HIGHER SCHOOL OF ECONOMICS
NATIONAL RESEARCH UNIVERSITY

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DISCLOSURES, ROLLOVER RISK, AND DEBT RUNS

Working Paper WP9/2021/01
Series WP9
Research of economics and finance

Moscow
2021

УДК 330.4
ББК 65.01
C21

Editor of the Series WP9
"Research of economics and finance"
Maxim Nikitin

Carre, S.*
C21 Disclosures, Rollover Risk, and Debt Runs [Electronic resource] : Working paper WP9/2021/01 / S. Carre ; National Research University Higher School of Economics. – Electronic text data (1 Mb). – Moscow : Higher School of Economics Publ. House, 2021. – (Series WP9 "Research of economics and finance"). – 66 p.

How do opacity and disclosure policies impact the likelihood of debt runs and economic efficiency? I construct a dynamic model where debt yields are endogenous and mapped explicitly to the degree of transparency, the regulatory disclosure regime and the state of the economy. I find that: opacity is desirable if and only if fundamentals are strong; transparency can be more efficient even when it entails more runs; the regulator should commit to disclosure except at large levels of opacity. There is a rich interaction between debt and beliefs dynamics and equilibrium outcomes: short-term yields may remain low while risk builds up, and a disclosure regime might consistently induce better beliefs but imply larger financing costs.

УДК 330.4
ББК 65.01

Keywords: dynamic debt runs; opacity; disclosure policy

JEL classification: G21; G28

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* I am grateful to my former advisor Pierre Collin-Dufresne for advice and support. I would also like to thank Semyon Malamud, Martin Oehmke, Hongda Zhong, Ansgar Walther and Andreas Uthemann for valuable comments, as well as Franck Gabriel, Damien Klossner and participants to seminar presentations at ETH Zürich, AFFI Valence, SFI Research days, EPFL, USI Lugano, Cergy-Pontoise, LSE, LBS, McGill, ICEF, Queen Mary University London. The usual disclaimer applies.

1 Introduction

Financial institutions issuing short-term debt collateralised by long-term assets are exposed to bank run phenomena: creditors may demand to withdraw their funds and trigger costly liquidations. Debt runs are prominent features of financial crises: during the turmoil of 2007-2008, runs hit the asset-backed commercial paper market, the repo market, money market mutual funds and banks such as Northern Rock and Bear Stearns.[1]

Many institutions managing opaque assets struggled throughout this period (Gorton (2008)). This ignited a debate among both academics and policy makers about the impact of opacity on financial fragility. One line of thinking, represented by Gorton and Ordoñez (2014) and Dang, Gorton, Holmström, and Ordoñez (2017), advocates that opacity is actually a desirable characteristic of the financial sector and should be fostered. On the other hand, the policy responses to the crisis seemed to go in a different direction: regulators engaged in a considerable effort to both gather and disclose more information about banks. This was evidenced by the start of the Supervisory Capital Assessment Program (SCAP) in February 2009, a massive effort to submit all major banking institutions in the United States to thorough stress tests. Gathering and disclosing information are two distinct decisions: the regulator may collect information to have the option to reduce opacity by releasing it to the public, but prefer ex post not to do so. In fact, there were concerns that fully releasing the results of the SCAP stress tests might have a destabilising effect.[2]

The present paper aims at answering the following questions: how does the accessibility of information impact the resilience of financial institutions to debt runs? under which circumstances should the regulator strive to collect information regularly? if the regulator has incentives to withhold information in some states, should he commit ex ante to a policy of full

[1] Gorton and Metrick (2012) document the run on the repo market, and Covitz, Liang, and Suarez (2013) investigate the run on the ABCP market.

[2] Bernanke (2010) mentions these concerns; see also Goldstein and Sapra (2014).

disclosure?

To do so, I modify the discrete time dynamic debt runs model of Acharya, Gale, and Yorulmazer (2011) by allowing the bank's assets to be opaque and information release to be strategic. My model features an uninsured financial institution ("bank") trying to roll over its short-term debt until its assets mature. The bank cannot communicate credibly. Instead, creditors have to rely on regulatory disclosures when deciding whether to renew their credit to the bank. Because the bank assets can be complex and investigation is costly, the regulator may not be able to constantly assess the soundness of the bank: the frequency at which he can and wishes to obtain bank-specific information defines the degree of transparency in the model. Opacity is defined as the opposite of transparency. In a regime of commitment (*mandatory* disclosure), the regulator conveys truthfully any information he has to the bank's creditors. Absent commitment (*voluntary* disclosure), the regulator finds it optimal to only release good news.

Because of the simple structure chosen for the bank's asset process, I am able to characterise analytically the interest rates demanded by creditors to roll over the bank's debt, and the states in which they instead decide to run. Runs are assumed to entail deadweight liquidation costs proportional to the fundamental value of the asset at the run time. Inefficiency is then defined as the expected liquidation costs.

Constructing a dynamic framework where the cost of debt is endogenous allows to uncover two channels that would not be apparent in a model with a single rollover date. First, I capture a *funding cost channel*: a signal provided to a creditor has a contemporaneous effect (it will trigger a run with some probability today), but also impacts the required interest rate. Hence, it affects future debt levels and thereby future incentives to run. The efficiency of an opacity level and a disclosure policy depend on both the direct and the indirect effect. Second, the model recognises that the disclosure policy of the regulator impacts the beliefs dynamics, which in turn impact future rollover decisions. When the regulator does not commit to disclosure, short-term funding costs are lower in good times. However, the lack of commitment generates depressed beliefs as long as the bad state does not realise, potentially leading to a larger probability of bank failure at longer horizons. A model with a single rollover date would obliterate this *beliefs channel* and the costs it entails; when in fact, all the costs associated with non-commitment are due to the fact that it generates worse beliefs.

The interaction between the information structure, debt dynamics and beliefs dynamics is rich. Short-term debt yields are determined by the number of default states tomorrow under the given information structure, not by the expected value of the collateral computed under the beliefs generated by this structure: yields do not primarily reflect the current expected collateral quality. Two results of the paper relate to this fact. First, there need not necessarily be a warning sign of a run in the time-series of short-term returns: yields may remain low while risk builds in the background. Second, there are situations in which the expected collateral value is always larger under one disclosure regime, but the bank nevertheless faces larger financing costs under this regime, and therefore fails only in the seemingly better scenario.

At the policy level, the main results—obtained in the continuous-time version of the model—are the following. First, I find that opacity reduces run probability and inefficiency only when fundamentals are strong enough: in situations where the regulator believes that the economy is healthy and likely to remain so for a long time, collecting and releasing information about banks is detrimental; in other situations, the regulator wants to implement transparency. Second, opacity may decrease run probability but increase inefficiency: the objective of the regulator is not to minimise the probability of a bank failure, but rather the expected costs associated with liquidation. Under transparency, runs may occur more frequently but they are concentrated on bad banks, for which liquidation is less inefficient. Third, voluntary disclosure is more efficient than mandatory disclosure except at extremely large levels of opacity: this implies that the regulator should commit to disclose stress test results unless access to information is extremely difficult. Thus, my model shows that whether stress test results should be systematically disclosed depends on the degree of asset opacity.

Relation to the literature. The game-theoretic study of bank runs traces back to the seminal paper of Diamond and Dybvig (1983): in the bad equilibrium, agents "panic" about the run decision of others, leading to an outcome where all creditors run on the bank and force an inefficient liquidation. Building on the global games literature pioneered by Carlsson and van Damme (1993) and Morris and Shin (1998), Rochet and Vives (2004) and Goldstein and Pauzner (2005) provide bank run models where the equilibrium is unique and runs arise as the result of both a coordination failure and concerns about the fundamentals. In these models, the coordination problem comes from the fact that creditors are dispersed and must decide simultaneously whether to withdraw their funds. Models of dynamic debt runs provide a related but distinct approach. There, the coordination problem is intertemporal in the sense that an agent may withdraw his funds because of concerns about future rollover decisions of other creditors. He and Xiong (2012) and Schroth, Suarez, and Taylor (2014) provide such models and use them to quantify the impact of factors such as maturity mismatch, leverage and liquidation costs on run likelihood, with a focus on the 2007 run on ABCP.

As Acharya, Gale, and Yorulmazer (2011), my paper highlights the importance of the specific nature of the information structure to the outcome of the rollover problem. In a broader framework, Kamenica and Gentzkow (2011) show how one can optimally design information structures (*i.e.* select signals[3]) to maximise non-linear functions of some agent's beliefs, what they call Bayesian persuasion. Finding the optimal opacity level and disclosure policy in the present model can be seen as a Bayesian persuasion problem, because it means choosing ex ante which signals about the fundamental to show to investors, and the non-linear function of their belief is the rollover decision. Papers linking explicitly the Bayesian persuasion approach to the

[3] By "selecting signals" one means of course selecting *ex ante* a random variable, rather than being able to show or conceal the realisation of a given signal.

research on stress tests include Quigley and Walther (2017), Goldstein and Leitner (2018), and Inostraza and Pavan (2020).

While the models of dynamic debt runs mentioned above assume full information, there is also a significant body of literature on banking under opacity. Alvarez and Barlevy (2014) develop a network model of banking where imposing mandatory disclosure of losses can only improve welfare when contagion concerns are strong. Faria e Castro, Martinez, and Philippon (2017) study the interaction between the fiscal capacity of the government and optimal disclosure policies. When deposit insurance can be provided at a low social cost, a disclosure policy that would be suboptimal absent insurance because of the run risk it implies may become desirable. In a model of coordination failures *à la* Goldstein and Pauzner (2005), Bouvard, Chaigneau, and de Motta (2015) investigate how a regulator endowed with perfect information about aggregate and idiosyncratic shocks on the banking sector should communicate with the public. My model does not distinguish between these shocks, but introduces the possibility that the regulator herself has no information: this generates a different commitment problem. Additionally, their model features a single rollover date and therefore does not capture the funding cost channel and the beliefs channel described above. Finally, Monnet and Quintin (2017) map the need for transparency to the degree of a bank's asset liquidity and show that opacity is preferable when secondary markets are shallow.

My paper also bears a connection with the series of papers by Gorton and Pennacchi (1990), Dang, Gorton, and Holmström (2013), Dang, Gorton, and Holmström (2015), Gorton and Ordoñez (2014) and Dang, Gorton, Holmström, and Ordoñez (2017). These authors focus on the notion of *information sensitivity*. A security is information insensitive when agents have no incentive to acquire costly signals about it. Because of their capped payoff, debt contracts are natural candidates for information insensitivity, and more so if collateral is opaque. If, in addition, the expected value of collateral is large enough, debt is risk-free and of constant value: it can be used as money.

Therefore bank should be "secret keepers" (Dang, Gorton, Holmström, and Ordoñez (2017)). Deterring information acquisition with opaque collateral also ensures that information is always symmetrical. This prevents market freezes due to adverse selection issues (Dang, Gorton, and Holmström (2015)). One can similarly define the information sensitivity status of debt in my model and map this status to the current state of the world, the degree of opacity, and the disclosure regime.

2 The Model

Time is discrete: $t = 0, 1, 2, \ldots$.[4] The model features an uninsured financial institution ("bank") whose short-term debt must be refinanced by successive creditors until its asset reaches maturity.[5]

2.1 The Bank

2.1.1 Asset side

The bank holds a long-term asset. For tractability purposes, its maturity is modelled as a random time ζ_ϕ. ζ_ϕ is assumed to be independent of all other variables and geometrically distributed with parameter $\phi \in (0,1)$: the expected maturity is $\mathbb{E}[\zeta_\phi] = \frac{1}{\phi}$. At time ζ_ϕ, the asset delivers its payoff, agents receive their payments, and the world ends. The asset does not pay anything before maturity.

The asset side of the bank is modelled by a Markov chain $(y_t)_{t\geq 0}$ with two states: $y^G > y^B$. The meaning of y_t is the following: if maturity occurs

[4]The solution in continuous-time is presented in section 4.3.1.

[5]A significant part of the short-term debt of financial institutions is not insured, and even bank deposits are typically insured only up to some limit. Moreover, ex-post liquidity assistance may not be systematical but contingent to some criteria (see for instance Santos and Suarez (2019)). For simplicity, I consider uninsured debt, but it is straightforward to amend the model solution to the case where the institution is bailed out with some exogenous probability when a run occurs.

at time t ($\zeta_\phi = t$), the asset payoff is y_{ζ_ϕ}. Assume that the asset is initially in the good state: $y_0 = y^G$. The transition matrix of (y_t) is

$$\Lambda = \begin{pmatrix} \lambda^{GG} & 1-\lambda^{GG} \\ \lambda^{BG} & 1-\lambda^{BG} \end{pmatrix}. \tag{1}$$

λ^{GG} represents the probability to stay in the good state from one period to the next, while λ^{BG} can be interpreted as a recovery probability. Under the conditions $\lambda^{GG} > \frac{1}{2}$ and $\lambda^{BG} < \frac{1}{2}$, we have

$$V^G \equiv \mathbb{E}[y_{\zeta_\phi} | y_t = y^G, t < \zeta_\phi] > \mathbb{E}[y_{\zeta_\phi} | y_t = y^B, t < \zeta_\phi] \equiv V^B. \tag{2}$$

(2) means that being in the state y^G before maturity signals a high expected payoff at maturity, so y^G is indeed the "good state".

2.1.2 Liability side

The initial capital structure of the bank is taken as given. The bank has raised an amount D_0 of short-term (*i.e.* one-period) debt D_0.[6] Equity is the residual claim and is owned by the banker. Since the asset does not pay anything before maturity, short-term debt must be refinanced: to do so, the bank has access to a pool of potential short-term creditors (see section 2.2). No other sources of financing are available.

