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We consider the problem of a decision-maker who seeks for advice from several experts. The experts have reputation concerns which generate incentives to herd on the prior belief about the state of the world. We address the following question: Should the experts be allowed to exchange their information before providing advice (“collective expertise”) or not (“independent expertise”)? We show that collective expertise is more informative than independent expertise under low prior uncertainty about the state and less informative otherwise. We also argue that collective expertise gains advantage as the number of experts grows.

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1 Introduction

Decision-makers routinely rely on expert advice, and often there are multiple experts available. In this paper we address the following question: Should experts be given the opportunity to share their information before talking to the decision-maker? A peer review process in academic journals is a typical example where experts (referees) cannot talk to each other, as they are simply not aware of each other's identity. We call this design "independent expertise". In many other contexts, instead, the experts are organized as a team and expected to converge to a shared opinion. For example, panels of experts are routinely formed to evaluate various public policies, with the explicit aim of aggregating their views into a unique, final recommendation. Advisory boards in corporations, universities, and other organizations would be another example of this kind.[1] We call this design "collective expertise".

In these and many other examples, the experts care about being considered smart. Such reputation concerns are the key friction in our paper. As argued in a series of papers by Ottaviani and Sørensen (2001, 2006a, 2006b), reputation concerns can make the advisors herd on the prior belief, and, consequently, lead to loss of information for the decision-maker.

We show that, due to aggregation of information *prior* to advice, collective expertise is better at predicting *which* state of the world is more likely. However, it fails to provide the decision-maker with information on the individual signals of experts, which is valuable when it is also important to know *how* likely the more likely state is.

[1] Yet another example would be faculty recruitment committees that discuss job applicants and then make a recommendation whether the person should be hired or not to the dean/provost/rector.

The experts receive informative *non-verifiable* signals about the state. The informativeness of an expert's signal depends on his ability, which is unknown to anyone, including himself. The objective of each expert is to maximize the decision-maker's posterior belief about his ability once the state is revealed. The experts care about their *absolute* reputation, that is, they do not care about their relative standing in the eyes of the decision-maker.

We first consider a setting with two ex-ante identical[2] experts and compare two communication schemes. Under "independent expertise", each expert sends a report to the decision-maker without knowing anything about the other expert's signal. Under "collective expertise", the experts share their signals before submitting a joint report. Regardless of the communication scheme, all reports (including reports between the experts) are non-contractible "cheap talk" messages. Because there is no conflict of interest between the experts, under collective expertise we can focus on equilibria in which the experts report truthfully to each other, agree on which message to send to the decision-maker, and this message is anonymous, in the sense that it does not specify which expert got which signal.

The potential benefit of signal-sharing between the experts is the alleviation of the herding-on-the-prior incentives. This effect materializes when each expert's signal is weaker than the prior (so that herd behavior results under independent expertise), but the combination of two signals contradicting the prior generates sufficiently strong evidence against the prior.

The potential cost of signal-sharing is that it aggravates the herding

[2]This is not crucial. If the two experts have different expected ability, our results still hold, provided that the experts are not too different from each other. If they are too different, the organization of expertise loses relevance. See Section 7 and the online appendix.

incentives when the experts receive opposite signals.[3] Two opposite signals of the same strength just leave the experts' beliefs at the prior, which implies that they will herd on the prior *regardless* of its strength. In fact, we show that a fully revealing equilibrium never exists under collective expertise, and the partially informative equilibrium that exists for the widest range of priors has the following structure: When both experts have received signals countering the prior, this fact is revealed; all other vectors of signals are pooled.

Thus, the main conclusion of our model is that collective expertise dominates independent expertise when there is sufficiently low prior uncertainty about the state (i.e., for strong prior beliefs), whereas independent expertise dominates for sufficiently high prior uncertainty (i.e., when the prior beliefs are weak). Since independent expertise provides no information when the prior is strong and perfect information when the prior is weak, the two expertise schemes are always Blackwell-ranked: collective expertise provides better information for weak priors and independent expertise provides better information for strong priors. As a consequence, the optimal expertise scheme for the decision-maker depends on neither the decision problem she needs to solve, nor on her preferences.

As an extension, we also consider a different information structure with continuous signals. For strong priors, our main finding is confirmed: collective expertise dominates independent expertise, because of its anti-herding effect. For weak priors, independent and collective expertise may generate

[3]A famous example of this kind of "groupthink failure" is the decision at NASA of launching the space shuttle Challenger. A share of experts were of the opinion that the shuttle was not ready to fly, but their views were hidden by the final aggregate recommendation that the shuttle was fit to fly.

information partitions that cannot be Blackwell-ranked, therefore which one is better may depend on the decision problem and the decision-maker's preferences. We argue that collective expertise is likely to be preferable when the decision-maker only needs to know which state is more likely (e.g., when the decision set is binary and the decision-maker wants to match her decision to the state, with the same cost of a mistmatch in both states), and independent expertise when how likely is the more likely state matters too (e.g, when there are decisions that are optimal only under high uncertainty).

We also extend our model to the case of more than two experts. Our insights go through, with the following qualification. As the number of experts grows, collective expertise, because of increasing potential confidence of the experts, is able to provide information for lower and lower levels of prior uncertainty, and this information, by the law of large numbers, becomes more and more accurate regarding the true state. Thus, collective expertise becomes less *dis*advantageous for low priors, and gains more advantage for high priors. This result suggests that, if the expertise scheme has to be set up before the prior is known[4], collective expertise is more likely to dominate independent expertise (in expectation) as the number of experts grows.

The rest of the paper is organized as follows. The next section reviews the related literature. Section 3 sets up a model with two homogeneous experts. Sections 4 and 5 analyze independent and collective expertise, respectively, in this setup. In Section 6, we extend our analysis to the case of more than two experts. Section 7 discusses other extensions. The Appendix contains the proofs for Section 5 (except for the proof of Lemma

[4]E.g., because it needs to be designed as an institution to be applied in all circumstances (like the refereeing process in an academic journal), or because some public information may arrive after the expertise scheme is set up.

7 relegated to the online appendix). In the online appendix (available at https://sites.google.com/site/sergeystepan/), we present the proofs omitted from the main paper, show that our insights extend to the cases of continuous signals, two heterogeneous experts, and argue that our solution under collective expertise is robust to the communication protocol.

2 Related literature

Our paper joins the literature that explores how information aggregation and decision-making can be improved in the presence of reputation concerns. Ottaviani and Sørensen (2001) examine the role of the order of speech in a public debate among reputation-concerned experts. Prat (2005) studies the effects of transparency of decisions on the actions of a reputation-concerned decision-maker. Catonini and Stepanov (2019) show how the decision-maker can improve extraction of information from reputation-concerned experts by asking for advice only in certain circumstances.

In the setting of a committee of reputation-concerned experts, Levy (2007) studies whether *secret* rather than *open* voting helps to mitigate the members' incentives to vote against the decision that is more likely to be optimal. Differently from us, Levy considers a binary-decision problem and a different information structure (the experts know their own types), which leads to a different type of inefficiency – "contrarian" voting rather than herding on the prior. That is, Levy and us look at different types of problems and different tools to address them: Instead of letting the experts share their information, secret voting garbles the experts' votes in a way that improves their incentives to vote for the right decision.

This paper looks at how the adverse effects of reputation concerns can be alleviated by the optimal organization of expertise. In this sense, it is close to the work by Ottaviani and Sørensen (2001). The fundamental distinction of our work from Ottaviani and Sørensen (2001) is that in our study, under collective expertise, the experts exchange their information *privately* and *before* reporting to the decision-maker, whereas in the latter paper they speak *sequentially* and *publicly*. Hence, in Ottaviani and Sørensen (2001), advisors cannot coordinate their reporting behavior based on their joint information, whereas in our paper they can, and this is crucial. For example, with identical advisors and public reporting, sequential advice cannot improve over independent reporting: If the first advisor herd on the prior, so will all others; if the first advisor does not herd, no one will herd under independent reporting either. In contrast, private exchange of signals can alleviate herding, as our model shows.

Another related paper is Fehrler and Janas (2020). They examine the choice between delegating a decision to a group of reputation-concerned experts and consulting them individually while keeping the decision-making power. Differently from our paper, the key tradeoff in Fehrler and Janas (2020) is between information aggregation and information acquisition; the issue of herding-on-the-prior is absent in their setting. Delegation improves information aggregation but hinders information acquisition. When the cost of information acquisition is low, incentivizing it is not a concern, and, hence, delegation is better. However, when this cost is sufficiently high, it becomes crucial to motivate information acquisition, and individual consultation dominates. Notice that in Fehrler and Janas (2020), delegation (a counterpart of our collective expertise) never fails to aggregate information. This is because, in their setting, if an expert is informed he is perfectly informed, and then the team with such an expert never makes mistakes. In contrast, with our *imperfect* signals, information aggregation is hindered by the herding-on-the-prior incentives and may turn out to be even worse under collective expertise than under independent consultations.

Also Andina-Diaz and Garcia-Martinez (2020) consider a problem of decision-making by multiple reputation-concerned experts. However, they study a different kind of distortion, which arises when experts are evaluated in relative terms and the state is not always observed at the end of the game. There is no role for information sharing between the experts in the paper.

In a companion paper (Catonini and Stepanov (2022)), under the same information structure as in this paper, we compare collective expertise to delegating all information collection (i.e., collecting all binary signals) to a single expert. We show that, regardless of the number of experts, collective expertise conveys not more than a bipartition of the experts' information. In contrast, a single expert responsible for all signals may provide more than a bipartition. The intuition is that, while the lone expert carries the responsibility for all the information and may cautiously "abstain" regarding which state is more likely, the experts in the team always take a stand towards one of the states, because they can "blame" the other members in case of a mistake.

There are works on eliciting information from multiple advisors in a Crawford and Sobel (1982) type of setting (e.g., Gilligan and Krehbiel (1989), Krishna and Morgan (2001a,b). Battaglini (2002), Ambrus and Takahashi (2008), McGee and Yang (2013), Wolinsky (2002), Gradwohl and Feddersen (2018), Feddersen, and Gradwohl (2020)). Due to a different nature of communication distortions, this literature is orthogonal to the "reputational cheap talk" literature. Moreover, most of this literature does not address the central question of our work: Should experts be allowed to talk to each other before reporting to the decision-maker?[5]

The only exception, to our knowledge, is Wolinsky (2002).[6] Wolinsky considers the problem of a decision-maker who wants to aggregate decision-relevant information that is disseminated among a number of experts. The decision is binary, and so each expert's piece of information (0 or 1). The experts care about the decision, and both for the experts and for the decision-maker the preferred decision depends on the sum of the experts' pieces of information. However, the experts are biased: For some values of this sum, their preferred decision is 0, while the decision-maker's is 1. Because of this, if the decision-maker asks each individual expert to reveal his piece of information, the expert will focus on the case when his advice is pivotal and will pretend that his information is 0 also when it is 1 (1 is verifiable but 0 is not). If instead subgroups of experts share their information before providing advice, informative equilibria arise: A subgroup of experts with many 1's will suggest to the decision-maker to take decision 1, because the increased weight of their advice on the final decision makes it pivotal also in situations where the experts prefer decision 1.

