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On the effects of income heterogeneity in monopolistically competitive markets*

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Abstract

This paper studies the effects of consumer income heterogeneity on monopolistically competitive product markets and individual welfare in the context of non-homothetic preferences. When expenditure of richer individuals is less sensitive to price change compared to poorer ones, a mean-preserving contraction of income distribution makes firm revenue less sensitive to price changes. This entices firms to charge higher prices. As a result, new firms enter the market, broadening product diversity. General equilibrium effects have a negative impact on poorer individuals and, in specific circumstances, on richer individuals. Furthermore, reduced income inequality may shift the market equilibrium further away from optimal product diversity. In open economies, lower income inequality in a country creates a price divergence between countries and decreases trade volumes and values. Those general equilibrium effects are shown to be quantitatively non negligible.

Keywords: Monopolistic competition, nonhomothetic preferences, income inequality, pricing, welfare, optimal product diversity, trade.

JEL codes: D43, L16, F12, F14, R13.

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1 Introduction

The current era of globalization has heralded dramatic increases in income inequality and reaches historic highs in some countries. While the rise of the number of billionaires and workers in the 'gig economy' have increased the income spread, many governments have contemplated to implement a wide variety of redistributive policies to mitigate income inequality. What implications do these trends and policies have for product markets and their efficiency? How large are the effects and what are their impacts on individuals' wellbeing? Imperfect product markets are affected by these shocks through changes in firms' prices, markups, entry and product diversity. Whereas standard models of monopolistic competition are widely applied to explain the effect of various economic shocks on those changes, they have not been applied to study the impacts of income inequality. Yet, these recent trends and policies about inequalities justify to account for income heterogeneity as one of the most important features that economists must bring to their research agenda.

In this paper, we investigate the impact of income heterogeneity on imperfect product markets and individual welfare. To the best of our knowledge, the causal effect of income heterogeneity on product markets and welfare has not been discussed extensively in the literature. In particular, does lower income inequality lead to a rise or fall in market prices, product diversity, and firm output? How do those changes affect different income groups? How quantitatively important are those effects?[1]

These questions are important for several reasons. First, they show the limitations of the representative consumer approach, as market outcomes depend not only on average income but also on the entire income distribution. Second, as income distribution impacts firms' market power, it produces additional distortions to the standard product quantity versus diversity trade-off. Third, the answers to these questions shape policy recommendations as a redistribution policy may create general equilibrium effects that alter product prices and diversity and accentuate the welfare gaps between income groups.

To address these questions, we develop a general equilibrium model in which individuals differ in their incomes while their preferences are nonhomothetic. Individuals consume a set of varieties produced by a monopolistically competitive sector in which they work. Relying on Mrazova and Neary's (2017) framework, we study the effects of income inequality on prices, product diversity, trade structure and individual welfare. Such effects strongly depend

[1] Tarasov (2009) addresses similar questions but within a very different framework with two income groups and indivisible goods as in Matsuyama (2000).

on the type of preferences assumed in economic models. To clarify the direction of general equilibrium effects, we concentrate on preferences such that individual demand elasticity falls and love for variety rises with consumption. This assumption combines the conditions for demand subconvexity (Mrazova and Neary, 2017) and aligned preferences (Dhingra and Morrow, 2019). Demand subconvexity matches the empirical fact that markups decrease with market size (Syverson, 2007; De Loecker *et al.*, 2016), while an increasing love for variety is claimed to be the most plausible case (Vives, 2001, ch. 6). We highlight that product market outcomes and individual welfare crucially depend on the level of income inequality. We furthermore extend our framework to an open economy and discuss the consequences of a country's income inequality for trade patterns and distant markets. Finally, in a quantification exercise, we assess the importance of general equilibrium effects of income redistribution.

We first show that market prices and product diversity are unaffected by income redistribution if consumers are endowed with the preferences described by Pollak (1971). Those preferences encompass the following commonly used utility functions: constant elasticity of substitution (CES) and its "translated" version, quadratic utility, utility with constant absolute risk aversion (CARA), and logarithmic utility. Under these preferences, income redistribution reshuffles individual consumption in a way that the firm's aggregate demand is invariant, and therefore, any income redistribution leaves product prices and firm entry unaffected. Hence, the first contribution of this paper is the extension of Pollak's (1971) argument to a general equilibrium framework with monopolistic competition.

However, the application of the above preferences in economic modeling raises several issues. First, these preferences are hardly supported by the data. Indeed, it is well known that the unit income elasticity of CES preferences is not empirically confirmed for many goods (Houthakker, 1957; Samuelson and Nordhaus, 2010, p. 93). Additionally, Pollak's preferences do not support any correlation between income distribution and product diversity, which contradicts empirical findings (e.g., Falkinger and Zweimüller, 1996). Second, starting from Murhy *et al.* (1989), the macroeconomic literature strongly emphasizes the link between income inequality and total demand for manufacturing products. Such a relationship is absent under Pollak's preferences. Finally, the latter relationship is also relevant in the trade context as empirical studies show a dependency between the demand for export goods and countries' levels of income inequality (Choi *et al.*, 2009; Dalgin *et al.*, 2008). This motivates us to study a set of preferences beyond Pollak's class.

To this end, we explore the conditions on preferences that shall induce variations in market prices, product diversity, and individual welfare after a change in income distribution. The effect hinges on the behavior of the convexity of the direct demand function (Aguirre *et al.*, 2010). When this convexity is an increasing function of consumption, the individual expenditure of low-income individuals is more sensitive to price changes than that of high-income individuals. Since firms' revenue aggregates individual expenditures, the changes in income and expenditures of low-income individuals contribute more to the changes in price sensitivity of the firms' revenue and, therefore, to their pricing decisions than similar changes among high-income individuals. The opposite pattern arises when the convexity of the direct demand function decreases with consumption.

We focus on a redistribution policy that conducts a mean-preserving contraction of income distribution. Such a policy typically reflects the effect of a progressive tax redistribution. We find that if the convexity of direct demand increases with consumption, the equilibrium price and product diversity rise while firm output falls. In that case, the price sensitivity of individual expenditure increases faster for poor individuals than it decreases for rich. These changes, therefore, make firm revenue less sensitive to price changes. Ultimately, the redistribution policy leads to a lower demand elasticity, which increases the market power of monopolistically competitive firms. In turn, this entices firms to charge higher markups and prices. As a result, new firms enter and augment the product diversity at the market. Finally, due to the business-stealing effect, firm output falls. Importantly, these changes in the product market generate negative general equilibrium effects on the welfare of poor individuals and may also be harmful to the richer ones. Under these circumstances, a policy targeting only individuals in the lowest income decile or only the poorest urban areas may lead to welfare losses for untargeted poor residents.

By contrast, if the convexity of demand is a decreasing function of consumption, the situation is the opposite. For instance, a reduction in income inequality makes firm revenue more sensitive to price change, thus increasing market demand elasticity and decreasing firms' market power. This leads to lower prices, markups, and number of varieties, while firms' output rises. Furthermore, the general equilibrium effect of such a redistribution policy is beneficial at least for the poor groups of consumers.

We also investigate the role of income heterogeneity on optimal product diversity. For a decreasing convexity of demand, we observe the standard pattern: the market always provides excessive varieties and insufficient output (Dixit and Stiglitz, 1977). However, for an increasing convexity of demand, stronger income heterogeneity reduces product diversity, thus, shifts the market outcome towards the socially optimal level of product diversity. Therefore, there exists a level of income heterogeneity such that the market delivers socially optimal product diversity.

Furthermore, for income heterogeneity beyond this level, the market provides an insufficient level of product diversity. The intuition is in line with the preceding discussion. Stronger income heterogeneity makes poor individuals even poorer and entices them to reduce their expenditure on each variety. Since individual expenditure of low-income individuals is more sensitive to price change, firms' revenue also becomes more sensitive to prices, which forces them to set lower prices. As a result, firms earn lower profits and a fraction of them exit the market, while survivors compensate for the fall in revenue by increasing their output. This shows that discussions on market regulations that correct for product diversity may be incomplete if they do not take into account the effect of income inequality.

The questions under consideration are also extremely relevant in international trade contexts. Since Jones (1965), researchers have studied the impact of trade patterns on various income groups. However, the literature is scant regarding the reverse effect of consumer heterogeneity on the patterns of trade and distant product markets. For instance, income redistribution within one country may affect the prices and entry decisions of firms in other countries and may also raise or reduce individuals' welfare levels there. Thus, we extend our setting to an open economy where countries trade their products. We discuss the effects of within-country income redistribution on trade and welfare in all countries. We find that under the increasing convexity of direct demand, lower income inequality in a country leads to higher market prices and wider product diversity in its local market. In the other country, both domestic and imported good prices decrease while product diversity expands. We therefore shed light on the price divergence between countries that stems from income inequality rather than trade costs and/or home market bias as emphasized in the literature. Furthermore, trade volumes and values are shown to fall. All residents of the foreign country gain from the contraction of the domestic income distribution. In the home country, the utility of poor individuals may, however, drop. Thus, poorer individuals are more negatively affected by general equilibrium effects than richer individuals. These findings show how within-country income inequality shapes trade patterns and welfare levels in both countries.

Finally, we propose a quantification exercise calibrated to the US economy. In this exercise, we retain the preference classes that are compatible with existing empirical estimates of the elasticities of demand and pass-through. Despite these constraints on the preferences and parameters, the exercise supports demand functions with both increasing and decreasing convexity and therefore allows us to present cases with opposite general equilibrium effects. We then study the effect of a redistributive transfer from the top to the bottom income decile. We show that general equilibrium effects are quantitatively non-negligible in both closed and open economies. For instance, a transfer involving 1.5% of total US income causes welfare changes up to 0.3% as measured by consumption equivalent. The impacts on product prices, diversity and output is stronger.

Literature review. This paper relates to several strands of literature. First, the paper is linked to the literature studying product markets in the monopolistic competition framework with separately additive preferences (Spence, 1976; Dixit and Stiglitz, 1977; Kuhn and Vives, 1999) and with applications to trading countries (Neary, 2004; Zhelobodko *et al.*, 2012, among others). Close to this paper, Foellmi and Zweimüller (2004) demonstrate that market behavior is governed by the properties of the convexity of direct demand. However, beyond the paper by those authors, we discuss a comprehensive set of properties and quantitative assessments related to individual welfare, social optimum and trading countries. The paper revisits a subset of the demand structures proposed in Bulow and Pfleiderer (1983), Mrazova and Neary (2017, 2019), Mrazova *et al.* (2019) and Nakamura and Zerom (2010). Among them are demand functions with constant superelasticity, translog, constant proportional pass-through and constant elasticity of marginal revenue. This paper shows that those demand structures yield contrasting properties of the convexity of direct demand and therefore lead to opposite conclusions about the general equilibrium effects of income heterogeneity on product markets and welfare. Such contrasting effects may even take place within the same class of preferences for different parameterizations. Finally, the convexity of direct demand plays a key role in third-degree price discrimination (Aguirre *et al.*, 2010; Cowan, 2012; Holmes, 1989). In contrast to our paper, this partial equilibrium literature shows that the properties of this convexity shape the welfare and output effects of market segmentation.

Second, this paper contributes to the literature on optimal product diversity by introducing the dimension of income heterogeneity. Dixit and Stiglitz (1977) first show that the market delivers optimal product diversity only under CES preferences, while preferences with decreasing elasticity of utility yield too much product diversity and insufficient output levels. Dhingra and Morrow (2019) expand this analysis in the context of firms with heterogeneous costs. We extend this literature along another dimension, i.e., income heterogeneity. In addition to the standard trade-off between product diversity and consumed quantity (Spence, 1976), our analysis reveals a new potential source of distortion: the unequal allocation of consumption among heterogeneous individuals that shapes firms' market power.

Third, there has been a long discussion on the impact of income inequality on aggregate demand through marginal propensities to consume (see Pigou, 1920; Keynes, 1936). Although a strand of this literature emphasizes the independence of aggregate demand from income distribution (Friedman, 1957, and followers), the other finds a negative relationship between demand and income inequality (Dynan *et al.*, 2004). In this paper, we uncover a very different mechanism that relates income inequality to aggregate demand through the entry/exit of firms into/from the market. To be precise, if poor individuals' expenditures are more sensitive to price change, an increase in income inequality leads to a higher aggregate demand for each variety. When the income of poor individuals falls, firm revenues become more sensitive to prices, which entices firms to set lower prices. This situation pushes a fraction of firms out of the market. As a result, surviving firms produce more.

Finally, this paper relates to the trade literature devoted to income heterogeneity. The first set of papers relies on a monopolistic competition framework to investigate the impact of trade liberalization on within-country income inequality. For example, Yeaple (2005) shows how trade widens the income gap between skilled and unskilled workers. Trade liberalization increases the skill premium paid by exporting firms using "high-tech" technologies in the context of a workforce with heterogeneous skills. Egger and Kreickemeier (2009), Helpman *et al.* (2010) and Felbermayr *et al.* (2011) explain the rise of within-country income inequality after trade liberalization through labor market imperfections and the presence of unemployment. Another set of papers discusses the relationship of between-country income heterogeneity on trade patterns. Fieler (2011) encompasses both per-capita income inequality and size differences into the Ricardian model with CES preferences and discusses their impact on trade flows. Using a Ricardian framework, Matsuyama (2000) studies the impact of income redistribution within a country on the wages and well-being of residents in both countries. Bertoletti *et al.* (2018) study trade patterns in the context of countries with heterogeneous per-capita incomes and preferences with income effects. Behrens and Murata (2012) contribute to both sets of papers, as they show that the impact of trade liberalization on the distribution of individual welfare depends on each country's relative per-capita income. This study is close to our paper, as it assumes within-country income heterogeneity. However, because this paper discusses CARA preferences, which belong to the Pollak class, market and trade properties hinge only on the countries' average income and their relative position in the world income distribution. We deviate from these two strands of trade literature by studying the role of the within-country income distribution on market and welfare outcomes within both countries.

The rest of the paper is organized as follows. Section 2 develops the baseline model and identifies the equilibrium in a closed economy. Section 3 studies the impact of income redistribution on market outcome and welfare while Section 4 discusses the socially optimal allocation. Section 5 extends the framework to the case of two countries. Section 6 quantifies the general equilibrium effects for different demand systems. Section 7 concludes. The appendices include mathematical details.

2 Model

The economy includes a mass L of individuals. Each individual h is endowed with a $s_h > 0$ labor units that are distributed with a cumulative distribution function $G : [s_0, s_1] \to [0, 1]$, where $0 < s_0 < s_1$ and $G' > 0$. Until Section 5, we normalize wage per labor unit to one, so that s_h stands for individual h income. When it doesn't lead to confusion, we denote the integral over individuals' income $\int_{s_0}^{s_1} \mathrm{d}G(s_h)$ as $\int \mathrm{d}G$; that is, we omit the integration boundaries and references to income s_h. The average individual income is then given by $s = \int s_h \mathrm{d}G$. In what follows, a variable without subscript h denotes its average over individual incomes.

