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Pricing, Market Power, and Friction in a Finite Market: The Role of Capacities *

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Abstract

This paper proposes a model of a finite two-sided market with a limited arbitrary number of products per seller, where buyers are involved in a directed search for the appropriate purchase. The effect of friction, discovered for the models with a single product per seller, remains, though the competition intensifies. We derive an analytical formula for the case of an equal number of products for every seller and deduce that the equilibrium price decreases with the growth of availability and drops to marginal costs when two sellers are able to serve the whole set of buyers. However, the seller's utility is a bell-shaped function of the number of products. This produces the controversial impact of market concentration on the various equilibrium characteristics. For the general model with different capacities across sellers, we formulate equilibrium conditions on prices, and clarify how the market power of a particular seller depends on its capacity. Numerical analysis is also applied to the related problem of endogenous capacities.

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1 Introduction

Directed search theory has developed a deep understanding of the endogenous matching of buyers and sellers in finite markets, optimal pricing schemes, and, as a result, market inefficiency (Wright et al. 2021). Two sides of a classic market, i.e. buyers or sellers, or firms and employers, are aimed to be matched in pairs for making a deal (purchase of a valuable product, hiring a productive worker) at a reasonable price. Strategic buyers make a non-correlated choice from the list of sellers' posted prices and, because of the finiteness of sellers and products they have, some buyers remain unserved. In the benchmark models, sellers generally have one unit of homogeneous product, and the attention is paid to the mechanics of the friction process, the resulting failure of effective matching, and the redistribution of bargaining power between the two market sides determined by their sizes.

In this paper, we assert that the real capacity of a seller, which is the important dimension of the finite market, has been poorly accounted for in previous studies, and fill this gap. If sellers have more than one product, this increases the availability of products for buyers and relaxes the competition for every seller. From the practical point of view, the setting with many products is more applicable since sellers, for instance operating in a marketplace, often have some reserve of a given product, whose capacity depends on the size of the firm. Even if the total number of products in the market exceeds the total demand for this product, this does not necessarily mean that the cheapest seller alone can satisfy the whole market. Not only does the number of market participants influence the equilibrium characteristics, like prices, matching probabilities, and utilities, but the distribution of products across sellers also affects a finite market. In this article, we determine the exact condition for the sellers' capacities under which the market demonstrates friction: if at least two sellers can individually serve all buyers, then the friction disappears and the market is indistinguishable from that without any capacity constraints.

A few studies implicitly incorporate greater capacities into the finite market model. The original early paper by (Burdett, Shi, and Wright 2001) considers an extension with some firms having a capacity of two versus others with a capacity of one. They analytically solve the small market 2 buyers $\times$ 2 sellers, and then extend their result to a large market with fixed buyer-seller ratio. Nevertheless, they learn nothing about intermediate finite cases and, what is more important for us, for a larger number of products per seller.

A similar setting, but in the labor market framework with firms and employers, was developed in (Lester 2010), which refers to firms with two vacancies as large firms and with one vacancy as small firms. The market mechanics is the same as in (Burdett, Shi, and Wright 2001) and in the current paper, but again the author focuses on the infinite market with fixed buyer-seller ratio. He endogenizes the choice of capacity for a firm, allows free entry of firms, and deduces a higher expected utility, total and per vacancy, of large firms. But here we see that the further extension of the vacancy set is not necessarily profitable.

Sellers with larger finite capacities are introduced in (Geromichalos 2012), but the author introduces an alternative mechanism where a seller posts the conditions of a contract "realized capacity+price", and the maximum capacity is given. Buyers also pay a submission fee, which is paid irrespective of the success of a purchase from a particular seller. The author focuses on efficiency problems of equilibria and on an infinitely large market. This paper puts forward an important problem of seller's optimal capacity which does not cover the whole demand.

The similar to (Geromichalos 2012) models have been developed for the labor market (Tan 2012), (Hawkins 2013), (Jacquet and Tan 2012). Wage-vacancy contracts, where wages may depend on the number of hired workers or may be different across workers hired by the same firm, fit the reality of the hiring process in classical labor markets. Another issue accounted for in these studies is the difference between the productivity of small and large firms, introduced in the models. However, they all consider large markets with a relaxed strategic interaction of workers, while in the finite case both friction and strategic behavior are non-negligible. Moreover, for a buyer-seller market, the assumption that sellers are able to post a menu of prices for the same products at a given moment does not generally fit the organization of the marketplace.

In this paper, we consider the finite market with n_s sellers and n_b buyers, which do not necessarily grow to infinity. We start with symmetric sellers with equal capacities $1 \leqslant h \leqslant n_b$. Sellers simultaneously post prices, and then buyers independently send their requests for a purchase to a particular seller. If the number of requests exceeds the number of products the seller has, some buyers remain unserved. The probability of being served is more complicated than in the case of a unique product per seller, as it depends both on the capacity and the actual demand. We derive an analytical solution of a buyers' game and, consequently, the equilibrium price in a sellers' game. This is a direct extension of formula (45) from (Wright et al. 2021).

This generalized formula has immediate implications for understanding the toughness of competition in the market. It also allows the matching probability and the seller utilities as functions of h to be calculated. For small $h \leq 2$, it was previously shown that firms with a greater capacity get larger utility, but this tendency changes with the growth of product reserve for each seller. Analyzing the comparative statics, we deduce that, because of the intensification of competition with the growth of availability and resulting price decrease, the potential to sell more products does not compensate for loses from the low price. This means that the total utility of the firm is a bell-shaped function of h. Thus, there exists an optimal seller capacity in the symmetric setting, and this capacity is not so large, such that some deficit is optimal for sellers.

Our results contribute to understanding the influence of market concentration on competition. We deduce that this influence is controversial and depends on the relation between the number of buyers and total number of products on the market. Although in a more concentrated market, friction decreases, under a large demand the equilibrium price grows, but at low demand the competition intensifies such that the price drops.

The exact representation of symmetric buyer equilibrium behavior allows passing to the maximization problem for sellers in the case of non-equal capacities. To ensure the existence and uniqueness of the solution, we apply the approach from (Kim and Camera 2014). We also improve the related recursive approach by (Camera and Kim 2013) when adopting it to the solution of the buyers' subgame. This explains why the numerical analysis is correct and well-defined.

The numerical solution of the general model demonstrates the difference in the pricing behavior between large and small sellers in terms of their capacities. We show that the large firms enjoy extra market power and propose higher prices than the small firms. This is because small sellers must compete more aggressively in order to sell a few products, while large sellers may sell less often per unit, but with greater marginality. Large sellers attract buyers not by low prices, but by greater availability and less friction. But again there is a limit of capacity which is profitable for a particular seller.

Relying on numerical solutions, we elaborate the setting with endogenous capacities of sellers and associate their choice with a separate simultaneous game. The equilibria for several special cases demonstrate that arbitrary large capacities are generally not the best choice and far from what we may observe on the market. Generally, rational sellers do not try to attract all buyers unilaterally, but also their total supply is larger than that under collusion.

This study enriches the setting of a classic finite market by extending possible seller capacities and explains the decisions of sellers and the classic relationship of "price – matching efficiency – firm size" clearer. It clarifies the nature of buyers' competitive behavior which in turn affects sellers' expectation of demand, and highlights the influence of product distribution across sellers.

The paper is organized as follows. Next session introduces the model and important functions for the analysis. Section 3 solves the symmetric model and clarifies the dependence of the equilibrium on the parameters. Section 4 considers the general heterogeneous case, presents the limits of analytical solution, and analyzes some numerical properties and the setting with endogenous capacities. Section 5 concludes.

2 The finite market model

2.1 The model

Assume we have the finite market with $n_s \geqslant 2$ sellers and $n_b \geqslant 2$ buyers. Every seller has $1 \leqslant h_i \leqslant n_b, i \in \{1, \ldots, n_s\}$ units of homogeneous indivisible products which he values as $c \geqslant 0$. Every buyer aims to buy one unit of product, the common buyer valuation is $u \geqslant c$.