[5] Although some of these models compare sequential and simultaneous communication, see Hori (2009), Li (2010), Li, Rantakari, and Yang (2016).

[6] Rather than studying ex-post information-sharing, Elliott, Golub, and Kirilenko (2014) consider sharing *technologies* for generating recommendations to the decision-maker in a setup where two experts have different attitude to type I versus type II errors. The authors show that allowing for such sharing can harm the decision-maker, because the resulting expansion in the sets of technologies available to each expert may make the experts switch to suboptimal choices of recommendation-generating procedures from the decision-maker's perspective.

All in all, the information structure, the nature of distortions in communication, and, most importantly, the channel through which information sharing among experts improves the informativeness of communication all differ with respect to our work. In our model, the beliefs about the state are the key determinant of the effect of reputation concerns on the experts' reporting behavior, and information sharing acts through changing these beliefs. In contrast, there are no reputation concerns in Wolinsky's paper, and merging experts in teams acts through changing the "pivotality" of experts: It helps them to coordinate on disclosing a critical mass of information that is sufficiently influential to be willingly (but coarsely) transmitted to the decision-maker.

Finally, there are works on deliberation in committees (see Austen-Smith and Feddersen (2009) for a survey). These papers however do not examine whether committee members *should* be allowed to share their information before voting or not.[7] Instead, they are focused on distortions (in both information sharing and voting outcomes) created by divergence of preferences, reputation concerns and strategic voting considerations, and how such distortions can be alleviated through the design of optimal voting rules (Coughlan (2000), Austen-Smith and Feddersen (2005, 2006), Visser and Swank (2007), Gerardi and Yariv (2007)) deliberation rules (Van Weelden (2008)) and transparency regulations (e.g., Meade and Stasavage (2008), Swank and Visser (2013), Fehrler and Hughes (2018), Henry and Louis-Sidois (2020)).

[7]An exception is Ali and Bohren (2019). In their setup, committee members' losses from type I and type II errors are different from those of the principal designing a committee. The authors show that banning deliberation can benefit the principal if she can choose the equilibrium the committee members play at the voting stage or if she can use non-monotone or non-anonymous social choice rules.

3 Model

A decision-maker wants to learn about an unknown, binary state of nature $\omega \in \{0, 1\}$, with common prior

$$p := \Pr(\omega = 0).$$

Without loss of generality, we assume that $p > 1/2$.[8] The decision-maker can consult two experts. The experts are ex-ante identical, and each of them can be of two types, Good and Bad with commonly known prior probability $\Pr(t_i = G) = q \in (0, 1),\ \forall i \in \{1, 2\}$.[9] The experts' types are uncorrelated and unknown to anyone, including the experts themselves. Each expert receives a private non-verifiable signal $\sigma_i \in \{0, 1\}$. Independently of the state, an expert's signal is correct with probability either g or $b < g$, depending on his type:

$$g := \Pr(\sigma_i = \omega | t_i = G) > b := \Pr(\sigma_i = \omega | t_i = B) \geq 1/2.$$

Conditional on the state, the experts' signals are independent. We let $\sigma := (\sigma_1, \sigma_2)$. The expected precision of an expert's signal is denoted by

$$\rho := qg + (1 - q)b.$$

The timing of the game is as follows:

[8] We exclude $p = 1/2$ from consideration, as a trivial degenerate case: Under $p = 1/2$, reputation concerns create no misreporting incentives, and there is full information revelation under either expertise scheme.

[9] In the online appendix, Section 4, we argue that our results for the two experts model are robust to introducing heterogeneity between the experts, in terms of prior ability.

1. The nature draws the state ω and the types of the experts.

2. The experts receive their private signals.

3. The experts communicate their information to the decision-maker, according to a given *expertise scheme*.

4. The state is revealed and the reputation of the experts is updated.

The focus of our work is the expertise scheme employed in stage 3. Under *independent expertise*, each expert sends a non-contractible message to the decision-maker (from any abstract message space). Under *collective expertise*, communication is as follows. First, the experts talk with each other. Then they send a single message to the decision-maker. For concreteness, we fix the following communication protocol: first, the two experts simultaneously send a message to each other; second, they simultaneously propose a message m for the decision-maker (from any abstract message space M), and if they propose two different messages, one of the two messages is selected at random with exogenous probabilities. We allow these probabilities to be zero and one, in which case we can think of communication to the decision-maker as "delegated reporting".

This communication protocol, in a simple way, allows us to examine the central feature of collective expertise in our framework – the ability of the experts to condition their reports on their aggregate information. At the end of Section 5 we argue that our results are robust to different modes of communication between the experts and the decision-maker.

Each expert cares only about his reputation, which is modelled as the decision-maker's ex-post belief about the expert's type. Thus, an expert's payoff is:

$$u_i(message, \omega) = \Pr(t_i = G | message, \omega), \ \forall i \in \{1, 2\},$$

where $message$ is either m_i or m depending on the expertise scheme.

4 Independent expertise

Under independent expertise, an expert's reporting behavior does not depend on the reporting strategy of the other expert. This is because (1) the experts learn nothing about each others' signals prior to reporting, and (2) the state is eventually revealed, thus making the other expert's report redundant in forming the decision-maker's belief about an expert's type.

Hence, each expert behaves as if he were a single expert. Consequently, we can just apply Lemma 1 from Ottaviani and Sørensen (2001), which deals precisely with the case of a single expert in a setup with two states, two expert types and a binary expert's signal. Given our notation and the assumption that $p > 1/2$, their lemma can be re-formulated as follows:

Lemma 1 *Under independent expertise, the following is true:*
- *When $p \leq \rho$, the experts report their true signals in the most informative equilibrium.*
- *When $p > \rho$, there exists no equilibrium with informative reporting.*

Ottaviani and Sørensen (2001) prove the result for the case of a binary message space; we extend their proof to the general case of an arbitrary message space in the online appendix.

The intuition behind the lemma is simple. An expert wants to maximize the decision-maker's posterior belief that he received the signal equal to the state. Since $p > 1/2$, an expert with signal 0 always believes that $\omega = 0$ is more likely. An expert with signal 1 believes that $\omega = 1$ is more likely exactly when $p < \rho$, and considers $\omega = 0$ more likely otherwise. Therefore, when $p < \rho$, reporting the true signal is the natural equilibrium. In contrast, when $p > \rho$, there is a temptation to "herd" on the prior, which destroys any informative communication.

5 Collective expertise

In this section, we show that no full information revelation is possible under collective expertise, but a partially informative equilibrium exists up to some $\overline{p} > \rho$. In particular, we prove that at most two informationally distinct (i.e., generating different beliefs about the state) messages are sent to the decision-maker in any equilibrium that satisfies a (arguably weak) selection criterion. We start from the "no full revelation" result. Let a "fully revealing mapping" be a mapping from the experts' signals to the messages to the decision-maker that truthfully reveals the number of zero signals received by the experts, for any pair of their signals.

Lemma 2 (no full revelation) *Consider a fully revealing mapping. Let $\overline{M}$ be the set of all messages revealing that $\sigma \in \{(0,1),(1,0)\}$, and let message m' reveal that $\sigma = (0,0)$. Suppose the experts have learned that $\sigma \in \{(0,1),(1,0)\}$. Then, for any $m \in \overline{M}$, one of the experts strictly prefers, and the other expert weakly prefers m' to m. Moreover, for any expert, there exists $m \in \overline{M}$ such that this expert strictly prefers m' to m.*

The intuition behind Lemma 2 is straightforward. Two contradictory signals leave the experts' belief at the prior, that is, believing that state 0 is more likely. Therefore, a message indicating that an expert may have received signal 1 is reputationally damaging for him in expectation.[10] Hence, say, expert 1 strictly prefers m' to m, unless m is sent only when $\sigma = (0, 1)$. In the latter case, he is indifferent, as both $(0, 0)$ and $(0, 1)$ mean he has definitely received 0, but then expert 2 strictly prefers m' to m. Moreover, if m is sent only under $\sigma = (0, 1)$, there must be another message from M which is sometimes sent under $\sigma = (1, 0)$, and, hence, expert 1 will strictly prefer m' to this other message. The last statement of the lemma follows.

The above lemma immediately implies that there is no fully revealing equilibrium under collective expertise. First, for full revelation, it is needed that any expert whose message reaches the decision-maker with a positive probability knows σ, otherwise his message to the decision-maker cannot be fully revealing. But then, for *any* such expert, there is a message from which he would strictly prefer deviating to claiming that $\sigma = (0, 0)$. Since, according to our communication protocol, at least one expert always has a chance to unilaterally affect the message to the decision-maker, the equilibrium breaks down.

Corollary 1 *Under collective expertise, a fully revealing equilibrium does not exist.*

We would like to stress that, by Lemma 2, the impossibility of full revelation does not rely on our specific game setting. First, under full revelation,

[10]Notice that the pairs of signals $(0, 1)$ and $(1, 0)$ are equally likely in any state. Hence, the ex-post belief that an expert who has sent $m \in \overline{M}$ has received signal 1 does not depend on ω. Thus, m cannot generate a favorable state contingency of reputation for the expert.

a deviation from any message sent under $\sigma \in \{(0,1),(1,0)\}$ to claiming $\sigma = (0,0)$ constitutes, at least, a weak Pareto improvement for the experts, so neither expert would want to block it. Second, the fact that for *any* expert there is a message from which he would strictly prefer to deviate means that we cannot achieve full revelation by delegating communication to an expert that would be indifferent in equilibrium. Hence, arguably, no meaningful communication protocol can sustain full revelation.[11]

If full revelation is impossible, which equilibria do we have? From now on, we apply the following *equilibrium selection* criterion: We assume that the experts never end up sending a message that one of them would not have chosen, knowing the true signal of the other. Formally:

Assumption 1 (selection criterion) *We will focus on equilibria that satisfy the following condition. For every signal profile $\sigma \in \{0,1\}^2$, for every equilibrium message m to the decision-maker that the experts send with positive probability under σ, the expected reputation of both experts conditional on σ is weakly greater after sending m than after sending any other equilibrium message.*

The goal of this assumption is to rule out equilibria in which the experts have any conflicts over equilibrium messages. Our experts do not compete with each other: each expert's payoff coincides with his own reputation, which does not depend on what the decision-maker learns about the other

[11]Contingent delegation of communication, in which expert 1 reports to the decision-maker when the mutually reported signals, $\widehat{\sigma}$, are $(0,1)$ and expert 2 reports when $\widehat{\sigma} = (1,0)$ could achieve full revelation in equilibrium, provided that the experts tell the truth to each other. However, it would be incompatible with truthtelling between the experts: An expert with signal 1 would prefer to lie and tell he got 0 to obtain the right to report when the other one reveals 1.

expert. Therefore, they are "on the same boat", and different preferences over messages can only be artificially induced by misinforming each other, or by an equilibrium message structure where the only available messages prevent them from conveying their shared opinion.