Short-term debt can stop being rolled over in two cases. (i) (*strategic default*) The bank can decide to default on the debt, in which case its asset is liquidated at a fraction of its current expected value. The strategic default time is denoted ζ_s. (ii) (*rollover freeze*) If debt is too high, there is no short-term debt contract that compensates adequately for default risk. No creditor accepts to roll over the debt, forcing the bank into liquidation. The time at

[6]Explicit motivations for short-term debt include Calomiris and Kahn (1991) and Diamond and Rajan (2001). Brunnermeier and Oehmke (2013) show how debt maturities can endogenously shorten in response to dilution concerns. Carré and Klossner (2018) provide a global games model for the short-term leverage choice of a bank whose debt provides liquidity but creates rollover risk.

which this happens is denoted ζ_z.

Important details on the liquidation procedure are given in section 2.4.3. Let $\zeta_\ell = \min\{\zeta_s, \zeta_z\}$ be the liquidation time. I will use the convention $\zeta_l = \infty$ when liquidation does not occur prior to maturity. Finally, define the end date as

$$\zeta_f = \min\{\zeta_\ell, \zeta_\phi\}. \tag{3}$$

It is convenient to introduce the following assumption.

Assumption 1. *$D_0 > V^B$.*

This condition ensures that the bank is insolvent when the bad state is revealed, which triggers liquidation.

2.2 Creditors

The bank has access to an unlimited pool of risk-neutral and competitive creditors.

I assume that all the short-term debt is held by a single investor at each period, and that the investor entering the debt contract at date t exits forever the pool of creditors after receiving his payment at $t + 1$.

Given an amount of debt to roll over at time t, the bank offers a contract with a promised repayment at time $t + 1$, the face value F. The risk-free rate is normalised to zero. Hence, since creditors compete to obtain the debt contract, the equilibrium face value is such that a creditor makes zero profit on average. If no face value satisfies the zero profit condition, liquidation occurs (*i.e.* ζ_z is reached). I use the convention $F = \emptyset$ in that case, since the bank cannot offer any acceptable face value.

2.3 The Regulator

A regulator who may obtain information about the bank's asset and can disclose them to creditors. When the regulator does not commit to reveal all its information, he selects his disclosure policy to minimise inefficiency.

Note that since creditors make zero expected profit in equilibrium, the regulator's objective is in fact to maximise the banker's equity value: see the equilibrium definition in section 2.5.3. The next section describes the information structure and provides details about the constraints under which the regulator operates.

2.4 Information Structure

2.4.1 Asset Opacity

I make the following assumptions. First, the bank observes (y_t) but cannot credibly communicate any information to investors. Second, at each time t, the regulator observes the current state of the chain, y_t, with probability p, independently of everything else.

It will be convenient to define the dummy variables

$$\omega_t = \begin{cases} 1 & \text{if the regulator observes } y_t \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

By assumption, $(\omega_t)_{t\geq 0}$ is an i.i.d. sequence of Bernoulli variables with parameter p. p characterises the degree of opacity of the asset. When $p = 1$, there is full information, while $p = 0$ corresponds to the case of a fully opaque asset.

Agents in the pool of creditors cannot make any direct observation and rely on the regulator's disclosures.

The motivation for this particular modelling of opacity is the following. One wants to capture the fact that it is not feasible for the regulator to monitor the bank at all times, because of the excessive costs this would imply. As Bernanke (2010) noted, "The SCAP represented an extraordinary effort on the part of the Federal Reserve staff and the staff of other banking agencies. In a relatively short time, the supervisors had to gather and evaluate an enormous amount of information".

Considering an exogenous ω_t allows to maintain tractability; and letting $p < 1$ incorporates the regulator's constraints into the model as desired.

2.4.2 Disclosure Regimes

At each time t, the regulator has the opportunity to disclose information to the pool of creditors after the realisation of ω_t. Disclosure takes the form of an announcement δ_t:

$$\delta_t = \begin{cases} \emptyset & \text{“I did not observe the asset value ”} \\ y^G & \text{“I observed the asset value and } y_t = y^G \text{”} \\ y^B & \text{“I observed the asset value and } y_t = y^B \text{”}. \end{cases} \tag{5}$$

I compare two disclosure regimes: *voluntary* and *mandatory*.

Under mandatory disclosure, the regulator is compelled by law to announce the truth. That is, he has been able to credibly commit to communicate any information he has. In that case, disclosure is mechanical:

$$\delta_t = \begin{cases} y_t & \text{if } \omega_t = 1 \\ \emptyset & \text{if } \omega_t = 0. \end{cases} \tag{6}$$

Under voluntary disclosure, the regulator can conceal news. That is, he can claim to be uninformed while he is. Formally, it means he can play $\delta_t = \emptyset$ when $\omega_t = 1$. However, if a state is announced, is must be accompanied with evidence. Hence, it is impossible to announce that a state has been observed when it is not the case. Formally, it means that $\delta_t = y^i$ implies $\omega_t = 1$ and $y_t = y^i$ for $i = G, B$. These assumptions on the voluntary disclosure regime are borrowed from Dye (1985).

The equilibrium under voluntary disclosure will feature a *sanitisation strategy*:[7] the regulator discloses the good state and conceals the bad state.

[7] See Shin (2003).

That is, he plays $\delta = \delta^S$, where

$$\delta_t^S \equiv \begin{cases} y^G & \text{if } \omega_t = 1 \text{ and } y_t = y^G \\ \emptyset & \text{otherwise} \end{cases} \tag{7}$$

is the sanitisation strategy.

Denote $(\mathcal{F}_t^I)_{t\geq 0}$ the filtration of the investors:

$$\mathcal{F}_t^I = \sigma\left((\delta_s)_{s\leq t}, \zeta_\ell \mathbb{I}_{\{\zeta_\ell \leq t\}}, \zeta_\phi \mathbb{I}_{\{\zeta_\phi \leq t\}}\right), \tag{8}$$

$(\mathcal{F}_t^R)_{t\geq 0}$ the filtration of the regulator:

$$\mathcal{F}_t^R = \sigma\left((\omega_s, y_s \mathbb{I}_{\{\omega_s = 1\}})_{s\leq t}, \zeta_\ell \mathbb{I}_{\{\zeta_\ell \leq t\}}, \zeta_\phi \mathbb{I}_{\{\zeta_\phi \leq t\}}\right), \tag{9}$$

and $(\mathcal{F}_t^B)_{t\geq 0}$ the filtration of the bank, which observes everything but cannot communicate information credibly.

A belief system of the investors $\mathcal{B}$ is a $(\mathcal{F}_t^I)$-adapted process $(q_t^{\mathcal{B}})$. $q_t^{\mathcal{B}} = \mathbb{P}(y_t = y^G | \mathcal{F}_t^I, \mathcal{B})$ is the probability to be in the good state at time t from the perspective of the investors.

2.4.3 Liquidation

If liquidation occurs at time t ($t < \zeta_\phi$), the value αV is recovered, where $\alpha \in [0, 1]$ and $V = \mathbb{E}[y_{\zeta_\phi} | \mathcal{F}_t^I]$ is the fundamental value of the asset computed under the outsiders' information set at time t.

In case of a strategic liquidation under asymmetric information, the liquidation decision has a signalling content and we need to specify the beliefs of outsiders. For simplicity, I focus on equilibria where the bank's decision to liquidate at t when $\delta_t = \emptyset$ is interpreted as the fact that the bank has observed the bad state ($y_t = y^B$):

I restrict the attention to equilibria where the belief system $\mathcal{B}$ is such that

$$\zeta_\ell = t,\ \delta_t = \emptyset \implies q_t^{\mathcal{B}} = 0.$$

When $\delta_t \neq \emptyset$, payoff-relevant information is symmetric because announcements of states are trustworthy. Hence, there is no signalling problem in that case.

$1-\alpha$ is a measure of illiquidity as it represents the fraction of asset value destroyed due to premature liquidation.[8] Because of the deadweight cost $(1-\alpha)V$, liquidation is never efficient in this model; the inefficiency is large for good banks (*i.e.* V high) and small for bad banks (*i.e.* V low).[9]

2.5 Equilibrium

2.5.1 Debt Dynamics

Assume we are at time $t < \zeta_f$ with a level of debt D_t. The bank has promised the face value D_{t+1} to the current creditor. The actual payment, $\tilde{D}_{t+1}$, satisfies:

$$\tilde{D}_{t+1} = \begin{cases} \min\{y_{t+1}, D_{t+1}\} & \text{if } \zeta_\phi = t+1 \\ \min\{\alpha V_{t+1}, D_{t+1}\} & \text{if } \zeta_\phi > t+1 \text{ and } \zeta_\ell = t+1 \\ D_{t+1} & \text{otherwise.} \end{cases} \tag{10}$$

In the first case, maturity occurs at time $t+1$ and the asset delivers the payoff y_{t+1}. In the second case, the bank is liquidated at the value αV_{t+1} where $V_{t+1} = \mathbb{E}[y_{\zeta_\phi} | \mathcal{F}^I_{t+1}]$. In the third case, the banker is able to roll its debt over. That is, she obtains the financing necessary to repay D_{t+1} in full.

[8] If the bank is the first-best user of the asset, transferring its control rights to another party reduces its value (Shleifer and Vishny (1992)).

[9] The typical situation in 3-dates models of runs is that premature liquidations are efficient when the fundamental is very low and inefficient otherwise. The common conclusion is that the deadweight cost of liquidating good banks is larger.

Lemma 1. *The break-even condition of lenders is equivalent to the property that $(\tilde{D}_{t\wedge\zeta_f})_{t\geq 0}$ is a $(\mathcal{F}_t^I)$-martingale.*

(All proofs are relegated to the Appendix).

2.5.2 Quantities of Interest

My goal is to understand how asset opacity and disclosure regimes impact the likelihood of debt runs and their inefficiency. In this section, I explain how these quantities are measured in the model. I then define formally the equilibrium.

The probability of a run is

$$\mathcal{P} \equiv \mathbb{P}(\zeta_\ell < \zeta_\phi). \tag{11}$$

Because lenders make zero profit on average, the bank bears the costs of inefficient runs. Optimality for the bank coincides with a social planner's optimality in the model, that is, maximising the expectation of the payoff U of the asset. This quantity is given by

$$U \equiv \alpha V_{\zeta_l} \mathbb{I}_{\{\zeta_\ell < \zeta_\phi\}} + y_{\zeta_\phi} \mathbb{I}_{\{\zeta_\phi \leq \zeta_\ell\}}. \tag{12}$$

Equivalently, the measure of *inefficiency* is the expected deadweight cost

$$\mathcal{I} \equiv \mathbb{E}[(1-\alpha) y_{\zeta_\ell} \mathbb{I}_{\zeta_\ell < \zeta_\phi}] = V^G - \mathbb{E}[U]. \tag{13}$$

Saying that the banker maximises equity value is equivalent to saying that she maximises $\mathbb{E}[U]$ or minimises $\mathcal{I}$.

We are now ready to define the equilibrium.

2.5.3 Equilibrium concept

Definition 1. *Given a belief system $\mathcal{B}$, a consistent bank policy is a promised face value schedule F and a time of strategic liquidation ζ_s such that* *i) F_t is Markov in $(D_t, q_t^{\mathcal{B}})$; $D_{t+1} = F_t$ and the process $(\tilde{D}_t)$ associated with (D_t) is a $(\mathcal{F}_t^I)$-martingale. F is required to satisfy*

(M) F_t is non-decreasing in D_t and non-increasing in q_t,[10]

(NP) $F \leq K$ for some constant $K > y^G$.

ii) ζ_s is $\mathcal{F}_t^B$-adapted, and, given F, it minimises $\mathcal{I}$.

An equilibrium is $(F, \zeta_s, \delta, \mathcal{B})$ such that

i) given $\mathcal{B}$, (F, ζ_s) is a consistent bank policy that minimises $\mathcal{I}$.

ii) δ is $\mathcal{F}_t^R$-adapted and given $(F, \zeta_s, \mathcal{B})$ it minimises $\mathcal{I}$.[11]

iii) the belief system $\mathcal{B}$ is compatible with the disclosure policy δ.[12]

The implicit assumption here is that the banker commits to an interest rate schedule at date 0. Otherwise, the banker conveys signalling information when offering a face value to creditors. In particular due to the specification of out-of-equilibrium beliefs, this comes with significant complications in the formalisation and solution of the game which are unrelated to the mechanisms I want to explore here.[13]

Requiring that F_t is Markov in (D_t, q_t) is to simplify the exposition. We could just demand that F_t is $(\mathcal{F}_t^I)$-adapted; but since q_t encapsulates all the relevant information about the asset payoff, the bank has nothing to gain to condition its face value to other $\mathcal{F}_t^I$-measurable variables.

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[10] Recall the convention $F = \emptyset$ when there is no acceptable face value. The meaning of the monotonicity condition is then that if $F(D_1) = \emptyset$ and $F(D_2) \in \mathbb{R}$, $D_1 > D_2$.

[11] Under mandatory disclosure, δ and $\mathcal{B}$ are mechanical and are *de facto* not equilibrium objects.

[12] The unique belief system compatible with a given disclosure policy δ will be denoted $\mathcal{B}(\delta)$.

[13] I emphasise that the goal of the paper is not to study a rich strategic disclosure problem; instead, this problem is made as simple as possible to focus on the comparison between a voluntary and a mandatory disclosure regime. Indeed, my stylised disclosure problem is sufficient to generate sharp and non-trivial differences between the equilibrium dynamics of these regimes.

Condition (NP) rules out Ponzi schemes, and, as usual, the constraint $F \leq K$ is never binding in equilibrium.[14] This is a consequence of the following useful lemma:

Lemma 2. *In a consistent bank policy, an insolvent bank is necessarily forced into liquidation.*

This is the standard result that insolvency implies illiquidity (of course, the converse is not true). Hence, since $K > y^G$, the bank would be ran upon before debt can reach K, so the constraint $F \geq K$ does not bind.

Figure 1 sums up the model setup graphically. When necessary, the set of parameters is denoted by Θ and Θ_{-x} stands for this set without the variable x.

3 Model Solution

The first step towards solving the model is to establish that the bank never wishes to force liquidation:

Lemma 3. *The bank never liquidates strategically in a consistent bank policy for $\alpha \in [0, 1)$: $\zeta_s = \infty$.*

(In the extreme case $\alpha = 1$, there is no cost associated with liquidation. Thus, when $y_t = y^G$ is observed, the bank is indifferent between holding the asset or liquidating it.) The intuition behind this result is the following. Since debt comes at a zero expected cost for the bank, the banker has no incentive to incur the deadweight liquidation cost today: she is always better off waiting.