2.1 Demands

Individuals consume a set of varieties $\omega \in [0, n]$ where n denotes the endogenous number of varieties (product diversity). Each individual s_h maximizes her utility

$$U(x_h) = \int_0^n u(x_h(\omega))\mathrm{d}\omega$$

subject to her budget constraint $\int_0^n p(\omega)x_h(\omega)\mathrm{d}\omega = s_h$, where $x_h(\omega)$ is her consumption of variety ω and $p(\omega)$ is the price for variety ω. The utility function is increasing and concave, $u''(x_h) < 0 < u'(x_h)$. Due to the concavity of utility, consumers purchase all available varieties. The first order condition yields the inverse demand function $p(\omega) = \lambda_h^{-1} u'(x_h(\omega))$, where λ_h is the consumer's budget constraint multiplier. For the sake of clarity, we temporarily drop the reference to ω and write the individual demand as

$$x_h \equiv v(\lambda_h p), \tag{1}$$

where v is the inverse function of $u'(x_h)$, which decreases with its argument.

Demand side statistics. This paper highlights the role of three statistics of the demand side. The first one is the price elasticity of the individual's demand given by

$$\varepsilon(x_h) \equiv -\frac{d\ln x_h}{d\ln p} = -\frac{\lambda_h p v'(\lambda_h p)}{v(\lambda_h p)} = -\frac{u'(x_h)}{x_h u''(x_h)} > 0, \tag{2}$$

which we refer to as *demand elasticity*. For conciseness, we denote its value for an individual with consumption x_h as $\varepsilon_h \equiv \varepsilon(x_h)$.

Following Mrazova and Neary (2017), we define a *subconvex* demand function as a function with a decreasing demand elasticity: $\varepsilon_h' < 0$. Subconvex demands feature the inverse relationship between market elasticity and average consumption which is in line with the empirical literature (Syverson, 2007; De Loecker *et al.*, 2016). In what follows, we rely on the subconvex demands. This assumption also matches Mion and Jacob's (2020) empirical findings.[2]

Using (2) and simplifying, we can state that the individual demand function is subconvex if and only if

$$\varepsilon_h' = -\frac{1}{x_h}\left(1 + \varepsilon_h - r_h\right) < 0, \tag{3}$$

where $r_h \equiv r(x_h)$ is the second statistics of interest with

$$r(x_h) \equiv -\frac{\mathrm{d}\ln v'\left(\lambda_h p\right)}{\mathrm{d}\ln p} = -\frac{\lambda_h p v''\left(\lambda_h p\right)}{v'(\lambda_h p)} = \frac{u'(x_h)u'''(x_h)}{\left(u''(x_h)\right)^2}. \tag{4}$$

Our results mainly hinge on the behavior of this statistics which measures the *convexity of direct demand function* (Aguire *et al.*, 2010; Mrazova and Neary, 2017).

Finally, we define the statistics for the love for variety as $1 - \eta(x_h)$ where

$$\eta(x_h) \equiv \frac{x_h u'(x_h)}{u(x_h)} \in (0, 1) \tag{5}$$

is the elasticity of utility function defined in Dixit and Stiglitz (1977). This represents the degree of preference for a variety as the proportion of social surplus not captured by revenues (Vives, 2001). Because u is concave and increasing, η_h lies between 0 and 1. The index $1 - \eta(x_h)$ is equal to zero in the absence of love for variety (because utility u is linear) and rises to one as the latter becomes stronger. As explained in Vives (2001), $1 - \eta(x_h)$ measures the preference for variety, namely, the utility gain from adding a new variety while holding quantity fixed. This statistics plays an important role in consumption behavior and welfare assessment. Furthermore, at some point, we shall make use of Dhingra and Morrow's (2019) definition of "aligned preferences" according to which individual demand elasticity $\varepsilon(x_h)$ and the elasticity of utility $\eta(x_h)$ move in the same direction. Hence, combined with subconvex demand, aligned preferences imply that individuals become less sensitive to price and more sensitive to product diversity when they consume more. This situation is considered more plausible in economic

[2] It can be shown that, for small enough income heterogeniety, subconvex demand functions generate a decreasing elasticity of pass-through. This is confirmed by Mion and Jacob (2020) using French data.

theory (Vives, 2001).[3] From now on, we assume that this condition holds.

2.2 Firms

Labor is the only production factor. Each firm produces a single variety ω and finds the price $p(\omega)$ that maximizes its profit $\pi(\omega) = L\int(p(\omega)-c)x_h(\omega)\mathrm{d}G - f$. In this expression, c and f are the firm's marginal and fixed labor requirements. Since demands are symmetric across varieties we omit the reference to ω. Plugging the demand function (1) into profit and differentiating, we obtain the first order condition for the producer problem:

$$\frac{\mathrm{d}\pi}{\mathrm{d}p} = (p-c)\int \lambda_h v'(\lambda_h p)\mathrm{d}G + \int v(\lambda_h p)\mathrm{d}G = 0.$$

After some algebra, using (1), the profit-maximizing price is given by

$$p = \frac{\varepsilon}{\varepsilon - 1}c, \tag{6}$$

where

$$\varepsilon \equiv \frac{\int x_h \varepsilon_h \mathrm{d}G}{\int x_h \mathrm{d}G} \tag{7}$$

is the market elasticity. The second order condition of the producer problem imposes

$$\frac{\mathrm{d}^2\pi}{\mathrm{d}p^2} = 2\int \lambda_h v'(\lambda_h p)\mathrm{d}G + (p-c)\int \lambda_h^2 v''(\lambda_h p)\mathrm{d}G < 0.$$

Using the definitions of ε_h and r_h and plugging the optimal prices (6), this condition takes the following form:

$$\int (2\varepsilon - r_h)\varepsilon_h x_h \mathrm{d}G > 0. \tag{8}$$

We make two remarks. First, in the absence of individuals' heterogeneity, $s_h = s$, consumption is homogenous, $x_h = x$, so that condition (8) collapses to $r < 2\varepsilon$, as in Zhelobodko *et al.* (2012). Second, condition (8) is always satisfied when $r_h < 0$ for all values of h. When $r_h > 0$, Appendix A shows that (8) holds under $r_h' > 0$. Other configurations must be checked on a case by case basis.

[3] The literature has focused on the benchmark preferences with constant elasticity of substitution (CES), defined by the utility function $u = x^{1-1/\sigma}$ with $\sigma > 1$ and yielding the three constant statistics $\varepsilon_h = \sigma$, $r_h = \sigma + 1$ and $\eta_h = 1 - 1/\sigma$. Their individual demand functions are neither sub- nor super-convex. As a result, subconvexity can be interpreted in reference to the CES demand functions: a demand function is subconvex at some arbitrary price and quantity levels if it is less convex at those levels than a CES demand function.

2.3 Equilibrium

An equilibrium is defined as the set of consumptions x_h, the price p, the number of firms n, and the firm output y that are consistent with the consumers' budget constraints

$$npx_h = s_h, \tag{9}$$

the firm's optimal price

$$p = \frac{\varepsilon}{\varepsilon - 1}c, \tag{10}$$

the zero-profit condition (free entry), the product and labor market clearing conditions

$$p = \frac{f}{y} + c, \qquad y = L\int x_h \mathrm{d}G, \qquad L\int s_h \mathrm{d}G = n\left(f + cy\right). \tag{11}$$

By the Walras law, one identity is redundant.

While all consumers must purchase all varieties,[4] a sufficient condition for the existence of a fixed point requires the market elasticity ε to fall as prices decrease and consumption levels rise, which holds if

$$\int \varepsilon'_h x_h s_h \mathrm{d}G < 0. \tag{12}$$

In particular, this holds under individual demand subconvexity $\varepsilon'_h < 0$ for every income level. In this case, higher prices decrease consumption levels, increase the demand elasticity of every individual and therefore raise the market elasticity that each firm faces. Furthermore, under subconvex demand, the demand elasticity at the lowest income must exceed one for $\varepsilon > 1$ to hold. The latter implies positive prices in equilibrium. We prove the following proposition in Appendix B.

Proposition 1 *Assume subconvex demands. Then, an equilibrium where all individuals consume all available varieties exists and is unique if the second order condition (8) holds and the demand elasticity at the lowest income exceeds one.*

[4] Consumer h purchases all varieties if her utility $nu(x_h)$ increases with the number of varieties n. Given her budget constraint $s_h = pnx_h$, this implies that $nu(s_h/pn)$ must rise with n, or equivalently, $u'(x_h)x_h/u(x_h) < 1$ where $x_h = s_h/pn$. This condition holds in the presence of love for variety as expressed in (5). As a result, consumers always purchase all goods. Furthermore, as in Behrens and Murata (2012), the lowest income consumers buy each available variety because their budget constraint $pnx_0 = s_0$ implies that her consumption level is non zero: $x_0 = s_0/(pn) > 0$.

3 Income inequality

The main tenet of this paper is to investigate the effects of income heterogeneity on product markets and welfare. Towards this goal, we consider small changes in distribution of individual income s_h. We will show later in this section that our results hold for arbitrary changes in distribution. Suppose that every individual with income s_h gets a new income $s_h + \mathrm{d}s_h$ where $\mathrm{d}s_h$ is an infinitely small change. We denote the individual income changes as the mapping $\widehat{s}_h \equiv \mathrm{d}\ln s_h = \mathrm{d}s_h/s_h$. This implies the following small changes in endogenous variables: $\widehat{x}_h = \mathrm{d}\ln x_h$, $\widehat{p} = \mathrm{d}\ln p$, $\widehat{y} = \mathrm{d}\ln y$, and $\widehat{n} = \mathrm{d}\ln n$ while the change in average income s is given by $\widehat{s} \equiv \mathrm{d}\ln s = \frac{1}{s}\int s_h \widehat{s}_h \mathrm{d}G$.[5]

Log-linearization of the equilibrium conditions (9)-(11) yields:

Budget	$\widehat{x}_h = \widehat{s}_h - \widehat{p} - \widehat{n}$
Price	$\widehat{p} = \frac{1}{x\varepsilon(\varepsilon-1)}\int(1+\varepsilon - r_h)x_h\widehat{x}_h\mathrm{d}G$
Entry	$\widehat{y} = -\varepsilon\widehat{p}$
Product market	$\widehat{y} = \frac{1}{x}\int x_h\widehat{x}_h\mathrm{d}G$
Labor market	$\widehat{n} = \widehat{s} - \frac{\varepsilon-1}{\varepsilon}\widehat{y}$

Table 1: Deviations around the equilibrium.

The first line shows that a rise in the individual's income raises her consumption x_h whereas higher prices and broader product diversity reduce it. From the second line, we see that the change in prices is caused by the changes in individual consumptions, which are not symmetric across consumers. If the change in income distribution increases the price, then, its partial effect on the individual consumption and the output of each firm is negative (first and third lines). Finally, a rise in aggregate income $\widehat{s}$ inflates the labor supply, triggers the entry of new firms and has a negative effect on individual consumption.

Using Table 1, changes in consumption, production and number of firms can be expressed as functions of changes in individual income and price (see Appendix C for details):

$$\widehat{p} = -\frac{1}{\Psi\varepsilon}\int r_h(\widehat{s}_h - \widehat{s})s_h\mathrm{d}G, \tag{13}$$

$$\widehat{x}_h = (\widehat{s}_h - \widehat{s}) - \varepsilon\widehat{p}, \qquad \widehat{y} = -\varepsilon\widehat{p}, \qquad \widehat{n} = \widehat{s} + (\varepsilon-1)\widehat{p}, \tag{14}$$

[5] That is, $\widehat{s} \equiv \mathrm{d}\ln\left(\int s_h\mathrm{d}G\right) = \left[\mathrm{d}\left(\int s_h\mathrm{d}G\right)\right]/\left(\int s_h\mathrm{d}G\right) = \left(\int \mathrm{d}s_h\mathrm{d}G\right)/s = \int\left(\frac{s_h}{s}\frac{\mathrm{d}s_h}{s_h}\right)\mathrm{d}G = \frac{1}{s}\int s_h\widehat{s}_h\mathrm{d}G$.

where

$$\Psi \equiv \int (2\varepsilon - r_h)\, s_h \mathrm{d}G \tag{15}$$

is positive under subconvexity of demand. The price change (13) reflects the feedback effect of the changes in the number of varieties on individuals' consumption baskets, which itself depends on the changes in the income distribution through the difference between individual and average income, $\widehat{s}_h - \widehat{s}$. There is no such feedback if individual incomes are changed in identical proportions ($\widehat{s}_h = \widehat{s} \neq 0$). The price change (13) makes apparent the role of the convexity of direct demand, r_h, as shown by (13).

3.1 Invariance to redistribution

We first determine the preferences for which changes in individual income distribution do not affect prices and, therefore, product diversity, and output. Those should keep the price change (13) equal to zero whatever the income distribution. Since $\int (\widehat{s}_h - \widehat{s}) s_h \mathrm{d}G = \int \widehat{s}_h s_h \mathrm{d}G - \widehat{s}s = 0$, r_h must be the same for all individuals independently of their income s_h. That is,

$$r(x_h) = \sigma + 1, \tag{16}$$

where $\sigma \in \mathbb{R}$. Using the definition of $r = u'u'''/\left(u''\right)^2$, we express the class of utility functions (up to affine transformations) that satisfy this condition (see Appendix D for details):

$$u(x_h) = \begin{cases} x_h\,(1 - x_h) & \text{if} \quad \sigma = -1 \\ 1 - e^{-x_h} & \text{if} \quad \sigma = 0 \\ \ln x_h & \text{if} \quad \sigma = 1 \\ \frac{x_h^{1-1/\sigma}}{1-1/\sigma} & \text{if} \quad \sigma \neq -1, 0, 1 \end{cases}$$

Each line respectively denotes the quadratic utility function, the utility with constant absolute risk aversion (CARA), the logarithmic utility and the CES preferences with the elasticity of product substitution equal to σ. They also encompass Stone-Geary preferences and "translated" CES utility functions, which are well-known affine transformations of those utility functions.

Condition (16) relates to Pollak's (1971) condition (3.10a) for demand functions that are *locally linear in income*. This imposes that the demand of each variety ω is linear in income at the equilibrium prices, i.e. $\mathrm{d}^2 x_h(\omega)/\mathrm{d}s_h^2 = \mathrm{d}^2 v(p(\omega)\lambda_h)/\mathrm{d}s_h^2 = 0$.[6] Under this condition,

[6] According to Pollak (1971), the demand for a variety ω is locally linear in income if it has the form $x_h(\omega) = A(\omega, p(\omega)) + B(\omega, p(\omega)) \cdot s_h$ where A and B are two functions of the variety ω and its price $p(\omega)$. Under this condition, preferences are homothetic with respect to a specific quantity profile $x_0(\omega) = x_0$ for all ω. See

changes in individual incomes reshuffle individual consumptions in a way that does not change market demands for each variety. As a result, prices and product diversity are also unaltered.

Proposition 2 *Market prices and product diversity are not affected by changes in individual income distribution if consumers are endowed with Pollak type preferences, which include CES, quadratic, CARA, and logarithmic utility functions.*

The validity of Pollak's preference has been empirically tested by checking income nonlinearity in demand functions. Empirical works often report income elasticities of the demand for manufacturing goods significantly different from 1,[7] which is *incompatible* with locally linear demand in income. This entices us to pay attention to other classes of demand functions.

