchase imported goods \( i_{T,t+1} \) at price \( p_t^{imp} \) and domestic goods \( i_{N,t+1} \) to produce aggregate investment goods \( i_{t+1} \) in accordance with the technology, represented by a CES aggregator:

\[
i_{t+1} = A^i[(\phi^i)^{\frac{\nu_i - 1}{\nu_i}} i_{N,t+1}^{\frac{1}{\nu_i}} + (1 - \phi^i)^{\frac{1}{\nu_i}} i_{T,t+1}^{\frac{\nu_i - 1}{\nu_i}}],
\]

(29)

The capital production technology includes an adjustment cost to investment. The production function takes the form:

\[
K_{t+1} = (1 - \tau)K_t + \varepsilon_t^{inv} i_{t+1} \left(1 - \frac{X}{2} \left(\frac{i_{t+1}}{i_t} - 1\right)^2\right),
\]

(30)

Capital producers sell new capital to wholesale producers. The profit is:

\[
\Pi_t^{cap} = p_t^{K} i_t^{inv} i_{t+1} \left(1 - \frac{X}{2} \left(\frac{i_{t+1}}{i_t} - 1\right)^2\right) - i_{N,t+1} - i_{T,t+1} p_t^{imp}
\]

(31)

Capital producers solve:

\[
\max_{i_{N,t+1},i_{T,t+1}} E_0 \sum_{t=1}^{\infty} (\beta_t^{-1})^t \lambda_t^h \Pi_t^{cap}
\]

(32)

In contrast to Pancrazi et al. (2016) we do not distinguish between the price of newly-produced capital and the price of previously-installed capital. In our setup capital producers have investment adjustment costs that do not depend on the stock of capital. So, the previous stock of capital has no effect on the cost of production of new capital. Moreover, there is not separate market for undepreciated capital as capital producing firms choose the level of investment and not the amount of capital they buy. The amplification effect of financial frictions on aggregate dynamics we find is through the interaction of expected default rates and the capital Euler equation, rather than through variations in the price of capital as in their paper.
3.4 Banking Sector

New-born banks are capitalized with equity ($e^\text{bank}_t$). They accept deposits from households ($d^\text{bank}_{t+1}$), extend secured ($\mu^\text{bank}_t, s$) and unsecured ($\mu^\text{bank}_t, u$) loans to firms.

The first period budget constraint of a bank is given by

$$\mu^\text{bank}_{t+1} = d^\text{bank}_{t+1} + e^\text{bank}_t - adj_t^b,$$

(33)

where $\mu^\text{bank}_{t+1} = \mu^\text{bank, s}_{t+1} + \mu^\text{bank, u}_{t+1}$, $adj_t^b$ - adjustment costs for bank, $adj_t^b = 0.5a^b_s(\mu^\text{bank, s}_{t+1} - \mu^\text{bank, s}_{ss})^2 + 0.5a^b_u(\mu^\text{bank, u}_{t+1} - \mu^\text{bank, u}_{ss})^2 + 0.5a^d_d(d^\text{bank}_{t+1} - d^\text{ss}_{ss})^2$.

The capital adequacy ratio is defined as the ratio of bank capital to risk weighted assets net of reserves ($rwa^\text{bank}_t$):

$$k^\text{bank}_t = \frac{e^\text{bank}_t}{rwa^\text{bank}_t} = \frac{e^\text{bank}_t}{(r(w^\text{bank, u} + 1)(1 - \delta^u_{t+1}) + (1 - \theta^w)\mu^\text{bank, u}_{t+1} + (1 + r^w^s_{t+1})\mu^\text{bank, s}_{t+1} - [(1 + r^d_{t+1})d^\text{bank}_{t+1}]}.$$

(34)

Banks then choose how much of secured and unsecured debt to lend out to firms:

$$\Pi^\text{bank}_{t+1} = \theta^w(1 + r^w^u_{t+1})(1 - \delta^w_{t+1})\mu^\text{bank, u}_{t+1} + (1 - \theta^w)(1 + r^w^u_{t+1})\mu^\text{bank, u}_{t+1} + (1 + r^w^s_{t+1})\mu^\text{bank, s}_{t+1} - [(1 + r^d_{t+1})d^\text{bank}_{t+1}],$$

(35)

where $r^w^u$ and $r^w^s$ are unsecured and secured lending rates. We also assume that only "unlucky" firms default on their unsecured borrowing.

Given $\{\delta^w_{t+1}, r^w^u_{t+1}, r^w^s_{t+1}, r^d_{t+1}\}$, banks maximize:

$$\max_{\mu^\text{bank, u}_{t+1}, \mu^\text{bank, s}_{t+1}, d^\text{bank}_{t+1}} \mathbb{E}_{t} [\beta^t (\Pi^\text{bank}_{t+1})^{1-\gamma^\text{bank}} - 0.5[k^\text{bank}_t - k^\text{bank}_{ss}]^2]$$

(36)

Bank profits are concave along the lines of De Walque et al. (2010) and Goodhart et al. (2005), and reflect the limited liability assumption we make. The penalty term
for deviations from the steady state level of the capital adequacy ratio reflects the desire of banks to maintain a target level which may be higher than the regulatory minimum. Although it is costly for banks to go below its target level, thus signaling a weakening balance sheet, going above the target is not desirable as it reflects assets not being utilized correctly. Ultimately the penalty reflects both the imposition of capital requirements and an agency conflict between bank managers and its owners.

3.5 Government

3.5.1 Fiscal authority

Government gets all revenue \( p_t^{O,dom} O_t \) from oil export \( O_t \). Government spends its funds on the domestically produced final goods \( G_t \) and imported goods \( G_t^{imp} \), can save or borrow through the domestic government bonds \( B_t^g \) and receives net taxes from agents in the economy.

The Government Budget Constraint:

\[
G_t + p_t^{imp} G_t^{imp} + B_t^g \frac{(1 + i_t)}{1 + \pi_t} = B_t^{g+1} + p_t^{O,dom} O_t + T^w
\]  

(37)

Our modeling of fiscal authority has a number of limitations. Firstly, we fix the government’s purchases of imported goods at the steady state level. In the steady state the value of government’s purchases of imported goods is determined as four percent of total government spending, which is in line with the Russian data. Secondly, the taxes collected by the government are kept at the constant level and do not vary over the business cycle. Thirdly, government borrowing is fixed at the steady state level and doesn’t vary over the business cycle. As government doesn’t form any reserves and doesn’t change its borrowing, all the changes in the government revenue, which are in our case essentially changes in oil revenue, transmit into the changes of domestic government spending.
3.5.2 Monetary authority

The Central Bank controls the interest rate $i_t^b$ according to the following rule:

$$\frac{1 + i_t^b}{1 + i_{ss}^b} = \left(1 + i_{t-1}^b\right)^{\rho_i} \left(1 + \pi_{t}^{cpi}\right)^{1+\rho_\pi} \left(\frac{GDP_t}{GDP_{ss}}\right)^{\rho_{gdp}} \varepsilon_t^i,$$  (38)

where $\varepsilon_t^i$ is a monetary policy shock that follows AR(1) process.

The CPI inflation is defined as:

$$1 + \pi_t^{CPI} = (1 + \pi_t) \frac{r_t^{cpi}}{r_t^{CPI}},$$  (39)

where $r_t^{CPI}$ is measured as:

$$r_t^{CPI} = p_t^{imp} T_t^{weight} + (1 - T_t^{weight}),$$  (40)

with $T_t^{weight}$ being defined as:

$$T_t^{weight} = \frac{C_{T,t}}{C_{T,t} + C_{N,t}}$$  (41)

Along with the represented above form of the Taylor rule we considered some other specifications. In particular, we considered the Taylor rule that doesn’t have a GDP component in it. If the estimated value of the parameter $\rho_{gdp}$ is close to zero, then it essentially means that the monetary authority doesn’t respond to the movement in GDP when setting the policy rate.

The other form of the Taylor rule that could be the one, which accounts only for the inflation of domestically produced goods instead of CPI inflation. However, given that the model is estimated on the Russian data, the use of the CPI inflation becomes more relevant as the Bank of Russia targets CPI inflation when conducting its policy.

For the macroprudential policy analysis the Taylor rule could be augmented by the component representing the ratio of current unsecured lending to its steady state level. This would result in a higher policy rate when there is an excessive unsecured lending in the economy.
The applied Taylor rule is the adjusted multiplicative form of the linear Taylor rule proposed in Taylor (1993). It is similar to the one used in Brzoza-Brzezina et al. (2013) and generally in line with the other estimated DSGE models including Adolfson et al. (2013) and Christiano et al. (2015).

### 3.6 Equilibrium

Given the exogenous shocks, equilibrium is a sequence of prices and quantities such that households, banks, and firms maximize, and all markets clear.

In particular, market clearing condition for labor requires:

\[ l_t^h = l_t^w \]  \hspace{1cm} (42)

Market clearing for secured loans:

\[ \mu_{t+1}^{bank,s} = \mu_{t+1}^{w,s} \]  \hspace{1cm} (43)

Market clearing for unsecured loans:

\[ \mu_{t+1}^{bank,u} = \mu_{t+1}^{w,u} \]  \hspace{1cm} (44)

Market clearing for deposits:

\[ d_{t+1}^h = d_{t+1}^{bank} \]  \hspace{1cm} (45)

Market clearing for domestic bonds:

\[ B_{t+1}^g = B_{t+1}^{g,h} \]  \hspace{1cm} (46)

Market clearing for domestic output:

\[ Y_t^{ret} = c_{N,t} + i_{N,t+1} + G_t + \theta^w \frac{\Omega^w}{1 + \psi} \left( \delta^w \mu_t^w (1 + r_t^{w,u}) \right)^{1 + \psi} + adj_t^h + adj_t^w + adj_t^b \]  \hspace{1cm} (47)
Household’s time-preference variable $\beta_t^h$ is defined as:

$$\beta_t^h = \beta^h \varepsilon_t^{\beta,h}. \quad (48)$$

Domestic price of an imported good is:

$$p_t^{imp} = Q_t p_t^{imp,*}, \quad (49)$$

where $p_t^{imp,*}$ is an international price of an imported good and we assume it to be constant and $Q_t$ is a real exchange rate.

Domestic price of commodity good (oil) is:

$$p_t^{o,dom} = Q_t p_t^{o,*}, \quad (50)$$

where $p_t^{o,*}$ is an international price of commodity good and it is defined as:

$$p_t^{o,*} = p_t^{o,*} \varepsilon_t^{p,o}. \quad (51)$$

So, the international price of oil is a product of some constant oil price $p_t^{o,*}$ and its shock process $\varepsilon_t^{p,o}$, which follows AR(1) process.

Interest rates on foreign bonds are also subject to a shock, which we call the “foreign interest rate shock”:

$$r_t^f = r_t^f + \varepsilon_t^{r,for}, \quad (52)$$

where $r_t^f$ is some constant interest rate on foreign bonds and $\varepsilon_t^{r,for}$ is a shock process for interest rate on foreign bonds that follows AR(1) process.

We assume that the technology levels of "lucky" and "unlucky" firms are correspondingly $\bar{A}_t^j$ and $A_t^j$.

$$A_t^j = A_t \bar{A}_t^j, \quad (53)$$

where $\bar{A}_t^j$ is some constant and
\[ A_j^i = A_t A_j^i, \]  
(54)

where \( A_j^i \) is some constant with \( \bar{A}^i > 1 > A_j^i \).