The market operates in two distinct stages. At the first stage, all sellers simultaneously and independently announce their prices, which do not change. Let p_j be the price posted by seller j for every unit of her products. A particular seller is not allowed to discriminate among buyers and propose menu of prices. So she is obligated to sell all her products at p_j. This restriction is natural in the setting of a marketplace, while for the dual setting of a labor market and hiring it can be more reasonable to introduce flexible wages contracts, since the anonymity of workers is a more questionable issue there.

At the second stage, buyers observe the whole list of prices from stage one. Every buyer independently of others chooses one seller and submits to her a request for the purchase of a unit of the product, at the given price. The strategic part of interaction stops at this moment. Then the "blind" matching mechanics works. If seller j obtains not more than h_j requests, then she sells to every requester a unit of product at price p_j. If seller j obtains more than h_j requests, then she is not able to serve all demand she met and needs to choose exactly h_j buyers from the set of requesters. Since all buyers are homogeneous, the choice is just equiprobable. This also can be interpreted as the seller serving first h_j buyers in a queue while their time rank is determined not strategically but randomly. Here is a potential place for friction among buyers.

All market participants behave rationally and strategically. Every buyer maximizes her expected gain $u - p$ from a successful purchase, accounting for the probability that it occurs. Every seller maximizes the expected sum of markups $p - c$ from all her successful sales.

2.2 Representation of utilities

Denote by $\mathcal{I} = \{1, 2, \ldots, n_b\}$ the set of buyers and by $\mathcal{J} = \{1, 2, \ldots, n_s\}$ the set of sellers. Let on the first stage sellers propose a price vector $\mathbf{p} = (p_1, \ldots, p_{n_s})$ and on the second stage buyers submits requests for the purchase with probabilities expressed by the matrix $\mathbf{\Gamma}$:

$$\underset{(n_b \times n_s)}{\mathbf{\Gamma}} = \begin{pmatrix} \gamma_{11} & \cdots & \gamma_{1n_s} \\ \vdots & \ddots & \vdots \\ \gamma_{n_b 1} & \cdots & \gamma_{n_b n_s} \end{pmatrix}$$

We denote an arbitrary row i of matrix $\mathbf{\Gamma}$ as $\gamma_i^b = (\gamma_{i1}, \ldots, \gamma_{in_s})$ and an arbitrary column j as $\gamma_j^s = (\gamma_{j1}, \ldots, \gamma_{jn_b})$.

We want to find a symmetric equilibrium $(\mathbf{p}, \mathbf{\Gamma})$ such that

- $\gamma_{i_1}^b = \gamma_{i_2}^b \ \forall i_1, i_2 \in \mathcal{I}$, that is for each particular seller it is true that all buyers submit to her a request with equal probabilities (because buyers are homogeneous), and

- $h_{j_1} = h_{j_2} \iff (p_{j_1} = p_{j_2}) \wedge \left(\gamma_{j_1}^s = \gamma_{j_2}^s\right) \ \forall j_1, j_2 \in \mathcal{J}$, that is iff two sellers have equal capacities, then in symmetric equilibrium they set equal prices and all buyers visits them with equal probabilities.

Denote by $\tilde{\gamma}^s = \left(\tilde{\gamma}_1^s, \ldots, \tilde{\gamma}_{n_s}^s\right)$ the vector of probabilities with which buyers submit requests to sellers in symmetric equilibrium that is the unique column values of matrix $\mathbf{\Gamma}$.

The key bricks in all equilibrium derivations are buyers and sellers expected utility and profit functions, respectively. First, let consider buyer $i \in \mathcal{I}$ and seller $j \in \mathcal{J}$ in the equilibrium. The expected utility of this buyer from submitting to seller j conditional on other buyer sending requests to this seller with probability $\tilde{\gamma}_j^s$ is equal to

$$\mathbb{E}\left[u_i(j)\right] = (u - p_j) \times \textit{Probability of being served by seller } j \ (\tilde{\gamma}_j^s; n_b, h_j). \tag{1}$$

Because at the second stage buyers make their choices independently, we can write the expected profit of seller j from posting price p_j in equilibrium as

$$\begin{aligned}\mathbb{E}\left[\pi_j(p_j, p_{-j})\right] = (p_j - c) \times n_b \times \tilde{\gamma}_j^s(p_j, p_{-j}) \times \\ \times \textit{Probability of being served by seller } j \ (\tilde{\gamma}_j^s; n_b, h_j).\end{aligned} \tag{2}$$

Note that the expected payoff of seller j depends on the other players strategies through $\tilde{\gamma}_j^s(p_j, p_{-j})$ term, which is the result of the equilibrium in the buyers' subgame (equilibrium $\tilde{\gamma}_j^s$).

When $h_j = 1$ it is easy to express $\mathbb{E}\left[\pi_j\right]$ directly as

$$\begin{aligned}&\mathbb{E}\left[\pi_j(p_j, p_{-j})\right] = \\ &\quad = (p_j - c) \times \textit{Probability that at least one buyer out of } n_b \textit{ come to seller } j = \\ &\qquad = (p_j - c) \times \left(1 - \left(1 - \tilde{\gamma}_j^s\right)^{n_b}\right).\end{aligned}$$

From the connection between the general form of buyers expected utility (1) and sellers expected profit (2) functions we can easily derive that

$$\textit{Probability of being served by seller } j \ (\tilde{\gamma}_j^s; n_b, h_j)\Big|_{h_j=1} = \frac{\left(1 - \left(1 - \tilde{\gamma}_j^s\right)^{n_b}\right)}{n_b \tilde{\gamma}_j^s}. \tag{3}$$

These functions and the connection between them are well-known and widely used in different proofs and derivations (a good example is (Wright et al. 2021)).

Things become much more complicated when we consider the *Probability of being served* function with an arbitrary parameter $1 \leqslant h_j \leqslant n_b$. Firstly, let's rewrite this function in an explicit form:

$$
\begin{aligned}
&\textit{Probability of being served by seller } j\ (\tilde{\gamma}_j^s; n_b, h_j) = \\
&= \sum_{k=0}^{n_b-1} (\textit{The probability that exactly } k \textit{ buyers out of the rest } n_b - 1 \textit{ come to seller } j) \times \\
&\qquad \times (\textit{The probability that buyer will be served conditional on} \\
&\qquad\qquad \textit{exactly } k \textit{ other buyers come to seller } j) = \\
&\qquad = \sum_{k=0}^{n_b-1} \left(C_{n_b-1}^k \times \left(\tilde{\gamma}_j^s\right)^k \times \left(1 - \tilde{\gamma}_j^s\right)^{n_b-1-k} \right) \times \min\left(\frac{h_j}{k+1}, 1 \right). \quad (4)
\end{aligned}
$$

It is problematic to get any meaningful analytical results using *The probability of being served* function in that form, that is why previous researches only covers the case where $h_j = 1\ \forall j \in \mathcal{J}$.

We solve that problem and find a way to transform *The probability of being served* function and express it in the exact form without explicit summations such that it can be used in the further analytical derivations. The first key result of this paper is presented in lemma 1.

Lemma 1 (Analytical expression for *The probability of being served* function)**.** *The probability of being served by seller j at symmetric equilibrium as a function of $\tilde{\gamma}_j^s$ and parameters n_b, h_j is calculated as*

$$
\begin{aligned}
&\textit{the probability of being served by seller } j\ (\tilde{\gamma}_j^s; n_b, h_j) = \\
&\qquad = \mathbb{P}(X_1 \leqslant h_j - 1) + \frac{h_j}{n_b \times \tilde{\gamma}_j^s} \times \mathbb{P}(X_2 > h_j),
\end{aligned}
$$

where X_1 and X_2 are independent random variables equal to the number of successes in the $n_b - 1$ and n_b, respectively, Bernoulli experiments with the probability of success $\tilde{\gamma}_j^s$.

Since after transformations the function under consideration is not defined at $\tilde{\gamma}_j^s = 0$, we put The probability of being served by seller j $(0; n_b, h_j) = 1$ explicitly.