By the virtue of log-linearization, any transformation of the individual income distribution is equivalent to a sequence of two transformations, one with mean preserving component and the other with proportional increase in all s_h which affects $\widehat{s}$ only. We next discuss the mean preservation and then move to arbitrary changes in the distribution.

3.2 Mean-preserving redistribution

Income redistribution generally implies allocation of transfers across individuals under the government budget constraint, imposing that all transfers sum to zero, which results in a mean-preserving redistribution of income. In this context, a decrease in income inequality across individuals can be obtained by a mean-preserving contraction in the income distribution, which is the focus of this subsection. We will see in the next subsection that it also corresponds to a progressive tax rate.

We first define and discuss the mean-preserving change of the individual income distribution in the context of small relative changes of income $\widehat{s}_h$. The argument readily extends to arbitrary mean-preserving changes.

3.2.1 Product markets

Since it keeps average income s constant, we set $\widehat{s} = 0$ in (13) and (14) and obtain

$$\widehat{p} = -\frac{\int r_h s_h \widehat{s}_h \mathrm{d}G}{\varepsilon \Psi}, \tag{17}$$

$$\widehat{x}_h = \widehat{s}_h - \varepsilon \widehat{p}, \qquad \widehat{y} = -\varepsilon \widehat{p}, \qquad \widehat{n} = (\varepsilon - 1)\widehat{p}. \tag{18}$$

also Mrazova and Neary (2017) for a relationship between the utility moments and Pollak preferences.

[7] The income elasticities range from 0.15 for urban residential water to 2.9 for cars (McCarthy, 1996). See a recent discussion with trade data in Hummels and Lee (2018).

The numerator of the right-hand side of (17) can be integrated by parts as

$$\int_{s_0}^{s_1} r(x_h)\widehat{s}_h s_h \mathrm{d}G = \left[r(x_h) \int_{s_0}^{s_h} \widehat{s}_l s_l \mathrm{d}G_l \right]_{s_0}^{s_1} - \int_{s_0}^{s_1} r'(x_h) \frac{\partial x_h}{\partial s_h} \left(\int_{s_0}^{s_h} \widehat{s}_l s_l \mathrm{d}G_l \right) \mathrm{d}s_h,$$

where the first term is zero because of mean preservation. Thus, (17) takes the form

$$\widehat{p} = \frac{1}{\varepsilon \Psi} \int_{s_0}^{s_1} r'(x_h) \frac{\partial x_h}{\partial s_h} \left(\int_{s_0}^{s_h} \widehat{s}_l s_l \mathrm{d}G_l \right) \mathrm{d}s_h$$

where $\partial x_h / \partial s_h > 0$ due to (9). A mean-preserving contraction implies the second-order stochastic dominance of the final distribution of income s_h. In terms of relative income changes $\widehat{s}_h$, it implies that $\int_{s_0}^{s} s_h \widehat{s}_h \mathrm{d}G \geq 0$ for all s_h (see Appendix E). The opposite holds for mean preserving spread. Therefore, the equilibrium price increases if $r'(x_h)$ is *positive* for all consumption levels x_h. Finally, this conclusion holds if we integrate over a set of infinitesimally small changes $\widehat{s}_h$. This gives the following proposition:

Proposition 3 *Assume a mean-preserving contraction of income distribution. Then, the equilibrium price and product diversity rise and the firm output falls if and only if* $r'_h > 0$. *The opposite result holds for a mean-preserving spread.*

The impact on total output, $Y \equiv ny$, is $\widehat{Y} = \widehat{n} + \widehat{y} = -\widehat{p}$, which moves in the opposite direction to prices. The impact on GDP, $G \equiv npy$, is however nil because $\widehat{G} = \widehat{p} + \widehat{n} + \widehat{y} = 0$. The GDP is also the sum of labor supplies or incomes. As a result, it is unaffected by a mean-preserving change of individuals' incomes.

Note that Foellmi and Zweimüller (2004) provide a proof for Proposition 3 conditional on the uniqueness of equilibrium. By contrast, we rely on subconvex demands to ensure existence and uniqueness of the equilibrium. Next, we discuss in detail the intuition and demand properties related to this result.

The direction of the effect of income redistribution depends on the sign of the statistics r'_h, which characterizes the increasing or decreasing pattern of the convexity of the direct demand function. It is however more intuitive to relate the statistic r_h to *price sensitivities of consumer expenditure and firm revenue*. Individual h's expenditure is given by $pv(\lambda_h p)$ and its sensitivity with respect to price by

$$\frac{\mathrm{d}}{\mathrm{d}p} \left[pv(\lambda_h p) \right] = v(\lambda_h p) + p\lambda_h v'(\lambda_h p) = x_h - x_h \varepsilon_h \tag{19}$$

where the second equality stems from (2). The price sensitivity of firm revenue is given by

$$\frac{\mathrm{d}}{\mathrm{d}p}\int pv(\lambda_h p)\mathrm{d}G = \int \frac{\mathrm{d}}{\mathrm{d}p}\left[pv(\lambda_h p)\right]\mathrm{d}G = \int \left(x_h - x_h\varepsilon_h\right)\mathrm{d}G, \tag{20}$$

which aggregates the individuals' effects of prices on their expenditures. This is negative at the equilibrium. How does the price sensitivity of consumer expenditure vary with redistribution? The effect of an infinitesimally small transfer Δx_h on (19) is given by $(x_h - x_h\varepsilon_h)' \cdot \Delta x_h$. Because $(x_h - x_h\varepsilon_h)' = 1 - \varepsilon_h - x_h\varepsilon_h' = 2 - r_h$, this effect takes a form of $(2 - r_h) \cdot \Delta x_h$. Its direction obviously depends on whether the statistics r_h rises or falls with income. For instance, if r_h rises with s_h, the expenditures of individuals with lower income are more reactive to price changes. To keep things simple, consider the transfer from a mass of rich consumers h' to the same mass of poor ones h: $\Delta s_h = -\Delta s_{h'} > 0$. Due to (9), we have $\Delta x_h = -\Delta x_{h'} > 0$. Then, the aggregate effect on the sensitivity to revenues is augmented by the amount $(r_{h'} - r_h) \cdot \Delta x_h$, which is positive if and only if the statistics r_h is an increasing function. In this case, the price sensitivity of revenue becomes less negative so that firm revenue becomes less sensitive to price change. As a consequence, firms raise their prices, as stated in Proposition 3.

A lower sensitivity of firm revenue to price corresponds to lower market elasticity which increases firms' market power. The latter allows firms to charge higher markups and prices. This, in turn, invites new entrants to the product market so that product diversity expands. Finally, business-stealing effect leads to a decrease in firm output.

Due to Proposition 2, Proposition 3 applies for classes of preferences beyond Pollak. To shed light on the behavior of the r_h' function, we examine properties of other known classes of subconvex preferences. For instance, we study: (i) function with constant super-elasticity of demand (CSED)[8]; (ii) additive version of Feenstra's (2003) translog functions (TLOG), (iii) demand function with constant revenue elasticity of marginal revenue (CREMR);[9] (iv) constant proportional pass-through (CPPT) with the property $\mathrm{d}\ln p/\mathrm{d}\ln c$ being constant; (v) constant (output) elasticity of marginal revenue (CEMR) demands; and (vi) an inverse "translated" CES (ITCES) function, which belong to the class of preferences investigated in Bulow and Pfleiderer (1983). We summarize preferences patterns in Table 2 (see Appendix F for details) where parameters α and β are positive scalars.

[8]Super-elasticity is defined as $\alpha \equiv \mathrm{d}\ln\varepsilon(x)/\mathrm{d}\ln x$.
[9]See important properties in Mrazova *et al.*, (2017).

	Demand functions	$r'_h > 0$
CSED	$p\left(x_h\right) = \frac{1}{\lambda_h} e^{-\frac{1}{\alpha\beta} x_h^{\alpha}}$	iff $\alpha > 1$
TLOG	$p(x_h) = \frac{1}{\lambda_h} \frac{\alpha+\beta \log x_h}{x_h}$	iff $\varepsilon(x_h) < 3/2$
CREMR	$p\left(x_h\right) = \frac{1}{\lambda_h x_h} (x_h - \beta)^{\frac{\alpha}{\alpha+1}}$	no
CPPT	$p(x_h) = \frac{1}{\lambda_h x_h} (x_h^{-\alpha} + \beta)^{-\frac{1}{\alpha}}$	iff $\alpha < 1$
CEMR	$p\left(x_h\right) = \frac{1}{\lambda_h x_h} (x_h^{\frac{\alpha}{1+\alpha}} - \beta)$	yes/no
ITCES	$p(x_s) = \frac{1}{\lambda_h} (x_h^{-\frac{\alpha}{1+\alpha}} - \beta)$	no

Table 2: Properties of demand systems.

Table 2 shows that those preferences have varying patterns of r_h. The general equilibrium effect of mean-preserving changes in income distribution therefore depends not only on the demand system but also on their parameterizations.

3.2.2 Welfare

The welfare impact of the above changes can be determined as follows. Because goods are symmetric, the welfare of an individual with income s_h is given by $U_h = nu(x_h)$. Log-linearization gives the relative welfare change $\widehat{U}_h = \widehat{n} + \eta_h \widehat{x}_h$, which rises with higher product diversity and consumption levels. Recall that love for variety is given by $1 - \eta_h$ so that η_h measures the dislike for variety. Thus, in the previous expression, consumption is weighed by the statistics on dislike for variety η_h, defined in (5). Product diversity has a higher weight compared to consumption for stronger love for variety. Under Pollak preferences, the welfare implication is trivial: an increase in an individual's income results in welfare gains through higher individual consumption solely.

Beyond Pollak, using (18), welfare changes can be expressed as

$$\widehat{U}_h = \widehat{s}_h \eta_h + \varepsilon \left(1 - \eta_h - \frac{1}{\varepsilon}\right) \widehat{p}. \tag{21}$$

The first term reflects the direct effect on utility from the change in individual income $\widehat{s}_h$ while the second term represents the total general equilibrium effect stemming from market changes. For the clarity of exposition, suppose that $r'_h > 0$ so that the mean-preserving contraction raises prices, $\widehat{p} > 0$. On top of the changes in price $\widehat{p}$, the general equilibrium effect depends on the sign of $1 - \eta_h - 1/\varepsilon$. Under the combination of subconvex demands and aligned preferences, $\varepsilon'_h < 0$ and $\eta'_h < 0$, individuals become less sensitive to price and more sensitive to the number of varieties when they become richer and consume more. Then, $\eta'_h < 0$ implies that there exists a consumption level $\bar{x}$ such that $1-\eta(x_h) \leq 1/\varepsilon$ if and only if $x_h \leq \bar{x}$, or equivalently $s_h \leq \bar{s} \equiv \bar{x}/(np)$, where $\bar{x}$ solves $1-\eta(\bar{x}) = 1/\varepsilon$. This creates a negative general equilibrium effect for the individuals with income lower than $\bar{s}$ and a positive effect for the others. The effect could be negative for all individuals if $\bar{s} > s_1$. However, the effect is never positive for all individuals because $\bar{s} < s_0$ does not hold. Indeed, some lines of computation show that

$$\frac{\mathrm{d}\eta(x_h)}{\mathrm{d}x_h} < 0 \iff 1-\eta(x_h) < \frac{1}{\varepsilon(x_h)}, \tag{22}$$

while $\varepsilon(x_0) > \varepsilon > \varepsilon(x_1)$ since $\varepsilon(x_h)$ is a decreasing function and $x_0 < x_1$. The last two sets of conditions imply that $1-\eta(x_0) < 1/\varepsilon(x_0) < 1/\varepsilon$. Therefore, the poorest individual with consumption x_0 is always harmed by negative general equilibrium effect.

This result has policy implications. If an income redistribution policy targets only a fraction of poor individuals, it may harm those who are not targeted. For example, if a redistribution policy transfers income from the highest to the lowest income decile, leaving other deciles unchanged, it leads to losses for middle income deciles due to the negative general equilibrium effect. Similarly, the general equilibrium effect of a redistribution policy may widen the welfare gap between the poorest and richest individuals if the latter are not affected by the transfers.

Proposition 4 *Assume subconvex demands and aligned preferences ($\varepsilon_h' < 0$ and $\eta_h' < 0$). If $r_h' > 0$, the general equilibrium effect of mean-preserving contraction of income distribution on welfare is negative at least for the poorest households. It decreases welfare of all income groups if and only if $1-\eta(s_1) < 1/\varepsilon$. The opposite holds for $r_h' < 0$.*

3.3 Generic redistribution

Consider now an arbitrary transformation in income distribution. This is equivalent to a sequence of two transformations: a transformation a with proportional change in all income levels and a transformation b that preserves its mean. Formally, this is defined as $\widehat{s}_h = \widehat{s}_h^a + \widehat{s}_h^b$ where $\widehat{s}_h^a = \widehat{s}$ is the average income change and $\widehat{s}_h^b$ is a mean-preserving change such that $\int_{s_0}^{s_1} \widehat{s}_h^b s_h \mathrm{d}G = 0$.

Under the first transformation, every individual is affected by the same proportional income change. Since $\widehat{s}_h^a = \widehat{s}$, (13)-(14) boil down to

$$\widehat{p}^a = \widehat{x}_h^a = \widehat{y}^a = 0 \quad \text{and} \quad \widehat{n}^a = \widehat{s}. \tag{23}$$

In words, any proportional change has no impact on prices and therefore consumption and firm output. It however affects the number of firms because it changes the total labor supply. Also, total output and GDP move in proportion to the change in average income because $\widehat{Y}^a = \widehat{n}^a + \widehat{y}^a = \widehat{s}$ and $\widehat{G}^a = \widehat{p}^a + \widehat{n}^a + \widehat{y}^a = \widehat{s}$.

Since the total change in variables is a sum of two components, the total changes in individual consumption, price and variety are as follows:

$$\widehat{x}_h = \widehat{x}_h^a + \widehat{x}_h^b = \widehat{x}_h^b, \qquad \widehat{p} = \widehat{p}^a + \widehat{p}^b = \widehat{p}^b, \qquad \widehat{n} = \widehat{n}^a + \widehat{n}^b = \widehat{s} + \widehat{n}^b. \tag{24}$$

Therefore, *the impact on prices and consumption is driven only by its mean-preserving change. The impact on product diversity results from both the mean-preserving change and proportional component.*

The welfare changes under the arbitrary transformation are given by $\widehat{U}_h = \widehat{n} + \eta_h \widehat{x}_h$. Using (18) and (24), we get

$$\widehat{U}_h = \widehat{s}^a + \eta_h \widehat{s}_h^b + \varepsilon(1 - \eta_h - 1/\varepsilon)\widehat{p}^b.$$

The only difference with mean preservation is the first term on the right-hand side, $\widehat{s}^a$. This reflects the positive general equilibrium effect of larger average income on firm creation and product diversity.