The real interest rate on domestic government bonds is defined as:

\[ 1 + r_t = \frac{1 + i_t}{1 + \pi_t}. \]  
(55)

We define real GDP as value of final goods and oil produced:

\[ GDP_t = Y_t^{ret} + p_{os, dom} O_t \]  
(56)

Aggregate real consumption in the model is defined as:

\[ cons_t = p_{imp} c_{T,t} + c_{N,t} \]  
(57)

In the data the procedure of calculating GDP and its components in constant prices includes two key approaches: the reevaluation of GDP and its components in the previous periods prices using the indexes of volume and through the direct division of current nominal values by the change in the price index. So, given that model variables are in real prices, consumption and GDP could be measured either in constant real prices or in changing real prices. In our model we measure real GDP in constant real prices, while we measure consumption in changing real prices.

### 3.7 Wedges and Financial Frictions

Below we consider two specifications of the model related to the two sources of financial inefficiency in the model: the collateral constraint, and the dead-weight cost of loss-given-default. In one specification, the “wedges” or inefficiencies arising from these frictions are time varying and generated from financial frictions. We call this the “endogenous financial frictions” case. The second case we call the “ex-
ogenous financial frictions” case. In this case the “wedges” are constant over the business cycle.

All derivations are in the appendix in Section 9.

The wedge from the collateral constraint was found not to be important, so we focus here on the wedge from the dead-weight cost of default. In the endogenous financial frictions case firms optimally choose the fraction \( \delta^w_t \) of the debt they want to default upon. In the exogenous financial frictions case firms do not optimize for the default rate. In this case default rate and the cost of default are fixed at constant level and don’t vary over the business cycle:

\[
\delta^w_t = \delta^{w,ss}, \quad \Omega^w_t (\delta^w_t \mu^w,u_t (1 + r^w,u_t))^2 = \Omega^{ss,w} (\delta^{w,ss} \mu^{w,ss} (1 + r^{w,ss,u}))^2,
\]

where \( \delta^{w,ss}, \Omega^{w,ss}, \mu^{w,ss}, r^{w,ss,u} \) are the steady state values of the corresponding variables.

In the endogenous financial frictions case, the optimality condition of the firm w.r.t. the default rate at time \( t \) is:

\[
\delta^w_t = \frac{1}{\Omega^w_t \mu^w,u_t (1 + r^w,u_t)} \tag{58}
\]

and results in the first order condition for debt of:

\[
\lambda^h_t (1 + r^w,u_{t+1}) \delta^w_t = \lambda^w_t (1 - a^{w,u} (\mu^w,u_t - \mu^{w,ss,u})) \tag{59}
\]

where \((1 + r^w,u_{t+1}) \delta^w_t\) is the wedge arising from the cost of defaulting. In the exogenous financial frictions case, we maintain this wedge at the steady-state level. This effectively means that although the loss-given default is constant over the business cycle, the premium or wedge associated with default is still priced in and varying. The first order condition in the exogenous case is:

\[
\lambda^h_t (1 + r^w,u_{t+1}) (1 - \delta^w_t) + (1 + r^w,u_{t+1}) \delta^w_t = \lambda^w_t (1 - a^{w,u} (\mu^w,u_{t+1} - \mu^{w,ss,u})) \tag{60}
\]

This allows us to disentangle the effect of variations in the rate of default (and hence the importance of incomplete markets), from the role of the wedge, and hence borrowing constraint.
When we linearize the optimality conditions for unsecured borrowing in the two cases, as is shown in Appendix II, the wedge between endogenous financial frictions case and exogenous financial frictions case becomes

\[
\frac{(\delta_{t+1}^{w})(r_{t+1}^{w,u}-r_{t+1}^{w,u,ss})}{1 + r^{w,u,ss}}.
\] (61)

This corresponds to the “investment wedge” in the terminology of Chari et al. (2007). The last equation shows that moving over the business cycle loss given default rates create a wedge for unsecured borrowing. When Equation 58 is substituted into 61 and recalling the definition of \(\Omega_{t}^{w}\), we can see that the wedge is a function of the debt-to-GDP ratio. It is by linking these variables to the investment wedge that we obtain a better model fit and allow the oil price shock to directly affect investment and hence GDP.

For collateral, the first order condition for secured borrowing in endogenous financial frictions case:

\[
\lambda_{t+1}^{s,u} \beta_{t}^{s,u} (1 + r_{t+1}^{w,s}) = \lambda_{t}^{w}(1 - a_{w,s}(\mu_{t+1}^{w,s} - \mu_{w,s}^{ss})) - \psi_{t}^{u}(1 + r_{t+1}^{w,s}),
\] (62)

while for the exogenous financial frictions case we assume the collateral constraint itself only binds at the steady state. The first order condition is:

\[
\lambda_{t+1}^{h} \beta_{t}^{h} (1 + r_{t+1}^{w,s}) = \lambda_{t}^{w}(1 - a_{w,s}(\mu_{t+1}^{w,s} - \mu_{w,s}^{ss})) - \psi_{t}^{w}(1 + r_{t+1}^{w,s}).
\] (63)

Here again we hold the wedge constant over the business cycle, but as it is additive, under a local approximation the wedge drops out so we effectively lose the constraint altogether.
4 Measurement

4.1 Observables

We estimate our model using Bayesian Estimation techniques for the two cases: endogenous financial frictions and exogenous financial frictions based on eight data series: GDP growth rates, household consumption growth rates, percentage change of CPI inflation, percentage change of interest rate, total loans growth rates, household domestic currency deposits growth rates, percentage change of ratio of non-performing loans to total loans and growth rates of international oil price. As the interest rate we use the data on Moscow interbank average credit rate (MIACR).

Our sample covers the period of Q2 2001 - Q2 2018. As the data sources we used the data from Federal State Statistics Service of the Russian Federation and Bank of Russia. In particular, the data for quarterly consumption and output were taken from Federal State Statistics Service of the Russian Federation. Other data series were taken from the internal database of Bank of Russia (some of them are available in the open source). The key descriptive statistics of the data used are represented in the table 1.

We transform the data in the following way:

\[ GDP_{t}^{obs} = \log(GDP_{t}) - \log(GDP_{t-1}) - E[\log(GDP_{t}) - \log(GDP_{t-1})] \]  

\[ cons_{t}^{obs} = \log(cons_{t}) - \log(cons_{t-1}) - E[\log(cons_{t}) - \log(cons_{t-1})] \]  


11data on deposits, loans and non-performing loans to loans for some periods could be found at https://www.cbr.ru/analytics/bnksyst/.

Monthly data on MIACR are available at https://www.cbr.ru/nd_base/mkr/mkr_monthes/.

Monthly data on CPI are available at http://www.gks.ru/bgd/free/b00_24/IssWWW.exe/Stg/d000/000717-10.HTM.
\[ p_{oil, t}^{\text{obs}} = \log(p_{oil, t}^{\text{oil, s}}) - \log(p_{oil, t-1}^{\text{oil, s}}) - E[\log(p_{oil, t}^{\text{oil, s}}) - \log(p_{oil, t-1}^{\text{oil, s}})] \] (66)

\[ \text{Loans}_{t}^{\text{obs}} = \log(\text{Loans}_{t}) - \log(\text{Loans}_{t-1}) - E[\log(\text{Loans}_{t}) - \log(\text{Loans}_{t-1})] \] (67)

\[ \text{Dep}_{t}^{\text{obs}} = \log(\text{Dep}_{t}) - \log(\text{Dep}_{t-1}) - E[\log(\text{Dep}_{t}) - \log(\text{Dep}_{t-1})] \] (68)

\[ \pi_{t}^{\text{cpi, obs}} = \pi_{t}^{\text{cpi}} - \pi_{t-1}^{\text{cpi}} - E[\pi_{t}^{\text{cpi}} - \pi_{t-1}^{\text{cpi}}] \] (69)

\[ i_{t}^{b, \text{obs}} = i_{t}^{b} - i_{t-1}^{b} - E[i_{t}^{b} - i_{t-1}^{b}] \] (70)

\[ \frac{\text{NPL}_{t}^{\text{obs}}}{\text{Loans}_{t}} = \frac{\text{NPL}_{t}}{\text{Loans}_{t}} - \frac{\text{NPL}_{t-1}}{\text{Loans}_{t-1}} - E[\frac{\text{NPL}_{t}}{\text{Loans}_{t}} - \frac{\text{NPL}_{t-1}}{\text{Loans}_{t-1}}] \] (71)

The transformations applied help us to remove both the trend and the mean from data series. This step is essential as the model variables are stationary.

### 4.2 Shocks

The model contains fourteen exogenous variables, six of them are structural shocks that follow AR(1) process and eight are measurement errors, one for every observable. The structural shocks included in the model are: international oil price shock, monetary policy shock, total factor productivity shock, shock to foreign bond interest rate premia and saver time-preference shock.

The international oil price shock \( \epsilon_{p,o, t}^{\text{obs}} \) follows AR(1) process:
\[
\log(\varepsilon_{t}^{P,o}) = \rho^{P,o} \log(\varepsilon_{t-1}^{P,o}) + \varepsilon_{t}^{P,o}, \quad (72)
\]

where \(\varepsilon_{t}^{P,o}\) is a size of the oil price shock in period \(t\) and \(\rho^{P,o}\) is a persistence of oil price shock.

Monetary policy shock process is defined as:

\[
\log(\varepsilon_{t}^{i}) = \rho^{mon} \log(\varepsilon_{t-1}^{i}) + \varepsilon_{t}^{mon}, \quad (73)
\]

where \(\varepsilon_{t}^{mon}\) is a size of the monetary policy shock in period \(t\) and \(\rho^{mon}\) is a persistence of the monetary policy shock.

Premia to the foreign bond interest rate is defined as:

\[
\varepsilon_{t}^{r,for} = \rho^{r,for} \varepsilon_{t-1}^{r,for} + \varepsilon_{t}^{r,for}, \quad (74)
\]

where \(\varepsilon_{t}^{r,for}\) is a size of the foreign bond interest rate premia in period \(t\) and \(\rho^{r,for}\) is a persistence of the shock to the foreign bond interest rate premia.

The technology level \(A_{t}\) is also a shock process, defined as:

\[
\log(A_{t}) = \rho^{a} \log(A_{t-1}) + \varepsilon_{t}^{a}, \quad (75)
\]

where \(\varepsilon_{t}^{a}\) is a size of the TFP shock in period \(t\) and \(\rho^{a}\) is a persistence of the TFP shock.

Household’s time-preference shock is defined as:

\[
\log(\varepsilon_{t}^{\beta,h}) = \rho^{\beta,h} \log(\varepsilon_{t-1}^{\beta,h}) + \varepsilon_{t}^{\beta,h}, \quad (76)
\]

where \(\varepsilon_{t}^{\beta,h}\) is a size of the time-preference shock in period \(t\) and \(\rho^{\beta,h}\) is a persistence of time-preference shock.

Investment shock process is defined as:

\[
\log(\varepsilon_{t}^{inv}) = \rho^{inv} \log(\varepsilon_{t-1}^{inv}) + \varepsilon_{t}^{inv}, \quad (77)
\]
where $\epsilon_{t}^{inv}$ is a size of the investment shock in period $t$ and $\rho^{inv}$ is a persistence of the investment shock.

The rest of the shocks are the measurement errors that correspond to each of the observables:

$\epsilon_{p,o}^{me}, \epsilon_{GDP}^{me}, \epsilon_{cons}^{me}, \epsilon_{\pi}^{me}, \epsilon_{ib}^{me}, \epsilon_{l}^{me}, \epsilon_{NPL}^{me}, \epsilon_{dep}^{me}.$

The measurement errors are mean-zero with a variance set to 10% of the variance of the corresponding data series. By doing this we follow the approach used in Adolfson et al. (2013). As a result, each of the observables could be explained by no more than six shocks: five structural shocks and the corresponding measurement error.