[14] In the sense that the banker never actually sets F at K. But of course the constraint binds in a dynamic sense since it rules out Ponzi schemes. Also note that absent requirement (NP), there is a Ponzi equilibrium where each lender is simply betting against maturity, *i.e.* hoping he is not the last in line (this is made possible by the random maturity assumption). The actual asset value is irrelevant in that case. See *e.g.* Blanchard and Watson (1982).

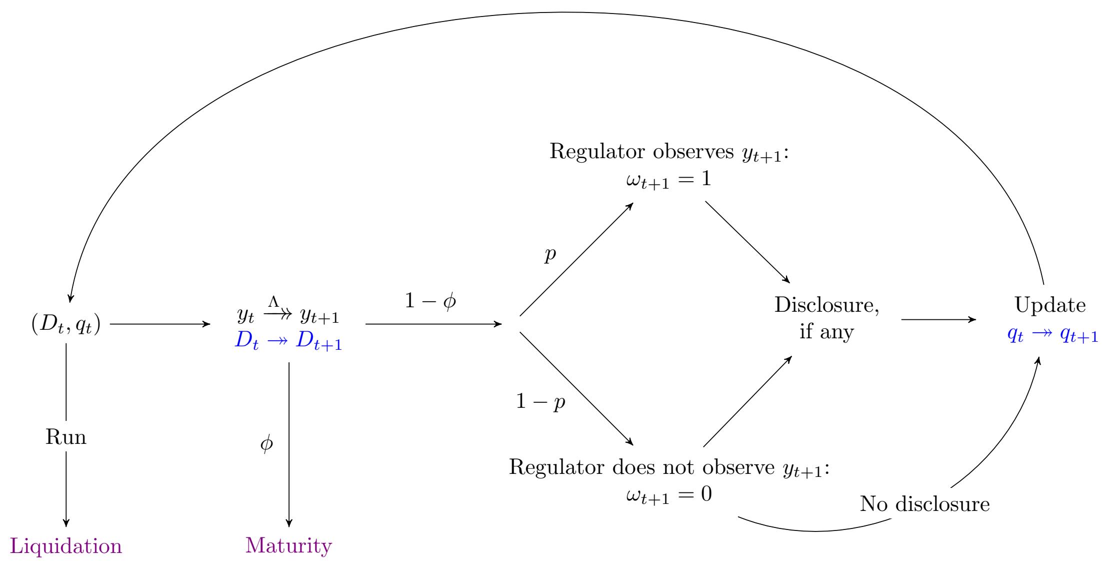

Figure 1: Graphical representation of the model.

3.1 Voluntary Disclosure

I now characterise the policy of the regulator in the voluntary disclosure case.

Lemma 4. *The regulator follows the sanitisation strategy δ^S (defined in (7)) in equilibrium.*

This result is very intuitive. When the regulator observes the good state, it is clearly in his best interest to communicate it to investors. When the regulator observes the bad state, it is always best to conceal it. Even if creditors understand that the regulator may be hiding information, their updated belief about the probability of the good state cannot be worse than if the regulator had revealed the bad state.

3.1.1 State Variables

Suppose we are at time $t < \zeta_\phi$ and current debt is D. From Lemma 4, we know that the regulator discloses only $\emptyset$ or y^G in equilibrium. Let τ be the time elapsed since the last disclosure of y^G:

$$\delta_{t-\tau} = y^G, \delta_{t-\tau+1} = \emptyset, \delta_{t-\tau+2} = \emptyset, \ldots, \delta_t = \emptyset. \tag{14}$$

Given the stationarity of the problem, the data of (D, τ) contains all the relevant information for decision making and we can select (D, τ) as the state variable:

Remark 1. *Any face value schedule F in a consistent bank policy is Markov in (D, τ). Due to Lemmas 3 and 4, what remains to be determined in order to find the equilibrium is which $F(D, \tau)$ are compatible with a consistent bank policy, and which one maximises the banker's equity value.*

The full characterisation of the equilibrium is in section 3.2.5. The next sections explain how to get there.

3.1.2 Beliefs Dynamics

I begin by determining the beliefs dynamics under δ^S, *i.e.* the compatible belief system $\mathcal{B}(\delta^S)$. The probability to be in state y^G, under $\mathcal{F}_t^I$, sums up the outsiders' beliefs: denote it q. Initially we have $q = 1$, and immediately after any disclosure $q = 1$ as well, because disclosure only occurs when the regulator observes y^G. Now assume no disclosure at $t = 1$. Either the state was bad and observed (probability $p(1 - \lambda^{GG})$) or the state was not observed (probability $1 - p$). So non-disclosure happens with probability $1 - p + p(1 - \lambda^{GG})$. And non-disclosure in the good state happens with probability $(1 - p)\lambda^{GG}$. Hence, the probability to be in state y^G after one non-disclosure period is

$$q_1 = \frac{(1-p)\lambda^{GG}}{1 - p + p(1 - \lambda^{GG})}. \tag{15}$$

And the probability to be in state y^G at $t = 2$ is

$$\gamma_1 = q_1\lambda^{GG} + (1 - q_1)\lambda^{BG}. \tag{16}$$

Recall that τ is the time elapsed since the last disclosure. Let

$$q_k(t) = \mathbb{P}(y_t = y^G | \tau = k, \zeta_\phi > t) \tag{17}$$

be the value of q after k periods of non-disclosure and

$$\gamma_k(t) = \mathbb{P}(y_{t+1} = y^G | \tau = k, \zeta_\phi > t) \tag{18}$$

be the probability to be in state y^G tomorrow after k periods of non-disclosure. These quantities only depend on t to the extent that t must be smaller than the maturity time. Hence, we can drop the dependency in t. Also for notational simplicity, the subscript k will be denoted τ. Using Bayesian updating, as in the case $k = 1$ detailed above, we obtain recursively:

$$q_{\tau+1} = \frac{(1-p)\gamma_\tau}{1-p+p(1-\gamma_\tau)}, \tag{19}$$

$$\gamma_\tau = q_\tau \lambda^{GG} + (1-q_\tau)\lambda^{BG}. \tag{20}$$

To each τ corresponds one q_τ; Figure 2 provides a graphical representation. Note that q_τ decreases to a limit weight q_V^*, which bears an economic interpretation, discussed in section 3.2.2.

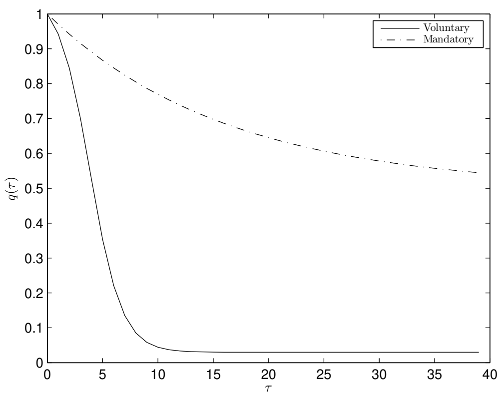

Figure 2: Probability q_τ to be in state y^G after τ periods of non-disclosure.

When no information arrives, outsiders' perceived probability to be in the good state decreases and goes to a limit weight. When disclosure is voluntary, the downgrade is much faster because the regulator is increasingly likely to be concealing bad news. The limit weight on state y^G is lower in that case.

3.1.3 Fundamental Value

Let $V(q)$ be the fundamental value of the asset when the probability to be in state y^G is q. Let $\mathbf{y} = (y^G \; y^B)^{\mathbf{T}}$ be the vector of states, and $\mathbf{q} = (q \; 1-q)$ be the vector of weights on the two states. By assumption the asset has not matured at time $t = 0$, and the probability of the maturity being $\zeta_\phi = t + 1$ for $t \geq 0$ is $(1 - \phi)^t \phi$. At time $t + 1$, the weights on the 2 states are given by the vector $\mathbf{q}\Lambda^{t+1}$, so the expected asset value conditional on $t + 1 = \zeta_\phi$ is $\mathbf{q}\Lambda^{t+1}\mathbf{y}$. Therefore

$$V(q) = \sum_{t\geq 0} \mathbb{E}[y_t | \zeta_\phi = t{+}1]\mathbb{P}(t{+}1 = \zeta_\phi) = \sum_{t\geq 0}(1{-}\phi)^t\phi\mathbf{q}\Lambda^{t+1}\mathbf{y} = \phi\mathbf{q}\Lambda(\mathrm{Id}_2{-}(1{-}\phi)\Lambda)^{-1}\mathbf{y}. \tag{21}$$

Note that V is affine in q:

$$V(q) = qV^G + (1 - q)V^B, \tag{22}$$

where V^G and V^B are found by specifying respectively $\mathbf{q} = (1\ 0)$ and $\mathbf{q} = (0\ 1)$ in (21). It will be convenient to be able to express V as a function of τ, the time since last disclosure:

$$V_\tau \equiv V(q_\tau). \tag{23}$$

3.1.4 Debt Capacity

Definition 2. *The debt capacity is the maximal amount of debt financing that can be obtained by pledging the assets under management as collateral. Under a consistent bank policy, it depends on the state τ and is defined by*

$$C(\tau) = \inf\{D \geq 0, F(D, \tau) = \emptyset\}. \tag{24}$$

By definition, if debt exceeds debt capacity during the lifetime of the asset, it is no longer possible to find investors to roll debt over. In my model, this forces a premature liquidation, because no other sources of financing are available: a run occurs. Hence, debt capacity coincides here with a *run threshold*.

Definition 3. *The fair pricing function in state τ, m_τ, is the mapping that associates to any promise F the expectation of the actual payment, under the* *creditors' information. In a consistent bank policy,*

$$m_\tau(F(D, \tau)) = D \tag{25}$$

holds in state (D, τ)*.*

Of course, m_τ, an inverse of F, is also an equilibrium object and remains to be determined, jointly with the debt capacities. In general, we have the following relationship:

$$C(\tau) = \sup_{F \geq 0} m_\tau(F). \tag{26}$$

That is, today's debt capacity is the maximum amount of financing that can be raised from promising F tomorrow.

The first key observation towards the analytical characterisation of debt capacities is the following:

Lemma 5. *Assume we are in state* τ *and let* $\chi_1, \ldots, \chi_k$ *be the possible states of the world tomorrow, and* $C(\chi_i)$ *the maximum available financing in state* χ_i*. Then today's debt capacity satisfies*

$$C(\tau) = \max\{m_\tau(C(\chi_1)), \ldots, m_\tau(C(\chi_k))\}. \tag{27}$$

This means that we do not need to consider *all* promises, as suggested by equation (26), but only the maximal viable promises in tomorrow's states of the world. The intuition is the following. When the banker increases the face value from F to $F + dF$, two cases are possible. If the states χ_i in which there is default are unchanged, then the expected repayment under $F + dF$ must be larger: $m_\tau(F + dF) > m_\tau(F)$. By contrast, if the increase in face value creates an additional default state, the expected repayment *decreases* because of the deadweight liquidation cost. Hence, as F increases, $m_\tau(F)$ increases, except when a new default state is created, in which case it jumps downwards. When is χ_i a default state? It is precisely when the face value is larger than $C(\chi_i)$. Hence Lemma 5.

I now proceed and describe tomorrow's states of the world in my model (from the point of view of outsiders). There are always four:

- χ_1: the asset has just matured ($\zeta_\phi = t+1$), in the good state y^G.
- χ_2: the asset has just matured ($\zeta_\phi = t+1$), in the bad state y^B.
- χ_3: the asset has not matured ($\zeta_\phi > t+1$), and a disclosure was made ($\tau = 0$).
- χ_4: the asset has not matured ($\zeta_\phi > t+1$), no disclosure was made ($\tau \to \tau + 1$).

From Lemma 5, we obtain

$$C(\tau) = \max\{m_\tau(C(0)), m_\tau(C(\tau+1)), m_\tau(y^G), m_\tau(y^B)\}. \tag{28}$$

The second key observation in determining the debt capacities is that there is only "one kind of good news": the observation of y^G. In order to sustain today's debt capacity, one must promise a face value that will be paid in better states of the world, because in worst states, less financing is available than today. Hence, we can directly map the $C(\tau)$ to $C(0)$, and $C(0)$ to y^G: through a simple choice of asset process, we have been able to obtain an analytically tractable functional equation for debt capacity. The following derivations make these intuitions formal.

From condition (M), $C(\tau+1) \leq C(\tau)$, and since $m_\tau(F) \leq F$ always holds, equation (28) reduces to:

$$C(\tau) = \max\{m_\tau(C(0)), m_\tau(y^G), m_\tau(y^B)\}, \tag{29}$$

for $\tau \geq 1$ and

$$C(0) = \max\{m_0(y^G), m_0(y^B)\}. \tag{30}$$

y^B is the worst state of the world, so in equilibrium the banker can make a risk-free promise:

$$m_\tau(y^B) = y^B. \tag{31}$$

Let us now deal with the pricing of bonds with face value y^G and $C(0)$, respectively.

- In case the asset matures tomorrow, there will be full payment in the good state (state χ_1) and payment of y^B in state χ_2. Otherwise, there will be liquidation, since $C(\tau) < y^G$. The liquidation value will be either V_0 (in state χ_3) or $V_{\tau+1}$ (in state χ_4). So

$$m_\tau(y^G) = \phi(\gamma_\tau y^G + (1-\gamma_\tau)y^B) + \alpha(1-\phi)(p\gamma_\tau V_0 + (1-p\gamma_\tau)V_{\tau+1}). \quad (32)$$

- Payments in states χ_1 to χ_4 are respectively $C(0)$, y^B, $C(0)$, $\alpha V_{\tau+1}$. So

$$m_\tau(C(0)) = \phi(\gamma_\tau C(0) + (1-\gamma_\tau)y^B) + (1-\phi)(p\gamma_\tau C(0) + \alpha(1-p\gamma_\tau)V_{\tau+1}). \quad (33)$$

Notice that m_τ was *a priori* unknown. But equation (32) provides a necessary expression for $m_\tau(y^G)$, which determines $C(0)$ thanks to (30) and (31). In turn, $m_\tau(C(0))$ is determined by equation (33). (29) concludes the characterisation of the debt capacities in all states. We have thus proven the following.