Proof of lemma 1 is in Appendix. Here and further we will denote *Probability of being served by seller j* $(\tilde{\gamma}_j^s; n_b, h_j)$ function as $\zeta(\tilde{\gamma}_j^s; n_b, h_j)$ just for ease of notation. Now we can state some useful properties of $\zeta(\tilde{\gamma}_j^s; n_b, h_j)$ function.

Corollary 1 (Properties of $\zeta(\tilde{\gamma}_j^s; n_b, h_j)$ function)**.** *The following relation holds*

$$\frac{\partial \zeta(\tilde{\gamma}_j^s; n_b, h_j)}{\partial \tilde{\gamma}_j^s} = \begin{cases} \begin{cases} \frac{1-n_b}{2}, \text{ if } h_j = 1, \\ 0, \text{ if } 1 < h_j \leqslant n_b, \end{cases} & \text{if } \tilde{\gamma}_j^s = 0; \\ -\frac{h_j}{n_b \times \tilde{\gamma}_j^s} \times \mathbb{P}(X_2 > h_j), \text{ if } 0 < \tilde{\gamma}_j^s \leqslant 1. \end{cases}$$

$\zeta(\tilde{\gamma}_j^s; n_b, h_j)$ *is a monotonically decreasing function with continuous derivative.*

Corollary 2 (Properties of $\tau(\tilde{\gamma}_j^s; n_b, h_j)$ function)**.** *Let consider the function $\tau(\tilde{\gamma}_j^s; n_b, h_j) = n_b \times \tilde{\gamma}_j^s \times \zeta(\tilde{\gamma}_j^s; n_b, h_j)$ that express the expected number of seller deals when each buyer can come to that seller independently with equal probability. This function is a key part of the expression for the expected profit of a seller. The following relation holds*

$$\frac{\partial \tau(\tilde{\gamma}_j^s; n_b, h_j)}{\partial \tilde{\gamma}_j^s} = n_b \times \mathbb{P}(X_1 \leq h_j - 1), 0 \leqslant \tilde{\gamma}_j^s \leqslant 1.$$

$\tau(\tilde{\gamma}_j^s; n_b, h_j)$ *is a monotonically increasing and concave function with a continuous derivative.*

Note that function $\zeta(\tilde{\gamma}_j^s; n_b, h_j)$ with its explicit expression and properties can be used not only in the framework that we consider in our paper, but also in many other frameworks under the Bayesian equilibrium setting.

Finally, we can use our first key result for practical purposes. In the next section, we apply it to the derivation of analytical expressions for the equilibrium characteristics in the symmetric directed search model where all sellers have equal capacities h.

3 The symmetric equilibrium

3.1 The analytical solution

Let us focus on the symmetric framework where each firm has exactly $1 \leqslant h \leqslant n_b$ units to sell. We aim to find a symmetric equilibrium $(\mathbf{p}, \mathbf{\Gamma})$ such that $p = p_1 = \ldots = p_{n_s}$, $\gamma_{ij} = \frac{1}{n_s}$ $\forall i \in \mathcal{I}, j \in \mathcal{J}$. To ensure there are no incentives to deviate and $(\mathbf{p}, \mathbf{\Gamma})$ is a subgame perfect Nash equilibrium, the following conditions must be met (Wright et al. 2021):

- $p = \text{argmax}_{p^d} \mathbb{E}[\pi^d(p^d, p^{nd})]$, where p^d is the price of deviating seller and $p^{nd} = p$ are prices of all other non-deviating sellers;
- $\tilde{\gamma}^{s,d}(p^d, p^{nd})$ constitutes an equilibrium in the buyers subgame for any p^d, p^{nd};
- on the equilibrium path $\gamma_{ij} = \frac{1}{n_s}$, while after a deviation buyers submit requests for the purchase to a deviating seller with probability $\tilde{\gamma}^{s,d}$ and to all other sellers with probability $\tilde{\gamma}^{s,nd} = \frac{1-\tilde{\gamma}^{s,d}}{n_s-1}$.

We apply a similar technique to (Wright et al. 2021), by balancing the probability to be served for a buyer and the probability to make a deal for the seller. By the reasons from (Kim and Camera 2014), it is worth focusing on the symmetric price equilibrium. As an immediate corollary, in equilibrium, in buyers' subgame every seller will be chosen with equal probability.

Theorem 1. *In a symmetric subgame perfect equilibrium, all sellers post equal prices*

$$p^{ds} = \frac{u \cdot \mathbb{P}(X_2 > h) + c \cdot \frac{n_b}{n_s} \cdot \frac{1}{h} \cdot \left(1 - \frac{1}{n_s}\right) \cdot \mathbb{P}(X_1 \leqslant h-1)}{\mathbb{P}(X_2 > h) + \frac{n_b}{n_s} \cdot \frac{1}{h} \cdot \left(1 - \frac{1}{n_s}\right) \cdot \mathbb{P}(X_1 \leqslant h-1)}, \tag{5}$$

where X_1 and X_2 are independent random variables equal to the number of successes in the $n_b - 1$ and n_b, respectively, Bernoulli experiments with the probability of success $1/n_s$. This equilibrium is a unique non-coordinated one.

Proof. Suppose that some seller deviate from initial price vector $\mathbf{p}$ and consider the symmetric equilibrium with $\tilde{\gamma}^{s,d}(p^d, p^{nd})$. The expected profit of the deviant seller is equal to

$$\mathbb{E}\left[\pi^d\left(p^d, p^{nd}\right)\right] = \left(p^d - c\right) \times n_b \times \tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right) \times \zeta\left(\tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right); n_b, h\right),$$

where $\zeta(\cdot)$ is the probability of being served, as mentioned before.

The expected utility of a buyer from visiting a deviant seller in equilibrium is equal to

$$\mathbb{E}\left[u^d\left(p^d, p^{nd}\right)\right] = \left(u - p^d\right) \times \zeta\left(\tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right); n_b, h\right)$$

At the same time, the expected utility of a buyer from visiting a non-deviant seller in equilibrium is equal to

$$\mathbb{E}\left[u^{nd}\left(p^d, p^{nd}\right)\right] = \left(u - p^{nd}\right) \times \zeta\left(\tilde{\gamma}^{s,nd}\left(p^d, p^{nd}\right); n_b, h\right)$$

The deviant seller maximizes her profit after deviation from the symmetric price vector

$$\mathbb{E}\left[\pi^d\left(p^d, p^{nd}\right)\right] = \left(p^d - c\right) \times n_b \times \tilde{\gamma}^{s,d} \times \zeta(\tilde{\gamma}^{s,d}; n_b, h) = \left(p^d - c\right) \times \tau(\tilde{\gamma}^{s,d}; n_b, h) \to \max_{p^d}$$

FOC implies (using properties from corollary 1)

$$\frac{\partial\, \mathbb{E}\left[\pi^d\left(p^d, p^{nd}\right)\right]}{\partial p^d} = \left(p^d - c\right) \times n_b \times \mathbb{P}(X_1 \leqslant h-1) \times \frac{\partial \tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right)}{\partial p^d} + n_b \times \tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right) \times \zeta\left(\tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right); n_b, h\right) = 0. \tag{6}$$

In a symmetric equilibrium in the buyers' subgame, buyers should be indifferent between purchasing from deviant and non-deviant sellers.

$$\mathbb{E}\left[u^d\left(p^d, p^{nd}\right)\right] = \mathbb{E}\left[u^{nd}\left(p^d, p^{nd}\right)\right] \iff \left(u - p^d\right) \times \zeta\left(\tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right)\right) - \left(u - p^{nd}\right) \times \zeta\left(\tilde{\gamma}^{s,nd}\left(p^d, p^{nd}\right)\right) = 0.$$

From this equation, we take an implicit derivative of $\tilde{\gamma}^{s,d}$ with respect to p^d:

$$- \zeta\left(\tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right)\right) + \left(u - p^d\right) \times \frac{d\zeta\left(\tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right)\right)}{d\tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right)} \times \frac{\partial \tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right)}{\partial p^d} - \\ - \left(u - p^{nd}\right) \times \frac{d\zeta\left(\tilde{\gamma}^{s,nd}\left(p^d, p^{nd}\right)\right)}{d\tilde{\gamma}^{s,nd}\left(p^d, p^{nd}\right)} \times \left(-\frac{1}{n_s - 1}\right) \times \frac{\partial \tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right)}{\partial p^d} = 0$$