### 4.3 Measurement equations

We specify the measurement equations for our observables in the following form:

$$GDP_{t}^{obs} = (\log(GDP_{t}^{model}) - \log(GDP_{t-1}^{model})) + \epsilon_{GDP,t}^{me} \quad (78)$$

$$cons_{t}^{obs} = (\log(const_{t}^{model}) - \log(const_{t-1}^{model})) + \epsilon_{cons,t}^{me} \quad (79)$$

$$p_{t}^{p,o,*}^{obs} = (\log(p_{t}^{p,o,*}^{model}) - \log(p_{t-1}^{p,o,*}^{model})) + \epsilon_{p,o,t}^{me} \quad (80)$$

$$Loans_{t}^{obs} = (\log(\mu_{bank,t+1}) - \log(\mu_{bank,t})) + \epsilon_{l,t}^{me} \quad (81)$$

$$Dep_{t}^{obs} = (\log(d_{bank,t+1}) - \log(d_{bank,t})) + \epsilon_{dep,t}^{me} \quad (82)$$

$$\pi_{t}^{cpi,obs} = \pi_{t}^{cpi,model} - \pi_{t-1}^{cpi,model} + \epsilon_{\pi,t}^{me} \quad (83)$$

$$\hat{b}_{t}^{obs} = \hat{b}_{t}^{model} - \hat{b}_{t-1}^{model} + \epsilon_{\hat{b},t}^{me} \quad (84)$$
\[
\frac{NPL_t^{obs}}{Loans_t} = \frac{NPL_t^{model}}{Loans_t^{model}} - \frac{NPL_{t-1}^{model}}{Loans_{t-1}^{model}} + \varepsilon_{NPL,t}, \tag{85}
\]

where \(\text{var}_t^{model}\) is a corresponding variable from the model and \(\varepsilon_{\text{var},t}\) is a corresponding measurement error.

5 Parameterization

We set some of the parameter values a priori. These values are given in the table 2. Household’s time-preference parameter \(\beta\) is set to yield in the steady state an annual risk-free rate of about 9.4% which corresponds to the average Russian government bond yield for the period we consider. Loss given default value \(\delta_w\) is also set in accordance with the Russian data. Capital requirement for banks \(k_{bank}\) corresponds to the Russian capital requirement for big banks. The depreciation rate \(\tau\) is set to yield an annual depreciation rate of 10%. The fraction of firm’s that default \(\theta_w\) is calibrated to the Russian banks’ statistics on firms’ default. Other parameters are calibrated to yield the steady state ratio of aggregate consumption to GDP of about 54% as well as the steady state size of the oil sector in the economy of about 39%. Given that oil revenue is the main source of government’s income in our setup, the steady state level of government spending to GDP is 39%.

The parameter values that we use for our calibration are close to those used or estimated in other models of the Russian economy. For instance, the depreciation rate corresponds to the rate used in Malakhovskaya and Minabutdinov (2014). As follows from Malakhovskaya and Minabutdinov (2014), estimated value of household risk aversion for Russian economy is 1.015. In Polbin (2014) the estimated mean value of household risk aversion is close to its prior value of 1.19.
<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^h )</td>
<td>0.977</td>
</tr>
<tr>
<td>( \theta^h )</td>
<td>1</td>
</tr>
<tr>
<td>( \gamma^h )</td>
<td>1</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>1.5</td>
</tr>
<tr>
<td>( \phi^h )</td>
<td>0.35</td>
</tr>
<tr>
<td>( \nu^c )</td>
<td>0.97</td>
</tr>
<tr>
<td>( \phi^i )</td>
<td>0.5</td>
</tr>
<tr>
<td>( \nu^i )</td>
<td>0.97</td>
</tr>
<tr>
<td>( \beta_{bank} )</td>
<td>0.977</td>
</tr>
<tr>
<td>( \xi_{bank} )</td>
<td>1</td>
</tr>
<tr>
<td>( \delta^w )</td>
<td>0.5</td>
</tr>
<tr>
<td>( k_{bank} )</td>
<td>0.115</td>
</tr>
<tr>
<td>( \bar{r}^w )</td>
<td>1</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.025</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.33</td>
</tr>
<tr>
<td>( \text{coll} )</td>
<td>0.7</td>
</tr>
<tr>
<td>( \theta_w )</td>
<td>0.05</td>
</tr>
<tr>
<td>( \theta_c )</td>
<td>3</td>
</tr>
<tr>
<td>( \epsilon_w )</td>
<td>3</td>
</tr>
</tbody>
</table>

**Calibrated ratios**

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C/GDP )</td>
<td>0.54</td>
</tr>
<tr>
<td>( B^f/GDP )</td>
<td>0.24</td>
</tr>
</tbody>
</table>

| Table 2: Calibrated Parameters and Ratios |
6 Estimation

Table 3 shows the results of the Bayesian Estimation of the model for the two cases: endogenous financial frictions and exogenous financial frictions. The main difference in the estimation lies in the higher investment shock standard deviation and adjustment costs, in particular, banks’ and firms’ adjustment costs to secured lending and capital producers’ adjustment costs to investment. These three adjustment costs are much higher in the exogenous financial frictions case which means that they add additional frictions proportional only to the quantity of debt into the model to match the data.
### Table 3:Estimated parameters for endogenous and exogenous financial frictions cases

The central result of our estimation is presented in Table 4 where we conduct Bayesian model comparison between the endogenous and exogenous financial frictions cases.

From this table we can see that the marginal likelihood$^{12}$ for the model with endogenous financial frictions is higher (1092 vs 759). This is the likelihood of the

$^{12}$ Laplace approximation of the log data density.
<table>
<thead>
<tr>
<th>Marginal (log) likelihood</th>
<th>Endogenous case</th>
<th>Exogenous case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1092</td>
<td>759</td>
</tr>
</tbody>
</table>

Table 4: Marginal (log) Likelihood for Endogenous and Exogenous financial frictions cases

data given the model. With equal priors on the model, the Bayes factor is $e^{235}$ which gives almost 100% probability that the model with endogenous financial frictions is superior.

7 Quantitative results

7.1 Theoretical moments

The simulated standard deviation and correlation of the variables used in estimation are presented in Table 5. When we compare it with the empirical counterpart in Table 1, we can summarize our stylized facts below where the number in brackets indicates the simulated value

1. Strong positive correlation of output and consumption .66 (.61).
2. Strong positive correlation of output with oil price .52 (.36).
3. Excess volatility of consumption over output 2.1/1.47 (3.26/2.02).
4. Positive correlation between GDP growth and loans .61 (.12).
5. Negative correlation of GDP growth and interest rates -.53 (-.05).
6. Negative correlation between Loan growth and NPLs -.69 (-.02).

7.2 Historical decomposition

Figures 4a to 5h show historical decomposition of the observed data series by shocks for the endogenous and exogenous financial frictions models. Overall the
<table>
<thead>
<tr>
<th>Std</th>
<th>2.02</th>
<th>3.26</th>
<th>12.31</th>
<th>3.82</th>
<th>5.36</th>
<th>1.51</th>
<th>0.90</th>
<th>2.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP, q/q growth, %</td>
<td>1</td>
<td>0.61</td>
<td>0.36</td>
<td>0.12</td>
<td>0.05</td>
<td>0.08</td>
<td>-0.03</td>
<td>-0.05</td>
</tr>
<tr>
<td>Consumption, q/q growth, %</td>
<td>0.61</td>
<td>1</td>
<td>-0.29</td>
<td>0.26</td>
<td>-0.44</td>
<td>0.05</td>
<td>-0.02</td>
<td>-0.04</td>
</tr>
<tr>
<td>Oil price, q/q growth, %</td>
<td>0.36</td>
<td>-0.29</td>
<td>1</td>
<td>-0.16</td>
<td>0.84</td>
<td>0.19</td>
<td>-0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Real loans, q/q growth, %</td>
<td>0.12</td>
<td>0.26</td>
<td>-0.16</td>
<td>1</td>
<td>0.19</td>
<td>-0.02</td>
<td>-0.16</td>
<td>-0.42</td>
</tr>
<tr>
<td>Real deposits, q/q growth, %</td>
<td>0.05</td>
<td>-0.44</td>
<td>0.84</td>
<td>0.19</td>
<td>1</td>
<td>0.16</td>
<td>-0.06</td>
<td>-0.06</td>
</tr>
<tr>
<td>NPL to loans, quarterly, %</td>
<td>0.08</td>
<td>0.02</td>
<td>0.19</td>
<td>-0.02</td>
<td>0.16</td>
<td>1</td>
<td>0.22</td>
<td>0.78</td>
</tr>
<tr>
<td>CPI, quarterly, %</td>
<td>-0.03</td>
<td>-0.04</td>
<td>-0.04</td>
<td>-0.16</td>
<td>-0.06</td>
<td>0.22</td>
<td>1</td>
<td>0.33</td>
</tr>
<tr>
<td>Interest rate, quarterly, %</td>
<td>-0.05</td>
<td>-0.10</td>
<td>0.04</td>
<td>-0.42</td>
<td>-0.06</td>
<td>0.78</td>
<td>0.33</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5: Business cycle moments Q2 2001- Q2 2018

endogenous financial frictions model is able to capture more of the dynamics of the data by the oil shock series, which is especially the case for loans, deposits and non-performing loans to total loans.

Figures 5e and 5f show that deposits are well matched by the oil price shock in the case of endogenous financial frictions, while in the exogenous financial frictions case the dynamics is matched through the relatively large contributions of different shocks. The superiority of the endogenous financial frictions wedges model is best seen in Figures 5g and 5h where the endogenous frictions model can explain most of the fluctuations in non-performing loans by oil price shocks while the exogenous frictions model requires measurement errors.

Other studies on Russia provide a more moderate presence of the oil price shock
in economic dynamics. For example, Polbin (2014) builds a New Keynesian model with a number of frictions and shows that the oil price shock has the main role in explanation of Great recession in Russia. Kreptsev and Seleznev (2017) build a large-scale DSGE model with the banking sector and the financial frictions along the lines of Bernanke et al. (1999) and show that GDP is explained well by the oil price shocks during Great recession, while during crisis episode of 2015 GDP was affected by oil price shocks to a lesser extent. In our paper, the oil price shock explains a large component of both the crisis episodes of 2008-2009 and 2015.
Figure 4: Historical decomposition (1)
ME: Measurement Error
(a) CB interest rate for endogenous financial frictions case

(b) CB interest rate for exogenous financial frictions case

(c) Total loans for endogenous financial frictions case

(d) Total loans for exogenous financial frictions case

(e) Deposits for endogenous financial frictions case

(f) Deposits for exogenous financial frictions case

(g) Non-performing loans as a share of total loans for endogenous financial frictions case

(h) Non-performing loans as a share of total loans for exogenous financial frictions case

Figure 5: Historical decomposition (2)
ME: Measurement Error
7.3 Error variance decomposition

Table 6 shows the percentage of the variation of each variable explained by a particular shock. Obs refers to the transformation of the variable used for the estimation as shown in Section 4.3. The Obs rows allow us to see how large the measurement errors are, and as most are around 10% or less, we can see that we have fit the data relatively well. What is of interest to understand business cycle dynamics is the Mod rows. These refer to the variable in levels for NPL Loans, πcpi and i while for the others it is the growth rates. We can see that in the endogenous financial frictions case 32.8% of the variation in GDP is explained by the oil price shock (εp,o) compared to 63.7% for the TFP shock (εa), while in the exogenous financial frictions case the contribution of oil and TFP is 6.2% and 44%.