Proposition 1. *The equilibrium debt capacities in the voluntary disclosure case are characterised analytically by equations* (29) *to* (33).

3.1.5 Endogenous Bond Yields

Proposition 1 says that we know the debt capacities in all states, and so we also know the default states, which means that the equilibrium fair pricing functions m_τ are determined. We are then in a position to obtain the following.

Proposition 2. *The equilibrium face value schedule is characterised by*

$$F(D, \tau) = \min\{F \geq 0, m_\tau(F) = D\}. \quad (34)$$

(Details and the explicit expression for m_τ are in Appendix A.6). The gross bond yield in state τ is $R(D,\tau) \equiv F(D,\tau)/D$.

3.2 Mandatory Disclosure

The model solution under mandatory disclosure is both similar and simpler: there is no disclosure policy and information is symmetric. I quickly repeat the analysis above in order to obtain the debt capacities and bond yields under mandatory disclosure.

3.2.1 Beliefs Dynamics

Let again denote q the probability of the asset being in state y^G (now the same for the bank and outsiders) and τ be the time since the last disclosure of y^G. As in the voluntary disclosure case, there is a correspondence between τ and q. The updating rule is modified. Here, $q(\tau = 0) = 1$ and

$$q_{\tau+1} = q_\tau \lambda^{GG} + (1 - q_\tau)\lambda^{BG}. \tag{35}$$

Figure 2 provides a graphical representation.

Since asset observability is now independent from asset value, the weights on states after τ periods without observation are simply given by the iterated transition matrix, $\Lambda^{\tau+1}$. q_τ decreases to the stationary weight

$$q_M^* = \frac{\lambda^{BG}}{1 + \lambda^{BG} - \lambda^{GG}}, \tag{36}$$

which is above the limit q_V^* of q_τ in the voluntary disclosure case. The intuition is that with mandatory disclosure, no information does not mean a higher chance of bad news being concealed.

I again define γ_τ as the probability to be in state y^G tomorrow given τ periods of non-disclosure. Here, we simply have

$$\gamma_\tau = q_{\tau+1}. \tag{37}$$

3.2.2 The Stationary Weights

There is an economic intuition behind q_M^*, which represents (up to an affine transformation) the asymptotic expected value of collateral, as the economy becomes information-less. If even in the information-less economy, agents accept to roll over debt because the expected value of collateral is high enough—q_M^* large enough—it is pointless to gather information. It would even be inefficient, since the (rare) bad banks would be inefficiently closed. This is one message of Gorton and Ordoñez (2014). But even if q_M^* is large enough, the expected value of the bond collateral in the information-less economy can be insufficient to ensure information insensitivity when disclosure is strategic. Indeed, the absence of information under voluntary disclosure is worse news than under mandatory disclosure. Formally, we have the following:

Lemma 6. *For any opacity parameter $p \in (0, 1)$, the stationary weights (the probability to be in the good state when the time since the last disclosure becomes large) satisfy $q_V^* < q_M^*$. As a consequence, the expected value of the bond collateral in the information-less economy is smaller in the voluntary disclosure case.*

3.2.3 Fundamental Value

The formula (22) for $V(q)$ still holds, using the probability q to be in the good state that obtains under mandatory disclosure. I still denote $V_\tau \equiv V(q = q_\tau)$ the fundamental value of the asset after τ periods without disclosure.

3.2.4 Debt Capacity and Endogenous Bond Yields

Using the same method as before, we obtain the parallel of Propositions 1 and 2:

Proposition 3. *The equilibrium debt capacities in the mandatory disclosure case are characterised analytically by equations (29) to (31), where the op-*

erator m_τ is modified (see Appendix A.7 for its explicit expression). Again, the equilibrium face value schedule satisfies

$$F(D,\tau) = \min\{F \geq 0, m_\tau(F) = D\}. \tag{38}$$

Figure 3 provides a graphical representation.

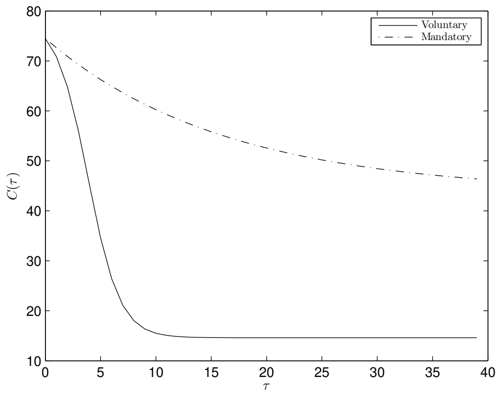

Figure 3: Debt capacities under both regimes.

3.2.5 Equilibrium characterisation

Collecting the results obtained so far, we can exhibit the equilibrium:

In the voluntary disclosure case, the equilibrium is $(F, \zeta_s = \infty, \delta^S, \mathcal{B}(\delta^S))$ where F is given in Proposition 2. In the mandatory disclosure case, the equilibrium is $(F, \zeta_s = \infty)$ where F is given in Proposition 3.

Section 4.3.1 reports the model solution in the limit of continuous-time, *i.e.* when debt has vanishing maturity.

4 Results

4.1 Opacity, Information Sensitivity and Rollover Risk

4.1.1 Notions of Information Sensitivity

The notion of *information sensitivity* is at the heart of a series of papers: Gorton and Pennacchi (1990), Dang, Gorton, and Holmström (2013), Dang, Gorton, and Holmström (2015), and Gorton and Ordoñez (2014). A security is information-insensitive when agents accept to trade it without paying to obtain a costly signal about it, and has a high information sensitivity when agents are ready to spend a lot to obtain a signal. Debt is a natural candidate to information insensitivity because its payoff is constant over all the range of non-default states.

Adverse selection. In the papers of Dang et al., this property is desirable mainly because it allows to sidestep adverse selection issues. Debt is liquid because agents are not concerned that the next buyer knows more about the collateral than they do. In this context, opacity is efficient since it makes debt information-insensitive in more states of the world.

Pooling. In Gorton and Ordoñez (2014), opacity permits the pooling of firms with good collateral with firms with bad collateral. If the average quality of collateral is high enough, firms obtain credit from lenders who do not verify firm-specific collateral quality. This financing is invested in positive NPV projects, and opacity is therefore desirable: it provides insurance to banks in terms of their access to financing. To the contrary, when information about a firm's collateral is cheap, debt becomes information-sensitive: lenders verify collateral quality and lend only conditional on good news. Firms with bad collateral are deprived of credit and welfare is lower.

One can also define the notion of information sensitivity in my model:

Definition 4 Let (D, τ) be the state today, and $F(D, \tau)$ the promised face value due tomorrow. I say that debt is *information-insensitive* if the full repayment of $F(D,\tau)$ does not imply disclosure tomorrow. To the contrary, debt is *information-sensitive* if the absence of disclosure tomorrow entails a run.

Endowed with this definition, it will be easier to understand how the information structure—the degree of transparency and the disclosure policy—impact rollover risk and the price of debt.

4.1.2 Rollover Risk, Funding Costs and the Information Structure

In this section, we back up formally the following claims.

- Transparency increases funding costs in good times; the reverse holds in bad times. As long as debt remains information-insensitive, there are less default states under opacity. This can backfire as conditions deteriorate: when debt becomes information-sensitive, the release of good news is required to avoid a bank failure, but this release is unlikely under opacity.

- Voluntary disclosure implies lower funding costs than mandatory disclosure as long as debt remains information-insensitive. However, we will see (Lemma 7) that voluntary disclosure also induces more pessimistic beliefs that mandatory disclosure as long as the bad state does not realise.

Figure 4 plots the gross yields $R(D,\tau) = F(D,\tau)/D$ after $\tau = 1$ period of non-disclosure in the voluntary disclosure case. $R(.,\tau)$ exhibits upwards jumps, which correspond the the creation of an additional default state, as explained in section 3.1.4. The jump points define regions, labelled II, IS, P and L in the Figure, with the following economic interpretation.

In the information-insensitive region (II), debt is safe: the face value satisfies $F(D,\tau) \leq C(\tau+1)$: it is below tomorrow's debt capacity if there

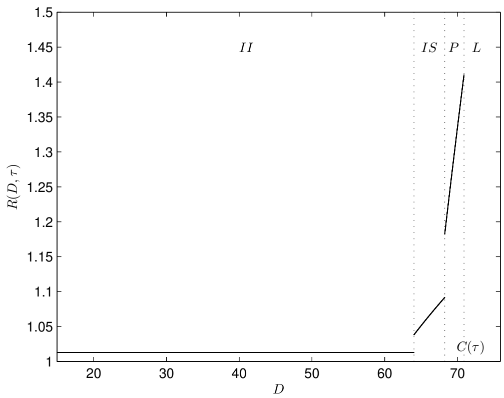

Figure 4: Bond yields as a function of debt for $\tau = 1$ under voluntary disclosure. $y^G = 100$, $y^B = 0$, $p = 50\%$, $\lambda^{GG} = 1 - \lambda^{BG} = 97\%$, $\phi = 15\%$, $\alpha = 85\%$.

is no disclosure. Hence, unless the asset matures tomorrow in the bad state, debt will necessarily be rolled over. In the II region, debt is money-like.

The information-sensitive (IS) region corresponds to face values $F(D, \tau)$ between $C(\tau + 1)$ and $C(0)$: those are higher than tomorrow's debt capacity if there is no disclosure. Hence, a run will occur tomorrow in the case no disclosure is made. Since $F(D, \tau) \leq C(0)$, however, a run will not occur if the bank discloses y^G tomorrow. Avoiding liquidation is contingent on the disclosure of good news.

The region P (for "pre-liquidation") corresponds to face values $F(D, \tau)$ above $C(0)$: liquidation will happen tomorrow unless the project matures in state y^G. This means that the bank can survive for one more period but not more. In order to provide incentives for lenders to stay in the game, the bank has to offer very high yields.

Finally, the liquidation region L corresponds to levels of debt where a run occurs today, for lack of an admissible face value to roll debt over: $D > C(\tau)$.

Figure 5 uses the same parameter values as Figure 4. The bank can now

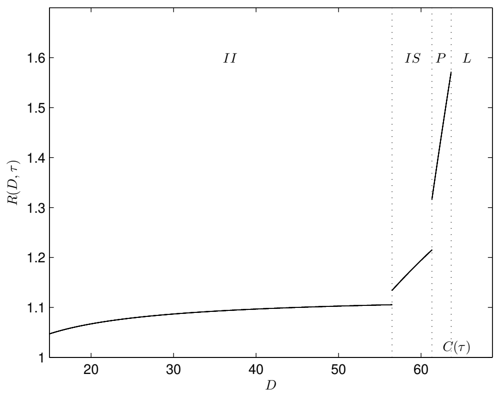

Figure 5: Bond yields as a function of debt for $\tau = 7$ under mandatory disclosure.

survive long periods of non-disclosure (here $\tau = 7$) because investors know that the regulator is genuinely uninformed. When the probability to fall into the bad state is low, the asset still has a good chance to be in state y^G after several non-disclosure periods.

I now present a series of analytical results implied by the expression of yields found in section 3. In turn, a first set of economic conclusions are derived from these results.

Proposition 4. *Let the superscript $[p]$ designate a variable relative to the model solution for opacity parameter p. The following holds:*

(a) (safer information-insensitive debt) *If regions II and IS both exist, short-term debt is less risky in the II region.*

(b) (opacity and information sensitivity) *For high opacity, i.e. small values of p, debt cannot be information-sensitive. In the voluntary disclosure case, when $p \to 1$, the information-insensitive region shrinks: for any τ, (D, τ) can not be in the II zone for p close enough to 1.*

(c) (bond yield discontinuity) *Bond yields are discontinuous in the value of debt for a given τ. As debt reaches the information-sensitivity threshold, yields jump upward.*

(d) (opacity component of short-term spreads) For a given (D, q), short-term spreads can vary with the opacity level:
(d1) *If (D, q) is at the right of the information-sensitive region for opacity parameters p_1 and p_2 with $p_1 < p_2$ then $F^{[p_1]}(D, q) > F^{[p_2]}(D, q)$.*
(d2) *If (D, q) is in the information-insensitive region for opacity parameters p_1 and p_2 with $p_1 < p_2$ then $F^{[p_1]}(D, q) \leq F^{[p_2]}(D, q)$, with equality if and only if disclosure is voluntary.*

(e) (disclosure component of short-term spreads) *For a given (D, q), voluntary disclosure provides lower yields in the V-II region: $F^V(D, q) < F^M(D, q)$.*

Points **(a)** and **(b)** confirm that information-insensitive debt is safer, and that opacity can increase the size of the information-insensitive region. This does not mean, however, that opacity is always desirable. Indeed, when $p = \mathbb{P}(\omega_t = 1)$ is small, disclosures are rare, and τ is large on average, meaning that q_τ often takes low values, and the debt level may escape the II zone. By contrast, under transparency (p close to 1), the II region is tiny and a single period of non-disclosure can trigger default; but non-disclosure is very rare.

Point **(d)** contains the prediction that there is an opacity component in short-term spreads. Spreads are primarily linked to future rollover decisions, not to the asset fundamental value. But rollover decisions occur at each node of the asset tree, whose structure depends on disclosures. Therefore opacity matters: the model predicts that in good times (D low, or q high) transparency increase spreads, while in crisis (at the right of the IS zone) transparency decrease spreads.

Point **(e)** contains the prediction that there is a disclosure component in short-term spreads and states that for a given belief about the current state of the world, voluntary disclosure allows the bank to borrow at better terms as long as debt is information-insensitive under this disclosure regime. This does not mean, however, that voluntary disclosure is always desirable. Indeed, voluntary disclosure produces more pessimistic beliefs than mandatory disclosure as long as the bad state does not realise: see Lemma 7 in the next section, where we formalise the comparison between the two disclosure regimes.