The above analysis can be applied to the assessment of tax reforms. Decomposition of welfare changes shows that the effect of tax reforms must be broken down between the effects of tax revenue and tax progressivity. Suppose indeed that the government collects a tax revenue $T \mathrm{d}\xi$ by applying an average tax rate $\tau_h \mathrm{d}\xi$ to individual h where $\mathrm{d}\xi > 0$ is an infinitesimally small scalar, $\tau_h \equiv \tau(s_h)$ is the average tax rate and the tax revenue is proportional to $T \equiv \int s_h \tau_h \mathrm{d}G > 0$. The tax paid is given by $s_h \tau_h \mathrm{d}\xi$ so that the individual's net revenue is equal to $s_h (1 - \tau_h \mathrm{d}\xi)$. The tax is progressive if the average tax rate increases with income, $\tau_s' > 0$, regressive otherwise and neutral on income distribution if $\tau_s' = 0$. In this context, the relative changes in average and individual incomes are given by $\widehat{s} = -(T/s)\,\mathrm{d}\xi$ and $\widehat{s}_h = \tau_h \mathrm{d}\xi$. The first transformation a is a multiplicative shift of income given by $\widehat{s}^a = \widehat{s} = (-T/s)\,\mathrm{d}\xi < 0$. It corresponds to a neutral tax policy that raises the tax revenues $T\mathrm{d}\xi$. It reduces the utility of all individuals proportionally by the same amount. The second transformation b is given by $\widehat{s}_h^b = \widehat{s}_h - \widehat{s}^a = -(\tau_h - T/s)\,\mathrm{d}\xi$ which is a mean-preserving contraction of income distribution if the tax rate is progressive.[10] This second transformation is the general equilibrium effect

[10] Indeed, $\widehat{s}_h^b$ is a mean-preserving contraction of income distribution if $\int_{s_0}^{s} \widehat{s}_h s_h \mathrm{d}G \geq 0$; that is, if $\int_{s_0}^{s} (\tau_h - T/s)\, s_h \mathrm{d}G \leq 0$. Since the left-hand side of the last expression is nil at $s_h = s_0$ and $s_h = s_1$, it is

implied by tax progressivity. By Proposition 4, a progressive marginal tax reform increases the equilibrium price $\widehat{p}^b$ and reduces the welfare of poor groups of households if $r_h' > 0$. This shows that the overall general equilibrium effect of this tax policy worsens the welfare of - at least - the lowest income group. For a large enough multiplicative shifter $\widehat{s}^a$, those effects are negative for all income groups. This discussion shows that, besides a direct tax effect on income, there is an additional negative general equilibrium effect going through product market. Furthermore, the general equilibrium effect on welfare can be negative despite the progressive tax scheme reducing income inequality.

4 Social optimum

In this section we compare the market equilibrium and socially optimal allocation. Since firms have the same cost structure, they have the same output in the social optimum. Therefore, we study the problem of a utilitarian planner who chooses the individual consumption levels and the mass of varieties that maximize the total welfare W under the resource constraint:

$$W \equiv Ln \int u(x_h)\mathrm{d}G, \quad \text{s.t.} \quad L \int s_h \mathrm{d}G = n(f + cL \int x_h \mathrm{d}G).$$

The resource constraint balances the total endowment of labor units with their use in the production system.

Pointwise maximizing the Lagragian function of this problem and eliminating the Lagrange multiplier give the following first-order condition

$$u'(x_h) = \frac{cL \int u(x_l)\mathrm{d}G_l}{f + cL \int x_l \mathrm{d}G_l}.$$

Since the right hand side is independent of individual income s_h, each individual receives the same consumption quantity $x_h = x^o \ \forall h$. This is because the planner considers the total endowment rather than individual endowments of labor units. Plugging this value into the last equation and using the definition (5) of $\eta(x_h)$, the optimal consumption level is given by

$$1 - \eta(x^o) = \frac{f}{f + cLx^o}. \tag{25}$$

The planner therefore balances love for variety with the share of the cost of creating a variety

must be negative if and only $\tau_h' > 0$. The second transformation b is therefore a mean-preserving contraction of income distribution when the tax is progressive.

in the total cost. Because $\eta(x) \in (0, 1)$, equation (25) always accepts an interior solution x^o. The optimal number of firms is given by

$$n^o = \frac{Ls}{f + cLx^o}. \tag{26}$$

We now compare this allocation to the market equilibrium. Towards this goal, we simplify and rewrite the market equilibrium equations (10)-(11) as

$$\frac{1}{\varepsilon} = \frac{f}{f + cLx} \quad \text{and} \quad n = \frac{Ls}{f + cLx}, \tag{27}$$

where n is the equilibrium number of firms, ε is the equilibrium demand elasticity and $x \equiv \int x_h \mathrm{d}G$ is the equilibrium *average* consumption. Comparing (27) to (25) implies that $x^o > x$ and $n^o < n$ if and only if

$$1 - \eta(x^o) < 1/\varepsilon. \tag{28}$$

Since $x = y/L$, condition (28) implies that, at the equilibrium, firms produce insufficient output while the market provides too much product diversity. Intuitively, too many varieties are produced in equilibrium if consumers express weak love for variety $1-\eta(x^o)$. Furthermore, lower demand elasticity ε relaxes this condition and raises the number of equilibrium varieties further because it increases firms' market power, profits and incentives to enter the market. Condition (28) nevertheless depends on two statistics, x^o and ε, the first being optimal consumption level while the second being market elasticity at the equilibrium. The following proposition clarifies the properties of aligned preferences that satisfy condition (28) (see Appendix G for the proof).

Proposition 5 *Under subconvex demands and aligned preferences ($\varepsilon_h' < 0$ and $\eta_h' < 0$), the market equilibrium implies excessive entry and under-production if r_h remains constant or decreases with income levels s_h. If r_h increases, the market equilibrium implies excessive entry and under-production for low income heterogeneity, but results in insufficient entry and over-production under high heterogeneity of income.*

Proposition 5 concurs with the previous welfare statements about monopolistically competitive industries producing for homogeneous consumers. In particular, Dixit and Stiglitz (1977) highlight the presence of excess entry with homogenous individuals ($s_h = s$) and decreasing η_h. In the same vein, Dhingra and Morrow (2019) demonstrate the presence of excess entry for workers with homogenous incomes and aligned preferences ($\eta_h' < 0$). This results from homogenous workers having the same consumption ($x = x_h$) and same demand elasticities

$\varepsilon(x) = \varepsilon(x_h) = \varepsilon$ while aligned preferences imply $1 - \eta(x) \leq 1/\varepsilon(x)$, which is consistent with condition (28) (see Appendix G). Proposition 5 extends the property of excess entry under income heterogeneity *only for* $r_h' < 0$. In this case, stronger income heterogeneity strengthens condition (28).

However, if r_h is an increasing function, higher income heterogeneity relaxes the condition (28). Therefore, there exists a level of income heterogeneity such that the *market equilibrium yields the optimal product diversity.* When income heterogeneity further increases beyond this threshold, n falls below n^o, so that the market provides too few varieties while each firm's output is too high.

To illustrate this result, consider two income groups, s_1 and s_2, and preferences with constant super-elasticity where parameter $\alpha = 2$. In this case, $r_h' > 0$ (see first line in Table 2). Therefore, a mean-preserving spread of income distribution increases the market elasticity ε so that the right-hand side of condition (28) decreases and eventually becomes lower than its left-hand side. In particular, it happens when income dispersion among two groups is sufficiently large, $s_2/s_1 = 3$.

This discussion shows that the representative consumer concept is not an innocuous assumption for providing the market regulation policy.

5 Trade

The monopolistic competition framework is widely applied in trade models, in particular, with a combination of CES preferences. Whereas trade patterns are neutral to income heterogeneity under the CES, they may significantly differ under various forms of nonhomothetic preferences. To capture the sole effects of income distribution, we focus our discussion on two symmetric countries, home and foreign, with identical preferences, populations and cost structures. By doing so, we exclude possible effects of other types of country asymmetries. We study the impacts of changes in income distribution of a country on the trade patterns as well as on the markets and individual welfare in both countries. This exercise differs from the closed economy by the existence of separate (home and foreign) markets for each variety and labor force. Thus, changes in income distribution in a country give rise to asymmetric economic outcomes in two countries.

Population sizes of both countries are denoted by L while the distributions of individual incomes are denoted by G and $G^* : [s_0, s_1] \rightarrow [0, 1]$, where the asterisks refer to the variables of the foreign country. A home country individual consumes a set of home and foreign varieties $\omega \in$

$[0, n]$ and $\omega^* \in [0, n^*]$ where n and n^* are the masses of varieties produced in each country. She purchases the quantities $x_h(\omega)$ and $i_h(\omega^*)$ of the domestically produced and imported varieties at the home prices $p(\omega)$ and $p_i(\omega^*)$. She maximizes her utility $\int_0^n u(x_h(\omega))\mathrm{d}\omega + \int_0^{n^*} u(i_h(\omega^*))\mathrm{d}\omega^*$ subject to her budget constraint $\int_0^n p(\omega)x_h(\omega)\mathrm{d}\omega + \int_0^{n^*} p_i(\omega^*)i_h(\omega^*)\mathrm{d}\omega^* = s_h w$ where s_h is her individual income and w is the home wage per labor unit. The first-order conditions lead to inverse demand functions $p(\omega) = \lambda_h^{-1}u'(x_h(\omega))$ and $p_i(\omega^*) = \lambda_h^{-1}u'(i_h(\omega^*))$, where λ_h is her budget constraint multiplier. As before, by symmetry of varieties, we can drop the variety indices ω and ω^*. A consumer in the foreign country makes a similar choice of local and import consumption (x_h^*, i_h^*) given the prices (p^*, p_i^*) she faces there.

Under monopolistic competition and market segmentation, the home firm chooses its local and export prices, p and p_i^*, that maximizes its profit

$$\pi = L\int (p - cw)\, x_h \mathrm{d}G + L\int (p_i^* - cw)\, i_h^* \mathrm{d}G - fw.$$

The optimal prices are given by

$$p = \frac{\varepsilon}{\varepsilon - 1}cw \quad \text{and} \quad p_i^* = \frac{\varepsilon_i^*}{\varepsilon_i^* - 1}cw,$$

where

$$\varepsilon = \frac{\int x_h \varepsilon(x_h)\mathrm{d}G}{\int x_h \mathrm{d}G} \quad \text{and} \quad \varepsilon_i^* = \frac{\int i_h^* \varepsilon(i_h^*)\mathrm{d}G}{\int i_h^* \mathrm{d}G},$$

while $\varepsilon(x_h)$ and $\varepsilon(i_h^*)$ are the price elasticities of the individual's demand at domestic and foreign markets. The prices must be positive so that $\varepsilon > 1$ and $\varepsilon_i^* > 1$. Similar definitions and properties hold for foreign producers (p^*, p_i, ε^* and ε_i).

The trade equilibrium is defined as the set of variables that are consistent with the consumer choice between local and imported goods, the optimal prices set by firms for local and export markets, the firm's optimal entry decision and the clearing conditions of product and labor markets. Equilibrium conditions for the home country are presented in Table 3. Symmetric conditions hold for the foreign country.

Budget	$npx_h + n^* p_i i_h = s_h w$
	$p/p_i = u'(x_h)/u'(i_h)$
Optimal price	$p = \frac{\varepsilon}{\varepsilon-1} cw$
	$p_i^* = \frac{\varepsilon_i^*}{\varepsilon_i^*-1} cw$
Entry	$(p - cw)\, y + (p_i^* - cw)\, y_i^* = fw$
Product market	$y = L \int x_h \mathrm{d}G$
	$y_i^* = L \int i_h^* \mathrm{d}G$
Labor market	$L \int s_h \mathrm{d}G = n\,(f + c\,(y + y_i^*))$

Table 3: Domestic trade equilibrium conditions.

Market clearing conditions imply that trade balance is satisfied, i.e. $p_i^* y_i^* n = p_i y_i n^*$.

When countries are symmetric in their income distribution, the system collapses to similar equilibrium conditions as for the closed economy. Therefore, the symmetric equilibrium exists under the same equilibrium conditions as in the closed economy (see Appendix H for details).

5.1 Mean-preserving redistribution

We now consider a small mean-preserving contraction of income distribution in the home country. As before, we denote the individual income changes by $\widehat{s}_h \equiv \mathrm{d}\ln s_h = \mathrm{d}s_h/s$ while $\widehat{s} \equiv \frac{1}{s}\int \widehat{s}_h s_h \mathrm{d}G = 0$ under mean preservation. We assume no change in individual income distribution in the foreign country and normalize its wage to one so that $\widehat{s}_h^* = \widehat{s}^* = \widehat{w}^* = 0$. Equilibrium conditions can be log-linearized around the symmetric equilibrium with $G = G^*$ (see Appendix I). Denoting $\Upsilon \equiv \Psi + s\,(\varepsilon - 1)^2 > 0$, we solve them and get the following changes in prices, outputs, masses of firms and home wage:

$\widehat{p} = \widehat{p}_i = -\frac{1}{2\Psi\varepsilon}\left(\varepsilon + \frac{\Psi}{\Upsilon}\right)\int r_h s_h \widehat{s}_h \mathrm{d}G$	$\widehat{p}^* = \widehat{p}_i^* = \frac{1}{2\Psi}\frac{\varepsilon-1}{\varepsilon}\frac{\Psi - s(\varepsilon-1)}{\Upsilon}\int r_h s_h \widehat{s}_h \mathrm{d}G$
$\widehat{x}_h = \widehat{i}_h = \widehat{s}_h + \widehat{y}$	$\widehat{x}_h^* = \widehat{i}_h^* = \widehat{y}^*$
$\widehat{y} = \widehat{y}_i = \frac{1}{2\Psi}\left(1 + \frac{\Psi}{\Upsilon}\right)\int r_h s_h \widehat{s}_h \mathrm{d}G$	$\widehat{y}^* = \widehat{y}_i^* = \frac{1}{2\Psi}\frac{s(\varepsilon-1)^2}{\Upsilon}\int r_h s_h \widehat{s}_h \mathrm{d}G$
$\widehat{n} = \widehat{n}^* = -\frac{1}{2\Psi}\frac{\varepsilon-1}{\varepsilon}\int r_h s_h \widehat{s}_h \mathrm{d}G$	$\widehat{w} = 0$

Table 4: Deviations around the trade equilibrium.

Note that the relative wage w is not affected by the small individual income redistribution ($\widehat{w} = 0$) because trade is initially symmetric between the two countries. This implies that the terms of trade are not affected.

Under subconvex demands, we have $\Psi > 0$ and $\Psi - s(\varepsilon - 1) = -\int \varepsilon_h' x_h s_h \mathrm{d}G > 0$. Given that $\varepsilon > 1$, all coefficients in the above expressions are positive so that the direction of changes is governed by the sign of $\int r_h s_h \widehat{s}_h \mathrm{d}G$. This term is negative for a mean-preserving contraction if and only if r_h is an increasing function of consumption. For the sake of exposition, we assume $r_h' > 0$ in the subsequent paragraphs.

Let us first examine the effect on the home market. Table 4 shows that a mean-preserving contraction raises the home prices and the number of locally produced goods, p and n, while it reduces firm output y and has a negative general equilibrium effect on the individual consumptions (second term in $\widehat{x}_h$). This is consistent with the redistribution effects discussed for the closed economy in sub-section 3.2. Furthermore, the effects are exactly the same on the goods imported to the home country (higher p_i and n^*, lower y_i and i_h). Hence, income redistribution leads to the same composition of the consumption of local and imported varieties in home.