Once we rule out these artificial conflicts of interest, we can focus on a particularly simple class of equilibria, without loss of generality for information aggregation. First, given any equilibrium that satisfies our refinement, if the experts reveal their true signals to each other, they do not change their mind about which message(s) to propose. Therefore we can focus on equilibria with *truthtelling* between the experts. Given this, under our refinement, the experts can propose the same message, or the same messages with the same probabilities. Therefore, we can focus on equilibria with *agreement* on the message to send. Finally, it is without loss of generality for information aggregation to focus on equilibria with *anonymous* messages, that is, where signal profiles $(0, 1)$ and $(1, 0)$ generate the same distribution over messages. The argument for anonymity is that for every non-anonymous equilibrium message, by symmetry the experts can also agree to send its "mirror image", and a mix of these two messages will produce an anonymous message that conveys the same information to the decision-maker. The following lemma formalizes these findings.

Lemma 3 *For any equilibrium that satisfies our selection criterion, there exists an equilibrium that delivers exactly the same information to the decision-maker and satisfies the following properties on the equilibrium path:*

1. *the experts reveal to each other their true signals;*

2. *for any signal profile, the experts propose the same messages with the*

same probabilities;

3. the messages to the decision-maker are anonymous.

Following Lemma 3, we are going to consider only equilibria with *truthtelling*, *agreement*, and *anonymity.* These properties, apart from being natural, are also very convenient, because they allow us to treat the team of experts as a single player possessing all experts' information. Indeed, in any such equilibrium, each equilibrium message to the decision-maker generates identical expected reputation for the two experts, conditional on their joint information. Hence, for each expert, a unilateral deviation to proposing a different *equilibrium* message is profitable if and only if the joint deviation is profitable. Notice that, given the cheap-talk nature of communication, unilateral deviations to (possibly non-anonymous) non-equilibrium messages are not necessary to consider: If there is no profitable deviation to an equilibrium message, any deviation to a non-equilibrium message can be ruled out by picking proper off-path beliefs (e.g., those generated by some equilibrium message). Given this, there is also no need to consider unilateral deviations to lying to the other expert, as lying may only result in a deviation to another message to the decision-maker, given the experts' joint information.

Hence, for the rest of this section, we will treat the team of experts as a single player and say "signal-type (x, y)" when referring to a team with signals (x, y).

Now we are ready to examine possible equilibria. First, we show that any equilibrium partitions the set of signal profiles into at most two *ordered* subsets (possibly with a common boundary), equivalent to only two messages being sent to the decision-maker. "Ordered" means that any profile of signals in one of the subsets contains a weakly higher number of zero signals than any profile in the other subset. A message can then be interpreted as a statement that the signal profile belongs to a certain element of the bipartition, with the qualification that a threshold profile can randomize between the two messages.

Formally, let us call messages informationally equivalent if they generate the same decision-maker's belief about the experts' signals (hence, about the state). An expert is obviously indifferent between any two informationally equivalent messages. Hence, we can treat all informationally equivalent messages as a single message, without loss of generality.

Lemma 4 *In any equilibrium, at most two informationally distinct messages, m^0 and m^1, are sent, and there is a threshold l, such that m^0 is never sent when the experts received less than l zeros, and m^1 is never sent when the experts received more than l zeros.*

Thus, if we consider *informative* equilibria without randomization by the threshold signal-type, two possible partitions arise, in which either

- signal-types $\{(0,0),(0,1),(1,0)\}$ always send m^0, and signal-type $(1,1)$ always sends m^1;

- signal-types $\{(0,1),(1,0),(1,1)\}$ always send m^1, and signal-type $(0,0)$ always sends m^0.

In addition, there can be informative equilibria with randomization:

- one in which signal-types $\{(0,0),(0,1),(1,0)\}$ always send m^0, and signal-type $(1,1)$ randomizes between m^1 and m^0 (in this case, l of the lemma is equal to 0);

- one in which signal-types $\{(0,1),(1,0),(1,1)\}$ always send m^1, and signal-type $(0,0)$ randomizes between m^0 and m^1 (in this case, $l = 2$);

- one in which signal-types $(0,0)$ and $(1,1)$ always send m^0 and m^1, respectively, while signal-types $\{(0,1),(1,0)\}$ mix between m^0 and m^1 (in this case, $l=1$).

Let us first examine the equilibrium generating partition $(\{(0,0),(0,1),(1,0)\},(1,1))$ without randomization.

Lemma 5 *The equilibrium* $(\{(0,0),(0,1),(1,0)\},(1,1))$ *exists if and only if* $p \leq \overline{p}$, *where* $\overline{p} = \dfrac{\rho^2(2-\rho)}{1-\rho+\rho^2} > \rho$. *Moreover, when* $p = \overline{p}$, $\Pr(\omega = 1|\sigma = (1,1)) > 1/2$.

Signal-type $(0,1)$ (or $(1,0)$) would never want to deviate to reporting $(1,1)$: As an expert believes that $\omega = 0$ is more likely, he would not want to be perceived as having received signal 1. In contrast, signal-type $(1,1)$ may want to deviate to reporting $m^0 \equiv \{(0,0),(0,1),(1,0)\}$, if the prior is sufficiently biased towards $\omega = 0$. It will clearly do so when the prior is so strong that $\Pr(\omega = 0|\sigma = (1,1)) > 1/2$: As an expert considers $\omega = 0$ more likely, he does not want to be perceived as having received signal 1. When $\Pr(\omega = 0|\sigma = (1,1)) < 1/2$, an expert has a trade-off. By revealing his signal, he will essentially "bet" on the more likely state. However, deviating to m^0 does not imply "betting" on the less likely state, because m^0 does not imply that an expert necessarily received signal 0. This "imprecise" message has the advantage of "favorable" state-contingent interpretation by the decision-maker. When the realized state is 1, the decision-maker assigns a higher probability to (the experts having received) $(0,1)$ or $(1,0)$, compared to when the realized state is 0. Similarly, when the realized state is 0, the decision-maker assigns a higher probability to $(0,0)$ compared to when the realized state is 1. As a result, the value of the prior at which the experts are indifferent between deviating and not, $\overline{p}$, is below p that makes $\Pr(\omega = 0|\sigma = (1,1)) = 1/2$. In other words, at $p = \overline{p}$ signal-type $(1,1)$ still believes that $\omega = 1$ is more likely.

The crucial thing is that $\overline{p} > \rho$. Two same signals combined are stronger than one. This allows to eliminate the herding-on-the-prior incentives of the experts, whenever both signals are 1, for a range of parameters where each expert separately would herd.

Let us now consider the equilibrium generating partition $((0,0), \{(0,1),(1,0),(1,1)\})$ without randomization.

Lemma 6 *The equilibrium* $((0,0), \{(0,1),(1,0),(1,1)\})$ *exists if and only if* $p \leq \dfrac{1+\rho}{3}$*, which is strictly below* ρ*.*

Here the threshold on p is determined by the incentive compatibility of signal-type $(0,1)$ (or $(1,0)$). Given that the prior is biased towards $\omega = 0$, signal-type $(0,0)$ is very confident that $\omega = 0$, and, thus, would never want to lie. In contrast, signal-type $(0,1)$ (or $(1,0)$) has a trade-off similar to the trade-off of signal type $(1,1)$ in the equilibrium of Lemma 5: betting on the more likely state by sending $m^0 \equiv (0,0)$ versus playing the "safer" strategy of staying pooled with the other two signal-types. Note that the threshold on p provided by Lemma 6, $\dfrac{1+\rho}{3}$, is smaller than ρ.

Finally, for equilibria with randomization between m^0 and m^1, we have the following:

Lemma 7 *Informative equilibria with randomization do not exist for* $p > \overline{p}$

Thus, such equilibria do not expand the set of priors where partial information revelation occurs under collective expertise. The analysis of this section implies the following fundamental result:

Proposition 1 *Irrespectively of which informative equilibrium is played under collective expertise (when an informative equilibrium exists), the following holds. When $p \leq \rho$, independent expertise results in more information transmitted to the decision-maker. When $p \in (\rho, \overline{p}]$, collective expertise results in more information transmitted to the decision-maker. For $p > \overline{p}$, both modes of expertise result in zero information transmission.*

The potential benefit of signal-sharing between the experts is the alleviation of the herding-on-the-prior incentives, when both experts receive a signal contradicting the prior. This benefit materializes when each expert's signal is weaker than the prior ($\rho < p$, so that herd behavior results under independent expertise), but two same signals combined are sufficiently stronger than the prior (at $p = \overline{p}$ signal-type $(1, 1)$ believes that $\omega = 1$ is more likely).

The potential cost of signal-sharing is that it aggravates the herding incentives, when the experts receive opposite signals. In such a case, the experts' belief remains at the prior, which implies herding on the prior *regardless* of its strength. As a result, only partial information revelation is possible under collective expertise.

Robustness to messaging rules Collective expertise, under our communication protocol, does not allow the two experts to send personal messages to the decision-maker. We find this feature realistic, because the whole point of forming the team is precisely to collect the shared opinion on which the experts converge. Besides this, any personal initiative of an expert to secretly convey a different opinion to the decision-maker may be frowned upon, and it may even imply that the expert has lied to the team. Nonetheless, in the online appendix (Section 2), we take this possibility seriously and analyze a modification of our setting in which the experts send separate messages to the decision-maker after talking to each other. In this setup, the equilibrium of Lemma 5 can be translated into an equilibrium where the two experts send the same message to the decision-maker. We show that such an equilibrium survives a two-senders adaptation of neologism proofness: A deviation to a "personal opinion" could make sense if preceded by a lie to the other expert, but it is equally rationalizable that the expert is lying to the decision-maker because he fears that the other expert lied to him.

The possibility to send separate messages would also allow the experts not to exchange any information and just convey to the decision-maker their personal opinions based on their own signals. In this way, collective expertise would subsume independent expertise, so Lemma 2 would fall through: for $p < \rho$, the experts could reveal their signals truthfully. However, this possibility would not change our insights: when independent expertise induces truthful reporting, it should still be preferred over collective expertise by the decision-maker, because it imposes that experts do not perturb each other's incentives by design (i.e., by not revealing the other expert's identity, as for referees of economics journals), rather than just hoping that the experts do not exchange information anyway. Indeed, the experts do not have any conflict of interest, so they can only gain from exchanging their signals truthfully.

Our focus on equilibria with *truthtelling*, *agreement*, and *anonymity* makes them also robust to the communication protocol between the experts and to the protocol to select the single message to send to the decision-maker. For example, we could also assume that the team of experts has to send a single letter, after an exchange of signals and a further discussion between themselves, where the discussion can be modeled through multiple rounds of alternating proposals, until both experts agree on the letter. It is easy to see that, after truthfully exchanging their information, our experts would immediately agree on a letter that reports the equilibrium message of our formal setup.

6 More than two experts

Our insights extend seamlessly to the case of more than two experts, with the qualification that collective expertise yields an informative bipartition of the signals space for higher and higher values of p, and provides more and more accurate information about the state.

For independent expertise, the behavior of each expert does not depend on the number of experts, hence Lemma 1 applies. This means that independent expertise becomes more and more informative as the number of experts grows for $p \leq \rho$, and remains completely uninformative otherwise.

Under collective expertise, full revelation is still impossible, for the same reason as in Section 5: Under any vector of signals that makes the experts believe that $\omega = 0$ is more likely, any of them would prefer claiming that all experts have received 0 rather than revealing that he might have received signal 1.[12]

At the same time, a larger number of experts makes it possible to achieve more extreme posterior beliefs about the state, which precludes herding on

[12]The formal proof (available upon request) is a straightforward extention of that of Lemma 2.

a “zero message” when sufficiently many experts received 1. This facilitates separation of signal-types with sufficiently many “ones” from the rest of signal-types.