As Figures 4 and 5 show, the IS zone is in general tiny, and can also not exist. In that case, the debt directly switches from being information-insensitive to being defaulted upon, making the trade off between short-term protection and long-term exposure even clearer. This occurs typically when the maturity of short-term debt is small compared to the expected time before the next observation of the asset. In this situation, disclosure is unlikely. An information-sensitive debt would therefore be defaulted upon with such a high probability that no information-sensitive contract is feasible.

The analysis so far indicates that in good times, opacity makes debt safer in the short term, at the cost of increasing exposure to runs at longer horizons; and that a similar tension exists regarding the comparison of the disclosure policies: voluntary disclosure allows to lower financing costs in the short term but can lead to significantly worse beliefs at longer horizons. In section 4.3, I study how these effects play out at the aggregate level, *i.e.* which ones affect the most efficiency and run probabilities.

4.2 Impact of the Disclosure Regime on Equilibrium Outcomes

4.2.1 Methodology

In order to understand how the nature of disclosure affects the dynamics of debt and our quantities of interest—run probability and our measure of inefficiency—we need to compare them all other things being equal. This is achieved in the following way. Consider some

$$\psi \equiv \left(\zeta_\phi, (y_t)_{t\leq\zeta_\phi}, (\omega_t)_{t\leq\zeta_\phi}\right). \tag{39}$$

ψ is the data of a maturity date ζ_ϕ, all the positions of the asset in the Markov chain before ζ_ϕ, and all the observability shocks before ζ_ϕ. We can now compute, for the same ψ, the equilibrium paths of debt $(D_t^V(\psi))$, $(D_t^M(\psi))$ and liquidation times (if any) $\zeta_\ell^V(\psi)$, $\zeta_\ell^M(\psi)$ in the cases of voluntary and mandatory disclosure, respectively.

The fundamental value of the asset is identical at all times across both scenarios. The same holds true for the information collected by the regulator. Moreover, if along ψ, $y_t = y^G$ for all t, the signals received by the creditors are also the same at all times across both scenarios (at any t, they received either $\delta_t = y^G$, announcement of the good state, or $\delta_t = \emptyset$, announcement that the asset has not been observed by the regulator). Even in that case, the debt and beliefs dynamics will be different across the two disclosure regimes considered. This can lead to dramatically different outcomes, as shown in the next section.

Hence, we are able to isolate effects due the disclosure policy by fixing a history ψ and computing the debt and beliefs dynamics along ψ in both disclosure regimes. Having defined formally the comparison between regimes, the following result, announced in section 4.1.2, is now clear:

Lemma 7. *Along any ψ, $q_t^V \leq q_t^M$ for any $t < \min\{\tau_f^V, \tau_f^M\}$ with equality only when $\omega_t = 1$ and $y_t = y^G$.*

The lemma states that voluntary disclosure consistently produces depressed beliefs as long as the bad state does not realise—in which case the bank fails under mandatory disclosure—because investors anticipate the possibility that the bank may conceal bad news. The only case where the beliefs are the same under both disclosure regimes is when the bank has just announced the good state.

We are also in a position to define the following:

Definition 4. *A voluntary disclosure-induced run (or credibility run) is a fundamental path ψ that produces a run when disclosure is voluntary but no run under mandatory disclosure. A mandatory disclosure-induced run is defined similarly.*

The alternative name "credibility run" for a voluntary disclosure-induced run comes from the fact that under voluntary disclosure, the regulator lacks credibility when she announces no observation of the asset, even when this is actually the case. Because she has not taken any commitment, the absence of news release is interpreted as very bad news by the creditors. In situations where no observations are made for a protracted period of time ($\omega_t = 0$ for several consecutive t), creditors rationally downgrade a lot their beliefs about the asset quality. This potentially leads to a run that would have been avoided under mandatory disclosure. Indeed, under mandatory disclosure, creditors are safe in the knowledge that the regulator is genuinely uninformed, and not trying to conceal bad news.

4.2.2 Voluntary disclosure-induced and Mandatory disclosure-induced Runs

Figures 7 and 8 (reported in Appendix C) plot two sample paths of debt for both disclosure regimes, and the associated beliefs dynamics: $q_t = \mathbb{P}(y_t = y^G|\mathcal{F}_t^I)$ is the probability to be in the good state under the creditors' information set. A black dot at time t indicates that the regulator has observed the asset at time t: $\omega_t = 1$. Along both sample paths, the asset actually always was in the good state: $y_t = y^G$ for all t. As mentioned in the previous section, this means that the fundamentals, the regulator's information, and the signals received by the creditors are identical in each example across the two disclosure regimes. All differences in outcomes are explained by the difference in the commitment decision of the regulator, which leads to different information structures and therefore different beliefs and debt dynamics.

Figure 7 depicts a credibility run. In the beginning, interest rates are lower under voluntary disclosure. This is because the bad state, should it occur, will not be revealed under voluntary disclosure, but will be revealed under mandatory disclosure. Hence, voluntary disclosure produces less default states and reduces the bank's cost of financing, leading to a slower growth of the stock of debt. But a run suddenly occurs: this is because news have not been released for a protracted period of time, leading to a sharp decline in the creditors' beliefs, as illustrated by the bottom panel: observe the plunge of q_t between periods $t = 10$ and $t = 14$. In turn, this strong decline in beliefs leads to a strong decline in debt capacities.

By contrast, under mandatory disclosure, the bank is resilient to long non-disclosure periods because creditors know that the regulator would be forced to reveal the bad state, had it been observed.

Prior to $\tau_\ell^V - 1$, the yields $\frac{D_{t+1}}{D_t}$ are lower under voluntary disclosure, but *it is under this disclosure regime that the bank undergoes a run.* This means that one cannot unconditionally map the current value of the short-term yield to the health of a financial institution: yields are to a large extent determined by the opacity of the collateral and the disclosure policy; and because they only reflect next period's rollover risk, they can remain low even when the probability of a run at a small horizon is very large.

Figure 8 shows a mandatory disclosure-induced run.

At $\tau_\ell^M = 52$, the bank undergoes a run under mandatory disclosure. Prior to that date, good news were regularly released, producing consistently large values of q_t and maintaining the information-insensitive status of debt under voluntary disclosure. Similar to point **(e)** in Proposition 4, the bank was therefore able to borrow at better terms under voluntary disclosure.

Across both disclosure regimes, the fundamentals and the signals are identical at all times, and the probability to be in the good state is in fact *always weakly larger under mandatory disclosure*, but it is nevertheless under this disclosure regime that the bank undergoes a run. *The critical channel here is the endogenous refinancing cost:* the funding cost channel. Mandatory disclosure produces an information structure that generates more default states in good times (even though it produces better *average* beliefs). This implies larger financing costs, and the stock of debt grows faster. This can lead to a run that only occurs under mandatory disclosure.

Finally, note that debt became information-sensitive at $t = 79$ under voluntary disclosure, consistent with a sharp decrease in q_t (see bottom panel of Figure 8). This corresponds to an upward "jump" in the stock of debt. A run was nevertheless avoided, because the required good news were indeed announced: $\omega_{80} = 1$.

The discussion in this section evidences that the two considered disclosure regimes can lead to dramatically different outcomes, but that none of them is uniformly better that the other. The question of which regime is more efficient on aggregate is studied in section 4.3.3.

4.3 Results at the Aggregate Level

4.3.1 Debt capacities and equilibrium dynamics in continuous-time

In the remainder of section 4, I focus on the limiting case of vanishing debt maturity, which gives rise to a continuous-time model. In this limit, debt capacities take a simple form and the stochastic process of the bank's debt is known explicitly. The reader uninterested in these derivations can move to section 4.3.2. Appendix B.1 clarifies why it is useful to consider the continuous-time limit to study the aggregate properties of the model and what are the pros and cons of working in discrete rather than continuous time.

In the limit of vanishing debt maturity, the Markov chain with transition matrix Λ becomes a continuous-time Markov chain with infinitesimal generator $A = \begin{pmatrix} -p_c & p_c \\ p_r & -p_r \end{pmatrix}$. $p_c dt$ (the equivalent of $1-\lambda^{GG}$) is the instantaneous probability to move from the good to the bad state; $p_r dt$ is the equivalent of λ^{BG}, and ϕdt is now the equivalent of ϕ. The bank only observes the chain at times $(\zeta_o^k)_{k\geq 1}$, which are exponentially distributed with parameter p_o (the equivalent of p). If the bank has to disclose the bad state at ζ_o, the game ends; we now consider times before the termination date ζ_f. The corresponding definition of the variable τ—time since the last observation of the good state—in the continuous-time version of the model is

$$\tau = t - \sup\{\zeta_o^k,\ \zeta_o^k \leq t,\ y_{\zeta_o^k} = y^G\}. \tag{40}$$

y^B is set to zero without loss of generality to simplify the exposition. The transition probabilities of the Markov chain with generator A at horizon t are

$$\begin{aligned} P_{GG}(t) &= \frac{p_r}{p_c + p_r} + \frac{p_c}{p_c + p_r} e^{-(p_c+p_r)t} & (41) \\ P_{BG}(t) &= \frac{p_r}{p_r + p_c} - \frac{p_r}{p_r + p_c} e^{-(p_c+p_r)t}, & (42) \end{aligned}$$

and the fundamental values $V^i = \mathbb{E}\left[y_{\zeta_\phi} | y_0 = y^i\right]$ satisfy

$$\begin{pmatrix} V^G \\ V^B \end{pmatrix} = \phi(\phi I - A)^{-1}\mathbf{y} == \frac{1}{\phi + p_c + p_r} \begin{pmatrix} \phi + p_r \\ p_r \end{pmatrix} y^G. \tag{43}$$

The fundamental value in state τ is then given by

$$V_\tau = q_\tau V^G + (1 - q_\tau) V^B \tag{44}$$

where (q_τ) depends on the disclosure regime and is given in Lemma 5.

Proposition 5. *In the limit of continuous-time, debt capacities take a simple form:*

$$C(\tau) = \alpha V_\tau$$

The beliefs dynamics are:

- *Mandatory disclosure:* $q_\tau = P_{GG}(t)$;
- *Voluntary disclosure:* $dq_\tau = (-p_c q_\tau + p_r(1-q_\tau) - p_o q_\tau(1-q_\tau))d\tau$, *i.e.* q_τ *solves a Ricatti equation and is therefore given by*

$$q_\tau = -\frac{1}{p_o}\frac{\kappa_1 k e^{\kappa_1 \tau} + \kappa_2 e^{\kappa_2 \tau}}{k e^{\kappa_1 \tau} + e^{\kappa_2 \tau}}. \tag{45}$$

The values of the constants k, κ_1, κ_2 *are given in the proof.*

Additionnally, in continuous-time, the stochastic process of the debt dynamics itself can be written as a compact analytical expression.[15] Let $\zeta_o^{k(t)} = \sup\{\zeta_o^k,\ \zeta_o^k \leq t,\ y_{\zeta_o^k} = y^G\}$.

Proposition 6. *For any time* t *before the final time* ζ_f*, the equilibrium debt dynamics under mandatory and voluntary disclosure are respectively given by*

$$\begin{aligned} D_t^{(M)} &= \frac{p_o}{p_o+\phi}\alpha V^B + \left(D_{\zeta_o^{k(t)}} - \frac{p_o}{p_o+\phi}\alpha V^B\right) e^{(\phi+p_o)a(t-\zeta_o^{k(t)}) + (\phi+p_o)\frac{a}{b}\left(e^{-b(t-\zeta_o^{k(t)})}-1\right)} \\ D_t^{(V)} &= D_{\zeta_o^{k(t)}} \exp\left(\phi\left(t-\zeta_o^{k(t)}\right) - \phi\left(Q_{t-\zeta_o^{k(t)}} - Q_0\right)\right), \end{aligned}$$

where Q *is a deterministic function. Under full transparency or full opacity, these expressions simplify even further and yield the moments of interests—*

[15] Recall that this process is a compounding of the endogenous state-contingent bond yields that realize along the equilibrium path. Just as in discrete time, these yields are available in closed-form; but their expression is now simpler, which allows to obtain a more practical expression of (D_t).

run probability and efficiency—in quasi-closed form. (All analytical expressions are given in the proof.)

4.3.2 Impact of opacity on run probability and efficiency

In this section only, I abstract from the disclosure regime and ask whether full opacity is preferable to ful transparency. In these two polar cases, the disclosure regimes are equivalent. Using the closed-form announced in Proposition 6, I can compute exactly the measures of interest on a fine discretisation of the parameter space. Hence, the following result is numerical in the sense that we cannot check for *all* possible parameter values, not because our measures of interest are approximated.

The subscripts "FO" and "FT" will stand for "full opacity" and "full transparency", respectively.

Numerical Result 1. *For any parameter set Θ, there are $\left(\lambda^{GG}\right)^*(\Theta_{-\lambda^{GG}})$, $\left(\lambda^{BG}\right)^*(\Theta_{-\lambda^{BG}})$, $\phi^*(\Theta_{-\phi})$, $\alpha^*(\Theta_{-\alpha})$ and $D_0^*(\Theta_{-D_0})$ such that $\mathcal{I}_{FO} < \mathcal{I}_{FT}$ if and only if*

. $\lambda^{GG} > \left(\lambda^{GG}\right)^*$

. $\lambda^{BG} > \left(\lambda^{BG}\right)^*$

. $\phi > \phi^*$

. $\alpha > \alpha^*$

. $D_0 < D_0^*$.

The same result holds for the comparison of the run probabilities $\mathcal{P}_{FO}$ and $\mathcal{P}_{FT}$.

The intuition is the following. A large λ^{GG}, a large λ^{BG}, a large ϕ, a large α or a small D_0 all correspond to situation with good fundamentals: the probability that the asset matures in the good state is large, liquidation costs are low, or the initial stock of debt is a lot below debt capacity. In those cases, debt is more likely to be information-insensitive, so the drawbacks associated with opacity matter less.