The contribution of investment shocks to explain all the variables declines and in some cases dramatically when we move from the exogenous to endogenous case. For GDP it falls from 31.3% to 0.2% while for Loans (Deposits) it falls from 36.1% (60.0%) to 20.1% (7.7%). Our results are consistent with Justiniano et al. (2010) and Justiniano et al. (2011) who show the importance of investment shocks for explaining business cycle fluctuations. The role that financial frictions can play is emphasized in Justiniano et al. (2011), and here we can also see that the role of the investment shock in explaining fluctuations in non-performing loans (i.e. the spread for lending to firms) falls from 75.6% to 2.2%. The shock to the discount factor (εβ,h), criticized by Chari et al. (2007) and Chari et al. (2009) as not being truly structural, falls in its contribution to the variance of variables when moving from the exogenous financial frictions to the endogenous financial frictions case. In particular, for Deposits, the contribution of the discount rate shock goes from 29.5% in the exogenous financial frictions case to 1.9% in the endogenous case. Impor-
tantly, non-performing loans are explained mostly by oil price shocks (75%) which indicates that policies targeting financial stability should focus on the response of the policies to all variables under oil price shocks. Coupled with the better fit for the non-performing loans rate, the larger contribution of the observed shock series gives a clearer role for policy actions to depend on these shocks.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Endogenous Financial Frictions</th>
<th>Exogenous Financial Frictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\epsilon^{p,o}$</td>
<td>$\epsilon^a$</td>
</tr>
<tr>
<td>GDP</td>
<td>obs 31.6</td>
<td>61.2</td>
</tr>
<tr>
<td></td>
<td>mod 32.8</td>
<td>63.7</td>
</tr>
<tr>
<td>cons</td>
<td>obs 14.1</td>
<td>69.2</td>
</tr>
<tr>
<td></td>
<td>mod 14.5</td>
<td>71.4</td>
</tr>
<tr>
<td>Loans</td>
<td>obs 15.3</td>
<td>19.1</td>
</tr>
<tr>
<td></td>
<td>mod 16.5</td>
<td>20.5</td>
</tr>
<tr>
<td>NPL</td>
<td>obs 60.2</td>
<td>9.84</td>
</tr>
<tr>
<td></td>
<td>mod 75.0</td>
<td>11.6</td>
</tr>
<tr>
<td>$\pi^{cpi}$</td>
<td>obs 0.74</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>mod 3.10</td>
<td>5.37</td>
</tr>
<tr>
<td>$i_b$</td>
<td>obs 6.67</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td>mod 54.1</td>
<td>4.16</td>
</tr>
<tr>
<td>$p^{\pi,x}$</td>
<td>obs 86.7</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>mod 100</td>
<td>0</td>
</tr>
<tr>
<td>Dep</td>
<td>obs 69.1</td>
<td>9.50</td>
</tr>
<tr>
<td></td>
<td>mod 73.4</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Table 6: Error variance decomposition: endogenous and exogenous financial frictions cases

### 7.4 Impulse Response Functions

Figure 7 represent IRFs for a positive oil price shock, while Figure 6 represents IRFs for a positive TFP shock. We present only the model with endogenous finan-
cial frictions and only a few of the variables. Figure 8 compares the mean Bayesian IRFs to a positive oil price shock for the two cases, endogenous financial frictions and exogenous financial frictions.

In Figure 6, following a positive TFP shock firms increase their demand for the factors of production, resulting in an increase in real wages, capital and the price of the capital and production. As the price of capital increases, the collateral constraint is relaxed and the quantity of secured debt issued increases. When the price of capital falls back to its steady state value, firms switch their issuance of debt towards unsecured loans. Higher wages allow households to increase consumption, particularly towards the relatively cheaper domestic goods, as well as increasing equity investment in the banking system which is used to finance the additional loans to the production sector. The higher profitability of the production sector results in an improvement in credit conditions and a sharp decline in the rate of non-performing loans. Government consumption rises due to the depreciation of the exchange rate, increasing the domestic value of foreign oil revenues. The response of inflation reflects the lower real price of domestic output which dominates the depreciation of the currency, resulting in inflation declining and a decline in the nominal interest rate.

In Figure 7, a positive shock to the foreign oil price causes a sharp appreciation in the exchange rate. This is because the increase in foreign income stimulates demand for domestic goods while the exchange rate adjusts to reflect the substitution effect for imported goods and foreign savings, causing a corresponding large increase in imports. The stronger exchange rate causes a reduction in the cost of imported goods for capital goods, and hence a fall in the price of capital. This causes an increase in the production of domestic non-tradable goods. In contrast to a TFP shock where the price of capital increases but is offset by higher productivity, here the decline in the price of capital temporarily stimulates production but is not enough to create efficiency gains and higher total income. The decline in the price of capital reduces the ability to issue secured debt, and consequently, the higher demand for investment is financed through issuing unsecured debt. House-
holds switch from domestic savings in equity to foreign bonds which is used to finance imported consumption and resulting in lower labor supply in subsequent periods. This causes a decline in the production of domestic non-tradables in the medium term and is evidence of a Dutch-disease type effect in Russia: an increase in the tradable sector causes the non-tradable sector to contract via the price of inputs, here labor. The decline in the interest rate on unsecured debt causes credit conditions to improve and non-performing loans rate to decline.

This is a central mechanism in our model where expected default rates affect current interest rates on loans and hence the quantity borrowed and invested. In contrast, Figure 8 shows that the Dutch-disease type effect is very short-lived and muted in the model where financial frictions are held exogenous. Our evidence for a Dutch-disease type effect is consistent with Malakhovskaya and Minabutdinov (2014), but contrasts Kreptsev and Seleznev (2017) and Kozlovteva et al. (2019). This effect is pronounced in our model because of the strong substitution between domestic and foreign consumption goods driven by the high elasticity of the real exchange rate with respect to the dollar price of oil. One reason is that our foreign interest rate doesn’t depend explicitly on the dollar oil price as in the case of Kreptsev and Seleznev (2017) and Kozlovteva et al. (2019). This means that as our foreign interest rate does not decrease when oil price increases, households have a greater incentive to accumulate foreign assets and sustain their consumption of imports in the future. Another reason for our stronger Dutch-disease type effect is that oil revenue is given directly to government who spends it, and as a result aggregate demand directly depends strongly on the domestic price of oil which falls due to a strongly appreciating exchange rate. In practice government spending will not adjust as much, however in our model government spending substitutes for a hand-to-mouse consumer whose consumption depends directly on domestic currency oil revenues.

\footnote{In the original Dutch-disease, growth in the tradable sector causes an increase in demand for labor and hence higher wage, which causes the non-tradable to become unprofitable and contract. We find that the non-tradable sector contracts because the income effect due to the more profitable tradable sector causes a reduction in labor supply and higher wages.}
Figure 6: IRFs to a positive TFP shock for endogenous financial frictions case

Figure 7: IRFs to a positive oil price shock for endogenous financial frictions case
8 Optimal Simple Rules

In this section we consider a set of commonly proposed macroprudential policy rules and search for the combination of these policies that maximize welfare. We consider a Lean-Against-The-Wind type of Taylor rule, a Liquidity Coverage Ratio, a Countercyclical Capital Buffer, and a loan-to-value ratio. We restrict all the parameters we optimize over to be positive.

The Lean against the wind (LAW) rule is a modified Taylor rule represented by equation:

\[
\frac{1 + \beta}{1 + \beta_{ss}} = \left(1 + \beta_{t-1}\right) \rho_i \left(1 + \pi_{cpi}^{crs}\right)^{1+\rho_{\pi}} \left(\frac{GDP_{t}}{GDP_{ss}}\right)^{\rho_{gd}} \left(\frac{\mu_{bank,u}^t + 1 + \mu_{bank,u}^{ss}}{\mu_{bank,s}^t + \mu_{bank,s}^{ss}}\right)^{\zeta_{t}},
\]

(86)

In this type of Taylor rule policy rate depends not only on the previous period policy rate, current CPI inflation and GDP, but also positively reacts to the growth of unsecured debt in the economy. The parameters that are optimized are \(\rho_i\), \(\rho_\pi\), \(\rho_{gd}\), and \(\zeta\).
The Liquidity Coverage Ratio (LCR) in our model requires all banks keep the share \( r_{es_t} \) of deposits to the central bank each period and receive the same nominal amount next period. The dynamics of \( r_{es_t} \) is represented by equation:

\[
r_{es_t} = \left( \frac{\mu_{bank,u}^{t+1} + \mu_{bank,s}^{t+1}}{\nu_{ss}} \right)^\nu - 1. \tag{87}
\]

The LCR, as implemented in Basel 3, is considered as a tool for regulating liquidity, but also affects the banks’ internal and external return of funding. In Basel 3 the denominator is the cash outflows over 30 days. Here we take it as deposits as they are the main outflow in the second period of the life of the bank. \( \nu \) is optimized.

The CounterCyclical capital Buffer (CCyB) regards capital adequacy ratio \( \bar{k}_{bank} \) as a dynamic variable and regulates it based on the equation:

\[
\bar{k}_{bank}^{t} = \bar{k}_{bank}^{ss} \left( \frac{\mu_{bank,u}^{t+1} + \mu_{bank,s}^{t+1}}{\nu_{ss}} \right)^\eta. \tag{88}
\]

Higher aggregate loans lead to a higher capital requirement. This rule affects the internal profitability of lending by increasing the requirement for equity financing and ultimately affects the supply of loans. We optimize \( \eta \).

The Loan-to-value (LTV) macroprudential policy rule suggests collateral discount \( coll \) (equation (89)) to be dynamic and monetary authority regulates it in accordance with the law:

\[
coll_{t} = coll_{ss} \left( \frac{\mu_{bank,u}^{t+1} + \mu_{bank,s}^{t+1}}{\nu_{ss}} \right)^{-\chi}. \tag{89}
\]

When aggregate loans exceed their steady state value, the amount of capital that is collateralized decreases. As a result, firms are forced to finance a larger proportion of their expenditures on capital through equity. We optimize \( \chi \).

\[15\text{Reserve requirements exist in Russia, and the rule we consider can be equivalently considered as a countercyclical reserve requirement.}\]
We approximate the value function of the household (Equation 3) using a 2nd order Taylor expansion and numerically find the parameters that maximize the theoretical mean of the unconditional welfare. We search over the space of 7 parameters using a simulated annealing algorithm. The robustness of the results were checked with various starting values, all of which converged to a result in the neighborhood of those reported. The starting values used are given below together with the optimized ones. We compare the results of the endogenous financial frictions wedges case with the exogenous one in Table 7.

<table>
<thead>
<tr>
<th>Welfare</th>
<th>Parameters</th>
<th>LAW</th>
<th></th>
<th>LCR</th>
<th>CCyB</th>
<th>LTV</th>
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<tr>
<td></td>
<td>ρ_λ</td>
<td>ρ_π</td>
<td>ρ_{gdp}</td>
<td>ζ</td>
<td>ν</td>
<td>η</td>
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<tr>
<td>Starting Values</td>
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<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Endogenous Baseline</td>
<td>-67.440</td>
<td>0.433</td>
<td>3.018</td>
<td>0.116</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Endogenous Optimal</td>
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<td>0.021</td>
<td>1.001</td>
<td>0.056</td>
<td>10.379</td>
<td>6.537</td>
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<tr>
<td>Exogenous Baseline</td>
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<td>2.948</td>
<td>0.167</td>
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<tr>
<td>Exogenous Optimal</td>
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<td>8.624</td>
<td>5.159</td>
<td>2.269</td>
<td>0.110</td>
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Table 7: Optimized parameters for Policy

The results in Table 7 show that the optimal simple rule in the endogenous case has a greater emphasis on macroprudential policy. The coefficients for the credit-to-GDP term in the Taylor rule, the LCR and the CCyB are very large compared to the exogenous financial frictions wedge case. In contrast, inflation and GDP are much more important in the exogenous case. Given the cyclicality of the wedges from financial frictions in the endogenous case, these results reflect the dependence of inflation and GDP growth on credit conditions, which in turn, can be addressed by countercyclical macroprudential policies.