Specifically, if we adopt the definition of a bipartitional equilibrium from Lemma 4,[13] we obtain the following.

Proposition 2 *Let n be the number of experts. For every $n \geq 2$, there exists $\overline{p}_n > \rho$ tending to 1 as $n \to \infty$ such that an informative bipartitional equilibrium exists for all $p \in \left(\frac{1}{2}, \overline{p}_n\right]$.*

Moreover, a higher number of experts generates more extreme posteriors “in general”, making thereby the resulting bipartition more and more informative. As the number of experts goes to infinity, the proportion of signals equal to ω in a profile of the experts' signals will tend to ρ in probability. Consequently, even such a coarse message as “more than half of us have received signal 1” becomes a very precise evidence about the state. We formalize this argument in the following proposition:

Proposition 3 *For every $p \in \left(\frac{1}{2}, 1\right)$ and $\mu \in \left[\frac{1}{2}, 1\right)$, there exists $\overline{n} > 1$ such that for every $n \geq \overline{n}$, in any bipartitional equilibrium (m, m'), $\Pr(\omega = 1|m) > \mu$, $\Pr(\omega = 0|m') > \mu$.*

The results of this section imply the following: As the number of expert sufficiently grows, the advantage of collective expertise over independent expertise for $p > \rho$ increases, whereas its disadvantage for $p \leq \rho$ shrinks, as the loss of information under collective expertise diminishes and tends to

[13] In Catonini and Stepanov (2022) we show that, for any number of experts, all equilibria satisfying the properties of Lemma 3 are bipartitional.

zero at the limit. For reasons outside of our model, the expertise scheme may need to be designed as an institution to be applied in all circumstances (for example, think of the refereeing process in an academic journal). Also, even for a given problem, some public information may arrive after the expertise scheme is set up, thereby effectively moving the prior. So, imagine the expertise scheme is to be established before the public belief p is realized, and there is some prior distribution over p. Then the results of these section entail that collective expertise gains advantage over independent expertise as the number of experts sufficiently grows.

7 Other extensions

7.1 Decision problem

As suggested at the end of the previous section, although the two expertise schemes can be Blackwell-ranked for a given prior belief, it may also be interesting to examine which scheme is better when the prior is "a priori" unknown. Apart from the number of experts and the distribution of p, the answer to this question would generally depend on the decision problem and the preferences of the decision-maker.

Notice that, when an informative equilibrium under collective expertise exists, the partition of Lemma 5 always correctly predicts which state is more likely, given the prior and experts' information.[14] Hence, provided that such an equilibrium is selected, collective expertise (weakly) dominates

[14] In the companion paper, Catonini and Stepanov (2022), we show that, for any number of experts, there exists an equilibrium that satisfies this property for all $p < \overline{p}_n$, where $\overline{p}_n$ is the same as in Proposition 2.

independent expertise regardless of p, when the decision-maker needs to know only which state is more likely. This would be the case, for example, if the decision-maker has to take a binary decision $a \in \{0, 1\}$, and receives a payoff of 1 if the decision matches the state ($a = \omega$), or 0 if it does not ($a \neq \omega$).

Often, however, it is important to know not just which state is more likely, but also how likely it is. This is the case, for instance, if the decision-maker can also take a "safe action" that gives constant payoff s independently of the state, besides the two actions above. Another example would be when a is continuous, and the decision-maker's utility is $-(\omega - a)^2$, in which case the optimal decision equals the expected state.[15] Then, the informational advantage of independent expertise under weak priors becomes payoff-relevant, and the more important the information about the actual uncertainty is, the more likely is that independent expertise becomes optimal in expectation.

7.2 Continuous signals

While many decision-relevant states are binary in nature, the signals about such states are often more complicated than binary. Ottaviani and Sørensen (2006a) consider a class of continuous signals called "multiplicative linear experiments" characterized by a conditional pdf function being linear in the product of the signal, the state, and the type (precision) of an expert.[16]

In a single expert setting, they show that all informative equilibria are

[15] Yet another example is when the cost of mistmatching the decision to the state is different in different states. Then, if, say, a mistake in state 0 is more costly, and state 0 is just slightly more likely than state 1, $a = 1$ will still be optimal.

[16] Formally, the conditional pdf of the expert's signal is $f(s|x,t) = tg(s|x) + (1-t)h(s) = t\frac{(1+sx)}{2} + (1-t)\frac{1}{2} = \frac{1}{2}(1 + stx)$, where x is the state, s is the signal, and t is the expert's type.

binary (Proposition 4), such that message m^0 is sent whenever the expert's signal is below a certain threshold, and m^1 is sent whenever the signal exceeds the threshold. Furthermore, the authors show that no informative equilibrium exists when the prior is sufficiently concentrated on one state (Proposition 8), i.e., in terms of our model, when p is greater than a certain threshold, call it ρ^{mle}. In light of this result, independent expertise, as in our baseline setting, ceases to provide any information for a sufficiently strong prior. At the same time, as we show in the online appendix (Section 3), collective expertise is able to transmit an informative bipartition up to stronger priors. So, the positive anti-herding effect of collective expertise is confirmed: as in the baseline model, it dominates independent expertise for $p > \rho^{mle}$.

On the other hand, for $p < \rho^{mle}$, independent expertise and collective expertise are not Blackwell-ranked. Then, like in subsection 7.1, which one is better depends on the decision problem and the decision-maker's preferences. Suppose the decision-maker just wants to match the state with an action from a binary set, with the benefit from matching independent of the state (like in the first example of subsection 7.1). Then she needs to know only which state is more likely, given the prior and the two signals of the experts. The bipartition (of posterior beliefs) transmitted by collective expertise could be closer to achieving this goal than the combination of the bipartitions of each expert's signal generated by independent expertise. For example, take p very close to 1/2. Under independent expertise, the bipartition for each expert would split the beliefs, conditional on the expert's signal, almost at 1/2.[17] Then, if the decision-maker receives message m^0 from one expert and message

[17] Ottaviani and Sørensen (2006a), Proposition 6.

m^1 from the other expert, she has no clear recommendation about what to do, and, hence, she will often take suboptimal decisions (conditional on the experts' actual information). In contrast, one can conjecture that collective expertise would generate a bipartition of beliefs conditional on *both* experts' information with a split almost at $1/2$, thereby clearly indicating what state is more likely, given the prior and the experts' signals.

However, sometimes the optimal decision depends not only on what state is more likely but also on the strength of the decision-maker's belief (e.g., as in the examples with more than two possible actions from subsection 7.1). In such a case, she would like to learn also how likely the more likely state is. Then, if collective expertise results in a binary equilibrium, independent expertise is likely to be better (under weak priors), as it results in three pairs of individual messages, (m^0, m^0), $\{(m^0, m^1), (m^1, m^0)\}$, (m^1, m^1), with each pair being more informative about the level of actual uncertainty than binary messages under collective expertise. As in the baseline model, the key mechanism for achieving this result is prevention of coordinated reporting.

7.3 Other methods of organizing expertise

Consider first the case of two experts. One alternative way to organize expertise would be to ask the experts sequentially and publicly, so that the second expert hears what the first one says. This mechanism is considered in Ottaviani and Sørensen (2001). While it may be optimal with experts of heterogeneous ability, such a mechanism is weakly inferior to independent expertise when experts are identical. The behavior of the expert who speaks first is obviously the same as under independent expertise. If he herds, so will the second expert under both public sequential speech and independent reporting. If the first experts tells the truth, the second one (being identical) also tells the truth under independent expertise but herds under sequential communication when the first one reports zero.

When the number of experts is more than three, one could think of organizing them into subgroups of two or more experts, with each team reporting independently to the decision-maker – a combination of collective and independent expertise. In general, such a scheme can improve over collective expertise for a range of priors where independent expertise fails, because it is potentially capable of transmitting more information: If each subgroup plays a bipartition within itself, the resulting structure will be finer than a bipartition in the whole space of signals. Yet, as Section 6 suggests, the whole team is capable of generating any informative communication for a wider set of priors than any subteam. Hence, for strong enough priors, collective expertise should dominate. All in all, the described "partially collective" expertise, while refining the results, does not affect our qualitative conclusions.

7.4 Heterogeneous experts

What if the experts have different prior ability? In the online appendix we argue that, unless the heterogeneity is too high, all the qualitative results of the model with identical experts hold through, but the difference between independent and collective expertise diminishes as the heterogeneity grows.

First, full revelation becomes possible for weak enough priors even under collective expertise, provided that the stronger expert is assigned the role of a deputy who first receives information from a weaker expert and then reports to the decision-maker. The necessary and sufficient condition for full revelation under collective expertise is that the weaker expert's signal is stronger than the prior (hence he reports the truth being unaware of the other expert's signal), whereas the stronger expert's signal is stronger than the prior combined with the signal of the weaker expert (hence, he reports his signal and reveals the weaker expert's report).

Second, equilibrium $(m^0, (1, 1))$ becomes more difficult to sustain, because message m^0 becomes both more attractive for the stronger expert and less attractive for the weaker expert. This is due to what can be called "shifting the blame" effect: If the only information the decision-maker has is that at least one of the experts received a signal confirming the prior, then, for any realization of the state, she will rationally assign a higher probability to the weaker expert receiving a wrong signal, compared to the stronger one. Hence, deviations from $(1, 1)$ to m^0 for the stronger expert, and from m^0 to $(1, 1)$ for the weaker expert, become more attractive.

7.5 Private information on competence

Another potential extension is to let the experts know more about their types than the decision-maker does. Suppose there are still two underlying unknown competence-types, but an expert has additional information about himself, which generates one of two possible "perception-types": "optimistic" and "pessimistic", with higher and lower beliefs about own competence relative to the prior belief, respectively. Private information about types adds signaling incentives to the game: the optimistic type with signal 1 may want to reveal his true signal even when he considers state 0 more likely, and the pessimistic type then has a temptation to mimic the optimistic one instead of complete herding on 0. Yet, as the prior on state 0 grows, signaling considerations weaken. It is easy to show that, provided that the experts are not too informed about their own competence, the incentives to “guess” the state correctly eventually dominate, and there will be no informative reporting for priors on the state above a certain threshold.[18] Then the logic of combined signals of several experts reducing their herding incentives works in the same way as in our baseline model, and, thus, collective expertise generates information transmission for a wider set of priors.

Appendix

Preliminaries to proofs

Let

$$x := \Pr(t = G|\sigma_i = \omega);\ y := \Pr(t = G|\sigma_i \neq \omega)$$

denote the expected reputation of expert i conditional on having received correct and incorrect own signal respectively.