What is the welfare impact in the home country? A home individual has an equilibrium utility $U_h = nu(x_h) + n^* u(i_h)$, which yields a relative welfare change equal to $\widehat{U}_h \equiv \frac{1}{2}(\widehat{n} + \eta_h \widehat{x}_h) + \frac{1}{2}\left(\widehat{n}^* + \eta_h \widehat{i}_h\right)$, where the weights $1/2$ are the symmetric contributions of local and imported varieties to her utility. Applying the result in Table 4 leads to

$$\widehat{U}_h = \eta_h \widehat{s}_h - \frac{1}{2\Psi}\left[1 - \frac{1}{\varepsilon} - \left(1 + \frac{\Psi}{\Upsilon}\right)\eta_h\right]\int r_h s_h \widehat{s}_h \mathrm{d}G. \tag{29}$$

Individuals are directly affected by the changes in their incomes $\widehat{s}_h$ and indirectly through the general equilibrium effect of the income redistribution (second term). Individuals with weaker love for variety (higher η_h) are more likely to face a negative general equilibrium effect on their welfare, as it is the case in the closed economy. Under a combination of subconvex demands and aligned preferences, this negative general equilibrium effect will harm at least the poorest individuals.

Now we discuss the effect on the foreign market. Table 4 shows that the home income redistribution decreases the prices of domestic goods and imported goods, p^* and p_i^*. This leads to a divergence in home and foreign market prices: in particular, prices become relatively higher in the country with lower income inequality. This point is remarkable as *country price differences here are caused by differences in income distribution* and not by the presence of trade costs and/or home bias as emphasized in the literature. Furthermore, the fall in foreign prices entices foreigners to increase their spending on wider ranges of goods, n and n^*, but consume smaller quantities, x_h^* and i_h^*.

Foreign residents are better off because both domestic and imported prices decrease in the foreign market while product diversity expands. Their gains are however distributed unequally across individuals with different incomes. To be precise, the change in the welfare of foreign individuals is given by

$$\widehat{U}_h^* = \widehat{n}^* + \eta_h^* \widehat{x}_h^*.$$

The first term on the right-hand side is positive while the second one is negative. Therefore, under subconvex demands and aligned preferences, welfare gains will be lower for poorer individuals because their value of η_h^* is higher.

Note finally that a decrease in home income inequality reduces the production scales of all home and foreign firms to both local and export markets, y, y_i^*, y^*, and y_i^*. Hence, a reduction in home income inequality fosters the creation of new varieties worldwide, at the expense of the production and consumption of each of them. In other words, the home income redistribution stimulates extensive margins and mitigates intensive margins. Moreover, Table 4 shows that import volumes y_i and y_i^* fall. The values of traded goods also fall because the changes in export and import values are equal to

$$\widehat{p}_i + \widehat{y}_i = \widehat{p}_i^* + \widehat{y}_i^* = \frac{\varepsilon - 1}{2\Psi\varepsilon} \int r_h s_h \widehat{s}_h \mathrm{d}G < 0.$$

Proposition 6 summarizes this discussion.

Proposition 6 *Assume subconvex demands and two initially symmetric countries. Then, if $r_h' > 0$, a mean-preserving contraction of the home income distribution raises all home prices and diminishes foreign prices. It fosters creation of new varieties and reduction of firm production scales in every country. All foreigners gain. The general equilibrium effect of a home mean-preserving contraction reduces the welfare of at least the individuals with the weakest love for variety. Trade volumes and values fall. The opposite holds for $r_h' < 0$.*

6 Quantification

In the previous sections, we have shown that the general equilibrium effects of income distribution depend on how the statistic r_h varies with income. Table 2 has nevertheless demonstrated that the pattern of this statistic strongly depends on the assumptions on preferences and their parametrization. Whereas the above theoretical study can help determine the absence of such effects and their directions, it does not shed light on the amplitude of those effects. The main purpose of this section is therefore to quantify the general equilibrium effects of income redistribution on product market and individuals' welfare.

Towards this aim, we calibrate our model to the US industry and income distribution. We use a total employment of 148 billion workers and a total number of 2,22 billion firms with more than 5 employees and compute the average employment per firm of 66 workers (US census data, 2015). The average income is 56,516 USD (year 2018). We normalize the quantities of goods such that variable cost is equal to one while we set the fixed cost consistently with the above calibration values and the equilibrium conditions (9)-(11).[11] The worker population is divided in deciles of after-tax incomes (so that the distribution $G(s_h)$ is a discontinuous function with 10 steps). The lowest and highest deciles' incomes are 2,832 USD and 172,358 USD respectively.

6.1 Calibration and demand selection

We first explore how the demand systems in Table 2 match existing market statistics. Each demand system includes two parameters (α, β) to match with two empirical statistics. The first obvious statistic to match is the market elasticity ε. This has been estimated in many studies: it ranges between 6 and 11 and there seems to be a consensus amongst researchers for an estimate about 7 (Head and Ries, 2001; Head and Mayer, 2004; Bergstrand *et al.*, 2013). The second statistic that we propose to match is the pass-through elasticity, defined as

$$\mathcal{E}_{\mathrm{pt}} \equiv \frac{\mathrm{d}\log p}{\mathrm{d}\log c} = 1 + \frac{\mathrm{d}\log}{\mathrm{d}\log c}\left(\frac{\varepsilon}{\varepsilon-1}\right).$$

In our context of income heterogeneity, we differentiate (7) and get

$$\mathcal{E}_{\mathrm{pt}} = \frac{\varepsilon(\varepsilon-1)x}{2\varepsilon(\varepsilon-1)x - \int (r_h - 2)\,\varepsilon_h x_h \mathrm{d}G}. \tag{30}$$

The pass-through elasticity has been estimated in the range between 0.3 and 0.8. For instance, using trade macro data and exchange rates shocks, Campa and Golberg (2005) suggest average values of 0.46 and 0.64 in the short and long term. Amiti *et al.* (2019) also suggest 0.6 based on Belgian micro-level manufacturing data. Using Indian firm-level production data, De Loecker *et al.* (2016) find it in the range $[0.3, 0.4]$ while Mion and Jacob (2020) find values about 0.8 with French manufacturing firm information. To reflect this disparity, we will match two target pairs of values $(\varepsilon, \mathcal{E}_{\mathrm{pt}}) = (7, 0.4)$ and $(\varepsilon, \mathcal{E}_{\mathrm{pt}}) = (7, 0.6)$.

[11] Solving (9)-(11), one gets $p = \varepsilon/(\varepsilon-1)$, px =(employment per firm*average income)/(total employment), n =(total employment)/(employment per firm), and f =(employment per firm*average income)/ε. Those values are consistently adjusted for the elasticity ε, which is determined by the demand parameters (α, β).

To match those target elasticities, we use equations (10) to (11) to compute the equilibrium price, number of firms and fixed costs as a function of the market demand elasticity ε. Using equation (9), we compute the consumption of each decile x_h as a function of ε. From (7), $\varepsilon = \int x_h \varepsilon_h \mathrm{d}G / \int x_h \mathrm{d}G$ is itself a function of individual elasticities ε_h weighted by the equilibrium consumption x_h. We solve for the fixed point to recover the equilibrium market demand elasticity ε, which is then used to get the equilibrium price p and consumption levels x_h. We make sure that the equilibrium exists by checking condition (8).

The preferences proposed in Table 2 add two restrictions to the calibration process. Some utility functions are indeed defined on supports that do not include zero consumption and/or are not concave functions everywhere on their supports. In the context of income heterogeneity, this implies that strong income discrepancies might not be possible for calibration because consumption levels of the lowest income individuals would lie below the support where utility is defined and concave. Furthermore, the absence of concavity implies that the lowest income individuals may not express love for variety. In particular, condition (5) may not be maintained so that low income individuals refrain from consuming all varieties and the fixed point computation does not lead to an equilibrium.

We first take an extensive set of random draws for the parameter pairs (α, β) and apply them to each demand class in Table 2. We then search for the parameter values that match the target $(\varepsilon, \mathcal{E}_{\mathrm{pt}})$. Figure 1 summarizes the sets of elasticity pairs $(\varepsilon, \mathcal{E}_{\mathrm{pt}}) \in (1, 8) \times (0, 1)$ that are supported by parameters (α, β) for each of the six preference classes presented in Table 2. We briefly discuss each one. First, constant super-elasticity demands (CSED) are displayed in the (background) white color. Figure 1 shows that they support any elasticity pair with two parameters so that they match the target elasticity values.

Second, inverse translog demands (TLOG) are displayed by the red curve. They yield demand elasticities that are lower than 2 and cannot match the target pairs of elasticity values. The reason is that the number of US firms implies a high product diversity and, consequently, low consumption levels, while inverse translog demands have low individual elasticity at low consumption levels. In what follows, we exclude this demand system from our quantification exercise.

Third, demands with constant revenue elasticity of marginal revenue (CREMR) are displayed by the black area. They are supported by parameters only for pass-through elasticities close to 1 and cannot support the target pairs of elasticity values. Those utility functions are not concave everywhere and therefore do not guarantee that lower income individuals consume all available goods. We also exclude those from our quantification exercise.

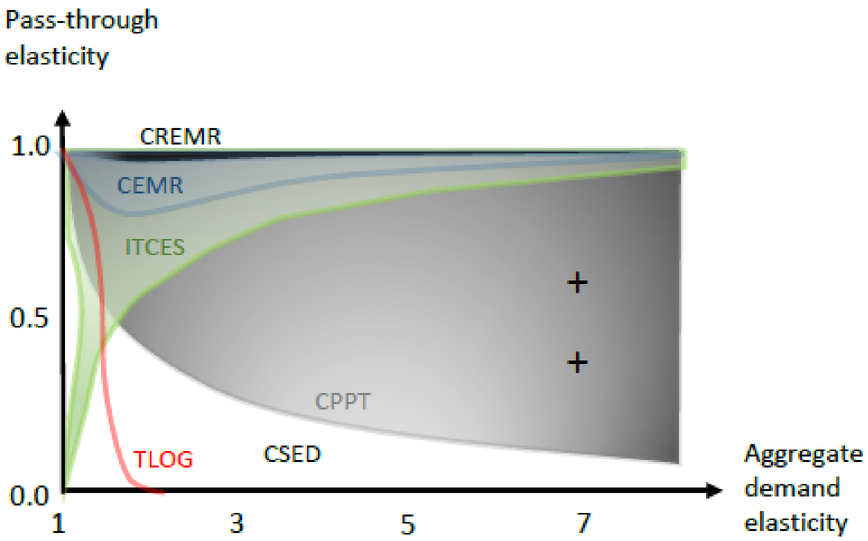

Figure 1: Feasible demand and pass-through elasticities

Fourth, demands with constant proportional pass-through (CPPT) are presented with gray color. They support pairs of large enough elasticities ε and $\mathcal{E}_{\mathrm{pt}}$, and in particular the target pairs of elasticities. In general, they are suited to reproduce economies with demand elasticity ε larger than 3 and elasticity of pass-through larger than 0.4, which is consistent with empirical studies. Note that $\mathcal{E}_{\mathrm{pt}} \leq 1/2$ if and only if $\alpha \geq 1$ (see Appendix F).

Fifth, demands with constant elasticity of marginal revenue (CEMR) support a set of elasticities displayed by the blue area. They only support pass-through elasticities larger than 0.8 and therefore do not encompass the target pairs of elasticities. As the CREMR utility, those functions hardly guarantee that lower income individuals consume all available goods. Finally, demands with inverse translated CES (ITCES) are presented in green color. They support low demand elasticities and high pass-through elasticity. Figure 1 shows that they do not support the target pairs of elasticities and are unsuited for the calibration exercise.

To sum up, only CSED and CPPT are well suited to reproduce our target values of demand and pass-through elasticities in the context of a production economy and income distribution like the US. As Figure 1 shows, these demand systems are robust to reasonable changes in the target values. The other demand systems produce either insufficient demand elasticities or excessive pass-through elasticities, or they may be incompatible with the assumption that all consumers buy all available varieties. Note that they might be better suited to replicate the economies with lower income inequality than the US'.

6.2 Income redistribution

We now examine the effect of income distribution on economic variables and individual welfare. To make things simple, we simulate the redistribution from the top to the bottom decile that raises the latter by 300%. This process represents a mean-preserving contraction of the income distribution and increases the bottom decile income to 11 328 USD, which is slightly lower than the income of the second to bottom decile. The total transfer involves about 1.5% of total income. We make demand systems comparable by fixing the elasticities of market demand and pass-through to the target values $(\varepsilon, \mathcal{E}_{\mathrm{pt}}) = (7, 0.4)$ and $(7, 0.6)$.

The effects of this redistribution are presented in Table 5 for the CPPT and CSED demand systems. The two top rows present the demand parameters α and β matching the target elasticities before income redistribution while the third and fourth rows report the target values. The next three rows give the percent change in price, number of firms and firm output, compared to the initial situation. In order to preserve consistency among different demand systems, we report the welfare changes as 'consumption equivalent' for each decile. The consumption equivalent x_h^{eq} is defined as the consumption level that gives the same utility as the one obtained at the initial price and number of goods. In other words, x_h^{eq} is such that $n_0 u(x_h^{\mathrm{eq}}) = n_1 u(x_h^1)$ where subscripts 0 and 1 refer to the initial and final allocation, respectively.

The first column in Table 5 indicates the direct effect of redistribution; that is, the changes when prices and firms do not adjust to the redistribution. Other columns indicate the general equilibrium effects, net of the direct redistribution effect from top to bottom decile and for each set of preferences and parameter values. Magnitudes are reported in percentage points (%). The direct effect causes the bottom decile to gain 300% and the top decile to lose 4.95% of the consumption equivalent.