Figure 6 provides a graphical illustration: $\tilde{\mathcal{I}} = \frac{1}{1-\alpha}\mathcal{I}$ designates the expected value of the asset conditional on premature liquidation and preserves the ordering between opacity and transparency given by $\mathcal{I}$. Note that under $\mathcal{I}$ both curves agree at 0 when $\alpha = 1$, in which case there is no loss of value upon liquidation. For low α, the short-term protection of opacity lasts less because debt capacities are low. Moreover runs, when they occur on good banks, are particularly harmful in terms of efficiency. The reverse holds for α close to 1.

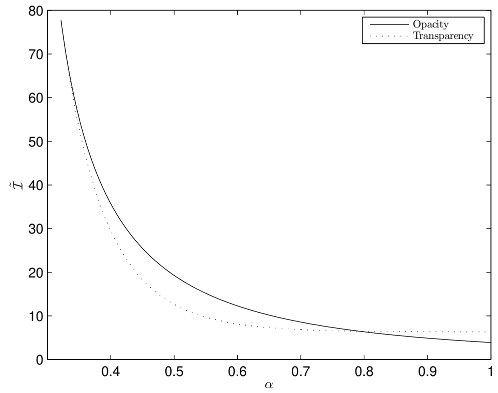

Figure 6: Inefficiency as a function of the liquidity parameter α.

Maximising efficiency does not imply minimising the likelihood of runs: one can have $\mathcal{I}_{FO} > \mathcal{I}_{FT}$ even if $\mathcal{P}_{FO} < \mathcal{P}_{FT}$. This is well illustrated by the following analytical result obtained in a special case:

Proposition 7. *Consider the continuous-time limit of the model and assume* $V^B = 0$.

When $\phi > 1 - \lambda^{GG}$*, there exists* α^* *such that for* $\alpha_{min} := \frac{D_0}{V^G} < \alpha < \alpha^*$, $\mathcal{P}_{FT}(\alpha) < \mathcal{P}_{FO}(\alpha)$ *and for* $\alpha > \alpha^*$, $\mathcal{P}_{FT}(\alpha) > \mathcal{P}_{FO}(\alpha)$.

When $\phi < 1 - \lambda^{GG}$, we always have $\mathcal{P}_{FT}(\alpha) > \mathcal{P}_{FO}(\alpha)$. However, $\mathcal{I}_{FT}(\alpha) < \mathcal{I}_{FO}(\alpha)$ for any interior α.

According to Proposition 7, there are parameters such that a bank undergoes more runs on average under transparency, but where the expected costs of premature liquidation are nevertheless larger under opacity. The intuition is that under opacity, a bank can be hit by a runs and nevertheless be healthy. With our assumption that runs on good banks are more costly, it follows that the average cost of a run under opacity is larger.

It is interesting to link this result to section 4.1.1, where we discussed the argument of Gorton and Ordoñez (2014) that opacity provides insurance to banks in their access to funding and may therefore be desirable. Proposition 7 says that opacity may improve access to financing in the sense that it lowers the probability that creditors refuse to refinance the debt, but be less efficient. Intuitively, the pooling of good and bad banks backfires, as some healthy institutions, for which credit is the most valuable, can be denied credit.

4.3.3 Comparison between mandatory and voluntary disclosure

We now return to the general case of interior levels of opacity. The main result of this section is the following; it is obtained by simulation of the continuous-time model (details are provided in Appendix B.2).

Numerical Result 2. *Mandatory disclosure is more efficient than voluntary disclosure for $p \in (p^* (\Theta_{-p}), 1)$. The average value of $\frac{p^*}{\phi}$ is below 2%.*

This result is striking: the average level of transparency above which mandatory disclosure is more efficient corresponds to a situation where the regulator approximately only has a 2% chance to observe the asset value once during its lifetime. In other words, unless we are at extreme levels of opacity, mandatory disclosure is more efficient.

When p is very small, the regulator is very unlikely to be informed, hence beliefs dynamics are virtually the same in both regimes. But under voluntary disclosure, the regulator can successfully conceal the bad state, should it realise and be observed. This can render voluntary disclosure more efficient; but clearly, the difference cannot be economically significant, since the states in which voluntary diclosure is useful have very low probability.

Otherwise, the two disclosure regimes stand in much sharper contrast: the disclosure policy matters economically. Numerical Result 2 states that in these cases, mandatory disclosure improves efficiency. This means that the beliefs channel outweighs the funding cost channel. Being able to borrow at better terms in good times and having the option to conceal bad news does not compensate for the fact that voluntary disclosure depresses investor's beliefs and therefore the debt capacities.

Because the average value of $\frac{p}{\phi}$ at which mandatory disclosure begins to be more efficient is extremely low and in light of the discussion above, Numerical Result 2 can be reformulated as: "whenever there is an economically meaningful difference between the two disclosure policies, mandatory disclosure is more efficient."

Conclusion

Opacity and disclosure regimes matter to the outcome of the rollover game because they shape the information tree, and therefore the short-term yields and the beliefs dynamics. Starting from the good state, opacity provides protection in the short run, but is likely to increase exposure at longer horizons—a tension which is amplified under voluntary disclosure. At the aggregate level, the model predicts that opacity reduces run probability and inefficiency only in situations where the fundamentals are strong anyways; that opacity may decrease run probability but increase inefficiency; and indicates that mandatory disclosure is more efficient than voluntary disclosure except at large levels of opacity.

Several extensions of the model appear interesting. First, relaxing the rigid structure of the bank's balance sheet should provide valuable additional insights; for instance, the bank may also have long-term debt, or use cash reserves to manage its risk of run. Second, one could introduce state-contingent regulation rather than fixing ex ante the disclosure regime. Third, the information structure could be refined by considering a richer set of signals about the current asset value, in order to hone the modelling of the regulator's strategy set. One could then reformulate the model as an explicit Bayesian persuasion problem and compare it to the existing Bayesian persuasion literature on stress tests. Finally, the bank could have access to several investment opportunities, and may have moral hazard incentives to engage into inefficient projects. Clearly, the bank's portfolio decision and the regulator's opacity and disclosure choices would affect each other, giving rise to a potentially rich interaction.

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A Proofs

A.1 Lemma 1

By definition of $(\tilde{D}_t)$, $\mathbb{E}\left[\tilde{D}_{(t+1)\wedge\zeta_f}|\mathcal{F}_t^I\right] = \mathbb{E}\left[\tilde{D}_{t+1}|\mathcal{F}_t^I\right] = \tilde{D}_t = \tilde{D}_{t\wedge\zeta_f}$ over $\{t < \zeta_f\}$. And $\tilde{D}_{(t+1)\wedge\zeta_f} = \tilde{D}_{t\wedge\zeta_f} = D_{\zeta_f}$ over $\{t \geq \zeta_f\}$. Therefore $\mathbb{E}[\tilde{D}_{(t+1)\wedge\zeta_f}|\mathcal{F}_t^I] = \tilde{D}_{t\wedge\zeta_f}$.

A.2 Lemma 2

From Lemma 1, $(\tilde{D}_{t\wedge\zeta_f})$ is a martingale, and it is bounded due to condition (NP). Therefore it is a closed martingale. Now consider a bank at t which is not forced into liquidation. This means that $\zeta_f > t$, and since $(\tilde{D}_{t\wedge\zeta_f})$ is a closed $(\mathcal{F}_t^I)$-martingale, $\mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^I] = D_t$. Now, note that $\zeta_f = \zeta_\ell \wedge \zeta_\phi$ and that

$$\tilde{D}_{\zeta_\ell}\mathbb{I}_{\{\zeta_\ell<\zeta_\phi\}} + \tilde{D}_{\zeta_\phi}\mathbb{I}_{\{\zeta_\ell\geq\zeta_\phi\}} \leq V_{\zeta_\ell}\mathbb{I}_{\{\zeta_\ell<\zeta_\phi\}} + y_{\zeta_\phi}\mathbb{I}_{\{\zeta_\ell\geq\zeta_\phi\}}. \tag{46}$$

Taking expectations and noting that the expectation of the right-hand side is the same average of maturity values y_{ζ_ϕ} as $V_t \equiv \mathbb{E}[y_{\zeta_\phi}|\mathcal{F}_t^I]$, we obtain

$$D_t = \mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^I] \leq V_t. \tag{47}$$

The bank is solvent.

A.3 Lemma 3

Assume voluntary disclosure (the proof in the mandatory disclosure case is included in this one). Consider a time $t < \zeta_f$. If $\delta_t = \emptyset$ and the banker decides to liquidate, she obtains the value $\alpha V^B < D_0 < D_t$ and her equity is worth zero. Now, if $\delta_t \neq \emptyset$, payoff-relevant information is symmetric:

$$\begin{aligned} V_t \equiv \mathbb{E}[y_{\zeta_\phi}|\mathcal{F}_t^I] &= \mathbb{E}[y_{\zeta_\phi}|\mathcal{F}_t^B] \\ \mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^I] &= \mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^B]. \end{aligned} \tag{48}$$

Since $(\tilde{D}_{t\wedge\zeta_f})$ is a closed martingale,

$$D_t = \tilde{D}_t = \mathbb{E}\left[\lim_{s\to\infty} \tilde{D}_{s\wedge\zeta_f}|\mathcal{F}_t^I\right] = \mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^I]. \tag{49}$$

Combining (48) and (49), we obtain

$$
\begin{aligned}
\alpha V_t - D_t &= \mathbb{E}[\alpha y_{\zeta_\phi} \mathbb{I}_{\{\zeta_\phi \le \zeta_\ell\}} + \alpha y_{\zeta_\ell} \mathbb{I}_{\{\zeta_\phi > \zeta_\ell\}} - \tilde{D}_{\zeta_f} | \mathcal{F}_t^I] \\
&= \mathbb{E}[\alpha y_{\zeta_\phi} \mathbb{I}_{\{\zeta_\phi \le \zeta_\ell\}} + \alpha y_{\zeta_\ell} \mathbb{I}_{\{\zeta_\phi > \zeta_\ell\}} - \tilde{D}_{\zeta_f} | \mathcal{F}_t^B] \\
&< \mathbb{E}[y_{\zeta_\phi} \mathbb{I}_{\{\zeta_\phi \le \zeta_\ell\}} + \alpha y_{\zeta_\ell} \mathbb{I}_{\{\zeta_\phi > \zeta_\ell\}} - \tilde{D}_{\zeta_f} | \mathcal{F}_t^B]. \qquad (50)
\end{aligned}
$$

The first line is the payoff from liquidating today. The last line is the $\mathcal{F}_t^B$-expected payoff of never liquidating strategically.

A.4 Lemma 4

First note that for $p \in (0,1)$, in a consistent belief system, $q_t = 0$ after disclosure of the bad state, $q_t = 1$ after disclosure of the good state, and $q_t \in (0,1)$ absent disclosure. Fix such a belief system and show that the sanitisation strategy is optimal. Due to discounting (Blackwell (1965)) we can focus on one-shot deviations. Note that under F and for a given D_0, the event tree is discrete. This is because there are always at most 4 possible states tomorrow, given the state today. Let us consider the choice between playing the sanitisation strategy δ^S at some node or something else, leaving the rest of the strategies unchanged. Let $\mathcal{O}$ be the event tree following playing δ^S at this node and $\mathcal{D}$ the event tree following the other move (the "deviation"). The deviation is either the regulator switching from concealing the bad state to disclosing it, or concealing the good state instead of disclosing it. The former case is equivalent to a strategic default enforced by the regulator, which is never optimal, similarly to Lemma 3. Thus, focus on the latter case and relabel $t = 0$ the deviation time (at which $y_0 = y^G$), and let $y_1, \ldots, y_n, \ldots$ and $J = \zeta_\phi$ be a possible realisation of future asset states and maturity. By condition (M) and induction, the face values $\tilde{F}_0, \ldots, \tilde{F}_n, \ldots$ associated with y_0 undisclosed and the realisations $y_1, \ldots, y_n, \ldots$ disclosed according to the sanitisation strategy satisfy $\tilde{F}_i \ge F_i$, where F_i are the face values in $\mathcal{O}$. Let j be the liquidation time in $\mathcal{D}$. Three cases are possible. (i) $j \le J - 1$ and there is liquidation at time j in $\mathcal{O}$: then there is liquidation at time j in both $\mathcal{O}$ and $\mathcal{D}$. Since due debt is higher in $\mathcal{D}$ ($\tilde{F}_{j-1} \ge F_{j-1}$), the residual claim is lower in $\mathcal{D}$. (ii) $j \le J - 1$ and there is no liquidation at time j in $\mathcal{O}$: there, debt is lower, and the arguments of the proof of Lemma 3 allow to conclude that the expected residual claim at time j conditional on $\zeta_\phi = J$ is higher in the original tree. (iii) $j \geq J$: the asset matures before liquidation both in $\mathcal{O}$ and $\mathcal{D}$. Since debt is lower in $\mathcal{O}$, the expected residual claim is higher in $\mathcal{O}$. Finally, note that the expected profit at date $t = 0$ is an average of expectations of the residual claim conditional on $y_0 = y^G, y_1, \dots, y_j, \zeta_\phi = J$. Cases (i), (ii) and (iii) above show that these quantities are higher in $\mathcal{O}$ for all J, j, $y_1, \dots, y_j$. Hence, it is optimal for the regulator to play the sanitisation strategy. The belief system consistent with this strategy is then the one given in section 3.1.2.