Representing $\frac{\partial \tilde{\gamma}^{s,d}\left(p^d, p^{nd}\right)}{\partial p^d}$ explicitly, we get

$$\frac{\partial \gamma^{s,d}\left(p^d, p^{nd}\right)}{\partial p^d} = \frac{\zeta\left(\gamma^{s,d}\left(p^d, p^{nd}\right)\right)}{\left(u - p^d\right) \times \frac{d\zeta\left(\gamma^{s,d}\left(p^d, p^{nd}\right)\right)}{d\gamma^{s,d}\left(p^d, p^{nd}\right)} + \left(u - p^{nd}\right) \times \frac{d\zeta\left(\gamma^{s,nd}\left(p^d, p^{nd}\right)\right)}{d\gamma^{s,nd}\left(p^d, p^{nd}\right)} \times \frac{1}{n_s - 1}}.$$

In a symmetric equilibrium, the maximum is reached at the symmetric price vector

$$\left.\frac{\partial \gamma^{s,d}\left(p^d, p^{nd}\right)}{\partial p^d}\right|_{p^d = p^{nd} = p} = \frac{\zeta\left(\frac{1}{n_s}\right)}{(u - p) \times \zeta'\left(\frac{1}{n_s}\right) \times \frac{n_s}{n_s - 1}}. \tag{7}$$

We substitute 7 into 6 and express p considering a symmetric equilibrium ($p = p_1 = \dots = p_{ns}$ and $\gamma_{ij} = \frac{1}{n_s}$). That gives exactly 5.

Finally, (Galenianos and Kircher 2012) prove and (Wright et al. 2021) highlighted that there are no asymmetric equilibrium where buyers use mixed and sellers use pure strategies for $h = 1$, but the arguments could be extended to an arbitrary h.

$\square$

Remark 1. The multiplier $\left(1 - \frac{1}{n_s}\right)$ (marked in blue) in (5) indicates a term reflecting the strategic effect among buyers. It disappears when we extend the formula from (Montgomery 1991) for h products per seller using the market utility approach.

Remark 2. It is easy to see that, when $h \geqslant n_b$, $\mathbb{P}(X_2 > h) = 0$ and $\mathbb{P}(X_1 \leqslant h - 1) = 1$, which yields $p^{ds} = c$. The intuition behind the result is similar to Bertrand's paradox.

After we derive the equilibrium in the symmetric setting, it make sense to analyze its characteristics.

Corollary 3. *The expected utility of a buyer in the equilibrium is equal to*

$$\mathbb{E}\left[u^{ds}\right] = (u - c) \times \left(\left(1 - \frac{1}{n_s}\right) \times \mathbb{P}\left(X_1 \leqslant h - 1\right)\right) \times \\ \times \frac{\left(n_b \times \frac{1}{h} \times \frac{1}{n_s} \times \mathbb{P}\left(X_1 \leqslant h - 1\right) + \mathbb{P}\left(X_2 > h\right)\right)}{\left(n_b \times \frac{1}{h} \times \frac{1}{n_s} \times \left(1 - \frac{1}{n_s}\right) \times \mathbb{P}\left(X_1 \leqslant h - 1\right) + \mathbb{P}\left(X_2 > h\right)\right)}.$$

This expression can be limited from above and below as:

$$\mathbb{E}\left[u^{ds}\right] \in \left((u - c) \times \left(1 - \frac{1}{n_s}\right) \times \mathbb{P}\left(X_1 \leqslant h - 1\right), (u - c) \times \mathbb{P}\left(X_1 \leqslant h - 1\right)\right). \tag{8}$$

The expected profit of each seller in the equilibrium is equal to

$$\mathbb{E}\left[\pi^{ds}\right] = (u-c) \times h \times \mathbb{P}\left(X_2 > h\right) \times \\ \times \frac{\left(n_b \times \frac{1}{h} \times \frac{1}{n_s} \times \mathbb{P}\left(X_1 \leqslant h-1\right) + \mathbb{P}\left(X_2 > h\right)\right)}{\left(n_b \times \frac{1}{h} \times \frac{1}{n_s} \times \left(1-\frac{1}{n_s}\right) \times \mathbb{P}\left(X_1 \leqslant h-1\right) + \mathbb{P}\left(X_2 > h\right)\right)}$$

and is limited from above and below as: [1]

$$\mathbb{E}\left[\pi^{ds}\right] \in \left((u-c) \times h \times \mathbb{P}\left(X_2 > h\right), (u-c) \times h \times \mathbb{P}\left(X_2 > h\right) \times \left(1 + \frac{1}{n_s - 1}\right)\right). \tag{9}$$

The expected total market surplus in the equilibrium is equal to

$$n_b \times \mathbb{E}\left[u^{ds}\right] + n_s \times \mathbb{E}\left[\pi^{ds}\right] = (u-c) \times n_b \times \zeta\left(\frac{1}{n_s}; n_b, h\right).$$

The expected number of deals in the equilibrium is equal to

$$\mathbb{E}\left[\textit{Number of deals}\right] = n_b \times \zeta\left(\frac{1}{n_s}; n_b, h\right).$$

Now we can use theorem 1 and corollary 3 to analyze the comparative statics and make some observations about equilibrium tendencies.

3.2 Comparative statics

Valuations. It is clear, that equilibrium price linearly increases with the growth of both buyer and seller valuations. The growth of seller value c shrinks the bargaining space and negatively affects the utilities of participants and their surpluses. On the other hand, the growth of buyer value u expands the market and, even though the price increases, the utilities of all market participants also increase. Neither values influence the expected number of deals and market efficiency, since they depend only on the sizes of market sides n_b and n_s and the capacities h.

The number of buyers. With the growth of buyer side n_b, they compete more intensely for products, and the sellers' market power increases, which means that the individual seller's utility and total seller surplus increase. The utility of a particular buyer decreases, nevertheless, the total surplus of buyers is a bell-shape function: when the number of buyers is low, a small growth in n_b does not lead to rapid growth of prices and, in sum, buyers win. But with the further growth of n_b, the price rise and the growing friction hit all buyers, and though a new buyer gets some positive utility, the total effect is negative (Fig. 1).

[1] We can see that as $n_s \to \infty$, in 8 and 9, the upper bound tends to the lower one and the asymptotic expression is quite simple and easy to interpret for both expressions of profits.

The number of sellers. Increasing the number of sellers intensifies seller competition, which leads to a price drop, and a rise in buyer utility and product surplus. The situation with sellers is similar to the previous case effect for buyers, i.e. the influence on their surplus is mixed. If the number of buyers is large enough in comparison with the total number of products in the market (from all sellers), the growth of the number of sellers always decreases their individual utility. However, the total seller surplus is bell-shaped. A little growth of a small seller market enforces the friction which does not allow prices to decrease too much, but with the further growth of the seller side the price decrease becomes so great that this does not compensate for the increased number of sellers. When the number of buyers is not large in comparison with h, then even a small growth of n_b leads to tougher competition among sellers for buyers and decreases the total seller surplus (Fig. 1).