16We follow the approach of papers such as Lambertini et al. (2013), Quint and Rabanal (2014), and Schulze and Tsomocos (2019) among others.

17For Russian data, Kozlovceva et al. (2019) extend the model of Kreptsev and Seleznev (2017) to find that leaning against the wind monetary policy serves the role of output stabilization.
a combination of the CCyB and the LCR, the former raises the amount of equity required to extend loans while the latter prevents the balance sheet from expanding by requiring a greater proportion of assets to be held as reserves at the Central Bank.\(^{18}\)

Our results in the endogenous case are consistent with much of the literature that advocates coordinated macroprudential and monetary policy (for example Angelini et al. (2014), Rubio and Carrasco-Gallego (2014)), the importance of LAW monetary policy (for example, Gourio et al. (2018)), and the importance of capital buffers and provisioning (Mendicino et al. (2018), Aguilar et al. (2019), Jiménez et al. (2017)). Our contribution is to describe how and to what extent countercyclical policy depends on the inclusion of the wedges from financial frictions, in a similar vein but richer framework than considered in Farhi and Werning (2016)).

9 Concluding Remarks

Since the Global Financial Crisis policy makers in emerging economies focused on novel, macroprudential tools to maintain both price and financial stability. These tools mitigate the domestic effects of external shocks. Since the effectiveness of policy tools depends on the shock, discerning which shocks drive business cycle dynamics becomes as important to understand as which financial frictions amplify them. Through the lens of an estimated financial frictions augmented, small open economy New Keynesian model, we show that the contribution of commodity price shocks to output fluctuations depends qualitatively and quantitatively on the inclusion of frictions in the intermediation of domestic loanable funds. When frictions in domestic credit markets are included in the endogenous structure of the model, the estimated contribution of the commodity price shock increases while that of in-

\(^{18}\)The loan-to-value ratio policy does not seem to be important. The proximate reason is that we only consider equilibria around a binding collateral constraint meaning fluctuations in collateral (capital) prices cannot have a large enough amplifying effect. However, as firms can also issue unsecured debt, our results indicate that it is the possibility of default on debt which should be targeted by policy, rather than the wedge arising from collateral per-se.
vestment shocks declines. This supports the suggestion of Justiniano et al. (2010) that the contribution of investment shocks may be a proxy for absent financial frictions in a model. We showed that the business cycle dynamics of the wedges that arise from these frictions allow us to capture the time series properties of financial variables better through the model thereby resulting in a better model fit and identification of shocks.

Our results complement the rich literature on shocks to the credit spread on foreign debt affecting domestic interest rates. We show that disruptions in the domestic banking system following a non-foreign interest rate shock can result in similar effects as a foreign interest rate shock. For the specific Russian case we estimate, commodity price shocks are amplified by these financial frictions. Macroprudential policy rules, in particular CCyB and LCR, were found to compliment each other while including credit-to-GDP in the Taylor rule substituted away from targeting inflation and GDP growth intensely. This reflects the dependence of inflation and real economic activity on financial intermediation and the necessity to target the inefficiencies that arise from it.
10 References


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Jiménez, Gabriel, Steven Ongena, José-Luis Peydró and Jesús Saurina (2017), ‘Macroprudential policy, countercyclical bank capital buffers, and credit supply: Evidence from the spanish dynamic provisioning experiments’, *Journal of Political Economy* 125(6), 2126–2177.


Roch, Francisco, FRoch@imf.org, Harald Uhlig and HUhlig@imf.org (2016), ‘The Dynamics of Sovereign Debt Crises and Bailouts’, *IMF Working Papers* **16**(136), 1.


Appendix I: Corporate Loans in Russia

<table>
<thead>
<tr>
<th>Type of Loan</th>
<th>Raiffeisen (2017)</th>
<th>Moscow Credit Bank (2016)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsecured loans</td>
<td>50.3 %</td>
<td>-</td>
</tr>
<tr>
<td>Guarantees</td>
<td>24.5%</td>
<td>-</td>
</tr>
<tr>
<td>Total uncollaterized</td>
<td>74.8 %</td>
<td>56.2%</td>
</tr>
<tr>
<td>Real estate</td>
<td>18.1%</td>
<td>16.9%</td>
</tr>
<tr>
<td>Other</td>
<td>7.1%</td>
<td>26.9%</td>
</tr>
<tr>
<td>Total collaterized</td>
<td>25.2%</td>
<td>43.8%</td>
</tr>
</tbody>
</table>

Table 8: Corporate loans in Russia: secured and unsecured

Appendix II: Optimality conditions

Household optimality conditions

F.O.C. for consumption of domestic goods $c_{N,t}$:

$$c_{N,t} = \phi^h (\lambda_t^h)^{-\nu_c} c_t^{1-\nu_c} (A^c)^{\nu_c-1} \tag{90}$$

F.O.C. for consumption of imported goods $c_{T,t}$:

$$c_{T,t} = (1 - \phi^h)(p_{t}^{imp} \lambda_t^h)^{-\nu_c} c_t^{1-\nu_c} (A^c)^{\nu_c-1} \tag{91}$$

F.O.C. for deposits $d_{t+1}^h$:

$$\lambda_t^h (1 + a^{h,d}(d_{t+1}^h - d_{ss}^h)) = \beta_t^h (1 + r_{t+1}) \lambda_t^h \tag{92}$$

F.O.C. for equity of a bank $e_{t}^{\text{bank}}$:

$$e_{t}^{\text{bank}} (1 + a^{h,b,e}(e_{t}^{\text{bank}} - e_{ss}^{\text{bank}})) = \frac{\lambda_{t+1}^{h}\beta_t^{h}}{\lambda_t^{h}} \Pi_{t+1}^{\text{bank}} \tag{93}$$
F.O.C. for holding of domestic bonds $B_t^{g,h}$:

$$
\lambda_t^h (1 - a^{h,b,g} (B_t^{g,h} - B_{ss}^{g,h})) = \beta_t^h \lambda_{t+1}^h (1 + r_{t+1}^b) \tag{94}
$$

F.O.C. for holding of foreign bonds $B_t^f$:

$$
\lambda_t^f (1 - a^{h,b,f} (B_t^f - B_{ss}^f)) = \beta_t^f \lambda_{t+1}^h (1 + r_{t+1}^f) \tag{95}
$$

F.O.C. for firm equity $e_{t,\text{total}}^w$:

$$
\lambda_t^h (1 + a^{h,w,e} (e_{t,\text{total}}^w - e_{ss,\text{total}}^w)) = \lambda_{t+1}^h \beta_t^h \frac{\Pi_{t+1}^w \theta^w + \Pi_{t+1}^w (1 - \theta^w)}{e_{t,\text{total}}^w} \tag{96}
$$

Wage setting problem derivation

In the wage rigidity setup demand for individual labor takes the form similar to the demand for individual firm output in the case of price stickiness. And so, demand for individual labor becomes a function of total labor demand, aggregate wage and individual wage. In particular, it takes the form:

$$
l_t^h (j) = \left( \frac{W_t(j)}{W_t} \right)^{1+\gamma^h} l_t^h \tag{97}
$$

Then the part of the household’s Lagrangian that is associated with the choice of labor can be represented as (note that for the time being nominal BC is used):

$$
\tilde{L} = -\theta^h \left( \frac{l_t^h(j)}{l_t^h} \right)^{1+\gamma^h} \frac{1}{1+\gamma^h} \tag{98}
$$
Given the demand for individual labor the previous expression can be written as:

$$\tilde{L} = -\theta h \left( \frac{(W_t(j))^{-\epsilon_w t_h}}{1 + \gamma h} \right)^{1 + \gamma h} + \lambda^h_0 (W_0(j)) (W_0(j))^{-\epsilon_w t_h} +$$

$$+ E_0 \sum_{t=1}^{\infty} \left( \beta (W_t(j))^{-\epsilon_w t_h} \right)^{1 + \gamma h} + \lambda^h (W_t(j)) (W_t(j))^{-\epsilon_w t_h})$$

(99)

Individual real wage can be expressed as:

$$w_t(j) = \frac{W_t(j)}{P_t}$$

(100)

Aggregate real wage can be expressed as:

$$w_t = \frac{W_t}{P_t}$$

(101)

Given that an individual can reset their nominal wage next period with probability $1 - \theta^{\mu w}$, real wage that individual gets at period $t + s$ if they are stuck with the wage they chose at time $t$ can be represented as:

$$w_{t+s}(j) = \frac{W_t(j)}{P_{t+s}} = \frac{W_t(j)}{P_{t}} \frac{P_{t}}{P_{t+s}} = w_t(j) \Pi_{t,t+s}^{-1}$$

(102)

where $\Pi_{t,t+s} = \prod_{m=1}^{s} \Pi_{t+m}$

Then, for the choice of real wage rate at time $t$ corresponding part of the Lagrangian will be:
\[
\tilde{L}_t = -\theta^h \left( \frac{w_t(j)\Pi_{t,t}^{-1}}{w_t} - \epsilon_w \rho_t^h \right) \frac{1 + \gamma^h}{1 + \gamma^h} \sum_{s} \frac{w_t(j)\Pi_{t,t}^{-1}}{w_t} + \lambda_t^h (w_t(j)\Pi_{t,t}^{-1})^{-\epsilon_w \rho_t^h}
\]
\[
+ \sum_{s=1}^{\infty} \left( \beta_{t+s-1}^h \theta_{w}^w \right)^{s} \left( -\theta^h \left( \frac{w_t(j)\Pi_{t,t}^{-1}}{w_{t+s}} - \epsilon_w \rho_t^h \right) \frac{1 + \gamma^h}{1 + \gamma^h} \sum_{s} \frac{w_t(j)\Pi_{t,t}^{-1}}{w_{t+s}} + \lambda_t^h (w_t(j)\Pi_{t,t+s}^{-1})^{-\epsilon_w \rho_t^h} \right)
\]

(103)

The FOC for \( w_t(j) \) becomes:

\[
\epsilon_w w_t(j)^{-\epsilon_w (1+\gamma^h)-1} \left( \theta^h (w_t^e)^{\epsilon_w (1+\gamma^h)} \Pi_{t,t}^{\epsilon_w (1+\gamma^h)} (L_t^h)^{1+\gamma^h} + \sum_{s=1}^{\infty} \left( \beta_{t+s-1}^h \theta_{w}^w \right)^{s} \left( \theta^h (w_{t+s}^e)^{\epsilon_w (1+\gamma^h)} \Pi_{t,s}^{\epsilon_w (1+\gamma^h)} (L_{t+s}^h)^{1+\gamma^h} \right) = (\epsilon_w - 1) w_t(j)^{-\epsilon_w} \left( \lambda_t (w_t^e)^{\epsilon_w \Pi_{t,t}^w - 1} (L_t^h) + \sum_{s=1}^{\infty} \left( \beta_{t+s-1}^h \theta_{w}^w \right)^{s} \lambda_{t+s}^h (w_{t+s}^e)^{\epsilon_w \Pi_{t,s}^{w-1} (L_{t+s}^h)} \right)
\]

(104)