A.5 Lemma 6

Voluntary disclosure case. In equations (19) and (20), we obtained the recursive relationship

$$q_{\tau+1} = \frac{(1-p)(q_\tau \lambda^{GG} + (1-q_\tau)\lambda^{BG})}{1 - p + p(1 - q_\tau \lambda^{GG} - (1-q_\tau)\lambda^{BG})}. \tag{51}$$

A standard sequence analysis reveals that (q_τ) decreases to the root of G that lies in $[0,1]$, with $G \equiv G_1 - G_2$ and

$$\begin{aligned} G_1(q) &\equiv q(1 - p + p(1 - q\lambda^{GG} - (1-q)\lambda^{BG})) & (52)\\ G_2(q) &\equiv (1-p)\lambda^{BG} + q(1-p)(\lambda^{GG} - \lambda^{BG}). & (53) \end{aligned}$$

This root is the stationary weight in case of voluntary disclosure, q_V^*, and satisfies

$$q_V^* = \frac{1 - (1-p)\lambda^{GG} - (2p-1)\lambda^{BG} - \sqrt{(\lambda^{BG})^2 + 2\lambda^{BG}(\lambda^{GG}(p-1) - 2p + 1) + (1 - (1-p)\lambda^{GG})^2}}{2p(\lambda^{GG} - \lambda^{BG})}. \tag{54}$$

Mandatory disclosure case. The expression of q_M^* in (36) results directly from considering the fixed point of (35). We now set out to obtain the inequality $q_V^* < q_M^*$. Since G can only be non-negative for $q \geq q_V^*$, it is sufficient to show that $G(q_M^*; p) \geq 0$ (with obvious notation) for any value of p. Direct calculation shows that

$$\frac{\partial}{\partial p} G(q_M^*; p) \tag{55}$$

has the same sign as $1 + \lambda^{BG} - 2\lambda^{GG}$. In particular, it is of constant sign and we obtain

$$G(q_M^*; p) \geq \min\{G(q_M^*; 0), G(q_M^*; 1)\}. \tag{56}$$

Since $G(q_M^*;0) = 0$ and $G(q_M^*;1) > 0$, we obtain $G(q_M^*;p) \geq 0$.

A.6 Proposition 2

We know that in a consistent bank policy, $m_\tau(F(D,\tau)) = D$. The banker picks the lowest F that satisfies this equation, because expected liquidation costs are increasing in F. Hence, in equilibrium,

$$F(D,\tau) = \min\{F \geq 0, m_\tau(F) = D\}. \tag{57}$$

In order to find F, we need to make m_τ explicit. If $F \leq y^B$, the promise of F is never defaulted upon: $m_\tau(F) = F$. If $F \leq C(\tau+1)$, there is one default state (state χ_2, see section 3.1.4) and

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)F. \tag{58}$$

If $F \in (C(\tau+1), C(0)]$, there are two default states (χ_2 and χ_3) and

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)(p\gamma_\tau F + \alpha(1-p\gamma_\tau)V_{\tau+1}). \tag{59}$$

If F belongs to $(C(0), y^G]$, there are three default states (χ_2, χ_3 and χ_4) and

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + \alpha(1-\phi)(p\gamma_\tau V_0 + (1-p\gamma_\tau)V_{\tau+1}). \tag{60}$$

A.7 Proposition 3

The probability of an announcement tomorrow is $p\gamma_\tau$. The probability of no announcement is $1-p$. Otherwise, state y^B is disclosed (probability $p(1-\gamma_\tau)$). First, if $F \leq y^B$, $m_\tau(F) = F$. If $y^B < F \leq C(\tau+1)$, then

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)((1-p(1-\gamma_\tau))F + p(1-\gamma_\tau)\alpha V^B). \tag{61}$$

If $C(\tau+1) < F \leq C(0)$, then

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)(p\gamma_\tau F + (1-p)\alpha V(\tau+1) + p(1-\gamma_\tau)\alpha V^B). \tag{62}$$

If $C(0) < F \leq y^G$, then

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + \alpha(1-\phi)(p\gamma_\tau V^G + (1-p)V(\tau+1) + p(1-\gamma_\tau)V^B). \tag{63}$$

A.8 Proposition 4

Proofs are presented in the voluntary disclosure case, and work identically in the mandatory disclosure case. I first need to introduce the

Lemma 8. *Let τ be a fixed integer and $0 < p^* < 1$. If $\alpha < 1$, there is $K_\tau > 0$ such that for all $p \leq p^*$,*

$$C^{[p]}(\tau) \geq \alpha V_\tau^{[p]} + K_\tau, \tag{64}$$

where the superscript $[p]$ designates a variable relative to the model solution under the opacity parameter p.

Proof. Promising y^G entails costly liquidation tomorrow unless the asset matures. Hence,

$$m_\tau^{[p]}(y^G) = \phi(1-\alpha)\left(\gamma_\tau^{[p]} y^G + (1-\gamma_\tau^{[p]})y^B\right) + \alpha V_\tau^{[p]}. \tag{65}$$

We obtain the result by setting $K_\tau = \phi(1-\alpha)\gamma_\tau^{[p^*]}$, noting that $C^{[p]}(\tau) \geq m_\tau^{[p]}(y^G)$. ■

We now come back to the proof of Proposition 4.

(a) Debt in the II region satisfies

$$D = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)F, \tag{66}$$

with $F \leq C(\tau+1)$. Thus, the inverse yield verifies

$$\frac{D}{F} \geq \phi\gamma_\tau + \phi(1-\gamma_\tau)\frac{y^B}{C(\tau+1)} + 1 - \phi. \tag{67}$$

Debt in the IS region satisfies

$$D = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)(p\gamma_\tau F + \alpha(1-p\gamma_\tau)V_{\tau+1}), \tag{68}$$

with $C(\tau+1) < F \leq C(0)$. From there,

$$\begin{aligned}\frac{D}{F} &\leq \phi\gamma_\tau + \phi(1-\gamma_\tau)\frac{y^B}{C(\tau+1)} + (1-\phi)\left(p\gamma_\tau + \alpha\frac{(1-p\gamma_\tau)V_{\tau+1}}{F}\right) \\ &\leq \phi\gamma_\tau + \phi(1-\gamma_\tau)\frac{y^B}{C(\tau+1)} + 1 - \phi, \end{aligned} \tag{69}$$

where the last inequality holds because of Lemma 8. We conclude by comparison with Equation (67).

(b) We first need to show that for p small, $m_\tau(C(0)) < m_\tau(C(\tau+1))$. This implies that promising face values between $C(\tau+1)$ and $C(0)$ does not allow to roll over other debt levels than the ones in the II zone: there is no IS zone. Given the expressions of $m_\tau(C(0))$ and $m_\tau(C(\tau+1))$, the desired inequality is equivalent to

$$p\gamma_\tau^{[p]}C(0) + \alpha(1-p\gamma_\tau^{[p]})V_{\tau+1}^{[p]} \leq C^{[p]}(\tau+1). \tag{70}$$

We conclude by letting $p \to 0$ and using Lemma 8. For the case $p \to 1$, recall that debt capacity is always below the fundamental value from Lemma 2. In the voluntary disclosure case, as p goes to 1, $q_{\tau+1}$ goes to 0, so the fundamental value goes to V^B. Now let $D > V^B$. We have

$$m_\tau^{[p]}(C^{[p]}(\tau+1)) < C^{[p]}(\tau+1) \leq V_{\tau+1}^{[p]} \to y^B, \tag{71}$$

hence D can not be in the II zone for p close enough to 1.

(c) is a consequence of the fact that $m(C(\tau+1)+\varepsilon) < m(C(\tau+1))$ for ε close to 0 and $\alpha < 1$. Recall that this is because the face value is only infinitesimally higher, but there will be default in one more state of the world (the non-disclosure state), meaning that the proportional cost $1-\alpha$ now applies to an additional, non-zero probability, state of the world.

(d) **(d1)** Let $p_1 < p_2$, τ_1, τ_2 such that

$$q_{\tau_1}^{[p_1]} = q_{\tau_2}^{[p_2]} = q. \tag{72}$$

The probability to be in state y^G tomorrow is $q' = \lambda^{GG}q + \lambda^{BG}(1-q) = \gamma_{\tau_1}^{[p_1]} =$

$\gamma_{\tau_2}^{[p_2]}$. Then, the probability to be in state y^G tomorrow conditional on no disclosure under parameter p_1 is $\frac{(1-p_1)q'}{1-p_1 q'}$. Using the expression of the yield in the IS region, we find

$$\begin{aligned} m^{[p_1]}(F) &= \phi(q'y^G + (1-q')y^B) \\ &+ (1-\phi)\left(q'p_1F + \alpha(1-p_1q')\left[\frac{(1-p_1)q'}{1-p_1q'}V^G + \frac{1-q'}{1-p_1q'}V^B\right]\right) \end{aligned} \tag{73}$$

From there,

$$m^{[p_1]}(F) - m^{[p_2]}(F) = (p_2 - p_1)q'(\alpha V^G - F), \tag{74}$$

which is negative for F close to $C(0)$ by Lemma 8. Given that $D = m^{[p_1]}(F^{[p_1]})$, we have $D < m^{[p_2]}(F^{[p_1]})$, from which we deduce that $F^{[p_1]} > F^{[p_2]}$. Indeed, (D, τ_2) belongs to the IS region under p_2, and $m^{[p_2]}(.)$ is increasing over this region, and must satisfy $D = m^{[p_2]}(F^{[p_2]})$.

(d2) This part of the proposition is clear from the expression of yields. There is equality in the voluntary disclosure case, and strict inequality in the mandatory disclosure, because increasing p increases the probability of having to disclose bad news.

(e) Let $F^V \equiv F^V(D,q)$ and q' be defined as above. We have

$$\phi\left(q'F^V + (1-q')y^B\right) + (1-\phi)F^V = D. \tag{75}$$

For $F \leq F^V$,

$$\begin{aligned} m^M(F) &\leq \phi\left(q'F^V + (1-q')y^B\right) + (1-\phi)\left((1-q')py^B + (1-(1-q')p)F\right) \\ &< \phi\left(q'F + (1-q')y^B\right) + (1-\phi)F \\ &\leq \phi\left(q'F^V + (1-q')y^B\right) + (1-\phi)F^V \\ &= D. \end{aligned} \tag{76}$$

Hence, $m^M(F) < D$ for $F \leq F^V$, implying that $F^M > F^V$.

A.9 Proposition 5

I treat the case of mandatory disclosure; the proof for voluntary disclosure is identical. When the maturity of debt vanishes, the probability that the asset matures before the next debt repayment debt goes to zero, and so does the probability of an announcement. Therefore, in equation (32), we must take the limit $\phi \to 0$ and $p \to 0$. Moreover, next period's fundamental value, denoted $V_{\tau+1}$ in discrete time, is nothing but V_τ in continuous time, as periods have vanishing length. Plugging these limits into (32), we obtain $m_\tau(y^G) = \alpha V_\tau$. The same observations applied to (33) lead to $m_\tau(C(0)) = \alpha V_\tau$. Given that $y^B = 0$, we can use (29) to conclude that $C(\tau) = \alpha V_\tau$.

The dynamics of beliefs under mandatory disclosure is standard. To obtain it under voluntary disclosure, observe that the *exact* continuous-time dynamics is obtained by only conserving first-order terms in the Bayesian updating equations (19) and (20). That is, we write

$$q_\tau + dq_\tau = \frac{(1 - p_o d\tau)\gamma_\tau}{1 - p_o d\tau \gamma_\tau} \tag{77}$$

$$\gamma_\tau = q_\tau(1 - p_c d\tau) + (1 - q_\tau)p_r d\tau. \tag{78}$$

Plugging the second equation into the first and keeping terms up to order $d\tau$ gives the claimed dynamics. We then know from the theory of ordinary differential equations that if we let $\varrho_1 = p_r p_0$, $\varrho_2 = -(p_c + p_r + p_0)$ and

$$\kappa_1 = \frac{\varrho_2 + \sqrt{\varrho_2^2 - 4\varrho_1}}{2} < 0,$$

$$\kappa_2 = \frac{\varrho_2 - \sqrt{\varrho_2^2 - 4\varrho_1}}{2} < \kappa_1,$$

we have

$$q_\tau = -\frac{1}{p_o}\frac{\kappa_1 k e^{\kappa_1 \tau} + \kappa_2 e^{\kappa_2 \tau}}{k e^{\kappa_1 \tau} + e^{\kappa_2 \tau}}. \tag{79}$$

Note that $\pi_\infty = -\frac{\kappa_1}{p_0} = \frac{-\varrho_2 - \sqrt{\varrho_2^2 - 4\varrho_1}}{2p_0}$, which is indeed the unique root of $\pi' = 0$ belonging to $(0, 1)$. k is determined by the condition $q_0 = 1$:

$$k = -\frac{p_o + \kappa_2}{p_o + \kappa_1}.$$

A.10 Proposition 6

First consider the case of full transparency. Away from the debt capacity, $C = \alpha V^G$, the only risk is the observation of the bad state, which happens with probability $p_c dt$. Hence

$$D_t = \alpha V^B + (D_0 - \alpha V^B)e^{p_c t}. \tag{80}$$

Let

$$t^1(\alpha) = \frac{1}{p_c} \ln \frac{\alpha(V^G - V^B)}{D_0 - \alpha V^B} \tag{81}$$

be the maximal time the bank can survive—even in the best scenario. Let ζ_c be the time of jump to the bad state. There are three cases: 1) t^1 realises before ζ_ϕ and ζ_c. This event has probability $e^{-(p_c+\phi)t^1}$. Then, liquidation happens at αV^G, repaying exactly creditors and leaving zero to the banker. 2) ζ_ϕ realises before t^1 and ζ_c. This event has probability $\frac{\phi}{p_c+\phi}\left(1 - e^{-(p_c+\phi)t^1}\right)$. Then, debt is repaid in full and y^G realises. 3) ζ_c realises first. This event has probability $\frac{p_c}{p_c+\phi}\left(1 - e^{-(p_c+\phi)t^1}\right)$. Then, liquidation occurs, at αV^B.

Hence

$$\mathcal{P} = 1 - \frac{\phi}{p_c + \phi}\left(1 - e^{-(p_c+\phi)t^1(\alpha)}\right) \tag{82}$$

$$\mathcal{I} = (1-\alpha)\left(e^{-(p_c+\phi)t^1(\alpha)}V^G + \frac{p_c}{p_c+\phi}\left(1 - e^{-(p_c+\phi)t^1(\alpha)}\right)V^B\right). \tag{83}$$

Now consider the case of full opacity. Away from C_t, the only risk is that maturity occurs, in the bad state, so

$$dD_t = \phi P_{GB}(t) D_t dt, \tag{84}$$

Hence

$$D_t = D_0 \exp\left(\phi a t + \frac{\phi a}{b}\left(e^{-bt} - 1\right)\right) \tag{85}$$

with $a = \frac{p_c}{p_c+p_r}$ and $b = p_c + p_r$. Let $t^0(\alpha)$ be the unique solution to $\alpha V_t = D_t$. There are two cases: 1) ζ_ϕ realises before t^0. Then with probability $1 - P_{GB}(\zeta_\phi)$, payoff realises at y^G and debt is fully repaid, and with probability $P_{GB}(\zeta_\phi)$, the realised payoff is $y^B = 0$. 2) ζ_ϕ realises after t_0. Then there is liquidation at αV_{t^0}.