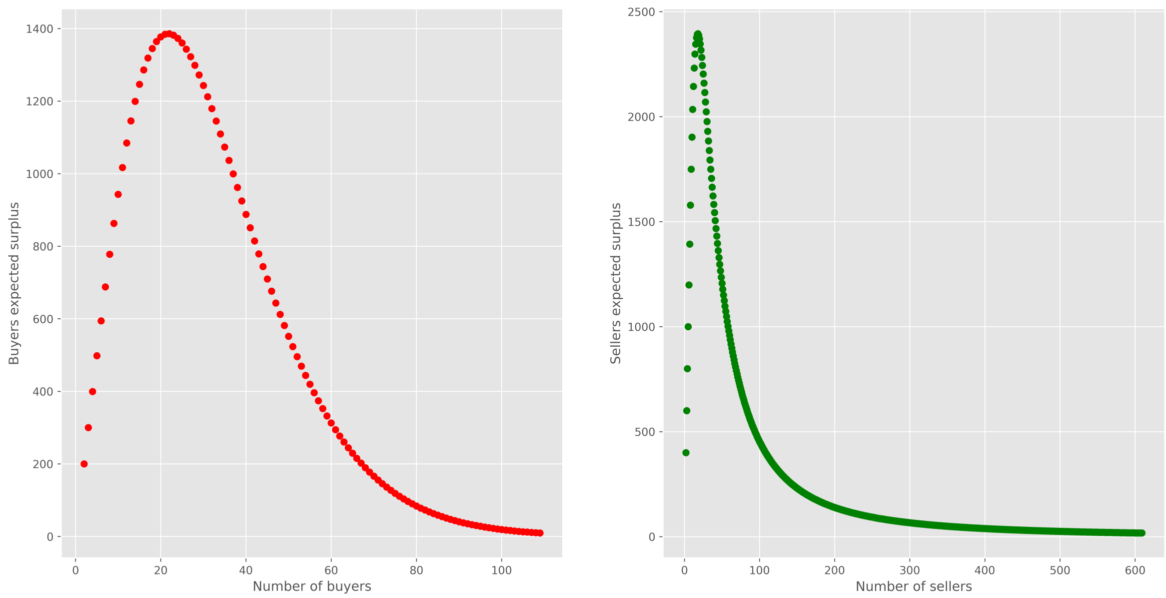

Figure 1: The total buyer surplus as a function of n_b, under $n_s = 10$, $h = 3$, $c = 100$, $u = 200$. The total seller surplus as a function of n_s, under $n_b = 60$, $h = 2$, $c = 100$, $u = 200$.

The number of products. The impact of the growth of number of products per seller is similar to, but not the same as, the impact of the n_s. For buyers, the tendency is straightforward and positively affect their utility. For sellers, two opposite effects take place: *products* compete more intensely and the returns from one unit decrease, but since every seller has more products, this *may be profitable*. The second effect prevails when the growth of h is small, while further growth brings down the price and the profit. So, the market with major sellers is less profitable for every seller than the market with minor sellers, and there exists an optimal firm size, i.e. the number of products for every seller (Fig. 2).

An interesting observation concerns the efficiency of market matching, which is the share of expected deals from the maximal possible number of deals, i.e. $\min\{hn_s, n_b\}$.

With a small growth in h, this share decreases, which means that the efficiency of the market matching mechanism fails. This is because it is more difficult for a seller to sell *all* her products when their number increases, even if it is relatively low in comparison with the demand. On the other hand, when the growth of h is significant, then selling all products is not possible, but we may care now about the probability of serving all buyers. Naturally, this probability increases, so the market becomes more efficient (Fig. 2).

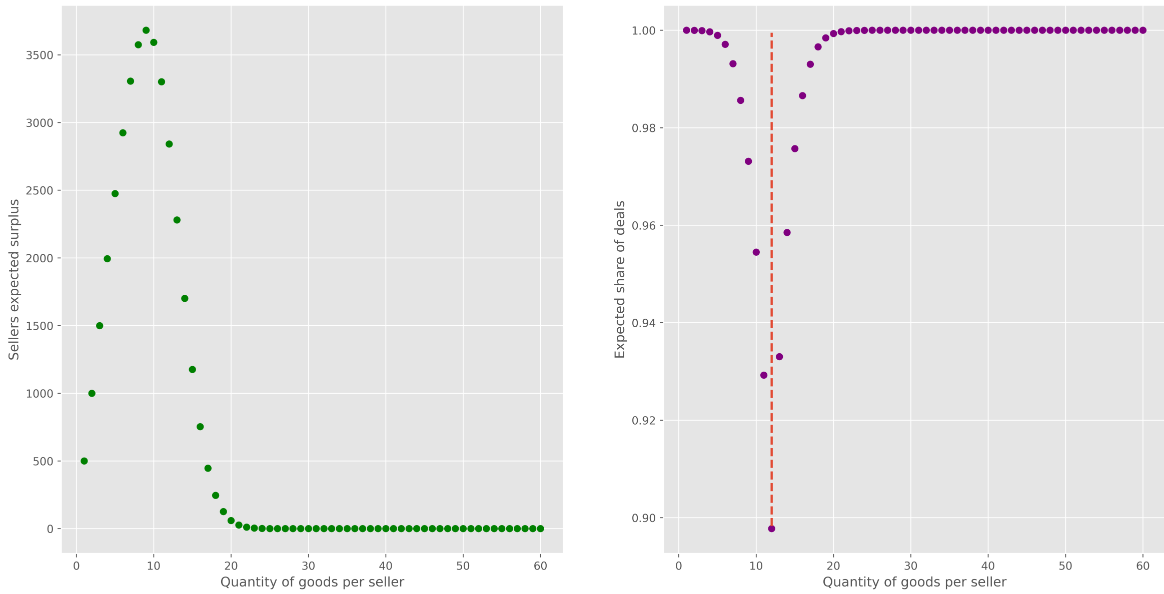

Figure 2: The total seller surplus and expected share of possible deals as a function of h, under $n_b = 60$, $n_s = 5$, $c = 100$, $u = 200$.

3.3 The market concentration

The same total number of product can be sold by a different number of sellers, such that markets with a smaller number of larger firms are referred to as more concentrated than markets with a large number of small firms. In the second case, sellers have less market power, however, there are high risks for buyers to request to the same seller and to lose from friction. In the first case, the competition among sellers is lower, but the friction is also lower since a large seller is able to serve many buyers. Thus, it is not clear in advance that a more concentrated, or monopolized, market is worse for consumers. Formula (5) allows a comparison of numerical equilibrium parameters of markets with the same number of products and buyers, but varying distributions of products among sellers. This clarifies the influence of market concentration and explains the consequences of monopolization for buyers and sellers.

We split a fixed number of products into all possible combinations of two integer multipliers, and assume that the first multiplier means the number of products per seller, while the second means the number of sellers. Increasing the number of products per seller means increasing the market concentration with monopolization as the limit. Numerical analysis demonstrates that equilibrium parameters depend on the relation between n_b and the total number of products (H). Several typical cases arise, which smoothly transform to each other.

- $n_b >> H$: the number of buyers exceeds the total number of products significantly. Here there is a large deficit of products and sellers do not compete tough for buyers under any distribution. With the growth of market concentration, the equilibrium price increases, the utility of every seller increases, and buyer's utility decreases. The friction diminishes not so great because of the total lack of products of the market, even if they are distributed by only two sellers or even monopolized. However, the drop in friction leads to the growth of the total number of deals. (for more details see Fig. 5 in Appendix).

- $n_b >\approx H$: the number of buyers slightly exceeds the total number of products. With the growth of market concentration, the equilibrium price increases, the utility of every seller increases, and the total number of deals increases. For the buyers, the growth of seller concentration has a mixed effect. If we start with a large market with a lot of sellers with a unit capacity, then a small growth of concentration is profitable for buyers since the gain from lowering friction dominates the small price growth; further, the price increase is harmful for buyers. (Fig. 6).

- $n_b << H$: the number of buyers is significantly lower than the total number of products. Then there is a deficit of buyers, and it increases with the growth of market concentration, since it is easier for buyers to choose the cheapest seller and to be served there. This intensifies competition and the equilibrium price decreases. Buyer utility and the total number of deals increase; for sellers, a small growth in market concentration is profitable, since the increased demand more than compensates for the drop in price. (Fig. 7).

- $n_b <\approx H$: the number of buyers is slightly lower than the total number of products. Then the effects are mixed and we can observe the transition from the case (2) to the case (3). With the growth in difference between n_b and H, the price switches from increasing ($H = 48$, $n_b = 47$) to decreasing ($H = 48$, $n_b = 20$) through bell-shaped form ($H = 48$, $n_b = 43$). Similar transformations hold for buyer and seller expected profits and surpluses (Fig. 8).