By denoting the optimal choice of \( w_t(j) \) at time \( t \) by \( w_t^f \) we get the following expression:

\[
w_t^{1+\epsilon_w \gamma^h} = \frac{\theta^h (w_t)^{\epsilon_w (1+\gamma^h)} \Pi_{t,t}^{\epsilon_w (1+\gamma^h)} (L_t^h)^{1+\gamma^h}}{\epsilon_w - 1} \lambda_t^h (w_t)^{\epsilon_w \Pi_{t,t}^w - 1} (L_t^h) + \sum_{s=1}^{\infty} \left( \beta_{t+s-1}^h \theta_{w}^w \right)^{s} \lambda_{t+s}^h (w_{t+s}^e)^{\epsilon_w \Pi_{t,s}^{w-1} (L_{t+s}^h)}
\]

(105)

Then expression for \( w^d \) can be represented as:

66
\[ w_t^{\gamma_h} = \frac{\epsilon_w}{\epsilon_w - 1} H_1, \]  
(106)

where \( \epsilon_w \) - elasticity of labor substitution.

\[ H_{1,t} = \theta^h w_t \epsilon_w \left(1 + \gamma_h\right) H_{1,1} + \beta^h \theta^p w \Pi_{t+1}^{1 + \gamma_h} H_{1,t+1}, \]  
(107)

where \( \theta^p \) - probability of saver household not to be able to adjust their wage rate next period.

\[ H_{2,t} = \lambda^h w_t \epsilon_w \left(1 + \gamma_h\right) H_{2,1} + \beta^h \theta^w \Pi_{t+1}^{1 - \gamma_h} H_{2,t+1} \]  
(108)

And labor wage rate dynamics follows (similar to the dynamics of inflation in case of price stickiness):

\[ w_t^{1 - \epsilon_w} = (1 - \theta^p w) w_t^{1 - \epsilon_w} + \theta^p w \Pi_{t+1}^{1 - \gamma_w} w_t^{1 - \epsilon_w} \]  
(109)

**Wholesale producer’s optimality conditions**

F.O.C. for labour in high state

\[ w_t = \frac{(1 - \alpha)^{P^w y_t} w_{t, high}}{P^w y_t} \]  
(110)

F.O.C. for labor in low state

\[ w_t = \frac{(1 - \alpha)^{P^w y_t} w_{t, low}}{P^w y_t} \]  
(111)

F.O.C. for secured borrowing in endogenous financial frictions case:

\[ \lambda_t^{h} \beta_t^h (1 + r^w_{t+1}) = \lambda_t^w (1 - a^w (\mu^w_{t+1} - \mu^w_{ss})) - \psi_t^w (1 + r^w_{t+1}) \]  
(112)
F.O.C. for secured borrowing in exogenous financial frictions case:

$$\lambda_{t+1}^{h} \beta^{h}_{t} (1 + r_{t+1}^{w,s}) = \lambda_{t}^{w} (1 - a^{w,s} (\mu_{t+1} - \mu_{ss}^{w})) - \psi_{ss}^{w} (1 + r_{ss}^{w}) \quad (113)$$

F.O.C. for unsecured borrowing in endogenous financial frictions case:

$$\lambda_{t+1}^{h} \beta^{h}_{t} (1 + r_{t+1}^{w,u}) = \lambda_{t}^{w} (1 - a^{w,u} (\mu_{t+1} - \mu_{ss}^{w})) \quad (114)$$

F.O.C. for unsecured borrowing in exogenous financial frictions case:

$$\lambda_{t+1}^{h} \beta^{h}_{t} ((1 + r_{t+1}^{w,u})(1 - \delta_{ss}^{w}) + \delta_{ss}^{w} (1 + r_{ss}^{w,u})) = \lambda_{t}^{w} (1 - a^{w,u} (\mu_{t+1} - \mu_{ss}^{w})) \quad (115)$$

F.O.C. for capital in endogenous financial frictions case:

$$\lambda_{t+1}^{h} \beta^{h}_{t} (\alpha p_{t+1}^{w} A_{t+1}^{w} (k_{t+1}^{w})^{\alpha - 1} (l_{t+1}^{w})^{1 - \alpha} + (1 - \tau)p_{t+1}^{K}) = \lambda_{t}^{w} p_{t}^{K} (1 + a^{w,k} (k_{t+1}^{w} - k_{ss}^{w})) - \psi_{t}^{w} \text{coll}(1 - \tau)p_{t+1}^{K}, \quad (116)$$

F.O.C. for capital in exogenous financial frictions case:

$$\lambda_{t+1}^{h} \beta^{h}_{t} (\alpha p_{t+1}^{w} A_{t+1}^{w} (k_{t+1}^{w})^{\alpha - 1} (l_{t+1}^{w})^{1 - \alpha} + (1 - \tau)p_{t+1}^{K}) = \lambda_{t}^{w} p_{t}^{K} (1 + a^{w,k} (k_{t+1}^{w} - k_{ss}^{w})) - \psi_{ss}^{w} \text{coll}(1 - \tau)p_{s+1}^{K}, \quad (117)$$

F.O.C. for default rate:

$$\Omega_{t} \frac{\text{cost}_{t}^{def}}{\delta_{t}^{w}} = \mu_{t}^{w,u} (1 + r_{t}^{w,u}), \quad (118)$$

where $\text{cost}_{t}^{def} = \left( \delta_{t}^{w} \mu_{t}^{w,u} (1 + r_{t}^{w,u}) \right)^{1+\psi}$

Capital producer’s optimality conditions

With respect to domestic investment component:
\[
1 = p_t^K \left(1 - \frac{\zeta}{2} \left( \frac{i_{t+1}}{i_t} - 1 \right)^2 - \varpi(\frac{i_{t+1}}{i_t} - 1) \left( \frac{i_{t+1}}{i_t} \right) \left( A^i \right) \left(1 - \frac{1}{N} \right) \left( \frac{\zeta}{1} \right) \right) + \beta_t^K \left( \frac{\lambda^{1+h}}{\lambda^h} \right) \left( p_{t+1}^K \right) \\
+ \beta_t^K \left( \frac{\lambda^{1+h}}{\lambda^h} \right) \left( p_{t+1}^K \right) \left( \frac{\zeta}{2} \left( \frac{i_{t+1}}{i_t} - 1 \right)^2 - \varpi(\frac{i_{t+1}}{i_t} - 1) \left( \frac{i_{t+1}}{i_t} \right) \left( A^i \right) \left(1 - \frac{1}{N} \right) \left( \frac{\zeta}{1} \right) \right)
\]

with respect to imported investment component:

\[
p_t^{imp} = p_t^K \left(1 - \frac{\zeta}{2} \left( \frac{i_{t+1}}{i_t} - 1 \right)^2 - \varpi(\frac{i_{t+1}}{i_t} - 1) \left( \frac{i_{t+1}}{i_t} \right) \left( A^i \right) \left(1 - \frac{1}{N} \right) \left( \frac{\zeta}{1} \right) \right) + \beta_t^K \left( \frac{\lambda^{1+h}}{\lambda^h} \right) \left( p_{t+1}^K \right) \\
+ \beta_t^K \left( \frac{\lambda^{1+h}}{\lambda^h} \right) \left( p_{t+1}^K \right) \left( \frac{\zeta}{2} \left( \frac{i_{t+1}}{i_t} - 1 \right)^2 - \varpi(\frac{i_{t+1}}{i_t} - 1) \left( \frac{i_{t+1}}{i_t} \right) \left( A^i \right) \left(1 - \frac{1}{N} \right) \left( \frac{\zeta}{1} \right) \right)
\]

**Bank’s optimality conditions**

With respect to deposits:

\[
E_{\left( \frac{\beta_t}{\Pi^{bank}_{t+1}} \right)} \left( 1 + r_{t+1}^d \right) = \lambda^{bank}_t \left( 1 - a^{b,d} (d_{t+1}^{bank} - d_{ss}^{bank}) \right)
\]

with respect to secured loans to firms:

\[
E_{\left( \frac{\beta_t}{\Pi^{bank}_{t+1}} \right)} \left( 1 + r_{t+1}^{w,s} \right) \left( k_{t+1}^{bank} - k_{ss}^{bank} \right) \left( r_{w}^{bank} \right) = \lambda^{bank}_t \left( 1 + a^{b,s} (\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}) \right)
\]

with respect to unsecured loans to firms:
E \left[ \frac{\beta_h}{(\Pi_{t+1}^{\text{bank}})^{\varsigma_{\text{bank}}}} \right] [(1 + r_{t+1}^{w,s}) (1 - \theta_w^u \delta_{t+1}^w)] + (k_t^{\text{bank}} - k_t^{\text{bank}}) r_t^w e_t^{\text{bank}} / r_{wa_t}^{\text{bank}} = \\
= \lambda_t^{\text{bank}} (1 + a^{b,u} (\mu_{t+1}^{\text{bank},u} - \mu_{ss}^{\text{bank},u}))
(123)

Log-linearized equations

F.O.C. for secured borrowing in endogenous financial frictions case:

\lambda_{t+1}^h \beta_t^h (1 + r_{t+1}^{w,s}) = \lambda_t^w (1 - a (\mu_t^{w,s} - \mu_{ss}^{w,s})) - \psi_t^w (1 + r_{t+1}^{w,s})
(124)

⇒ log(\lambda_{t+1}^h) + log(\beta_t^h) + log((1 + r_{t+1}^{w,s})) = log(\lambda_t^w (1 - a (\mu_t^{w,s} - \mu_{ss}^{w,s}))) - \psi_t^w (1 + r_{t+1}^{w,s})
(125)

⇒ log(\lambda_{t+1}^{h,ss}) + log(\beta_t^{h,ss}) + log((1 + r_{t+1}^{w,s,ss})) + log((1 + r_{t+1}^{w,s,ss})) + \\
= log(\lambda_t^{w,ss} (1 - a (\mu_{ss}^{w,s} - \mu_{ss}^{w,s}))) - \psi_t^{w,ss} (1 + r_{t+1}^{w,s,ss}) + \\
+ \frac{1}{\lambda_t^{w,ss} - \psi_t^{w,ss} (1 + r_{t+1}^{w,s,ss})} (\lambda_t^w - \lambda_t^{w,ss}) - a \lambda_t^{w,ss} - \\
- \psi_t^{w,ss} (1 + r_{t+1}^{w,s,ss}) (\mu_t^{w,s} - \mu_{ss}^{w,s}) - \frac{1 + r_{t+1}^{w,s,ss}}{\lambda_t^{w,ss} - \psi_t^{w,ss} (1 + r_{t+1}^{w,s,ss})} (\psi_t^{w} - \psi_t^{w,ss}) - \\
- \frac{1 + r_{t+1}^{w,s,ss}}{\lambda_t^{w,ss} - \psi_t^{w,ss} (1 + r_{t+1}^{w,s,ss})} (r_{t+1}^{w,s} - r_{t+1}^{w,s,ss})
(126)
\[ \Rightarrow \frac{\lambda_{h,t+1} - \lambda_{h,ss}}{\lambda_{h,ss}} + \frac{\beta_{h} - \beta_{h}}{\beta_{h}} + \frac{r_{w,s}^{w,s} - r_{w,s,ss}^{w,s}}{1 + r_{w,s,ss}^{w,s}} = \]

\[ = \frac{1}{\lambda_{w,ss} - \psi_{w,ss}(1 + r_{w,s,ss}^{w,s})} \left( \lambda_{t}^{w} - \lambda_{w,ss}^{w,s} \right) - a \lambda_{w,ss}^{w,s} \]

\[ - \psi_{w,ss}^{w,s}(1 + r_{w,s,ss}^{w,s})(\lambda_{t}^{w} - \lambda_{w,ss}^{w,s}) - \psi_{w,ss}^{w,s}(1 + r_{w,s,ss}^{w,s}) \left( \frac{1}{\lambda_{w,ss} - \psi_{w,ss}(1 + r_{w,s,ss}^{w,s})} \right) (127) \]