Let m be the message sent to the decision-maker and I – the information available to the expert. Then, the expert's expected reputation from message m conditional on I is:

$$\begin{aligned} R_i(m, I) = \Pr(\omega = 0|I)[\Pr(\sigma_i = 0|\omega = 0, m) \cdot x + \Pr(\sigma_i = 1|\omega = 0, m) \cdot y] \\ + \Pr(\omega = 1|I)[\Pr(\sigma_i = 0|\omega = 1, m) \cdot y + \Pr(\sigma_i = 1|\omega = 1, m) \cdot x] \\ = \alpha_i(m, I) \cdot x + \beta_i(m, I) \cdot y, \end{aligned}$$

[18]First, the pessimistic type starts herding on 0, and then, at a higher p, so does the optimistic one.

where

$$\alpha_i(m, I) := \Pr(\omega = 0|I) \Pr(\sigma_i = 0|\omega = 0, m) + \Pr(\omega = 1|I) \Pr(\sigma_i = 1|\omega = 1, m),$$

$$\beta_i(m, I) := \Pr(\omega = 0|I) \Pr(\sigma_i = 1|\omega = 0, m) + \Pr(\omega = 1|I) \Pr(\sigma_i = 0|\omega = 1, m).$$

It is easy to see that $\alpha_i(m, I) + \beta_i(m, I) = 1$. It is also straightforward to derive

$$x = \frac{qg}{\rho} > y = \frac{q(1-g)}{(1-\rho)}$$

Therefore, all comparisons of expected reputations are equivalent to comparing values of $\alpha_i(m, I)$:

$$R_i(m', I) > R_i(m'', I) \Leftrightarrow \alpha_i(m', I) > \alpha_i(m'', I), \text{ for any } I \text{ and any } m' \text{ and } m'' \tag{A.1}$$

Proofs

Proof of Lemma 2. As we have shown in "Preliminaries to proofs", comparing expected reputations is equivalent to comparing values of $\alpha_i(m, I)$ defined therein.

Let m be a message sent with *positive* probability from the team of experts when $\sigma = (1, 0)$, and let m' be a message fully revealing that $\sigma = (0, 0)$. Under a fully revealing mapping, message m can only be sent when either $\sigma = (1, 0)$ or $\sigma = (0, 1)$ (that is, it belongs to $\overline{M}$).

Pairs of signals $(0, 1)$ and $(1, 0)$ provide the same information about ω to the experts, let us denote this information by $\widetilde{\sigma}$. Clearly, $\Pr(\omega = 0|\widetilde{\sigma}\} = p$.

Consider expert 1. We want to show that $\alpha_1(m', \widetilde{\sigma}) > \alpha_1(m, \widetilde{\sigma})$. Since $\Pr(\sigma_1 = 1|m') = 0$ under a fully revealing mapping, irrespective of the realized state, we have

$$\alpha_1(m', \widetilde{\sigma}) = p$$

Now compute $\alpha_1(m, \widetilde{\sigma})$. Denote:

$$\mu := \Pr(m|\sigma = (1,0)),\ \ \nu := \Pr(m|\sigma = (0,1))$$

Using the fact that message m is never sent by signal-types $(0,0)$ and $(1,1)$, we can derive:

$$\Pr(\sigma_1 = 0|\omega = 0, m) = \frac{\Pr(\sigma_1 = 0 \cap m|\omega = 0)}{num. + \Pr(\sigma_1 = 1 \cap m|\omega = 0)}$$
$$= \frac{\Pr(\sigma = (0,1) \cap m|\omega = 0)}{num. + \Pr(\sigma = (1,0) \cap m|\omega = 0)} = \frac{\rho(1-\rho)\nu}{\rho(1-\rho)\nu + (1-\rho)\rho\mu} = \frac{\nu}{\nu + \mu},$$

$$\Pr(\sigma_1 = 1|\omega = 1, m) = \frac{\Pr(\sigma_1 = 1 \cap m|\omega = 1)}{num. + \Pr(\sigma_1 = 0 \cap m|\omega = 1)}$$
$$= \frac{\Pr(\sigma = (1,0) \cap m|\omega = 1)}{num. + \Pr(\sigma = (0,1) \cap m|\omega = 1)} = \frac{\rho(1-\rho)\mu}{\rho(1-\rho)\mu + (1-\rho)\rho\nu} = \frac{\mu}{\nu + \mu}$$

Hence,

$$\alpha_1(m, \widetilde{\sigma}) = p \cdot \frac{\nu}{\nu + \mu} + (1-p) \cdot \frac{\mu}{\nu + \mu}$$

Since, by assumption, $\mu > 0$, $\alpha_1(m, \widetilde{\sigma}) < p = \alpha_1(m', \widetilde{\sigma})$.

By symmetry,

$$\alpha_2(m, \widetilde{\sigma}) = p \cdot \frac{\mu}{\nu + \mu} + (1-p) \cdot \frac{\nu}{\nu + \mu}$$

and, obviously, $\alpha_2(m', \widetilde{\sigma}) = p$. Hence, if $\nu > 0$ expert 2 also strictly prefers m' to m, and if $\nu = 0$ he is indifferent.

If m is a message that is sent with a positive probability when $\sigma = (0, 1)$, the argument above applies without any changes, with experts 1 and 2 being swapped. Notice finally that, for any expert, there is always a message from $\overline{M}$ that is sent with a positive probability when this expert has signal 1. Hence, the last statement of the lemma follows. ■

Proof of Lemma 3. Fix an equilibrium that satisfies our refinement. Let $\mu : \{0, 1\}^2 \to \Delta(M)$ denote the map that associates each possible signal profile with the probabilities of messages induced by the equilibrium (note: we are not assuming that the messages are induced via truthtelling of the signal profile). Thus, we have the following:

(*) Conditional on every $\sigma \in \{0, 1\}^2$, every $m \in M$ with $\mu(m|\sigma) > 0$ gives to both experts a non-lower expected reputation than any other message.

Construct the mirror image $\mu' : \{0, 1\}^2 \to \Delta(M)$ of μ as follows: for each $m \in M$,

$$\begin{aligned} \mu'(m|(0,1)) &= \mu(m|(1,0)), \\ \mu'(m|(1,0)) &= \mu(m|(0,1)) \\ \mu'(m|\sigma) &= \mu(m|\sigma), \qquad \forall \sigma \in \{(0,0), (1,1)\}. \end{aligned}$$

By symmetry, also μ' satisfies (*). Note also that, for each $\omega = 0, 1$, by construction and by $\Pr((1, 0)|\omega) = \Pr((0, 1)|\omega)$,

$$\textstyle\sum_{\overline{\sigma} \in \{0,1\}^2} \mu(m|\overline{\sigma}) \Pr(\overline{\sigma}|\omega) = \sum_{\overline{\sigma} \in \{0,1\}^2} \mu'(m|\overline{\sigma}) \Pr(\overline{\sigma}|\omega). \qquad \text{(A.2)}$$

Consider now the map $\mu'' : \{0, 1\}^2 \to \Delta(M)$ such that, for every $\sigma \in \{0, 1\}^2$

and every $m \in M$,

$$\mu''(m|\sigma) = \frac{1}{2}\mu(m|\sigma) + \frac{1}{2}\mu'(m|\sigma).$$

By symmetry of μ' and μ'', μ'' conveys the same information about the state as μ. Note also that, for each $\omega = 0, 1$, we have

$$\begin{aligned}
\Pr(\overrightarrow{\sigma}|m, \omega; \mu'') &= \frac{\mu''(m|\sigma)\Pr(\overrightarrow{\sigma}|\omega)}{\sum_{\overline{\sigma}\in\{0,1\}^2}\mu''(m|\overline{\sigma})\Pr(\overline{\sigma}|\omega)} \\
&= \frac{\left(\frac{1}{2}\mu(m|\sigma) + \frac{1}{2}\mu'(m|\sigma)\right)\Pr(\sigma|\omega)}{\sum_{\overline{\sigma}\in\{0,1\}^2}\left(\frac{1}{2}\mu(m|\overline{\sigma}) + \frac{1}{2}\mu'(m|\overline{\sigma})\right)\Pr(\overline{\sigma}|\omega)} \\
&= \frac{1}{2}\Pr(\overline{\sigma}|m, \omega; \mu) + \frac{1}{2}\Pr(\overline{\sigma}|m, \omega; \mu'), \qquad \text{(A.3)}
\end{aligned}$$

where the last equality follows from (A.2). We want to show that μ'' is anonymous and satisfies the condition (*). It is anonymous because, for every $m \in M$,

$$\begin{aligned}
\mu''(m|(0,1)) &= \frac{1}{2}\mu(m|(0,1)) + \frac{1}{2}\mu'(m|(0,1)) = \\
&= \frac{1}{2}\mu'(m|(1,0)) + \frac{1}{2}\mu(m|(1,0)) = \mu''(m|(1,0)).
\end{aligned}$$

To see that it satisfies condition (*), proceed as follows. For every $\sigma \in \{0,1\}^2$,

$i = 1, 2$, and $m \in M$, we have

$$\begin{aligned}
&\textstyle\sum_\omega \Pr(\omega|\sigma) \Pr(t_i = G|m, \omega; \mu'') \\
&= \textstyle\sum_\omega \Pr(\omega|\sigma) \sum_{\overline{\sigma} \in \{0,1\}^2} \Pr(t_i = G|\overline{\sigma}, \omega) \Pr(\overline{\sigma}|m, \omega; \mu'') \\
&= \textstyle\sum_\omega \Pr(\omega|\sigma) \sum_{\overline{\sigma} \in \{0,1\}^2} \Pr(t_i = G|\overline{\sigma}, \omega) \left(\frac{1}{2} \Pr(\overline{\sigma}|m, \omega; \mu) + \frac{1}{2} \Pr(\overline{\sigma}|m, \omega; \mu') \right) \\
&= \frac{1}{2} \textstyle\sum_\omega \Pr(\omega|\sigma) \sum_{\overline{\sigma} \in \{0,1\}^2} \Pr(t_i = G|\overline{\sigma}, \omega) \Pr(\overline{\sigma}|m, \omega; \mu) \\
&\quad + \frac{1}{2} \textstyle\sum_\omega \Pr(\omega|\sigma) \sum_{\overline{\sigma} \in \{0,1\}^2} \Pr(t_i = G|\overline{\sigma}, \omega) \Pr(\overline{\sigma}|m, \omega; \mu') \\
&= \frac{1}{2} \textstyle\sum_\omega \Pr(\omega|\sigma) \Pr(t_i = G|m, \omega; \mu) + \frac{1}{2} \sum_\omega \Pr(\omega|\sigma) \Pr(t_i = G|m, \omega; \mu'),
\end{aligned} \tag{A.4}$$

where the second equality follows from (A.3). Now fix $\sigma \in \{0, 1\}^2$ and $\overline{m} \in M$ such that $\mu''(\overline{m}|\sigma) > 0$. Then, by construction of μ'', we have either $\mu(\overline{m}|\sigma) > 0$ or $\mu'(\overline{m}|\sigma) > 0$; without loss of generality, suppose $\mu(\overline{m}|\sigma) > 0$. Thus, $\overline{m}$ maximizes the experts' expected reputation under σ and μ. By symmetry, the same holds for the symmetric signal profile under μ'. Note that, given a signal profile, the expected reputations of an expert from all messages depend only on the posterior about the state, not on the exact signal profile. Then, $\overline{m}$ maximizes the experts' expected reputation also under σ and μ'. Therefore, when $m = \overline{m}$, each of the two terms in (A.4) is the maximum reputation of the experts under μ and μ'. Hence, $\overline{m}$ does no worse than any other message m under μ''.