In all cases, the growth of market concentration leads to the growth of market efficiency, such that a greater number of products will be sold and a greater number of buyers will be served. However, depending on the relation between the number of buyers and products, the growth of market concentration is not necessarily bad for buyers and good for sellers.

4 Heterogeneous sellers

After the extensive discussion of the symmetric framework, it is time to analyze the general case with heterogeneous sellers with different capacities. There are two important questions related to the current analysis: the first is about the existence and uniqueness of the equilibrium, and the second is about effective ways to find that equilibrium numerically.

In (Kim and Camera 2014), the authors demonstrate that in a certain type of directed search model there is a unique uncoordinated equilibrium. In fact, when $h = 1$, our setting is exactly that particular case. However, an arbitrary h does not change the essential fundamental properties of a key functions: $\mu(\cdot)$ and $\tau(\cdot)$. That is why it is possible to replicate all the steps from their proof and get the same results for our framework.

The answer to the question about effective ways of finding an equilibrium is not straightforward. In (Camera and Kim 2013), the authors propose the new recursive approach to find an equilibrium in the buyers' subgame for an arbitrary price vector. The drawback of their approach is the necessity to recursively solve J systems of equations of size $1, \ldots, J$. We significantly improve this approach for our setting, allowing which system we need to solve to be defined based on simple conditions without having to solve all other systems.

4.1 The effective way to find an equilibrium in the buyers' subgame

For this subsection, suppose that announced prices for the product are ordered as

$$c \leqslant p_1 \leqslant p_2 \leqslant \cdots \leqslant p_{n_s} \leqslant u$$

where $p_1 < u$.

The necessary condition for the buyers' symmetric mixed equilibrium $\tilde{\gamma}$ to exist is the indifference for all buyers to which seller they should send a request for purchasing a product. In other words, the following system of equations with corresponding boundaries must have a solution:

$$\begin{cases} (u - p_i) \times \zeta\left(\tilde{\gamma}_i; h_i\right) = (u - p_{i+1}) \times \zeta\left(\tilde{\gamma}_{i+1}, h_{i+1}\right), i \in \{1, \ldots, n_s - 1\} \\ \sum_{i \in \mathcal{J}} \tilde{\gamma}_i = 1 \\ \tilde{\gamma}_i \in (0, 1), \ \forall i \in \mathcal{J} \end{cases} \tag{10}$$

We can rewrite this system as:

$$\begin{cases} \zeta\left(\tilde{\gamma}_i; h_i\right) = \frac{u-p_{n_s}}{u-p_i} \times \zeta\left(\tilde{\gamma}_{n_s}, h_{n_s}\right), i \in \{1, \ldots, n_s - 1\} \\ \sum_{i \in \mathcal{J}} \tilde{\gamma}_i = 1 \\ \tilde{\gamma}_i \in (0,1), \forall i \in \mathcal{J}\,. \end{cases} \tag{11}$$

We show that system (11) has either exactly one solution or no solutions at all. We also derive the conditions under which such an equilibrium exists. Let us rewrite system (11) in the following way under the assumption that all inverse functions are correctly specified in corresponding points:

$$\begin{cases} \tilde{\gamma}_i = \zeta^{-1}\left(\frac{u-p_{n_s}}{u-p_i} \times \zeta\left(\tilde{\gamma}_{n_s}, h_{n_s}\right), h_i\right), i \in \{1, \ldots, n_s - 1\} \\ \sum_{i \in \mathcal{J}} \tilde{\gamma}_i = 1 \\ \tilde{\gamma}_i \in (0,1), \forall i \in \mathcal{J}\,. \end{cases} \tag{12}$$

Remember that function $\zeta(x; n_b, h)$ monotonically decreases and takes values from $\frac{h}{n_b}$ to 1 at points 1 and 0, respectively. Consequently, the inverse function $\zeta^{-1}(x; h)$ under restrictions in the third line of system (12) could take values from $\frac{h}{n_b}$ to 1 without including borders as its argument x.

To compute the inverse function $\zeta^{-1}\left(\frac{u-p_{n_s}}{u-p_i} \times \zeta\left(\tilde{\gamma}_{n_s}, h_{n_s}\right), h_i\right)$ on a nonempty set with respect to varying $\tilde{\gamma}_{n_s}$, it is necessary and sufficient that $\frac{u-p_{n_s}}{u-p_i} > \frac{h_i}{n_b} \iff h_i(u - p_i) < n_b(u - p_{n_s})$. Thus, to compute the inverse functions for all equations in system (11) on a nonempty set with respect to varying γ_{n_s}, it is necessary and sufficient that:

$$h_i(u - p_i) < n_b(u - p_{n_s}), \forall i \in \{1, \ldots, n_s\}\,. \tag{13}$$

Under conditions (13), we can start from point 0 and increase $\tilde{\gamma}_{n_s}$, decreasing at the same time $\frac{u-p_{n_s}}{u-p_i} \times \zeta\left(\tilde{\gamma}_{n_s}, h_{n_s}\right)$ until $\tilde{\gamma}_{n_s}$ increases to 1, or $\frac{u-p_{n_s}}{u-p_i} \times \zeta\left(\tilde{\gamma}_{n_s}, h_{n_s}\right)$ is equal to $\frac{h_i}{n_b}$. After further increasing $\tilde{\gamma}_{n_s}$, the inverse function $\zeta^{-1}\left(\frac{u-p_{n_s}}{u-p_i} \times \zeta\left(\tilde{\gamma}_{n_s}, h_{n_s}\right), h_i\right)$ stops accepting an argument from the set where its correctly defined. Therefore, the upper bound for $\tilde{\gamma}_{n_s}$ for each equation in system (11), under which all corresponding inverse functions are correctly defined, equals to

$$ub_i = \begin{cases} 1, \text{if } \frac{u-p_{n_s}}{u-p_i} \times \frac{h_{n_s}}{n_b} \geqslant \frac{h_i}{n_b} \\ x_i^* : \frac{u-p_{n_s}}{u-p_i} \times \zeta\left(x_i^*, h_{n_s}\right) = \frac{h_i}{n_b}, \text{otherwise} \end{cases} \tag{14}$$

Using the monotonicity of $\zeta(\cdot)$, we can define boundaries for correctly varying $\tilde{\gamma}_{n_s}$ in the following way

$$\tilde{\gamma}_{n_s} \in \begin{cases} \varnothing, \text{if } h_{\hat{i}}(u-p_{\hat{i}}) \geqslant n_b(u-p_{n_s}) \\ (0,x^*) : \frac{u-p_{n_s}}{u-p_{\hat{i}}} \times \zeta\left(x^*, h_{n_s}\right) = \frac{h_{\hat{i}}}{n_b}, \text{if } \begin{cases} h_{\hat{i}}(u-p_{\hat{i}}) < n_b(u-p_{n_s}) \\ \frac{u-p_{n_s}}{u-p_{\hat{i}}} \times \frac{h_{n_s}}{n_b} < \frac{h_{\hat{i}}}{n_b} \end{cases} \\ (0,1), \text{if } \begin{cases} h_{\hat{i}}(u-p_{\hat{i}}) < n_b(u-p_{n_s}) \\ \frac{u-p_{n_s}}{u-p_{\hat{i}}} \times \frac{h_{n_s}}{n_b} \geqslant \frac{h_{\hat{i}}}{n_b} \end{cases} \end{cases} \tag{15}$$

where $\hat{\imath} : h_{\hat{i}}(u-p_{\hat{i}}) \geqslant h_k(u-p_k), \forall k \in \mathcal{J}$.