F.O.C. for secured borrowing in exogenous financial frictions case:

\[ \lambda_{h,t+1}^{h} - \beta_{h} (1 + r_{w,s}^{w,s}) = \lambda_{t}^{w} \left( 1 - a \left( \mu_{t}^{w,u} - \mu_{w,ss}^{w,u} \right) \right) - \psi_{w,ss}^{w,s} (1 + r_{w,s}^{w,s}) (128) \]

\[ \Rightarrow \frac{\lambda_{h,t+1} - \lambda_{h,ss}}{\lambda_{h,ss}} + \frac{\beta_{h} - \beta_{h}}{\beta_{h}} + \frac{r_{w,u}^{w,u} - r_{w,u,ss}^{w,u}}{1 + r_{w,u,ss}^{w,u}} = \]

\[ = \frac{1}{\lambda_{w,u,ss} - \psi_{w,u,ss}(1 + r_{w,u,ss}^{w,u})} \left( \lambda_{t}^{w} - \lambda_{w,u,ss}^{w,u} \right) - a \lambda_{w,u,ss}^{w,u} \]

\[ - \psi_{w,u,ss}^{w,u} \left( 1 - \delta_{w,ss}^{w,u} \right) + \lambda_{t}^{w} \left( \frac{1}{\lambda_{w,u,ss} - \psi_{w,u,ss}(1 + r_{w,u,ss}^{w,u})} \right) \left( \frac{1}{\lambda_{w,u,ss} - \psi_{w,u,ss}(1 + r_{w,u,ss}^{w,u})} \right) \] (129)
\begin{align*}
\Rightarrow & \frac{\lambda_{t+1}^h - \lambda_{t+1}^{h,ss}}{\lambda_{t+1}^{h,ss}} + \frac{\beta_t^h - \beta_t^h}{\beta_t^h} + \frac{(1 - \delta_{ss}^w)(r_{t+1}^{w,u} - r_{t+1}^{w,u,ss})}{1 + r_{t+1}^{w,u,ss}} = \\
& \frac{(\lambda_t^w - \lambda_{t+1}^{w,ss})}{\lambda_{t+1}^{w,ss}} - a(\mu_{t+1}^{w,u} - \mu_{t+1}^{w,ss}) \quad (133)
\end{align*}

Collateral constraint of a firm in endogenous financial frictions case::

\begin{align*}
\mathbb{E}(1 + r_{t+1}^{w,s})^{\mu_{t+1}^{w,s}} \leq \text{coll}(1 - \tau)k_{t+1}^w \mathbb{E}p_{t+1}^K
\end{align*}

\begin{align*}
\Rightarrow & \frac{\mu_{t+1}^{w,s} - \mu_{t+1}^{w,s,ss}}{1 + r_{t+1}^{w,s,ss}} + \frac{\mu_{t+1}^{w,s} - \mu_{t+1}^{w,s,ss}}{\mu_{t+1}^{w,s,ss}} = \frac{k_{t+1}^w - k_{t+1}^{w,ss}}{k_{t+1}^{w,ss}} + \frac{p_{t+1}^K - p_{t+1}^{K,ss}}{p_{t+1}^{K,ss}} \quad (135)
\end{align*}

F.O.C. for $\delta_{t+1}^{w}$:

\begin{align*}
\Omega_t^{\text{costdef}} \Delta_t^{\text{def}} = \mu_{t-1}^{w,u}(1 + r_{t}^{w,u})
\end{align*}

\begin{align*}
\Rightarrow & \log(\Omega_t) + \log(\mathrm{cost}_{t}^{\text{def}}) - \log(\delta_{t}^{w}) = \log(\mu_{t-1}^{w,u}) + \log(1 + r_{t}^{w,u})
\end{align*}

\begin{align*}
\Rightarrow & \log(\Omega_t) + \log((\mu_{t}^{w,u}(1 + r_{t}^{w,u}))^{1+\psi}) - \log(\delta_{t}^{w}) = \log(\mu_{t-1}^{w,u}) + \log(1 + r_{t}^{w,u})
\end{align*}

\begin{align*}
\Rightarrow & \log(\Omega_t) + (1 + \psi)(\log(\delta_{t}^{w}) + \log(\mu_{t-1}^{w,u}) + \log(1 + r_{t}^{w,u})) - \log(\delta_{t}^{w}) = \\
& = \log(\mu_{t-1}^{w,u}) + \log(1 + r_{t}^{w,u}) \quad (139)
\end{align*}
\[ \Rightarrow \log(\Omega_t) + (1 + \psi)(\log(\delta^w_t) + \log(\mu^w_{t-1}) + \log(1 + r^w_{t-1})) - \log(\delta^w_t) = \log(\mu^w_{t-1}) + \log(1 + r^w_{t-1}) \] 

(140)

\[ \Rightarrow \log(\Omega_t) + \psi(\log(\delta^w_t) + \log(\mu^w_{t-1}) + \log(1 + r^w_{t-1})) = 0 \] 

(141)

\[ \Rightarrow \frac{\Omega_t}{\Omega_{ss}} - \frac{\delta^w_t}{\delta_{ss}^w} + \psi \frac{\mu^w_{t-1} - \mu^w_{ss}}{\mu_{ss}^w} + \psi \frac{r^w_{t-1} - r^w_{ss}}{1 + r^w_{ss}} = 0 \] 

(142)

F.O.C. for secured loans:

\[ \frac{\beta^h}{(\Pi^{bank}_{t+1})_{ss}}(1 + r^w_{t-ss}) + \left( k^w_{bank} - k^w_{bank} \right) \frac{\epsilon^{bank}_{t}}{R_{t}^{bank}} A^{bank}_{t} = \lambda^{bank}_{t} \left( 1 + a^{h,s}(\mu^w_{t+1} - \mu^w_{ss}) \right) \] 

(143)

\[ \Rightarrow \frac{\beta^h}{(\Pi^{bank}_{t+1})_{ss}}(1 + r^w_{t-ss}) + \left( \frac{\epsilon^{bank}_{t}}{R_{t}^{bank}} A^{bank}_{t} - k^{bank}_{bank} \right) \frac{\epsilon^{bank}_{t}}{R^{bank} \left( \mu^w_{t+1} + \mu^w_{ss} \right)} = \lambda^{bank}_{t} \left( 1 + a^{h,s}(\mu^w_{t+1} - \mu^w_{ss}) \right) \] 

(144)

\[ \Rightarrow \frac{\beta^h}{(\Pi^{bank}_{t+1})_{ss}}(1 + r^w_{t-ss}) + \left( \frac{\epsilon^{bank}_{t}}{R^{bank} \left( \mu^w_{t+1} + \mu^w_{ss} \right)} - k^{bank}_{bank} \right) \frac{\epsilon^{bank}_{t}}{R^{bank} \left( \mu^w_{t+1} + \mu^w_{ss} \right)} = \lambda^{bank}_{t} \left( 1 + a^{h,s}(\mu^w_{t+1} - \mu^w_{ss}) \right) \] 

(145)
⇒ \log \left( \frac{\beta_{h}}{(\Pi_{t+1}^{bank})_{ss}} (1 + r_{w,s}^{ss}) + \left( \frac{(e_{t}^{bank})^2}{r_{w}^{bank,s} + \mu_{t+1}^{bank,s}} - \frac{k^{bank} e_{t}^{bank}}{\mu_{t+1}^{bank,s} + \mu_{t+1}^{bank,u}} \right) \right) = \\
= \log (\lambda_{t}^{bank} (1 + a^{h,s}(\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}))) \quad (146)

⇒ - \beta_{h} (1 + r_{w,s}^{ss}) \left( \frac{(\Pi_{t+1}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} - \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} \right) + \\
+ \frac{\beta_{h} (1 + r_{w,s}^{ss})}{(\Pi_{ss}^{bank})^{\bar{\nu}}} \left( \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} - \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} \right) + \\
+ \frac{2 (e_{t}^{bank})^2}{r_{w}^{bank,s} + \mu_{t+1}^{bank,u}} - \frac{k^{bank} e_{t}^{bank}}{(\mu_{ss}^{bank} + \mu_{t+1}^{bank,s})^2} + \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} \left( \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} - \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} \right) + \\
+ \left( \frac{-2 (e_{t}^{bank})^2}{r_{w}^{bank,s} + \mu_{t+1}^{bank,u}} \right) + \frac{(e_{t}^{bank})^2}{r_{w}^{bank,s} + \mu_{t+1}^{bank,u}} + \frac{k^{bank} e_{t}^{bank}}{(\mu_{ss}^{bank} + \mu_{t+1}^{bank,s})^2} + \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} \left( \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} - \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} \right) + \\
+ \left( \frac{-2 (e_{t}^{bank})^2}{r_{w}^{bank,u} + \mu_{t+1}^{bank,u}} \right) + \frac{(e_{t}^{bank})^2}{r_{w}^{bank,u} + \mu_{t+1}^{bank,u}} + \frac{k^{bank} e_{t}^{bank}}{(\mu_{ss}^{bank} + \mu_{t+1}^{bank,u})^2} + \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} \left( \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} - \frac{(\Pi_{ss}^{bank})^{\bar{\nu}}}{(\Pi_{ss}^{bank})^{\bar{\nu}}} \right) + \\
= \frac{\lambda_{t}^{bank}}{\lambda_{ss}^{bank}} + a^{h,s}(\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}) \quad (147)
\[
\Rightarrow - \frac{\beta_h (1 + r_{w,s}^u)}{(\Pi_{t+1}^{\text{bank}})_{\text{bank}}} \left( (\Pi_{t+1}^{\text{bank}})_{\text{bank}} - (\Pi_{s}^{\text{bank}})_{\text{bank}} \right) + \\
\left\{ \frac{\beta_h (1 + r_{w,s}^u)}{(\Pi_{s}^{\text{bank}})_{\text{bank}}} (1 + r_{w,s}^u) \right\} + \\
\frac{\beta_h (1 + r_{w,s}^u)}{(\Pi_{s}^{\text{bank}})_{\text{bank}}} \left( r_{t+1}^{w,s} - r_{s}^{w,s} \right) + \\
\frac{e_{\text{bank}} - e_{s}^{\text{bank}}}{(\Pi_{s}^{\text{bank}})_{\text{bank}}} (1 + r_{s}^{w,s}) - \\
\frac{\mu_{t+1}^{\text{bank},u} - \mu_{s}^{\text{bank},u}}{(\Pi_{s}^{\text{bank}})_{\text{bank}}} (1 + r_{s}^{w,s}) = \\
\lambda_{\text{bank}} - \lambda_{s}^{\text{bank}} + a_{b,s} (\mu_{t+1}^{\text{bank},s} - \mu_{s}^{\text{bank},s})
\] (148)

\[
\Rightarrow - \frac{(\Pi_{t+1}^{\text{bank}})_{\text{bank}} - (\Pi_{s}^{\text{bank}})_{\text{bank}}}{(\Pi_{s}^{\text{bank}})_{\text{bank}}} (1 + r_{s}^{w,s}) + \\
\left\{ \frac{\beta_h (1 + r_{w,s}^u)}{(\Pi_{s}^{\text{bank}})_{\text{bank}}} (1 + r_{s}^{w,s}) \right\} + \\
\frac{\beta_h (1 + r_{w,s}^u)}{(\Pi_{s}^{\text{bank}})_{\text{bank}}} \left( r_{t+1}^{w,s} - r_{s}^{w,s} \right) + \\
\frac{e_{\text{bank}} - e_{s}^{\text{bank}}}{(\Pi_{s}^{\text{bank}})_{\text{bank}}} (1 + r_{s}^{w,s}) - \\
\frac{\mu_{t+1}^{\text{bank},u} - \mu_{s}^{\text{bank},u}}{(\Pi_{s}^{\text{bank}})_{\text{bank}}} (1 + r_{s}^{w,s}) = \\
\lambda_{\text{bank}} - \lambda_{s}^{\text{bank}} + a_{b,s} (\mu_{t+1}^{\text{bank},s} - \mu_{s}^{\text{bank},s})
\] (149)