Obviously, μ'' is also induced by an equilibrium where the experts reveal their true signal to each other. Moreover, under truthtelling, for each $\sigma \in \{0, 1\}^2$, $\mu''(\sigma)$ can be induced as follows: let each expert propose messages according to the distribution $\mu''(\sigma)$. Then, no matter the probabilities of whose proposed message is sent, the messages will arrive to the decision-maker according to μ''. So, there exists an (anonymous) equilibrium with truthtelling and agreement that induces μ''. ■

Proof of Lemma 4. In an anonymous equilibrium with truthtelling between the experts, the two experts have the same information and identical reputations for any m and realization of ω. Hence, their incentives are identical, and we can treat them as a single player with information σ, which we label "signal-type σ". By anonymity, we will treat the two signal-types $(0,1)$ and $(1,0)$ as one aggregate signal-type, so let σ^0, $\widetilde{\sigma}$, and σ^1 denote, respectively, $(0,0)$, $\{(0,1),(1,0)\}$, and $(1,1)$.

First of all, it can be shown that if two messages m and m' convey different information, they cannot yield the same reputations conditional on each state, i.e., $\Pr(G|m,\omega) \neq \Pr(G|m',\omega)$ at least for some ω. The formal proof of this statement can be found in the online appendix, Section 5.

Using this fact, we first show that equilibrium requires any such two messages to be "ordered" in the following sense: For each pair of informationally different equilibrium messages m, m', only one signal-type can be indifferent between them (and thus send both with positive probability), and if $\widetilde{\sigma}$ weakly prefers m, at least one of σ^0, σ^1 strictly prefers m (and thus will never send m'). Without loss of generality, let $\Pr(G|m,\omega=0) > \Pr(G|m',\omega=0)$. If $\Pr(G|m,\omega=1) \geq \Pr(G|m',\omega=1)$, m' cannot be an equilibrium message, so it must be that $\Pr(G|m,\omega=1) < \Pr(G|m',\omega=1)$. For each signal-type σ, the expected reputation from m reads

$$\Pr(G|m,\omega=0)\Pr(\omega=0|\sigma) + \Pr(G|m,\omega=1)\Pr(\omega=1|\sigma),$$

and likewise for m'. So, if σ is indifferent between m and m', a signal-type with more zero-signals strictly prefers m, because it assigns higher probability to $\omega = 0$ (and $\Pr(G|m, \omega = 0) > \Pr(G|m', \omega = 0)$), and vice versa for a signal-type with more one-signals. Moreover, if $\widetilde{\sigma}$ weakly prefers m, σ^0 strictly prefers m, by the same argument.

We will now show that there cannot be more than two reputationally distinct messages in equilibrium. Suppose there are three ordered equilibrium messages m^0, $\widetilde{m}$, m^1 sent with positive probability by σ^0, $\widetilde{\sigma}$, and σ^1 respectively. (We are implicitly allowing for even more than three messages being sent. As we will see, our argument does not rely on $\Pr\left(m \in \left\{m^0, \widetilde{m}, m^1\right\}\right) = 1$ for any signal-type.)

First, notice that due to the established monotonicity (messages being "ordered"), m^0 cannot be sent by σ^1: Since $\widetilde{m} \succsim m^0$ for $\widetilde{\sigma}$, it must be that $\widetilde{m} \succ m^0$ either for σ^1 or for σ^0; however, the latter is impossible as σ^0 sends m^0. Analogously, m^1 cannot be sent by σ^0.

Furthermore, equilibria where $\widetilde{m}$ is sent only by $\widetilde{\sigma}$ are ruled out by the following argument. As we have shown in "Preliminaries to proofs", comparing expected reputations is equivalent to comparing values of $\alpha_i(m, I)$, as defined therein. By playing $\widetilde{m}$, signal-type $\widetilde{\sigma}$ would then generate $\alpha_i(\widetilde{m}, \widetilde{\sigma}) = 1/2$, as, in such a case, $\Pr(\sigma_i = 0|\omega, \widetilde{m}) = 1/2$ for any ω. Instead, a deviation to m^0 would result in $\alpha_i(m^0, \widetilde{\sigma}) > 1/2$. This is because $\Pr(\omega = 0|\widetilde{\sigma}) > 1/2$ and $\Pr(\sigma_i = 0|\omega = 0, m^0) > 1/2$ (as m^0 is sometimes sent by σ^0 and never – by σ^1).

Hence, given that two different signal-types cannot be indifferent between the same two messages, only three types of equilibria remain possible, which can be characterized as follows:

1. $\widetilde{m}$ is sent with a strictly positive probability by both $\widetilde{\sigma}$ and σ^1 but not by σ^0, m^0 is never sent by σ^1 (but can be sent by $\widetilde{\sigma}$), m^1 is sent only by σ^1.

2. $\widetilde{m}$ is sent with a strictly positive probability by both $\widetilde{\sigma}$ and σ^0 but not by σ^1, m^1 is never sent by σ^0 (but can be sent by $\widetilde{\sigma}$), m^0 is sent only by σ^0.

3. $\widetilde{m}$ is sent with a strictly positive probability by all signal-types, m^0 is sent only by σ^0, m^1 is sent only by σ^1.

Let us first show that equilibria of type 1 do not exist. Our goal is to show that $\alpha_i(\widetilde{m}, \widetilde{\sigma}) \geq \alpha_i(m^0, \widetilde{\sigma})$ and $\alpha_i(\widetilde{m}, \sigma^1) = \alpha_i(m^1, \sigma^1)$ cannot hold simultaneously. Due do the anonymity of equilibria, $\alpha_i(m, \sigma)$ does not depend on i, so we can consider either expert; let it be expert 1, for concreteness.

Denote $\gamma := \Pr(m = m^0|\sigma = \sigma^0),\ \nu := \Pr(m = m^0|\sigma = \widetilde{\sigma}),\ \xi := \Pr(m = \widetilde{m}|\sigma = \widetilde{\sigma}),\ \mu := \Pr(m = \widetilde{m}|\sigma = \sigma^1)$.

Let us compute $\alpha_1(m^0, \widetilde{\sigma})$ and $\alpha_1(\widetilde{m}, \widetilde{\sigma})$:

$$\begin{aligned}
\alpha_1(m^0, \widetilde{\sigma}) &= \Pr(\omega = 0|\widetilde{\sigma}) \Pr(\sigma_1 = 0|\omega = 0, m^0) \\
&+ \Pr(\omega = 1|\widetilde{\sigma}) \Pr(\sigma_1 = 1|\omega = 1, m^0), \\
\alpha_1(\widetilde{m}, \widetilde{\sigma}) &= \Pr(\omega = 0|\widetilde{\sigma}) \Pr(\sigma_1 = 0|\omega = 0, \widetilde{m}) \\
&+ \Pr(\omega = 1|\widetilde{\sigma}) \Pr(\sigma_1 = 1|\omega = 1, \widetilde{m}).
\end{aligned}$$

Let us compute the ingredients of these expressions. First, $\Pr(\omega = 0|\sigma = \widetilde{\sigma}) = p$. Next,

$$
\begin{aligned}
\Pr(\sigma_1 = 0|\omega = 0, m^0) &= \frac{\Pr(\sigma_1 = 0 \cap m = m^0|\omega = 0)}{\Pr(m = m^0|\omega = 0)} \\
&= \frac{\Pr(\sigma = (0,0)|\omega = 0)\gamma + \Pr(\sigma = (0,1)|\omega = 0)\nu}{num. + \Pr(\sigma = (1,0)|\omega = 0)\nu} \\
&= \frac{\rho^2\gamma + \rho(1-\rho)\nu}{\rho^2\gamma + 2\rho(1-\rho)\nu} = \frac{\rho\gamma + (1-\rho)\nu}{\rho\gamma + 2(1-\rho)\nu}.
\end{aligned}
$$

$$
\begin{aligned}
\Pr(\sigma_1 &= 1|\omega = 1, m^0) = \frac{\Pr(\sigma_1 = 1 \cap m = m^0|\omega = 1)}{\Pr(m = m^0|\omega = 1)} \\
&= \frac{\Pr(\sigma = (1,0)|\omega = 1)\nu}{num. + \Pr(\sigma = (0,0)|\omega = 1)\gamma + \Pr(\sigma = (0,1)|\omega = 1)\nu} \\
&= \frac{\rho(1-\rho)\nu}{\gamma(1-\rho)^2 + 2\rho(1-\rho)\nu} = \frac{\rho\nu}{(1-\rho)\gamma + 2\rho\nu}
\end{aligned}
$$

$$
\begin{aligned}
\Pr(\sigma_1 &= 0|\omega = 0, \widetilde{m}) = \frac{\Pr(\sigma_1 = 0 \cap m = \widetilde{m}|\omega = 0)}{\Pr(m = \widetilde{m}|\omega = 0)} \\
&= \frac{\Pr(\sigma = (0,1)|\omega = 0)\xi}{num. + \Pr(\sigma = (1,0)|\omega = 0)\xi + \Pr(\sigma = (1,1)|\omega = 0)\mu} \\
&= \frac{\rho(1-\rho)\xi}{(1-\rho)^2\mu + 2\rho(1-\rho)\xi} = \frac{\rho\xi}{(1-\rho)\mu + 2\rho\xi}
\end{aligned}
$$

$$
\begin{aligned}
\Pr(\sigma_1 = 1|\omega = 1, \widetilde{m}) &= \frac{\Pr(\sigma_1 = 1 \cap m = \widetilde{m}|\omega = 1)}{\Pr(m = \widetilde{m}|\omega = 1)} \\
&= \frac{\Pr(\sigma = (1,0)|\omega = 1)\xi + \Pr(\sigma = (1,1)|\omega = 0)\mu}{num. + \Pr(\sigma = (0,1)|\omega = 1)\xi} \\
&= \frac{\rho(1-\rho)\xi + \rho^2\mu}{2\rho(1-\rho)\xi + \rho^2\mu} = \frac{(1-\rho)\xi + \rho\mu}{2(1-\rho)\xi + \rho\mu}.
\end{aligned}
$$

Hence,

$$\begin{aligned}\alpha_1(m^0,\widetilde{\sigma}) &= p\frac{\rho\gamma+(1-\rho)\nu}{\rho\gamma+2(1-\rho)\nu}+(1-p)\frac{\rho\nu}{(1-\rho)\gamma+2\rho\nu},\\ \alpha_1(\widetilde{m},\widetilde{\sigma}) &= p\frac{\rho\xi}{(1-\rho)\mu+2\rho\xi}+(1-p)\frac{(1-\rho)\xi+\rho\mu}{2(1-\rho)\xi+\rho\mu}\end{aligned}$$

Then, $\alpha_1(\widetilde{m},\widetilde{\sigma}) \geq \alpha_1(m^0,\widetilde{\sigma})$ writes as

$$p\left[\frac{\rho\gamma+(1-\rho)\nu}{\rho\gamma+2(1-\rho)\nu}-\frac{\rho\xi}{(1-\rho)\mu+2\rho\xi}\right] \leq (1-p)\left[\frac{(1-\rho)\xi+\rho\mu}{2(1-\rho)\xi+\rho\mu}-\frac{\rho\nu}{(1-\rho)\gamma+2\rho\nu}\right],$$

which after some algebra (it is straightforward to show that the expressions in the square brackets are positive) can be rewritten as

$$\frac{p}{1-p} \leq \frac{[(1-\rho)^2\gamma\xi+\rho(1-\rho)\gamma\mu+\rho^2\nu\mu]\cdot[\rho\gamma+2(1-\rho)\nu]\cdot[2\rho\xi+(1-\rho)\mu]}{[2(1-\rho)\xi+\rho\mu]\cdot[(1-\rho)\gamma+2\rho\nu]\cdot[\rho^2\gamma\xi+\rho(1-\rho)\gamma\mu+(1-\rho)^2\nu\mu]} \tag{A.5}$$