The second column reports the effects of the above redistribution with CPPT preferences matching $(\varepsilon, \mathcal{E}_{\mathrm{pt}}) = (7, 0.4)$. These elasticities are reached with the demand parameters $\alpha = 1.11$ and $\beta = 13.62$. As shown in Table 2, $\alpha > 1$ implies that r_h is an increasing function. The mean-preserving contraction of income distribution entices firms to increase their prices by 0.30%, decrease their production by 2.17% and, in the end, enter the market with an additional 1.85% of firms, as predicted by Proposition 3. Therefore, the general equilibrium effect leads to a reduction of the consumption equivalent between 0.31% and 0.05% from the first to the ninth decile and to a rise in the consumption equivalent for the top decile. Lower deciles are more negatively affected by the general equilibrium effect. This is because, by (21), the welfare weight $1 - \eta(s_h) - 1/\varepsilon$ takes less negative values as income rises and reverts to a positive value for top income individuals (see Proposition 4). This calibrated example confirms that general equilibrium effect may work in opposite directions for different income groups. Finally, recall that this income redistribution involves a transfer of 1.5% of the total US income. The changes in prices, production and product diversity have the same order of magnitude. The changes in consumption equivalent are slightly lower but still significant. Thus, *general equilibrium effects cannot be considered as negligible.*

	Direct effect	General equilibrium effects			
		CPPT		CSED	
α		1.11	0.82	1.06	0.76
β		13.61	3.88	0.10	0.39
ε		7	7	7	7
$\mathcal{E}_{\text{pt}}$		0.4	0.6	0.4	0.6
$\widehat{p}$ (%)	0	0.30	-0.26	0.18	-0.33
$\widehat{n}$ (%)	0	1.85	-1.52	1.08	-1.95
$\widehat{y}$ (%)	0	-2.17	1.76	-1.27	2.26
$\widehat{x}_1^{\text{eq}}$ (%)	300	-0.31	0.24	-0.18	0.30
$\widehat{x}_2^{\text{eq}}$ (%)	0	-0.29	0.21	-0.17	0.27
$\widehat{x}_3^{\text{eq}}$ (%)	0	-0.26	0.19	-0.16	0.25
$\widehat{x}_4^{\text{eq}}$ (%)	0	-0.24	0.17	-0.15	0.23
$\widehat{x}_5^{\text{eq}}$ (%)	0	-0.22	0.16	-0.13	0.21
$\widehat{x}_6^{\text{eq}}$ (%)	0	-0.20	0.14	-0.12	0.19
$\widehat{x}_7^{\text{eq}}$ (%)	0	-0.16	0.12	-0.10	0.16
$\widehat{x}_8^{\text{eq}}$ (%)	0	-0.12	0.09	-0.08	0.13
$\widehat{x}_9^{\text{eq}}$ (%)	0	-0.05	0.05	-0.03	0.07
$\widehat{x}_{10}^{\text{eq}}$ (%)	-4.95	0.16	-0.06	0.13	-0.09

Table 5: Effects of income redistribution in a closed economy.

The third column reports the effect with CPPT preferences and equilibrium elasticities $(\varepsilon, \mathcal{E}_{\text{pt}}) = (7, 0.6)$. The value of $\alpha = 0.82 < 1$ ensures that r_h is a decreasing function. In this case, the mean-preserving contraction of income has exactly the opposite effect. As stated by Proposition 3, income redistribution entices firms to reduce their prices and raise their production while entry falls. The general equilibrium effect of redistribution increases the consumption equivalent in all deciles except the top decile. The reason for the opposite behavior is the same to the previous paragraph.

The effects of income redistribution under CSED preferences are reported in columns 4 and 5. They have the same directions and similar amplitudes compared to the CPPT preferences. Differences in pass-through elasticity lead to more asymmetric effects. Again, those demands feature opposite behaviors of market aggregates and individual welfare according to the value of pass-through elasticity $\mathcal{E}_{\text{pt}} \in \{0.4, 0.6\}$. Since both values are supported by the empirical literature, this exercise highlights the importance of an accurate empirical assessment of the pass-through elasticity for the welfare impact of income redistribution.

To sum up, CPPT and CSED preferences yield similar and non negligible effects of income redistribution on prices, consumption and welfare. The direction of those effects depends crucially on the pass-through elasticity.

6.3 Trade

We finally study the quantitative impact of income redistribution in the presence of trade. Towards this aim, we divide the economy explored in sub-section 6.2 into two trading symmetric countries - home and foreign - and apply the same mean-preserving contraction of income redistribution in the home country only. This division strategy makes the open economy comparable to the above closed economy because it yields the same demand and pass-through elasticities at identical parameter values and also the same pattern of the statistics r_h around the symmetric equilibrium. So, what we study here is the effect of the division of a unique labor and product market into symmetric independent markets. Table 5 presents the prices, product diversity, firm output and individual welfare for the CPPT and CSED preferences calibrated for the target elasticities $(\varepsilon, \mathcal{E}_{\text{pt}}) = (7, 0.4)$ and $(7, 0.6)$. Rows and columns are organized as in the previous subsection.

For the sake of conciseness, let us consider CPPT preferences with $(\varepsilon, \mathcal{E}_{\text{pt}}) = (7, 0.4)$, which implies that $r'_h > 0$ (second column of Table 6). As predicted by theory, the mean-preserving contraction of the home income distribution raises all home prices and diminishes foreign prices. It also fosters the creation of new varieties and the reduction of firm production scales in each country. Compared to the closed economy, the home income redistribution raises home prices by 0.43% in the trade economy whereas it increased them only by 0.30% in the integrated market. Therefore the effect on home prices is about half as strong. Foreign prices move with a milder amplitude by 0.13% in the opposite direction. Hence, the home income redistribution leads to a price difference of 0.56% between the two countries. The home price hike allows home firms to dampen their output responses by a fall of 1.37% of production instead of 2.17% in the closed economy. By contrast, local product diversity rises by the same amount in both countries and the global product diversity reaches the same value as in the closed economy.

	Direct effect	General equilibrium effect							
		CPPT				CSED			
α		1.11		0.82		1.06		0.76	
β		13.61		3.88		0.10		0.39	
ε		7		7		7		7	
$\mathcal{E}_{\mathrm{pt}}$		0.4		0.6		0.4		0.6	
	home	home	foreign	home	foreign	home	foreign	home	foreign
$\widehat{p}$ (%)	0.	0.43	-0.13	-0.34	0.07	0.26	-0.09	-0.44	0.10
$\widehat{n}$ (%)	0.	0.92	0.92	-0.76	-0.76	0.54	0.54	-0.98	-0.98
$\widehat{y}$ (%)	0.	-1.37	-0.81	1.08	0.67	-0.81	-0.46	1.40	0.86
$\widehat{x}_1^{\mathrm{eq}}$ (%)	300.	-0.44	0.13	0.32	-0.09	-0.27	0.08	0.42	-0.12
$\widehat{x}_2^{\mathrm{eq}}$ (%)	0.	-0.43	0.14	0.31	-0.10	-0.26	0.08	0.4	-0.15
$\widehat{x}_3^{\mathrm{eq}}$ (%)	0.	-0.42	0.15	0.3	-0.11	-0.25	0.09	0.39	-0.15
$\widehat{x}_4^{\mathrm{eq}}$ (%)	0.	-0.41	0.16	0.28	-0.12	-0.25	0.10	0.38	-0.16
$\widehat{x}_5^{\mathrm{eq}}$ (%)	0.	-0.4	0.17	0.28	-0.13	-0.24	0.10	0.37	-0.17
$\widehat{x}_6^{\mathrm{eq}}$ (%)	0.	-0.38	0.18	0.27	-0.15	-0.23	0.11	0.36	-0.18
$\widehat{x}_7^{\mathrm{eq}}$ (%)	0.	-0.37	0.2	0.26	-0.15	-0.23	0.12	0.35	-0.19
$\widehat{x}_8^{\mathrm{eq}}$ (%)	0.	-0.35	0.22	0.25	-0.16	-0.21	0.13	0.33	-0.21
$\widehat{x}_9^{\mathrm{eq}}$ (%)	0.	-0.31	0.26	0.23	-0.18	-0.19	0.15	0.31	-0.24
$\widehat{x}_{10}^{\mathrm{eq}}$ (%)	-4.95	-0.21	0.36	0.18	-0.24	-0.11	0.24	0.23	-0.32

Table 6: Effects of home income redistribution in a open economy.

Since home consumers face higher home prices, the general equilibrium effect reduces their welfare. Table 6 shows that the second lowest decile of home workers reduce their consumption equivalent by 0.43% in the open economy instead of 0.29% in an integrated market. In the trade economy, the richest home individual however does not benefit from a positive general equilibrium effect as in the closed economy. Because foreigners face lower prices, they get higher welfare. It is evident that welfare effects are greater for the poorer home and the richer foreign individuals. Interestingly, the relative consumption equivalent losses of the poorest home individuals have the same magnitude as the corresponding gains of the richest foreigners. Finally, changes in trade values are given by $\widehat{p}_i + \widehat{y}_i = 0.43 - 1.37 = -0.94\%$. This is a significant change with regard to the transfer of 1.5% of total income in the home country. Similar effects can be observed for the CSED preference yielding the same elasticities. Opposite effects take place in economic contexts with pass-through elasticities $\mathcal{E}_{\mathrm{pt}}$ equal to 0.6. To sum up, in an open economy, income redistribution in a country significantly affects prices, output, individual welfare and import-export values in both countries.

7 Conclusion

In this paper, we study the role of income distribution on market aggregates, welfare and trade structure. Since the literature is scant on the effects of income distribution, our main contribution is determining the impact of income redistribution on product markets, individual welfare, and trade patterns.

We show that the general effects of income redistribution hinge on the behavior of the convexity of the direct demand function. Opposite patterns arise depending on whether this convexity is an increasing or decreasing function of individual consumption. Thus, to understand the direction of the general equilibrium effects of income redistribution, more empirical work is needed. However, estimations of the convexity of direct demand at the individual level would probably be a challenging task. On the bright side, for several known classes of demand systems, the behavior of this convexity is determined by the pass-through elasticity, in particular, whether it is larger or smaller than 0.5. What is more, different empirical studies report pass-through elasticities both above and below 0.5. More positively, our theoretical results suggest a one-to-one correspondence between general equilibrium effects and price variation. Thus, the estimation of the relationship between prices and income inequality could be informative in regards to the general equilibrium effects, which would permit us to quantify the general equilibrium effects of income inequality on product markets and individual welfare. Furthermore, such an estimation would allow policy makers to adjust their redistributive strategies, taking into account the general equilibrium consequences for households in different income groups.

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Appendices

Appendix A. Second order condition

In this Appendix, we discuss the second order condition (8), $\Lambda \equiv \int (2\varepsilon - r_h)\varepsilon_h x_h \mathrm{d}G > 0$. This clearly holds for $r_h < 0$. We now show that this condition holds for subconvex demands $\varepsilon_h' < 0$ with $r_h > 0$ and $r_h' > 0$.

Consider first the behavior of the function $\Psi \equiv \int (2\varepsilon_h - r_h)\, x_h \mathrm{d}G$ when $\varepsilon_h' < 0$. Using (3), we have $\Psi > 0$ because

$$\Psi = \int (\varepsilon_h - 1 - \varepsilon_h' x_h)\, x_h \mathrm{d}G = (\varepsilon - 1)x - \int \varepsilon_h' x_h^2 \mathrm{d}G > 0.$$

Furthermore, since $\varepsilon > 0$, we have $\varepsilon\Psi > 0$. Then,

$$\begin{aligned} \varepsilon\Psi &= \int (2\varepsilon\varepsilon_h - \varepsilon r_h) x_h \mathrm{d}G = \int (2\varepsilon - r_h)\varepsilon_h x_h \mathrm{d}G - \int (\varepsilon - \varepsilon_h) r_h x_h \mathrm{d}G \\ &= \Lambda - \int (\varepsilon - \varepsilon_h) r_h x_h \mathrm{d}G > 0, \end{aligned}$$

implies that

$$\Lambda > \int (\varepsilon - \varepsilon_h) r_h x_h \mathrm{d}G.$$

We now prove that the right-hand side of the latter condition, $\int (\varepsilon - \varepsilon_h) r_h x_h \mathrm{d}G > 0$, holds under the conditions $\varepsilon_h' < 0$ and $r_h' > 0$. We can rewrite this condition as $\int r(x_h) f(x_h) \mathrm{d}G > 0$ where $f(x_h) \equiv (\varepsilon - \varepsilon\,(x_h))x_h$. The function $f(x_h)$ is a continuous and increasing since $\varepsilon' < 0$. Note that $x_h = x(s_h)$ is continuous and increasing in s_h: $x'(s_h) > 0$. Integrating by parts gives

$$\begin{aligned} \int_{s_0}^{s_1} r(x(s_h)) f(x(s_h)) \mathrm{d}G(s_h) &= \left[r(x\,(s_h)) \int_{s_0}^{s_h} f(x\,(\xi)) \mathrm{d}G\,(\xi) \right]_{s_h = s_0}^{s_h = s_1} \\ &\quad - \int_{s_0}^{s_1} r'(x\,(s_h)) x'\,(s_h) \left(\int_{s_0}^{s_h} f(x\,(\xi)) \mathrm{d}G\,(\xi) \right) \mathrm{d}G\,(s_h)\,. \end{aligned}$$

The first term vanishes because

$$\int_{s_0}^{s_1} f(x\,(\xi)) \mathrm{d}G\,(\xi) = \int_{s_0}^{s_1} (\varepsilon - \varepsilon\,(x_h)) x_h \mathrm{d}G = \varepsilon x - \int \varepsilon_h x_h \mathrm{d}G = 0 \qquad (31)$$

since $\varepsilon = \int \varepsilon_h x_h \mathrm{d}G / x$ by (7). The second term is positive because the integral $\int_{s_0}^{s_h} f(x\,(\xi)) \mathrm{d}G\,(\xi)$ is negative. Indeed, because $f(x)$ is increasing in x and $x\,(\xi)$ is increasing in ξ, this integral is a convex function of s_h. Since it furthermore has zeroes at $s_h = s_0$ and $s_h = s_1$ by (31), this integral is strictly negative on the interval (s_0, s_1).

Appendix B. Fixed point

The fixed point can be shown as it follows. Note that, using (9), the market elasticity (7) at the equilibrium can be expressed as

$$\varepsilon = \frac{\int \varepsilon_h s_h \mathrm{d}G}{\int s_h \mathrm{d}G}. \tag{32}$$

The optimal price (10), entry and product market (11) conditions imply the following condition for the existence of an equilibrium:

$$\frac{\int x_h \varepsilon_h \mathrm{d}G}{\int x_h \mathrm{d}G} = \frac{cL}{f} \int x_h \mathrm{d}G + 1. \tag{33}$$

Using $z \equiv 1/(np)$ so that $x_h = s_h z$ and using $s = \int s_h dG$, the equilibrium condition writes as

$$\frac{1}{s} \int s_h \varepsilon(s_h z) \mathrm{d}G = \frac{cLs}{f} z + 1,$$

The right-hand side is a function of z that increases above one. The left-hand side lies above one for $z = 0$ if $\varepsilon(0) > 1$. A sufficient condition for an equilibrium is that the left-hand side decreases in z. That is,

$$\frac{d}{dz}\left(\frac{1}{s} \int s_h \varepsilon(s_h z) \mathrm{d}G\right) = \frac{1}{x} \int s_h \varepsilon_h' x_h \mathrm{d}G < 0.$$

The latter certainly holds true if every individual has a subconvex demand, $\varepsilon_h' < 0$.