F.O.C. for unsecured loans:

\[
\frac{\beta_h}{(\Pi_{t+1}^{\text{bank}})_{\text{bank}}} ((1 + r_{t+1}^{w,u})(1 - \theta_{w}^{u} r_{t+1}^{w}) + (k_t^{\text{bank}} - k_{t+1}^{\text{bank}}) r_{w}^{u} \frac{e_{\text{bank}}}{RWA_{\text{bank}}} = \\
\lambda_{t}^{\text{bank}} (1 + a_{b,u} (\mu_{t+1}^{\text{bank},u} - \mu_{s}^{\text{bank},u}))
\] (150)
\[
\Rightarrow \quad \left( \Pi_{t+1}^{\text{bank}} \right)_{\text{bank}} - \left( \Pi_{t+1}^{\text{ss}} \right)_{\text{bank}} + \frac{r_{t+1}^{w,u} - r_{t+1}^{w,ss}}{(1 + r_{t+1}^{w,ss})} - \theta^w \frac{\delta_{t+1}^w - \delta_{t+1}^w}{(1 - \theta^w \delta_{t+1}^w)} \]

\[
+ \frac{\varepsilon_{b}^{\text{bank}} - \varepsilon_{b}^{\text{bank}}}{(1 + r_{t+1}^{w,ss})(1 - \theta^w \delta_{t+1}^w)} \]

\[
- \frac{\alpha^h_{t+1}^{\text{bank},s} - \alpha^h_{t+1}^{\text{bank},s}}{(1 + r_{t+1}^{w,ss})(1 - \theta^w \delta_{t+1}^w)} \]

\[
= \lambda^{(1)}_{\text{bank}} + a^{h,s} \left( \mu_{t+1}^{\text{bank},s} - \mu_{t+1}^{\text{bank},s} \right) \] (151)

Taylor rule:

\[
\Rightarrow \quad \frac{1 + i_t^b}{1 + i_{t+1}^b} = \left( \frac{1 + i_{t-1}^b - 1 + i_{t+1}^b}{1 + i_{t+1}^b} \right) \rho_i \left( \frac{1 + \pi_{t+1}^{\text{cpi}}}{1 + \pi_{t+1}^{\text{cpi}}} \right) \left( 1 + \rho_g \left( \frac{GDP_t}{GDP_{t+1}} \right) \right) \varepsilon_t, \] (152)

\[
\Rightarrow \quad \log(1 + i_t^b) - \log(1 + i_{t+1}^b) = \rho_i \left( \log(1 + i_{t-1}^b) - \log(1 + i_{t+1}^b) \right) + \left( 1 + \rho_g \left( \log(1 + \pi_{t+1}^{\text{cpi}}) - \log(1 + \pi_{t+1}^{\text{cpi}}) \right) + \rho_g \left( \log(GDP_t) - \log(GDP_{t+1}) \right) \right) \] (153)

\[
\Rightarrow \quad \frac{i_t^b - i_{t+1}^b}{1 + i_{t+1}^b} = \rho_i \left( \frac{i_{t-1}^b - i_{t+1}^b}{1 + i_{t+1}^b} \right) + \left( 1 + \rho_g \left( \pi_{t+1}^{\text{cpi}} - \pi_{t+1}^{\text{cpi}} \right) + \rho_g \left( \frac{GDP_t - GDP_{t+1}}{GDP_{t+1}} \right) \right) \] (155)

**Wedges**

Linearized F.O.C.s give
F.O.C. for secured borrowing in endogenous financial frictions case:
\[
\frac{\lambda_{h+1}^t - \lambda_{h,ss}^t}{\lambda_{h,ss}^t} + \frac{\beta h_t}{\beta h} + \frac{\psi_{w,ss}^t}{1 + \psi_{w,ss,ss}^t} - \frac{\psi_{w,u,ss}^t}{1 + \psi_{w,u,ss,ss}^t} = \frac{1}{\lambda_{w,ss}^t - \psi_{w,ss}^t(1 + \psi_{w,u,ss,ss}^t)} (\lambda_w^t - \lambda_{w,ss}^t) - \\
\frac{a_{w,u}^t}{\lambda_{w,ss}^t - \psi_{w,u,ss}^t(1 + \psi_{w,u,ss,ss}^t)} \left( \mu_{w,u}^t - \mu_{w,ss}^t \right) - \frac{1}{\lambda_{w,ss}^t - \psi_{w,u,ss}^t(1 + \psi_{w,u,ss,ss}^t)} (\psi_w^t - \psi_{w,ss}^t) - \\
\frac{a_{w,u}^t}{\lambda_{w,ss}^t - \psi_{w,u,ss}^t(1 + \psi_{w,u,ss,ss}^t)} (r_{w,u,ss}^t - r_{w,u,ss,ss}^t) \tag{156}
\]

F.O.C. for secured borrowing in exogenous financial frictions case:
\[
\frac{\lambda_{h+1}^t - \lambda_{h,ss}^t}{\lambda_{h,ss}^t} + \frac{\beta h_t}{\beta h} + \frac{\psi_{w,ss}^t}{1 + \psi_{w,ss,ss}^t} - \frac{\psi_{w,u,ss}^t}{1 + \psi_{w,u,ss,ss}^t} = \frac{1}{\lambda_{w,ss}^t - \psi_{w,ss}^t(1 + \psi_{w,u,ss,ss}^t)} (\lambda_w^t - \lambda_{w,ss}^t) - \\
\frac{1}{\lambda_{w,ss}^t - \psi_{w,u,ss}^t(1 + \psi_{w,u,ss,ss}^t)} \left( \mu_{w,u}^t - \mu_{w,ss}^t \right) - \frac{1}{\lambda_{w,ss}^t - \psi_{w,u,ss}^t(1 + \psi_{w,u,ss,ss}^t)} (\psi_w^t - \psi_{w,ss}^t) - \\
\frac{a_{w,u}^t}{\lambda_{w,ss}^t - \psi_{w,u,ss}^t(1 + \psi_{w,u,ss,ss}^t)} \left( \mu_{w,u}^t - \mu_{w,ss}^t \right) \tag{157}
\]

So, the wedge\textsuperscript{19} between endogenous and exogenous financial frictions cases for secured borrowing becomes:
\[
\frac{1 + \psi_{w,u,ss}^t}{\lambda_{w,ss}^t - \psi_{w,u,ss}^t(1 + \psi_{w,u,ss,ss}^t)} (\psi_w^t - \psi_{w,ss}^t) + \frac{\psi_{w,u,ss}^t}{\lambda_{w,ss}^t - \psi_{w,u,ss}^t(1 + \psi_{w,u,ss,ss}^t)} (r_{w,u,ss}^t - r_{w,u,ss,ss}^t) \tag{158}
\]

F.O.C. for unsecured borrowing in endogenous financial frictions case:
\[
\frac{\lambda_{h+1}^t - \lambda_{h,ss}^t}{\lambda_{h,ss}^t} + \frac{\beta h_t}{\beta h} + \frac{\psi_{u,ss}^t}{1 + \psi_{u,ss,ss}^t} - \frac{\psi_{u,u,ss}^t}{1 + \psi_{u,u,ss,ss}^t} = \frac{a_{u,u}^t}{\lambda_{u,ss}^t - \psi_{u,u,ss}^t(1 + \psi_{u,u,ss,ss}^t)} (\mu_{u,u}^t - \mu_{u,ss}^t) \tag{159}
\]

\textsuperscript{19}The wedge is calculated as the difference between the F.O.C.s for the cases with endogenous and exogenous financial frictions
F.O.C. for unsecured borrowing in exogenous financial frictions case:

\[
\frac{\lambda_{t+1}^h - \lambda_{t,ss}^h}{\lambda_{t,ss}^h} + \frac{\beta_h^h - \beta_h}{\beta_h} + \frac{(1 - \delta_w^w)(r_{t+1}^{w,u} - r_{t,ss}^{w,u})}{1 + y_{t,ss}^{w,u}} = (\frac{\lambda_w^w - \lambda_{w,ss}^w}{\lambda_{w,ss}^w}) - d_u^w(\mu_{t}^{w,u} - \mu_{t,ss}^{w,u})
\] (160)

So, the wedge between endogenous and exogenous financial frictions cases for unsecured borrowing becomes:

\[
\frac{(\delta_w^{w+1})(r_{t+1}^{w,u} - r_{t,ss}^{w,u})}{1 + y_{t,ss}^{w,u}}
\] (161)
### Appendix III: steady state

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<th>Variable</th>
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<th>Value</th>
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Table 9: Steady state values of variables
Appendix IV: aggregate credit conditions

The empirical relevance of our credit conditions variable, $\Omega^w_t$, is constructed to be falsifiable. If it is not a valid description of the relevant dead-weight costs of default, then the estimated values of parameters $\omega$, $\gamma$ and $\psi$ should be estimated to be close to zero.

Suppose that $\omega, \gamma \to 0$. Then from equation (17), $\Omega^w_t \to \Omega^w_{ss}$.

$\Omega^w_{ss}$ is determined from equation:

$$\Omega^w_{ss}(\delta^w_{ss}(1 + r^w_{ss,u})\mu^w_{ss,u})\psi = 1.$$  \hspace{1cm} (162)

From equation (162) follows that as $\psi \to 0$, $\Omega^w_{ss} \to 1$.

Then from equation (118) we would have that:

$$(\delta^w_t(1 + r^w_{u,t})\mu^w_{u,t})\psi = 1.$$  \hspace{1cm} (163)

From (163), at $\psi = 0$, this optimally condition holds true which for all choices of $\delta^w_t$ and implies that $\delta^w_t$ stays close to its steady state level along a stable unique path.

However, as all the estimated values of these parameters are different from zero, we can say that both aggregate credit conditions variable and the cost of negotiating the debt are important for matching the movement of the observed data series.
Товарные циклы и финансовая нестабильность в развивающихся странах

Страны-экспортеры проявляют процикличность по отношению к ценам сырьевых товаров. Хотя финансовые фрикции могут усилить влияние шоков цен на сырьевые товары, не ясно как это происходит в действительности для стран – чистых экспортеров. Используя российские данные за 2001–2018 гг., мы оцениваем новокейнсианскую модель малой открытой экономики с банковской системой и отечественными фирмами, которые привлекают обеспеченные долговые обязательства и могут производить дефолт по своим необеспеченным внутренним долгам. Залоговое ограничение по обеспеченным кредитам и дефолт по необеспеченным создают фрикции финансового посредничества, которые эндогенно изменяются в течение делового цикла, усиливают влияние шоков цен на сырьевые товары и уменьшают важность инвестиционных шоков и шоков межвременных предпочтений в оценинной модели. При финансовых фрикциях оптимальная политика характеризуется денежно-кредитной политикой с меньшим ориентиром на показатели инфляции и ВВП. В данной ситуации на первый план выходит показатель соотношения общих кредитов к ВВП, оптимальное значение которого достигается посредством комбинации макропруденциальных инструментов.

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(на английском языке)