Let us compute now $\alpha_1(\widetilde{m},\sigma^1)$ and $\alpha_1(m^1,\sigma^1)$:

$$\begin{aligned}\alpha_1(\widetilde{m},\sigma^1) &= \Pr(\omega=0|\sigma^1)\Pr(\sigma_1=0|\omega=0,\widetilde{m})\\ &+ \Pr(\omega=1|\sigma^1)\Pr(\sigma_1=1|\omega=1,\widetilde{m}), \qquad \text{(A.6)}\\ \alpha_1(m^1,\sigma^1) &= \Pr(\omega=0|\sigma^1)\Pr(\sigma_1=0|\omega=0,m^1)\\ &+ \Pr(\omega=1|\sigma^1)\Pr(\sigma_1=1|\omega=1,m^1).\end{aligned}$$

Let us compute the ingredients of these expressions:

$$\Pr(\omega=0|\sigma^1)=\frac{\Pr(\sigma=\sigma^1|\omega=0)\Pr(\omega=0)}{num.+\Pr(\sigma=\sigma^1|\omega=1)\Pr(\omega=1)}=\frac{(1-\rho)^2p}{(1-\rho)^2p+\rho^2(1-p)} \tag{A.7}$$

$$\begin{aligned}
\Pr(\sigma_1 &= 0|\omega = 0, \widetilde{m}) = \frac{\Pr(\sigma_1 = 0 \cap m = \widetilde{m}|\omega = 0)}{\Pr(m = \widetilde{m}|\omega = 0)} \\
&= \frac{\Pr(\sigma = (0,1)|\omega = 0)\xi}{num. + \Pr(\sigma = (1,0)|\omega = 0)\xi + \Pr(\sigma = (1,1)|\omega = 0)\mu} \\
&= \frac{\rho(1-\rho)\xi}{2\rho(1-\rho)\xi + (1-\rho)^2\mu} = \frac{\rho\xi}{2\rho\xi + (1-\rho)\mu}.
\end{aligned}$$

$$\begin{aligned}
\Pr(\sigma_1 = 1|\omega = 1, \widetilde{m}) &= \frac{\Pr(\sigma_1 = 1 \cap m = \widetilde{m}|\omega = 1)}{\Pr(m = \widetilde{m}|\omega = 1)} \\
&= \frac{\Pr(\sigma = (1,0)|\omega = 1)\xi + \Pr(\sigma = (1,1)|\omega = 1)\mu}{num. + \Pr(\sigma = (0,1)|\omega = 1)\xi} \\
&= \frac{\rho(1-\rho)\xi + \rho^2\mu}{2\rho(1-\rho)\xi + \rho^2\mu} = \frac{(1-\rho)\xi + \rho\mu}{2(1-\rho)\xi + \rho\mu}.
\end{aligned}$$

$$\Pr(\sigma_1 = 0|\omega = 0, m = m_1) = 0, \; \Pr(\sigma_1 = 1|\omega = 1, m = m_1) = 1$$

Hence,

$$\begin{aligned}
\alpha_1(\widetilde{m}, \sigma^1) &= \frac{(1-\rho)^2 p}{(1-\rho)^2 p + \rho^2(1-p)} \frac{\rho\xi}{2\rho\xi + (1-\rho)\mu} \\
&+ \frac{\rho^2(1-p)}{(1-\rho)^2 p + \rho^2(1-p)} \frac{(1-\rho)\xi + \rho\mu}{2(1-\rho)\xi + \rho\mu}, \\
\alpha_1(m^1, \sigma^1) &= \frac{\rho^2(1-p)}{(1-\rho)^2 p + \rho^2(1-p)}
\end{aligned}$$

Then, $\alpha_1(\widetilde{m}, \sigma^1) = \alpha_1(m^1, \sigma^1)$ writes as:

$$(1-\rho)^2 p \frac{\rho\xi}{2\rho\xi + (1-\rho)\mu} + \rho^2(1-p)\frac{(1-\rho)\xi + \rho\mu}{2(1-\rho)\xi + \rho\mu} = \rho^2(1-p),$$

which can be rewritten as

$$\frac{p}{1-p} = \frac{\rho[2\rho\xi + (1-\rho)\mu]}{(1-\rho)[2(1-\rho)\xi + \rho\mu]} \tag{A.8}$$

For (A.5) and (A.8) to hold jointly, it must be that

$$\frac{[(1-\rho)^2\gamma\xi + \rho(1-\rho)\gamma\mu + \rho^2\nu\mu]\cdot[\rho\gamma + 2(1-\rho)\nu]}{[(1-\rho)\gamma + 2\rho\nu]\cdot[\rho^2\gamma\xi + \rho(1-\rho)\gamma\mu + (1-\rho)^2\nu\mu]} \geq \frac{\rho}{1-\rho},$$

which boils down to

$$(1-\rho)^3\rho(\gamma\xi + \nu\mu) + 2(1-\rho)^4\nu\xi \geq \rho^3(1-\rho)(\gamma\xi + \nu\mu) + 2\rho^4\nu\xi$$

Since $\rho > 1-\rho$, the left-hand side is always smaller than the right-hand side, i.e., this inequality never holds. Hence, equilibria of type 1 do not exist.

Notice now that equilibria of type 2 simply mirror those of type 1. Hence, the non-existence of type 2 equilibria is established in exactly the same way: we just need to swap σ^0 with σ^1, m^0 with m^1, and p with $1-p$.

Finally, consider equilibria of type 3. We will show that $\alpha_1(\widetilde{m}, \sigma^0) \geq \alpha_1(m^0, \sigma^0)$ and $\alpha_1(\widetilde{m}, \sigma^1) \geq \alpha_1(m^1, \sigma^1)$ cannot hold simultaneously. Denote $\delta := \Pr(m = \widetilde{m}|\sigma = \sigma^0)$, $\xi := \Pr(m = \widetilde{m}|\sigma = \widetilde{\sigma})$, $\mu := \Pr(m = \widetilde{m}|\sigma = \sigma^1)$.

Let us compute $\alpha_1(\widetilde{m}, \sigma^1)$ and $\alpha_1(m^1, \sigma^1)$. As m^1 is sent only by σ^1, the latter remains exactly the same as in equilibria of type 1, i.e.,

$$\alpha_1(m^1, \sigma^1) = \frac{\rho^2(1-p)}{(1-\rho)^2 p + \rho^2(1-p)}$$

The general formula for $\alpha_1(\widetilde{m}, \sigma^1)$ is given by (A.6), and $\Pr(\omega = 0|\sigma^1)$ is computed in (A.7) The remaining ingredients are:

$$\begin{aligned}
\Pr(\sigma_1 &= 0|\omega = 0, \widetilde{m}) = \frac{\Pr(\sigma_1 = 0 \cap m = \widetilde{m}|\omega = 0)}{\Pr(m = \widetilde{m}|\omega = 0)} \\
&= \frac{\Pr(\sigma = (0,1)|\omega = 0)\xi + \Pr(\sigma = (0,0)|\omega = 0)\delta}{num. + \Pr(\sigma = (1,0)|\omega = 0)\xi + \Pr(\sigma = (1,1)|\omega = 0)\mu} \\
&= \frac{\rho(1-\rho)\xi + \rho^2\delta}{2\rho(1-\rho)\xi + \rho^2\delta + (1-\rho)^2\mu}.
\end{aligned}$$

$$\begin{aligned}
\Pr(\sigma_1 &= 1|\omega = 1, \widetilde{m}) = \frac{\Pr(\sigma_1 = 1 \cap m = \widetilde{m}|\omega = 1)}{\Pr(m = \widetilde{m}|\omega = 1)} \\
&= \frac{\Pr(\sigma = (1,0)|\omega = 1)\xi + \Pr(\sigma = (1,1)|\omega = 1)\mu}{num. + \Pr(\sigma = (0,1)|\omega = 1)\xi + \Pr(\sigma = (0,0)|\omega = 1)\delta} \\
&= \frac{\rho(1-\rho)\xi + \rho^2\mu}{2\rho(1-\rho)\xi + \rho^2\mu + (1-\rho)^2\delta}.
\end{aligned}$$

Hence,

$$\begin{aligned}
\alpha_1(\widetilde{m}, \sigma^1) &= \frac{(1-\rho)^2 p}{(1-\rho)^2 p + \rho^2(1-p)} \frac{\rho(1-\rho)\xi + \rho^2\delta}{2\rho(1-\rho)\xi + \rho^2\delta + (1-\rho)^2\mu} \\
&\quad + \frac{\rho^2(1-p)}{(1-\rho)^2 p + \rho^2(1-p)} \frac{\rho(1-\rho)\xi + \rho^2\mu}{2\rho(1-\rho)\xi + \rho^2\mu + (1-\rho)^2\delta}
\end{aligned}$$

Thus $\alpha_1(\widetilde{m}, \sigma^1) \geq \alpha_1(m^1, \sigma^1)$ can be written as

$$\begin{aligned}
&(1-\rho)^2 p \frac{\rho(1-\rho)\xi + \rho^2\delta}{2\rho(1-\rho)\xi + \rho^2\delta + (1-\rho)^2\mu} \\
&\qquad + \rho^2(1-p)\frac{\rho(1-\rho)\xi + \rho^2\mu}{2\rho(1-\rho)\xi + \rho^2\mu + (1-\rho)^2\delta} \geq \rho^2(1-p),
\end{aligned}$$

or

$$p\frac{(1-\rho)^2[\rho(1-\rho)\xi+\rho^2\delta]}{2\rho(1-\rho)\xi+\rho^2\delta+(1-\rho)^2\mu} \geq (1-p)\frac{\rho^2[\rho(1-\rho)\xi+(1-\rho)^2\delta]}{2\rho(1-\rho)\xi+\rho^2\mu+(1-\rho)^2\delta},$$

or

$$\frac{p}{1-p} \geq \frac{\rho^2[\rho(1-\rho)\xi+(1-\rho)^2\delta][2\rho(1-\rho)\xi+\rho^2\delta+(1-\rho)^2\mu]}{(1-\rho)^2[2\rho(1-\rho)\xi+\rho^2\mu+(1-\rho)^2\delta][\rho(1-\rho)\xi+\rho^2\delta]} \qquad \text{(A.9)}$$

By symmetry (replacing p with $1-p$ and δ with μ), $\alpha_1(\widetilde{m},\sigma^0) \geq \alpha_1(m^0,\sigma^0)$ can be written as

$$(1-p)\frac{(1-\rho)^2[\rho(1-\rho)\xi+\rho^2\mu]}{2\rho(1-\rho)\xi+\rho^2\mu+(1-\rho)^2\delta} \geq p\frac{\rho^2[\rho(1-\rho)\xi+(1-\rho)^2\mu]}{2\rho(1-\rho)\xi+\rho^2\delta+(1-\rho)^2\mu},$$

or

$$\frac{p}{1-p} \leq \frac{(1-\rho)^2[\rho(1-\rho)\xi+\rho^2\mu][2\rho(1-\rho)\xi+\rho^2\delta+(1-\rho)^2\mu]}{\rho^2[\rho(1-\rho)\xi+(1-\rho)^2\mu][2\rho(1-\rho)\xi+\rho^2\mu+(1-\rho)^2\delta]} \qquad \text{(A.10)}$$