Hence

$$
\begin{aligned}
\mathcal{P} &= e^{-\phi t^0(\alpha)} && (86)\\
\mathcal{I} &= (1-\alpha)V_{t_0}e^{-\phi t^0(\alpha)}. && (87)
\end{aligned}
$$

In the general case, under mandatory disclosure, the debt dynamics is obtained by generalising the dynamics under full opacity, replacing the starting time 0 by $\zeta_o^{k(t)} = \sup\{\zeta_o^k,\ \zeta_o^k \le t,\ y_{\zeta_o^k} = y^G\}$. We obtain the expression given in the Lemma.

In the general case, under voluntary disclosure, the debt dynamics is

$$dD_\tau = \phi(1-q_\tau)D_\tau d\tau, \tag{88}$$

where q_τ is given by (79). By integration, we obtain

$$D_t = D_{\zeta_o^{k(t)}} \exp\left(\phi\left(t-\zeta_o^{k(t)}\right) - \phi\left(Q_{t-\zeta_o^{k(t)}} - Q_0\right)\right), \tag{89}$$

for $t < \zeta_f$, where

$$Q_t = -\frac{\kappa_1}{p_o(\kappa_1-\kappa_2)}\log\left(e^{(\kappa_1-\kappa_2)t}+k^{-1}\right) - \frac{\kappa_2}{p_o(\kappa_2-\kappa_1)}\log\left(e^{(\kappa_2-\kappa_1)t}+k\right). \tag{90}$$

A.11 Proposition 7

First, we need to show that for any $\alpha \in \left(\frac{D_0}{V^G}, 1\right)$, $\mathcal{I}_{FT}(\alpha) < \mathcal{I}_{FO}(\alpha)$. Under our assumptions, we have

$$t_1(\alpha) = \frac{1}{p_c}\ln\frac{\alpha V^G}{D_0} \tag{91}$$

and

$$\mathcal{I}_{FT}(\alpha) = (1-\alpha)e^{-(p_c+\phi)t_1(\alpha)}V^G. \tag{92}$$

Moreover, we know that $t_0(\alpha)$ is the unique solution to $f_\alpha(t) = g_\alpha(t)$, where

$$
\begin{aligned}
f_\alpha(t) &\equiv \alpha V^G e^{-p_c t} && (93)\\
g_\alpha(t) &\equiv D_0 e^{\phi t + \frac{\phi}{p_c}\left(e^{-p_c t}-1\right)}, && (94)
\end{aligned}
$$

and

$$\mathcal{I}_{FO}(\alpha) = (1-\alpha)V^G e^{-(p_c+\phi)t_0(\alpha)}. \tag{95}$$

Therefore, we need to show that $t_1 > t_0$ over $\left(\frac{D_0}{V^G}, 1\right)$. Since f_α decreases and g_α increases, it is sufficient to show that $f_\alpha(t_1(\alpha)) < g_\alpha(t_1(\alpha))$. This boils down to show

$$D_0 < D_0 \exp\left(\frac{\phi}{p_c} \ln \frac{\alpha V^G}{D_0} + \frac{\phi}{p_c}\left(\frac{D_0}{\alpha V^G} - 1\right)\right), \tag{96}$$

or

$$-\ln \frac{D_0}{\alpha V^G} + \frac{D_0}{\alpha V^G} - 1 > 0 \tag{97}$$

which holds true because $\ln x < x - 1$ for $x \in (0, 1)$.

Now, $\mathcal{P}_{FT} < \mathcal{P}_{FO}$ is equivalent to saying that $t_0(\alpha) < t(\alpha) \equiv -\frac{1}{\phi} \ln \mathcal{P}_{FT}(\alpha)$. As before, this is equivalent to $f_\alpha\left(t(\alpha)\right) < g_\alpha\left(t(\alpha)\right)$ for α small, or

$$\frac{\alpha V^G}{D_0} < \exp\left((\phi + p_c)t(\alpha) + \frac{\phi}{p_c}\left(e^{-p_c t(\alpha)} - 1\right)\right). \tag{98}$$

Noting that the expressions only depend on the ratio ϕ/p_c and $x = \frac{\alpha V^G}{D_0}$, we can assume w.l.o.g. that $p_c = 1$ and it is sufficient to study when the inequality

$$x < h(j(x)) \tag{99}$$

holds, with

$$\begin{aligned} j(x) &= -\frac{1}{\phi} \log\left(\frac{1}{1+\phi} + \frac{\phi}{1+\phi} e^{-(1+\phi)\log x}\right) & (100) \\ h(x) &= \exp\left((1+\phi)x + \phi\left(e^{-x} - 1\right)\right). & (101) \end{aligned}$$

j is increasing and

$$j^{-1}(y) = \exp\left(-\frac{1}{1+\phi} \log\left(\frac{\phi+1}{\phi} e^{-\phi y} - \frac{1}{\phi}\right)\right). \tag{102}$$

(99) is equivalent to $j^{-1}(y) < h(y)$, or, taking logs:

$$r(y) \equiv (1+\phi)y + \phi\left(e^{-y} - 1\right) > s(y) \equiv -\frac{1}{1+\phi} \log\left(\frac{\phi+1}{\phi} e^{-\phi y} - \frac{1}{\phi}\right), \tag{103}$$

with $0 \le y \le y_M \equiv \frac{1}{\phi}\log(1+\phi)$. Now note that $r(0) = s(0) = 0$, $r'(0) = s'(0) = 1$, $r''(0) = \phi$, $s''(0) = 1$, $r(y_M^-) < s(y_M^-) = +\infty$. Given these variations, it is now sufficient to show that $(r-s)''$ can only switch sign at most once. But

$$(r-s)'''(y) = -\phi e^{-y} - \phi^3 e^{\phi y} \frac{1+\phi+e^{\phi y}}{(1+\phi-e^{\phi y})^3} < 0 \tag{104}$$

for $0 \leq y < y_M$. If $\phi < p_c$, $r-s<0$ over $(0, y_M)$. When $\phi > p_c$, $r-s$ is positive in the neighborhood of 0 and negative close to y_M, so there exists y_0 with $(r-s)(y_0)=0$ and given the variations of $r-s$ given above, we have $r-s>0$ over $(0,y_0)$ and $r-s<0$ over (y_0,y_M). This concludes the proof.

B Additional Material

B.1 Discrete- versus Continuous-Time Specification

In the discrete time model, as long as $p>0$, the asset can be observed with positive probability between two successive rollover dates. This allows to make a distinction between information-sensitive and -insensitive debt and to obtain the results of section 4.1.1. Specifying the model in discrete time is also crucial in order to define properly a continuous-time setup where debt has vanishing maturity. Indeed, one cannot work with zero maturity debt; instead, one must first define the model for finite maturity and take a limit. It turns out, however, that using the continuous-time limit is more convenient to obtain results at the aggregate level. The reason is the following. Consider a small move of one model parameter. In discrete time, the event tree is discrete and there are two cases to consider: (i) the number of default nodes changes after the move, (ii) it doesn't, and only the probabilities to reach these nodes are modified. In case (i), the run probability and efficiency measures experience a jump, but not in case (ii). For instance, reducing p will decrease run probability until it creates an additional defaut state, when the probabilty jumps upwards. Thus, the comparative statics of the moments of interest are typically discontinuous and non-monotonic. The economic forces behind this phenomenon are important and generate valuable insights, including the results up to section 4.3. However, the technical consequence mentioned above is of little interest *in itself*, and not particularly desirable when it comes to uncover patterns at the aggregate level. Hence, it is better to focus on the continuous-time version of the model for section 4.3.

B.2 Numerics

Numerical Result 1 is obtained by using the analytical formulas of Proposition 6. For any parameter x, 1000 instances of the parameter set Θ_{-x} are generated randomly and the claims of the Numerical Result are verified along a 1000-point grid of admissible values of x.

Numerical Result 2 is obtained by running Monte-Carlo simulations (with $N = 2^{26}$ iterations) of the continuous-time model for 1000 parameter sets within the parameter space. p/ϕ varies in a uniformly spaced grid from 1% to 200% with 100 points. (A separate and much faster set of simulations is run for $p/\phi > 200\%$, as it turns out that mandatory disclosure largely dominates in that case; hence, much less iterations are needed). For each parameter set, the point p^* above which mandatory disclosure dominates is identified; the value 0 means that mandatory disclosure is always more efficient than voluntary disclosure along the considered grid. Then I output the mean value of p^*/ϕ, which is 0.0082. Given that the lowest p/ϕ considered is 1%, we have the conservative estimate $1.82\% < 2\%$ for the mean value of p^*/ϕ.

C Additional Figures

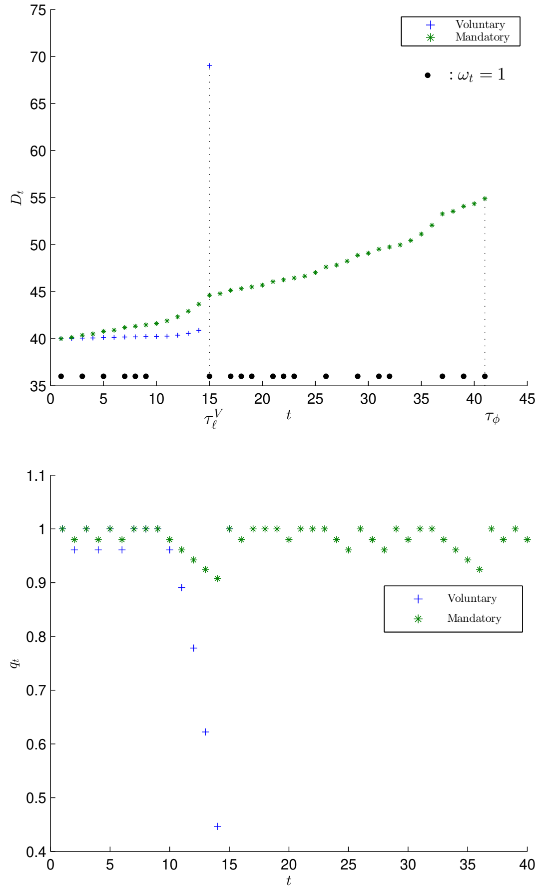

Figure 7: Debt and beliefs dynamics in a voluntary disclosure-induced run.

$$\phi = \lambda^{BG} = 1 - \lambda^{GG} = 2\%,\ y^G = 100,\ y^B = 0,\ p = 50\%,\ \alpha = 85\%.$$

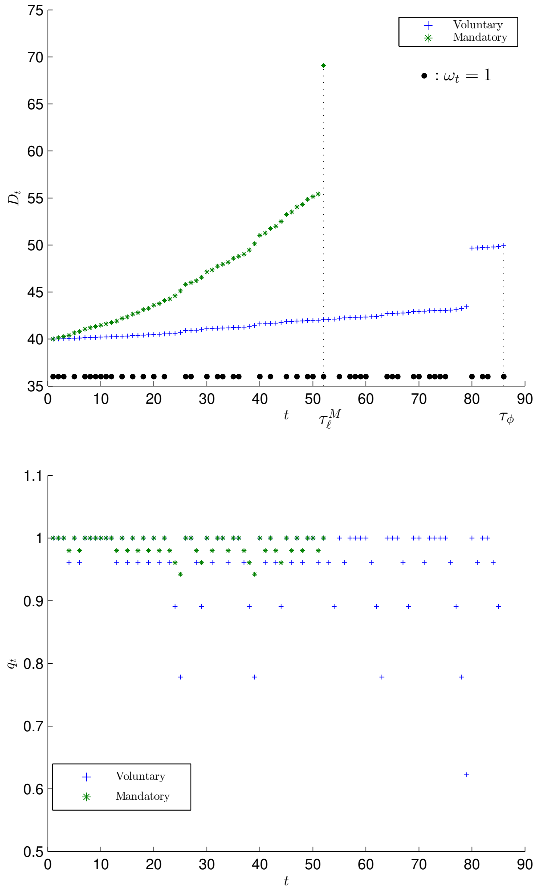

Figure 8: Debt and beliefs dynamics in a mandatory disclosure-induced run.

Same parameter values as above.

Карр, С.Ж.П.

Раскрытие информации, риск пролонгации и набеги на долг [Электронный ресурс] : препринт WP9/2021/01 / С.Ж.П. Карр; Нац. исслед. ун-т «Высшая школа экономики». – Электрон. текст. дан. (1 Мб). – М. : Изд. дом Высшей школы экономики, 2021. – (Серия WP9 «Исследования по экономике и финансам»). – 66 с. (На англ. яз.)

Как политика непрозрачности и раскрытия информации влияет на вероятность набегов на долг и экономическую эффективность? Я строю динамическую модель, в которой доходность долга является эндогенной и четко отображается в зависимости от степени прозрачности, режима раскрытия информации регулирующими органами и состояния экономики. Я нахожу, что: непрозрачность желательна тогда и только тогда, когда фундаментальные основы сильны; прозрачность может быть более эффективной, даже если она вызывает большее количество набегов; регулирующий орган обязан раскрывать информацию, за исключением случаев высокой степени непрозрачности. Существует тесная взаимосвязь между динамикой долга и убеждениями, а также результатами равновесия: краткосрочная доходность может оставаться низкой, в то время как риск накапливается, а режим раскрытия информации может последовательно вызывать более сильные убеждения, но влечет за собой более высокие расходы финансирования.

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Раскрытие информации, риск пролонгации и набеги на долг

(*на английском языке*)

Изд. № 2335