Put expression for $\tilde{\gamma}_i$ in the link equation in system (12) and consider the last expression as a function of $\tilde{\gamma}_{n_s}$.

$$F(\tilde{\gamma}_{n_s}) = \sum_{i=1}^{n_s-1} \zeta^{-1}\left(\frac{u-p_{n_s}}{u-p_i} \times \zeta\left(\tilde{\gamma}_{n_s}, h_{n_s}\right), h_i\right) + \tilde{\gamma}_{n_s} - 1. \tag{16}$$

Function $F(\tilde{\gamma}_{n_s})$ monotonically increases on the interval defined by system (15). Obviously, in the upper boundary of that interval $F(\tilde{\gamma}_{n_s})$ takes positive value. Consequently, to provide the existence and uniqueness of the solution of equation $F(\tilde{\gamma}_{n_s}) = 0$, it is necessary and sufficient to demand one more condition: $F(0) < 0$. In addition to system (15), necessary and sufficient conditions for the existence of the symmetric mixed non-coordinated equilibrium where all sellers are active could be represented as:

$$\begin{cases} h_{\hat{i}}(u-p_{\hat{i}}) < n_b(u-p_{n_s}) \\ \sum_{i=1}^{n_s-1} \zeta^{-1}\left(\frac{u-p_{n_s}}{u-p_i}, h_i\right) - 1 < 0 \end{cases} \tag{17}$$

where $\hat{\imath} : h_{\hat{i}}(u-p_{\hat{i}}) \geqslant h_k(u-p_k), \forall k \in \mathcal{J}$.

Finally, we need to note that if the conditions above are not satisfied, the mixed non-coordinated equilibrium where all sellers are active does not exist. Consequently, some firms with highest prices stay out of the market. Note that if several firms have equal prices then either all of them are active in equilibrium, or all of them are out of the market. It is also true that if a firm with a higher price is active in the market, then the all firms with lower prices must also be active. Thus, we can iteratively check conditions (17), excluding from consideration the firm with the highest prices until those conditions are satisfied. After that we can find a solution of equation (16) considering only those firms that must be active at the equilibrium according to the conditions we checked before.

4.2 The way to find an equilibrium in the sellers' game

As mentioned above, there exists a unique uncoordinated symmetric equilibrium in our general model. To find it numerically we need to solve the following system of equations:

$$
\begin{cases}
(u-p_i)\times\zeta\left(\tilde{\gamma}_i^s;n_b,h_i\right)=(u-p_{i+1})\times\zeta\left(\tilde{\gamma}_{i+1}^s;n_b,h_{i+1}\right),\ \forall i=\{1,\dots,n_s-1\}\\
\sum_{i=1}^{n_s}\tilde{\gamma}_i^s=1\\
\frac{\partial\pi_j(p_j,\mathbf{p}_{-j})}{\partial p_j}=0,\ \forall j\in\mathcal{J}\,.
\end{cases}
\tag{18}
$$

The first two lines of system (18) represent n_s equations that define the equilibrium in the buyers' subgame. The third line of system (18) represents n_s first order conditions for the sellers' profit functions which define the equilibrium in the sellers' game. We can derive this first order conditions in the explicit form and get:

$$
\begin{aligned}
\frac{\partial\pi_j\left(p_j,\mathbf{p}_{-j}\right)}{\partial p_j}&=n_b\times\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right)\times\zeta\left(\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right);n_b,h_j\right)+(p_j-c)\times n_b\times\frac{\partial\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right)}{\partial p_j}\times\\
&\times\left[\zeta\left(\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right);n_b,h_j\right)+\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right)\times\zeta'\left(\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right);n_b,h_j\right)\right]=\\
&=n_b\times\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right)\times\zeta\left(\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right);n_b,h_j\right)+\\
&+(p_j-c)\times n_b\times\frac{\partial\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right)}{\partial p_j}\times\mathbb{P}_j(X_1\leqslant h_j-1).
\end{aligned}
\tag{19}
$$

One sees that the final missing part here is the $\frac{\partial\tilde{\gamma}_j^s(p_j,\mathbf{p}_{-j})}{\partial p_j}$ function in the explicit form. After doing some math (see Appendix) we get:

$$
\begin{aligned}
\frac{\partial\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right)}{\partial p_j}&=\zeta\left(\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right);n_b,h_j\right)\times\frac{1}{(u-p_j)\times\zeta'\left(\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right);n_b,h_j\right)}\times\\
&\left[1-\frac{1}{(u-p_j)\times\zeta'\left(\tilde{\gamma}_j^s\left(p_j,\mathbf{p}_{-j}\right);n_b,h_j\right)}\times\left(\frac{1}{\sum_{j=1}^{n_s}\frac{1}{(u-p_j)\times\zeta'\left(\tilde{\gamma}_j^s(p_j,\mathbf{p}_{-j});n_b,h_j\right)}}\right)\right].
\end{aligned}
\tag{20}
$$

Now we can take this expression, put it into sellers' first order conditions (19) and solve the system 18 numerically. We get all the following results using function *fsolve* from *Python* package *scipy*.

The solution of the system derived above gives us the equilibrium in almost all cases, but there is one exception that we need to mention. If two or more sellers have exactly n_b products then any possibility of friction disappears, and we get clear Bertrand competition with equilibrium prices of the active sellers dropping to the marginal costs as a consequence. In all other cases, the equilibrium will be internal, i.e. all sellers will be active in the market and get nonzero expected profits.

4.3 Comparative statics in general case

The correctness of conditions (18) and the obvious lack of analytical solution motivates us to apply numerical solutions for understanding the patterns of equilibrium. The main property we observe in all cases is that the seller with a larger number of products sets higher prices and obtains higher profits, total and per product, than her competitors, because this seller attracts more buyers because of higher availability of products. However, unilaterally increasing the capacity of a particular seller, with fixed capacities of competitors, leads to non-monotonic consequences: a small advantage leads to a higher profit, but a large growth of capacity punishes the whole market and drops prices, which is not compensated for by the large market power.

For buyers, the effect is more predictable: the growth of capacity of any given seller, despite her price growth, increase buyer utility because of the higher probability of a successful deal and because competitors' prices drop. Buyer surplus growth is greater than potential seller loses, such that the total surplus increases with larger sellers. On the other hand, the total number of deals increases, but the probability of the average product being sold decreases. This last effect is true even for the large seller separately, even though her number of deals grows quicker than average.

The two figures below represent the noted relations. Fig. (3) shows the growth of capacity of the first seller when two other sellers are similar. Fig. (4) covers the completely asymmetric case.

4.4 Endogenous capacities

The natural implication of the ability to compute the market equilibrium is the idea of endogenizing the capacities. Let us consider as a zero stage of the market game the simultaneous and independent choice of capacities for all sellers in the market. It is reasonable that the set of pure strategies is limited from 0 to the number of buyers. So, in contrast to (Burdett, Shi, and Wright 2001), the equilibrium capacities may be different from 1 and 2. In the comparative statics section, we noted that the profit from strategically increasing the capacity is limited, but on the other hand it is better to have a larger capacity than competitors. This may lead to effects similar to the prisoners' dilemma or Bertrand competition, where participants are involved in a series of decisions resulting in Pareto dominated profiles. On the other hand, a seller's individual utility function is bell-shaped with respect to h, which make the capacity game intuitively close to Cournot competition.

The further analysis is completely numerical, and in order to avoid additional multiple cases we assume that choosing any number of products is costless for every seller, and normalize $c = 0$, $u = 1$. Note that bearing positive, not necessarily equal costs of adding the product may only shift equilibrium capacities down, but never makes them greater than in the costless case.