For (A.9) and (A.10) to hold jointly it must be that

$$\begin{aligned} & \frac{(1-\rho)^2[\rho(1-\rho)\xi+\rho^2\mu][2\rho(1-\rho)\xi+\rho^2\delta+(1-\rho)^2\mu]}{\rho^2[\rho(1-\rho)\xi+(1-\rho)^2\mu][2\rho(1-\rho)\xi+\rho^2\mu+(1-\rho)^2\delta]} \\ \geq\; & \frac{\rho^2[\rho(1-\rho)\xi+(1-\rho)^2\delta][2\rho(1-\rho)\xi+\rho^2\delta+(1-\rho)^2\mu]}{(1-\rho)^2[2\rho(1-\rho)\xi+\rho^2\mu+(1-\rho)^2\delta][\rho(1-\rho)\xi+\rho^2\delta]} \end{aligned}$$

or

$$\frac{(1-\rho)[(1-\rho)\xi+\rho\mu]}{\rho[\rho\xi+(1-\rho)\mu]} \geq \frac{\rho[\rho\xi+(1-\rho)\delta]}{(1-\rho)[(1-\rho)\xi+\rho\delta]},$$

which, after simple algebra, yields

$$(1-\rho)^4\xi^2+(1-\rho)^3\rho\xi(\mu+\delta) \geq \rho^4\xi^2+\rho^3(1-\rho)\xi(\mu+\delta).$$

Since $\rho > 1-\rho$, the left-hand side is always smaller than the right-hand side, i.e., this inequality never holds. Hence, equilibria of type 3 do not exist. ∎

Proof of Lemma 5. Suppose the experts have revealed their true signals to each other. Then, given anonymous equilibrium reporting strategies, and assuming that any off-path message is treated (belief-wise) as some equilibrium message by the decision-maker, the two experts have the same information and identical reputations for any m and realization of ω. Hence, their incentives are identical, and we can treat them as a single player with information σ, which we label "signal-type σ". By anonymity, we will treat the two signal-types $(0,1)$ and $(1,0)$ as one aggregate signal-type, so let σ^0, $\widetilde{\sigma}$, and σ^1 denote, respectively, $(0,0)$, $\{(0,1),(1,0)\}$, and $(1,1)$. Notice that once the incentive compatibility constraints of all signal-types are satisfied, truthtelling between the experts is equilibrium behavior: By lying, an expert cannot induce a report that would benefit him.

We need to check the incentive compatibility constraints of signal-types σ^1 and $\widetilde{\sigma}$. There is no need to check that for σ^0: If $\widetilde{\sigma}$ does not gain from deviating to reporting m^1, then neither does σ^0, as the latter assigns an even lower probability to $\omega = 1$. As we have shown in "Preliminaries to proofs", comparing expected reputations is equivalent to comparing values of $\alpha_i(m, I)$, as defined therein. Due do the anonymity of equilibria, $\alpha_i(m,\sigma)$ does not depend on i, so we can consider either expert; let it be expert 1, for concreteness.

Incentive compatibility of signal-type σ^1:

First, compute α_1 of signal-type σ^1 if he does not deviate.

$$\begin{aligned}\Pr\left(\omega=0|\sigma^1\right) &= \frac{\Pr(\sigma=\sigma^1|\omega=0)\Pr(\omega=0)}{num. + \Pr(\sigma=\sigma^1|\omega=1)\Pr(\omega=1)} \\ &= \frac{(1-\rho)^2 p}{(1-\rho)^2 p+\rho^2(1-p)}\end{aligned} \tag{A.11}$$

$$\Pr(\sigma_1=0|\omega=0,m^1)=0,\ \Pr(\sigma_1=1|\omega=1,m^1)=1$$

Thus,

$$\alpha_1\left(m^1,\sigma^1\right)=\Pr\left(\omega=0|\sigma^1\right)\cdot 0+\Pr\left(\omega=1|\sigma^1\right)\cdot 1=\frac{\rho^2(1-p)}{(1-\rho)^2p+\rho^2(1-p)} \tag{A.12}$$

Now, compute α_1 of signal-type σ^1 if he deviates to m^0.

$$\Pr\left(\sigma_1=0|\omega=0,m^0\right)=\frac{\Pr(\sigma_1=0\cap m=m^0|\omega=0)}{\Pr(m=m^0|\omega=0)}$$
$$=\frac{\Pr(\sigma\in\{(0,0),(0,1)\}|\omega=0)}{\Pr(\sigma\in\{(0,0),(0,1),(1,0)\}|\omega=0)}=\frac{\rho^2+\rho(1-\rho)}{\rho^2+2\rho(1-\rho)}=\frac{1}{2-\rho}$$

$$\Pr(\sigma_1=1|\omega=1,m^0)=\frac{\Pr(\sigma_1=1\cap m=m^0|\omega=1)}{\Pr(m=m^0|\omega=1)}$$
$$=\frac{\Pr(\sigma=(1,0)|\omega=1)}{\Pr(\sigma\in\{(0,0),(0,1),(1,0)\}|\omega=1)}=\frac{\rho(1-\rho)}{(1-\rho)^2+2(1-\rho)\rho}=\frac{\rho}{1+\rho}$$

Thus,

$$\alpha_1\left(m^0,\sigma^1\right)=\Pr(\omega=0|\sigma^1)\cdot\frac{1}{2-\rho}+\Pr(\omega=1|\sigma^1)\cdot\frac{\rho}{1+\rho}$$
$$=\frac{(1-\rho)^2p}{(1-\rho)^2p+\rho^2(1-p)}\cdot\frac{1}{2-\rho}+\frac{\rho^2(1-p)}{(1-\rho)^2p+\rho^2(1-p)}\cdot\frac{\rho}{1+\rho}$$

The expert will not deviate whenever

$$\alpha_1(m^1, \sigma^1) \geq \alpha_1(m^0, \sigma^1),$$

which yields

$$p \leq \frac{\rho^2(2-\rho)}{1-\rho+\rho^2} =: \overline{p}.$$

It is straightforward to show that $\overline{p} > \rho$, given that $\rho > 1/2$.

Let us show now that at $p = \overline{p}$, $\Pr(\omega = 1|\sigma^1) > 1/2$. Using (A.11),

$$\Pr(\omega = 1|\sigma^1) = \frac{\rho^2(1-p)}{\rho^2(1-p) + (1-\rho)^2 p}$$

$$\frac{\rho^2(1-\overline{p})}{\rho^2(1-\overline{p}) + (1-\rho)^2\overline{p}} > 1/2 \Leftrightarrow \frac{\rho^2}{(1-\rho)^2} > \frac{\overline{p}}{1-\overline{p}}$$

Substituting the expression for $\overline{p}$ into the last inequality yields $\rho > 1/2$ which is always true (as $g > b \geq 1/2$ and $q > 0$ by the assumptions of the model).

Incentive compatibility of signal-type $\widetilde{\sigma}$:

First, compute α_1 of signal-type $\widetilde{\sigma}$ if he does not deviate.

$$\begin{aligned}
\Pr(\omega = 0|\widetilde{\sigma}) &= \Pr(\omega = 0|\sigma = (1,0)) \\
&= \frac{\Pr(\sigma = (1,0)|\omega = 0)\Pr(\omega = 0)}{num. + \Pr(\sigma = (1,0)|\omega = 1)\Pr(\omega = 1)} \\
&= \frac{(1-\rho)\rho p}{(1-\rho)\rho p + \rho(1-\rho)(1-p)}
\end{aligned}$$

Using the expressions for $\Pr(\sigma_1 = 0|\omega = 0, m^0)$ and $\Pr(\sigma_1 = 1|\omega = 1, m^0)$

derived above, we obtain:

$$\begin{aligned}\alpha_1\left(m^0,\widetilde{\sigma}\right) = & \frac{(1-\rho)\rho p}{(1-\rho)\rho p+\rho(1-\rho)(1-p)}\cdot\frac{1}{2-\rho} \\ & +\frac{\rho(1-\rho)(1-p)}{(1-\rho)\rho p+\rho(1-\rho)(1-p)}\cdot\frac{\rho}{1+\rho}\end{aligned}$$

Now, compute α_1 of signal-type $\widetilde{\sigma}$ if he deviates to m^1.

$$\Pr(\sigma_1 = 0|\omega = 0, \sigma^1) = 0,\ \Pr(\sigma_1 = 1|\omega = 1, \sigma^1) = 1$$

Thus,

$$\alpha_1\left(m^1,\widetilde{\sigma}\right) = \Pr(\omega = 0|\widetilde{\sigma})\cdot 0 + \Pr(\omega = 1|\widetilde{\sigma})\cdot 1 = \frac{\rho(1-\rho)(1-p)}{(1-\rho)\rho p+\rho(1-\rho)(1-p)}$$

The expert will not deviate whenever $\alpha_1\left(m^0,\widetilde{\sigma}\right) \geq \alpha_1\left(m^1,\widetilde{\sigma}\right)$, which boils down to

$$\begin{aligned}(1+\rho)p & \geq (2-\rho)(1-p) \\ & \Leftrightarrow\ p \geq \frac{2-\rho}{3}\end{aligned}$$

As $\rho > 1/2$, the right-hand side is always below $1/2$. Thus, the incentive compatibility condition of signal-type $\widetilde{\sigma}$ is always satisfied. ■

Proof of Lemma 6. Since the equilibrium is symmetric to the one of the previous lemma, the incentive compatibility conditions are exactly the same as in the proof of the previous lemma, with the only difference that p has to be substituted with $1-p$.

Thus, the no-deviation condition of signal-type σ^0 is

$$1 - p \leq \frac{\rho^2(2-\rho)}{1-\rho+\rho^2} \equiv \overline{p} \Leftrightarrow p \geq 1 - \overline{p}$$

Since $\overline{p} > \rho > 1/2$, the condition is satisfied for all $p > 1/2$ (which is an assumption of the model).

The no-deviation condition of signal-type $\widetilde{\sigma}$ is

$$1 - p \geq \frac{2-\rho}{3} \Leftrightarrow p \leq \frac{1+\rho}{3},$$

which is smaller than ρ, as $\rho > 1/2$. ■

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Мы рассматриваем проблему человека, который обращается за советом к нескольким экспертам для принятия решения. Эксперты озабочены своей репутацией, что побуждает их придерживаться априорных представлений о состоянии мира при выдаче рекомендаций. Мы отвечаем на следующий вопрос: следует ли разрешать экспертам обмениваться информацией перед тем, как они дают рекомендации («коллективная экспертиза»), или нет («независимая экспертиза»)? Мы показываем, что коллективная экспертиза более информативна, чем независимая экспертиза, при низкой априорной неопределенности относительно состояния мира и менее информативна иначе. Мы также показываем, что коллективная экспертиза становится предпочтительнее по мере роста числа экспертов.

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