Appendix C. Log-linearization of closed economy equilibrium

We first log-linearize the FOC (10): $(p - c)/p = 1/\varepsilon$. Using the definition of ε, we write the latter as

$$(p - c) \int x_h \varepsilon(x_h) \mathrm{d}G = p \int x_h \mathrm{d}G$$

and totally differentiate it as

$$\mathrm{d}p \int x_h \varepsilon_h \mathrm{d}G + (p - c) \int (x_h \varepsilon(x_h))' \, \mathrm{d}x_h \mathrm{d}G = \mathrm{d}p \int x_h \mathrm{d}G + p \int \mathrm{d}x_h \mathrm{d}G,$$

Note that $(x_h\varepsilon(x_h))' = -1 + r_h$ by (3) and (4). Using $(p - c) = p/\varepsilon$ by (10) and $\hat{x}_h = \mathrm{d}\ln x_h = \mathrm{d}x_h/x_h$, this yields

$$\widehat{p} = \frac{\mathrm{d}p}{p} = \frac{\int (1+\varepsilon - r_h)\, x_h\hat{x}_h \mathrm{d}G}{\varepsilon(\varepsilon-1)x}.$$

Other conditions (9) and (11) are log-linearized in the same way and yield Table 1. Finally, we can replace $\hat{x}_h$ by its value in Table 1 and simplify the expression of $\widehat{p}$ as

$$\widehat{p} = \frac{\int (1+\varepsilon - r_h)\, x_h \left(\widehat{s}_h - \widehat{s}\right) \mathrm{d}G}{\varepsilon \int (2\varepsilon - r_h)\, x_h \mathrm{d}G}.$$

In the closed economy, the budget constraint $pnx_h = s_h$ gives the consumption levels and changes as $x_h = s_h/pn$ and $\hat{x}_h = \mathrm{d}\ln x_h = \mathrm{d}\ln s_h = \widehat{s}_h$. Note also that $\int (\widehat{s}_h - \widehat{s}) s_h \mathrm{d}G = \int \widehat{s}_h s_h \mathrm{d}G - \widehat{s} \int s_h \mathrm{d}G = \int \mathrm{d}s_h \mathrm{d}G - \widehat{s}s = 0$. Therefore, $\int (1+\varepsilon)\, x_h \left(\widehat{s}_h - \widehat{s}\right) \mathrm{d}G = (1+\varepsilon) \int \left(\widehat{s}_h - \widehat{s}\right) s_h \mathrm{d}G = 0$. The price change simplifies to

$$\widehat{p} = -\frac{1}{\varepsilon\Psi} \int r_h \left(\widehat{s}_h - \widehat{s}\right) s_h \mathrm{d}G,$$

where $\Psi \equiv \int (2\varepsilon - r_h)\, s_h \mathrm{d}G$. This gives (13). Note that Ψ is positive under subconvexity of demand.

Appendix D. Pollak preferences

We characterize the class of utility functions $u(x)$ that solve the differential equation $r(x) = u'(x)u'''(x)/\left(u''(x)\right)^2 = 1 + \sigma$ with $u'' < 0 < u'$. This identity is equivalent to

$$g'/g^2 = \sigma \quad \text{and} \quad u''/u' = g \tag{34}$$

where $g < 0$. We can sequentially solve the first differential equation for g and then the second one for u'. Since utility u is defined up an affine transformation, we report its simplest form.

Consider first $\sigma = 0$. Then, (34) is equivalent to $g' = 0$ and $u''/u' = -\alpha$ where $\alpha > 0$ is a first integration constant. This solves as $u' = \alpha e^{-\alpha(x-x_0)}$ for $x > x_0$ where $x_0 \in \mathbb{R}$ is another integration constant. The utility function u is the integral of the last expression. Since u is defined up an affine transformation, we report the subutility function $u(x) = 1 - e^{-\alpha(x-x_0)}$.

Consider then $\sigma = 1$. Then, (34) accepts the class of solutions $g = -\left(x - x_0\right)^{-1}$ and $u' = k_1 \left(x - x_0\right)^{-1}$ for $x > x_0$ and the integration constant $k_1 > 0$. The utility function u is the integral of the last expression. Since u is defined up an affine transformation, we can report utility function $u(x) = \ln\left(x - x_0\right)$ for $x > x_0 \in \mathbb{R}$.

Consider finally $\sigma \neq 1$, (34) accepts the class of solutions $g = -(x - x_0)^{-\frac{1}{\sigma}}$ and $u' = k_1 (x - x_0)^{1-\frac{1}{\sigma}}$ for $x > x_0$. The utility function u is the integral of the last expression. Since u is defined up an affine transformation and u must be an increasing function, we propose $u(x) = \text{sign}(\sigma - 1) \cdot (x - x_0)^{1-\frac{1}{\sigma}}$ for $x > x_0 \in \mathbb{R}$.

Note that, for $\sigma = -1$, we obtain an affine transformation of the quadratic utility function $u(x) = x(x - x_0)$ for $x > x_0 \in \mathbb{R}$.

Appendix E. Mean-preserving contraction

Consider an initial and final distribution $G^A(s_h)$ and $G^B(s_h)$. B is a mean-preserving contraction of A if and only if A is second-order stochastically dominated by B; that is, iff $\int_{s_0}^{s_h} \left[G^B(z) - G^A(z)\right] \mathrm{d}z \leq 0$ for all s_h. Consider an income mapping $m(s_h)$ such that $s_h^B = s_h^A + m(s_h^A)$, with $1 + m' > 0$ and m close to zero. We have $G^A(s_h) = G^B(s_h + m(s_h))$. So, $\int_{s_0}^{s_h} \left[G^B(z) - G^A(z)\right] \mathrm{d}z = \int_{s_0}^{s_h} \left[G^B(z) - G^B(z + m(z))\right] \mathrm{d}z \simeq -\int_{s_0}^{s_h} m(z)\mathrm{d}G^B(z)$. Hence, the income mapping $m(s_h)$ gives a change in the distribution such that resulting distribution B is a mean-preserving contraction of initial distribution A iff $\int_{s_0}^{s_h} m(z)\mathrm{d}G(z) \geq 0$ for all s_h. In the previous analysis, $m(s_h)$ is equal to $s_h^B - s_h^A$, which is equivalent to $\mathrm{d}s_h = \widehat{s}_h s_h$. So, the change in individual income $\widehat{s}_h$ is associated with a mean-preserving contraction of income distribution iff $\int_{s_0}^{s_h} \widehat{s}_h s_h \mathrm{d}G \geq 0$ for all s_h.

Appendix F. Demand properties

In this appendix we characterize the demand properties of the demand functions proposed in Table 2.

Demands with **constant super-elasticity** are given by $p(x_h) = e^{-\frac{1}{\alpha\beta}x_h^\alpha}/\lambda_h$ with $x \in \mathbb{R}^+$ and $\alpha, \beta > 0$. Note that, for $\alpha = 1$, this matches the demand function under CARA preferences. This implies that $\varepsilon(x_h) = \beta x_h^{-\alpha} > 0$ and $\varepsilon'(x_h) = -\alpha\beta x_h^{-\alpha-1} < 0$, i.e., individual demand is subconvex. Using (3), $r(x_h) = 1 + \varepsilon(x_h) + x_h\varepsilon'(x_h) = 1 + (1 - \alpha)\beta x_h^{-\alpha}$ so that $r(x_h)$ increases if and only if $\alpha > 1$. One computes $u(z) = \int_0^z e^{-z^\alpha} dx - u(0)$. One can numerically check that $\eta(x) = xu'(x)/u(x)$ is a decreasing function of x for all $x, \alpha > 0$. Using (30), the elasticity of pass-through takes the form

$$\mathcal{E}_{\text{pt}} = \frac{x\varepsilon(\varepsilon - 1)}{2\varepsilon(\varepsilon - 1)x + \int \left((\alpha - 1)\beta x_h^{-\alpha} + 1\right)\varepsilon_h x_h \mathrm{d}G}.$$

Thus, $\mathcal{E}_{\text{pt}} < 1/2$ if $\alpha > 1$. Therefore, one gets $\mathcal{E}_{\text{pt}} < 1/2$ and $r'(x_h) > 0$ if $\alpha > 1$.

Translog functions are given by $p(x_h) = (\alpha + \beta \log x_h)/(\lambda_h x_h)$ with $x \in (\exp(-\alpha/\beta), \infty)$ and $\alpha, \beta > 0$. This yields $p'(x_h) = -(\alpha + \beta \log x_h - \beta)/(\lambda_h x_h^2)$, which is negative for $x_h > \underline{x} \equiv \exp\left(1 - \frac{\alpha}{\beta}\right)$. Hence the domain of definition and concavity of $u(x_h)$ is $(\underline{x}, \infty)$. Furthermore, one computes $\varepsilon(x_h) = 1 + \beta/(\alpha + \beta \log x_h - \beta) > 1$ and $\varepsilon'(x_h) = -\beta^2/[x_h(\alpha + \beta \log x_h - \beta)^2] < 0$. Individual demand is therefore subconvex. Using (3), it can be checked that $r(x_h) = 1 + x_h \varepsilon'(x_h) + \varepsilon(x_h) = \varepsilon(x_h)(3 - \varepsilon(x_h))$ so that $r'(x_h) = (3 - 2\varepsilon(x_h))\varepsilon'(x_h)$, which is positive if and only if $\varepsilon(x_h) > 3/2$. Using the definition of $p(x_h)$, we have $u'(x_h) = (\alpha + \beta \log x_h)/x_h$, which integrates to $u(x_h) = \alpha \log x_h + \frac{\beta}{2} \log^2 x_h$. Thus, $\eta(x_h) = x_h u'(x_h)/u(x_h) = (\log x_h)^{-1} + (2\alpha/\beta + \log x_h)^{-1}$ is a decreasing function since it is a sum of two decreasing functions.

Consider the **CREMR** inverse demand function: $p(x_h) = (x_h - \beta)^{\frac{\alpha}{\alpha+1}}/(\lambda_h x_h)$, defined for $x_h \in (\beta, \infty)$ and α, $\beta > 0$. Thus, $p'(x_h) = -(x_h - \beta)^{-\frac{1}{\alpha+1}}(x_h - \underline{x})/(\lambda_h x_h^2(\alpha + 1))$, which is negative if $x_h > \underline{x} \equiv (\alpha + 1)\beta > \beta$. Hence the domain of definition and concavity of $u(x_h)$ is $(\underline{x}, \infty)$. The elasticity of demand is given by $\varepsilon(x_h) = 1 + \alpha x_h (x_h - \underline{x})^{-1} > 1$ and $\varepsilon'(x_h) = -\alpha(\alpha + 1)\beta(x_h - \underline{x})^{-2} < 0$. Individual demand is therefore subconvex. Furthermore, one computes $r(x_h) = 2 + \alpha x_h (x_h - 2\underline{x})(x_h - \underline{x})^{-2}$ and $r'(x_h) = 2\alpha(\alpha + 1)^2 \beta^2 (x_h - \underline{x})^{-3} > 0$. Our simulations also show that $\eta(x_h)$ may decrease or increase depending on the parameters of demand.

Consider **constant proportional pass-through** (CPPT) demand with $p(x_h) = \left(x_h^{-\alpha} + \beta\right)^{-\frac{1}{\alpha}}/(\lambda_h x_h)$ for $x \in \mathbb{R}^+$ and $\alpha, \beta > 0$. Its derivative is given by $p'(x_h) = -\beta\left(x_h^{-\alpha} + \beta\right)^{-\frac{1+\alpha}{\alpha}}/(\lambda_h x_h^2) < 0$. Elasticity of individual demand takes the form $\varepsilon(x_h) = 1 + x_h^{-\alpha}/\beta > 1$ and $\varepsilon'(x_h) = -\alpha x_h^{-\alpha-1}/\beta < 0$. Individual demand is therefore subconvex. Furthermore, $r(x_h) = 2 - (\alpha - 1)x_h^{-\alpha}/\beta$ and $r'(x_h) = (\alpha - 1)\alpha x_h^{-\alpha-1}/\beta$. Thus, $r'(x_h) > 0$ if and only if $\alpha > 1$ while $r'(x_h) < 0$. Using (30), the elasticity of pass-through takes the form

$$\mathcal{E}_{\text{pt}} = \frac{x\varepsilon(\varepsilon - 1)}{2\varepsilon(\varepsilon - 1)x + \frac{\alpha-1}{\beta}\int x_h^{1-\alpha}\varepsilon_h \mathrm{d}G}.$$

Therefore, $\mathcal{E}_{\text{pt}} \le 1/2$ if and only if $\alpha \ge 1$.

Consider the **CEMR** demand functions: $p(x_h) = \left(x_h^{\frac{\alpha}{\alpha+1}} - \beta\right)/(\lambda_h x_h)$ for $x \in (\beta, \infty)$ and $\alpha, \beta > 0$. Thus, $p'(x_h) = -\left(x_h^{\frac{\alpha}{\alpha+1}} - \underline{x}^{\frac{\alpha}{\alpha+1}}\right)/[\lambda_h x_h^2(\alpha + 1)] < 0$ if $x_h > \underline{x} \equiv [(\alpha + 1)\beta]^{\frac{\alpha+1}{\alpha}} > \beta^{\frac{\alpha+1}{\alpha}}$. Hence, those demands are defined and decreasing over the support $(\underline{x}, \infty)$. The elasticity of individual demand is given by $\varepsilon(x_h) = (\alpha + 1)\left(x_h^{\frac{\alpha}{\alpha+1}} - \beta\right)/\left(x_h^{\frac{\alpha}{\alpha+1}} - (\alpha + 1)\beta\right) > 1$ while $\varepsilon'(x_h) = -\alpha^2 \beta x_h^{-\frac{1}{\alpha+1}}\left(x_h^{\frac{\alpha}{\alpha+1}} - (\alpha + 1)\beta\right)^{-2} < 0$. This demand system is therefore subconvex. Furthermore, taking derivative of $r(x_h) = 1 + x_h \varepsilon'(x_h) + \varepsilon(x_h)$ shows that $r'(x_h) \ge 0$ if and only if $x_h \leq \overline{x}$ where $\overline{x} \equiv [(2\alpha+1)(\alpha+1)\beta]^{\frac{\alpha+1}{\alpha}} > \underline{x}$. Integrating $u'(x_h) = \left(x_h^{\frac{\alpha}{\alpha+1}} - \beta\right)/x_h$, we get $u(x_h) = (\alpha+1)x_h^{\frac{\alpha}{\alpha+1}}/\alpha - \beta \log x_h$. Thus, $\eta(x_h) = \left(x_h^{\frac{\alpha}{\alpha+1}} - \beta\right) / \left(\frac{\alpha+1}{\alpha}x_h^{\frac{\alpha}{\alpha+1}} - \beta \log x_h\right)$ which decreases for large values of x_h while it might increase for low enough x_h. One can show that $\eta(x_h)$ decreases for all x_h if $\beta > (\alpha+1)^{-1} \exp[(2\alpha+1)/(\alpha+1)]$ while it can increase for low values of x_h otherwise. Using (30), the elasticity of pass-through $\mathcal{E}_{\text{pt}}$ is larger than 0.5 if and only if α and/or β are small enough.

Consider finally the **inverse translated CES**, $p(x_h) = \left(x_h^{-\frac{\alpha}{\alpha+1}} - \beta\right)/\lambda_h$ for $x \in (\beta^{-\frac{\alpha+1}{\alpha}}, \infty)$ and $\alpha, \beta > 0$. This implies $p'(x_h) = -\frac{\alpha}{\alpha+1}\frac{1}{\lambda_h}x_h^{-\frac{2\alpha+1}{\alpha+1}} < 0$. The elasticity of individual demand is given by $\varepsilon(x_h) = \frac{\alpha+1}{\alpha}\left(1 - \beta x_h^{\frac{\alpha}{\alpha+1}}\right)$ and $\varepsilon'(x_h) = -\frac{\alpha+1}{\alpha}\beta x_h^{-\frac{1}{\alpha+1}} < 0$. This demand is sub-convex. We also have $r(x_h) = \frac{2\alpha+1}{\alpha}\left(1 - \beta x_h^{\frac{\alpha}{\alpha+1}}\right)$ and $r'(x_h) = -\frac{2\alpha+1}{\alpha}\beta x_h^{-\frac{1}{\alpha+1}} < 0$. Using $u'(x_h) = \left(x_h^{-\frac{\alpha}{\alpha+1}} - \beta\right)$, we integrate so that $u(x_h) = (\alpha+1)x_h^{\frac{1}{\alpha+1}} - \beta x_h$. Thus, $\eta(x_h) = \frac{x_h u'(x_h)}{u(x_h)} = \left(x_h^{\frac{1}{\alpha+1}} - \beta x_h\right) / \left[(\alpha+1)x_h^{\frac{1}{\alpha+1}} - \beta x_h\right]$ and $\eta'(x_h) = -\frac{\alpha^2}{\alpha+1}\beta x_h^{\frac{1}{\alpha+1}}\left[(\alpha+1)x_h^{\frac{1}{\alpha+1}} - \beta x_h\right]^{-2} < 0$. Therefore, $\eta(x)$ decreases for all values of x_h. Using (30), the elasticity of pass-through $\mathcal{E}_{\text{pt}} < 1/2$, when x_h is high enough, otherwise $\mathcal{E}_{\text{pt}} > 1/2$.

Appendix G. Excess entry

We here prove that, under aligned preferences $\eta'_h < 0$,the property $x^o > x$ and $n^o < n$ holds if $r(x_h)$ is monotone decreasing, $r'(x_h) < 0\ \forall h$, or if it is constant, $r'(x_h) = 0$. We also give an example of the same property and its opposite if $r'(x_h) > 0\ \forall h$. To avoid confusion, we here make explicit the reference to consumption levels x_h.

As a preliminary step, note that, as shown by (22), the condition for aligned preferences, $\eta'(x_h) < 0$, implies $1 - \eta(x_h) < 1/\varepsilon(x_h)$ for any $x_h \in [x_0, x_1]$ so that

$$1 - \eta(x) < \frac{1}{\varepsilon(x)} \tag{35}$$

holds where $x = \int x_h dG$ denotes the average consumption. Let us remind the notation for equilibrium elasticity $\varepsilon = \int \varepsilon(x_h)x_h dG / \int x_h dG$ and elasticity at average consumption $\varepsilon(x)$.

Step 1. We prove that $1/\varepsilon(x) < 1/\varepsilon$ if $r'(x_h) < 0\ \forall h$ and $1/\varepsilon(x) > 1/\varepsilon$ if $r'(x_h) > 0\ \forall h$. To this end, note that $(x_h\varepsilon(x_h))'' = \varepsilon'(x_h) + (x_h\varepsilon'(x_h))'$. Using (3), we have $(x_h\varepsilon'(x_h))' = r'(x_h) - \varepsilon'(x_h)$ so that $(x_h\varepsilon(x_h))'' = r'(x_h)$. The condition

$$\frac{1}{\varepsilon(x)} < \frac{1}{\varepsilon} \Longleftrightarrow \int x_h \varepsilon(x_h) dG < x\varepsilon(x)$$

holds if $x_h\varepsilon(x_h)$ is a strictly concave function of x_h; that is, if $r'(x_h) < 0$. This is false if $x_h\varepsilon(x_h)$ is a strictly convex function of x_h; that is, if $r'(x_h) > 0$. The condition holds with equality if $x_h\varepsilon(x_h)$ is an affine function of x_h; that is, if $r'(x_h) = 0$. The latter holds for all Pollak utility functions that satisfy $r'_h = 0$ and imply $\varepsilon(x) = \varepsilon$.

Step 2. We show that $x^o > x$ holds if $r'(x_h) < 0$ $\forall h$ or $r'(x_h) = 0$ $\forall h$. Using the properties of $\eta'(x_h) < 0$ and either $r'(x_h) < 0$ $\forall h$ or $r'(x_h) = 0$ $\forall h$, we get

$$1 - \eta(x) < \frac{1}{\varepsilon(x)} \leq \frac{1}{\varepsilon} \tag{36}$$

Now, suppose that $1 - \eta(x^o) > 1/\varepsilon$ holds so that $x^o < x$ holds. Since $1 - \eta(x_h)$ is an increasing function, this implies that $1 - \eta(x) > 1/\varepsilon$, which contradicts (36). Thus, $x^o > x$ if $r'(x_h) < 0$ $\forall h$ or $r'(x_h) = 0$ $\forall h$.

Step 3. Suppose $r'(x_h) > 0$ $\forall h$ so that $1 - \eta(x_h) < 1/\varepsilon(x_h)$ $\forall h$. Note that, under homogeneous consumers, we have $x_h = x$ and $\varepsilon = \varepsilon(x_h) = \varepsilon(x)$, so that $r'(x_h) > 0$ implies $1 - \eta(x) < 1/\varepsilon(x) = 1/\varepsilon$, and therefore $x^o > x$ and $n^o < n$. However, a mean preserving spread of income leads to a rise in x and a reduction in n while optimum values do not change. Therefore, when income heterogeneity is high enough, we can have $x^o < x$ and $n^o > n$. That is, the market equilibrium provides too few varieties while each firm over-produces.

Appendix H. Trade equilibrium

The monopolistic competitive equilibrium is defined as the set of variables $\{x_h,\, x_h^*,\, i_h,\, i_h^*\; p,\, p^*,\, p_i,\, p_i^*,\, y,\, y^*,\, y_i,\, y_i^*\; w,\, w^*,\, n,\, n^*\}$ that are consistent with the following relationships:

Consumer	$npx_h + n^*p_i i_h = s_h w$	$n^*p^*x_h^* + np_i^* i_h^* = s_h w^*$
	$p/p_i = u'(x_h)/u'(i_h)$	$p^*/p_i^* = u'(x_h^*)/u'(i_h^*)$
FOC	$p = \frac{\varepsilon}{\varepsilon-1}cw$	$p^* = \frac{\varepsilon^*}{\varepsilon^*-1}cw^*$
	$p_i^* = \frac{\varepsilon_i^*}{\varepsilon_i^*-1}cw$	$p_i = \frac{\varepsilon_i}{\varepsilon_i-1}cw^*$
Entry	$(p - cw)\,y + (p_i^* - cw)\,y_i^* = fw$	$(p^* - cw^*)\,y^* + (p_i - cw^*)\,y_i = w^*f$
Product	$y = L\int x_h \mathrm{d}G$	$y^* = L^*\int x_h^* \mathrm{d}G$
	$y_i^* = L^*\int i_h^* \mathrm{d}G$	$y_i = L\int i_h \mathrm{d}G$
Labor	$L\int s_h \mathrm{d}G = n\,(f + c\,(y + y_i^*))$	$L^*\int s_h \mathrm{d}G = n^*\,(f + c\,(y^* + y_i))$

Table F1: Trade equilibrium conditions

Under symmetry, we have $L = L^*$, $x_h = x_h^* = i_h = i_h^*$, $p = p^* = p_i = p_i^*$, $y = y^* = y_i = y_i^*$ $w = w^*$ and $n = n^*$. So, we can simplify the above conditions as

Consumer	$2npx_h = s_h w$
FOC	$p = \frac{\varepsilon}{\varepsilon-1} cw$
Entry	$2(p - cw)\, y = fw$
Product market	$y = L \int x_h \mathrm{d}G$
Labor market	$L \int s_h \mathrm{d}G = n(f + 2cy)$

Table F2: Symmetric trade equilibrium conditions

Those are the same equilibrium conditions as for the closed economy, except that we must substitute (y, n, L) for $(2y, 2n, 2L)$.

When countries are symmetric in their population and income distribution, this system collapses to the same equilibrium conditions as for the closed economy, except that we must substitute $(y,\, n,\, L)$ for $(2y,\, 2n,\, 2L)$. Therefore, the symmetric equilibrium exists under the same equilibrium conditions as in closed economy. In this case, revenues, costs and elasticities are related in the following way:

$$\frac{p - cw}{p} = \frac{1}{\varepsilon}, \quad \frac{cwy}{py} = 1 - \frac{1}{\varepsilon}, \quad \frac{2cwy}{f} = \varepsilon - 1, \quad \text{and } \frac{f}{2py} = \frac{1}{\varepsilon}.$$

Also symmetry guarantees that, as in closed economy, the following condition holds $x_h/x = s_h/s$.

Appendix I. Trade and income redistribution

Equilibrium conditions can be log-linearized about the symmetric equilibrium as follows:

Consumer	$\frac{1}{2}(\widehat{n} + \widehat{p} + \widehat{x}_h) + \frac{1}{2}\left(\widehat{n}^* + \widehat{p}_i + \widehat{i}_h\right) = \widehat{s}_h + \widehat{w}$	$\frac{1}{2}(\widehat{n}^* + \widehat{p}^* + \widehat{x}^*_h) + \frac{1}{2}\left(\widehat{n} + \widehat{p}^*_i + \widehat{i}^*_h\right) = 0$
	$\widehat{i}_h - \widehat{x}_h = \varepsilon_h(\widehat{p} - \widehat{p}_i)$	$\widehat{i}^*_h - \widehat{x}^*_h = \varepsilon_h(\widehat{p}^* - \widehat{p}^*_i)$
FOC	$\widehat{p} - \widehat{w} = \frac{1}{x\varepsilon(\varepsilon-1)} \int (1 + \varepsilon - r_h)\, x_h \widehat{x}_h \mathrm{d}G$	$\widehat{p}^* = \frac{1}{x\varepsilon(\varepsilon-1)} \int (1 + \varepsilon - r_h)\, x_h \widehat{x}^*_h \mathrm{d}G$
	$\widehat{p}^*_i - \widehat{w} = \frac{1}{x\varepsilon(\varepsilon-1)} \int (1 + \varepsilon - r_h)\, x_h \widehat{i}^*_h \mathrm{d}G$	$\widehat{p}_i = \frac{1}{x\varepsilon(\varepsilon-1)} \int (1 + \varepsilon - r_h)\, x_h \widehat{i}_h \mathrm{d}G$
Entry	$\frac{1}{2}\varepsilon(\widehat{p} + \widehat{p}^*_i) + \frac{1}{2}(\widehat{y} + \widehat{y}^*_i) = \varepsilon\widehat{w}$	$\frac{1}{2}\varepsilon(\widehat{p}^* + \widehat{p}_i) + \frac{1}{2}(\widehat{y}^* + \widehat{y}_i) = 0$
Product	$\widehat{y} = \frac{1}{x} \int x_h \widehat{x}_h \mathrm{d}G$	$\widehat{y}^* = \frac{1}{x} \int x_h \widehat{x}^*_h \mathrm{d}G$
	$\widehat{y}^*_i = \frac{1}{x} \int x_h \widehat{i}^*_h \mathrm{d}G$	$\widehat{y}_i = \frac{1}{x} \int x_h \widehat{i}_h \mathrm{d}G$
Labor	$0 = \widehat{n} + \frac{1}{2}\frac{\varepsilon-1}{\varepsilon}(\widehat{y} + \widehat{y}^*_i)$	$0 = \widehat{n}^* + \frac{1}{2}\frac{\varepsilon-1}{\varepsilon}(\widehat{y}^* + \widehat{y}_i)$

Table I.1: Log-linearization around symmetric trade equilibrium

We proceed in two steps.

Step 1. First, we show that $\widehat{w} = 0$. To this end, we take the difference of price changes in country 1 and get

$$\hat{p} - \hat{p}_i = \hat{w} + \frac{\int (1 + \varepsilon - r_h)\, x_h (\hat{x}_h - \hat{\imath}_h) dG}{(\varepsilon - 1)\varepsilon x}.$$

Combining it with the second line of Table I.1 leads to

$$\hat{p} - \hat{p}_i = \hat{w} - \frac{\int (1 + \varepsilon - r_h)\, x_h \varepsilon_h (\hat{p} - \hat{p}_i) dG}{(\varepsilon - 1)\varepsilon x},$$

or, after simplifications,

$$\hat{p} - \hat{p}_i = \frac{\hat{w}}{a},$$

where $a = \frac{\int (2\varepsilon - r_h)\varepsilon_h x_h dG}{(\varepsilon-1)\varepsilon x} > 0$ by the second order condition (8). By analogue, in country 2

$$\hat{p}^* - \hat{p}_i^* = -\frac{\hat{w}}{a}$$

Therefore,

$$\hat{\imath}_h - \hat{x}_h = (\hat{p} - \hat{p}_i)\varepsilon_h = \frac{\hat{w}\varepsilon_h}{a}, \qquad \hat{\imath}_h^* - \hat{x}_h^* = (\hat{p}^* - \hat{p}_i^*)\varepsilon_h = -\frac{\hat{w}\varepsilon_h}{a}.$$

Plugging $\hat{\imath}_s - \hat{x}_s$ into difference of firm outputs

$$\hat{y} - \hat{y}_i = \frac{\int x_h (\hat{x}_h - \hat{\imath}_h) dG}{x}$$

we obtain

$$\hat{y} - \hat{y}_i = -\frac{\hat{w}\varepsilon}{a},$$

while similar equations for country 2 yields

$$\hat{y}^* - \hat{y}_i^* = \frac{\hat{w}\varepsilon}{a}.$$

Combining entry conditions for both countries

$$\varepsilon\hat{p} + \varepsilon\hat{p}_i^* + \hat{y} + \hat{y}_i^* = 2\varepsilon\hat{w}, \qquad \varepsilon\hat{p}_i + \varepsilon\hat{p}^* + \hat{y}_i + \hat{y}^* = 0$$

leads to

$$\varepsilon(\hat{p} - \hat{p}_i) + \varepsilon(\hat{p}_i^* - \hat{p}^*) + \hat{y} - \hat{y}_i + \hat{y}_i^* - \hat{y}^* = 2\varepsilon\hat{w}.$$

Plugging the differences for price and output changes into the last equation, we get

$$\frac{\hat{w}}{a}\varepsilon + \frac{\hat{w}}{a}\varepsilon - \frac{\hat{w}\varepsilon}{a} - \frac{\hat{w}\varepsilon}{a} = 2\varepsilon\hat{w},$$

thus, $\hat{w} = 0$ which yield

$$\hat{p} = \hat{p}_i, \qquad \hat{p}_i^* = \hat{p}^*, \qquad \hat{\imath}_h = \hat{x}_h, \qquad \hat{\imath}_h^* = \hat{x}_h^*, \qquad \hat{y} = \hat{y}_i, \qquad \hat{y}^* = \hat{y}_i^*, \qquad \hat{n} = \hat{n}^*.$$

Step 2. The first two lines of Table I.1 take the form

$$\widehat{x}_h = \widehat{i}_h = \widehat{s}_h - (\widehat{n} + \widehat{p}), \qquad \widehat{x}_h^* = \widehat{i}_h^* = -(\widehat{n} + \widehat{p}^*).$$

Plugging those values in the other equations in Table I.1 and solving a linear system in the aggregate variable, yields closed-form solutions for the changes in prices, output, and product diversity as reported in Table 4.

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