The simplest capacity game is among 2 sellers and 2 buyers (Table 1). The matrix coincides with that from (Burdett, Shi, and Wright 2001) and two asymmetric equilibria may arise with three products in total in the market equilibrium. The seller who is

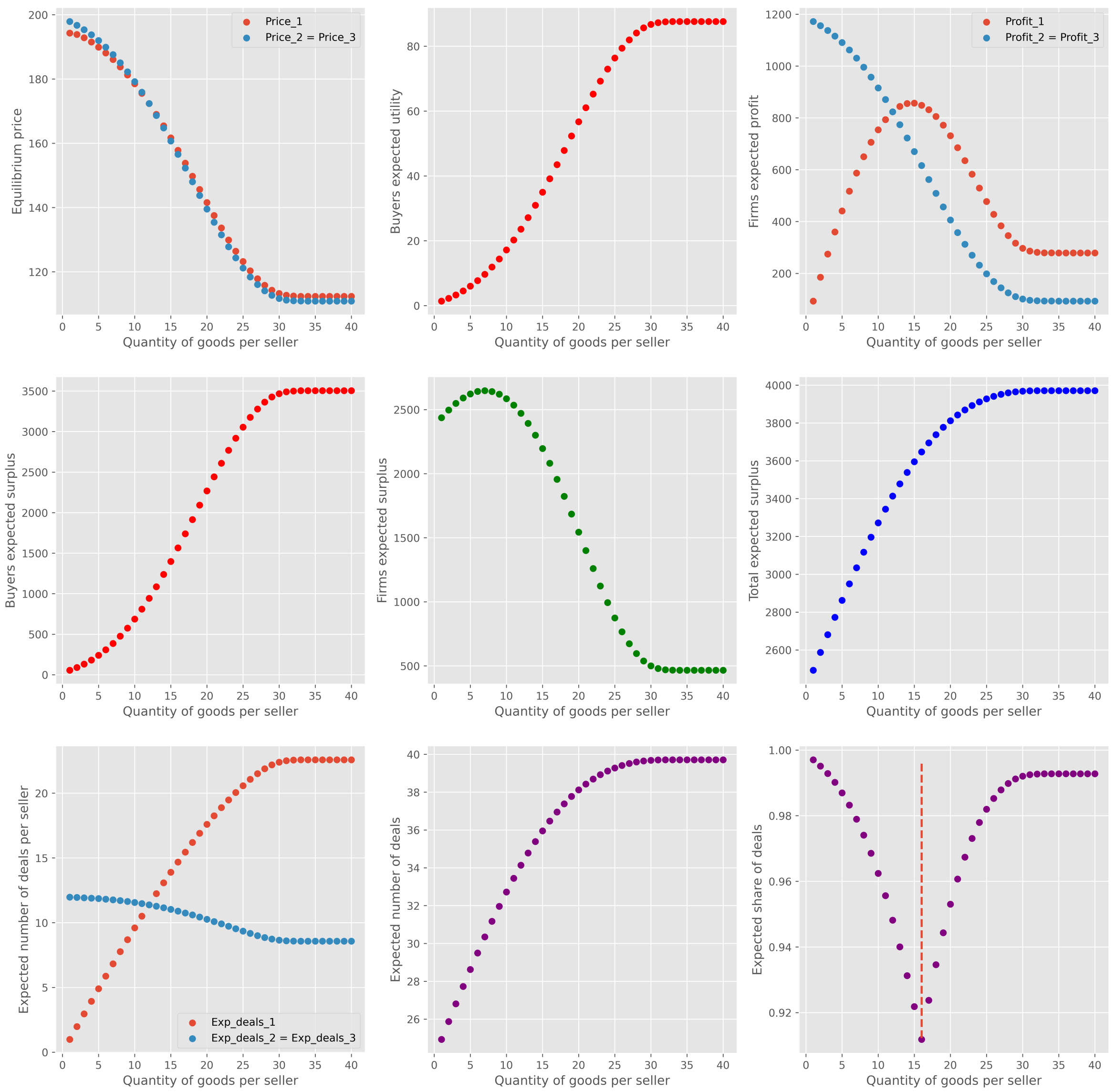

Figure 3: Equilibrium characteristics as a function of capacity of the first firm, under $n_b = 40$, $n_s = 3$, $h_2 = 12$, $h_3 = 12$, $c = 100$, $u = 200$.

able to serve the whole market obtains a larger profit, but still she is not a monopolist, since the smaller seller attracts buyers with positive probability, because of the lower price and despite the friction.

	0	**1**	**2**
0	(0; 0)	(0; 1)	(0; 2)
1	(1; 0)	(0.375; 0.375)	(0.0858; 0.4142)
2	(2; 0)	(0.4142; 0.0858)	(0; 0)

Table 1: Payoff matrix for sellers game with capacities: $n_b = 2$, $n_s = 2$, $u = 1$, $c = 0$.

For three buyers, one symmetric and two asymmetric equilibria hold, with four products in total (Table 2). In one equilibrium, both sellers restrict their capacity,

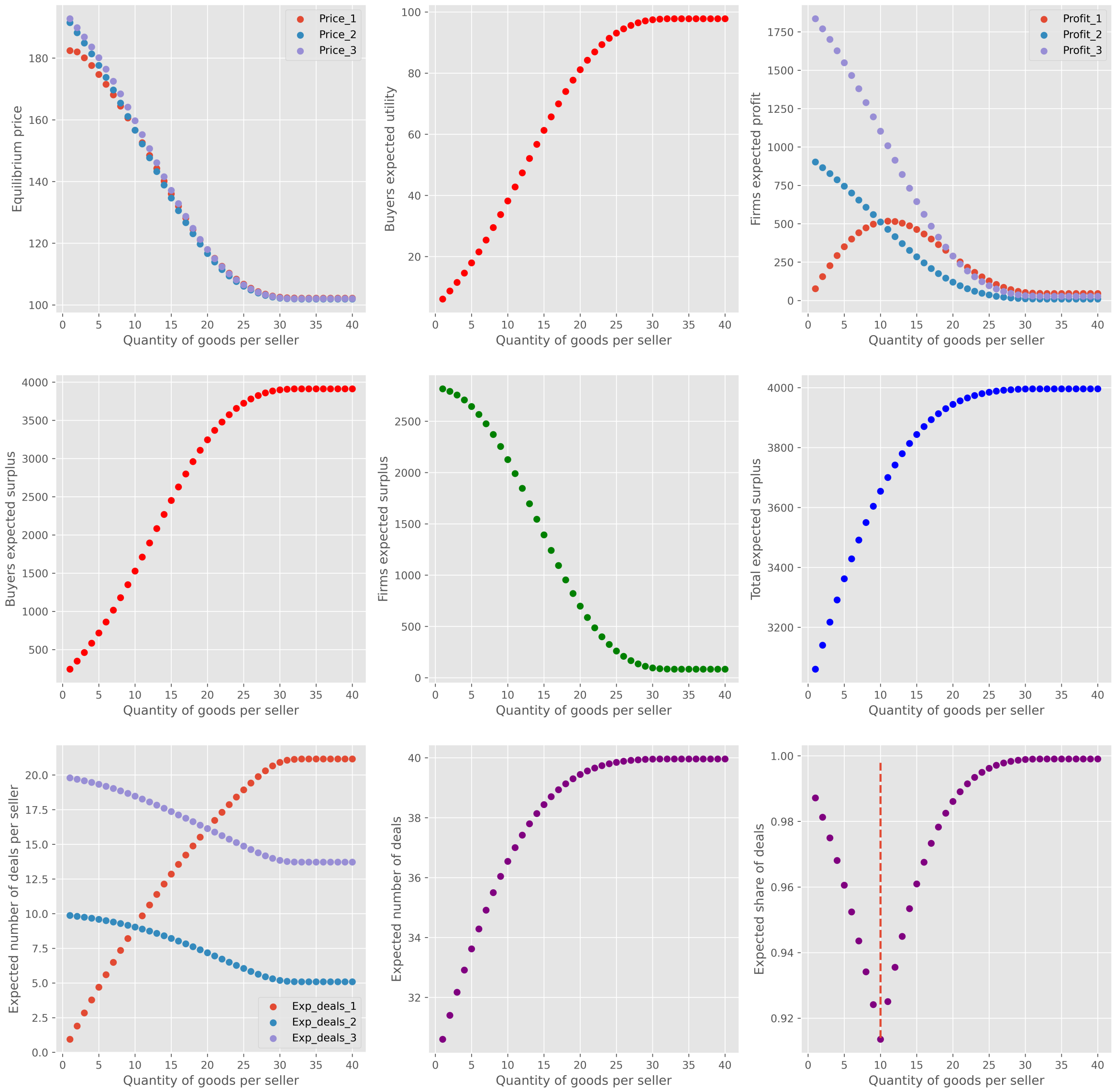

Figure 4: Equilibrium characteristics as a function of capacity of the first firm, under $n_b = 40$, $n_s = 3$, $h_2 = 10$, $h_3 = 20$, $c = 100$, $u = 200$.

while in the two others one seller tries to serve the whole market while her competitor challenges her monopoly status.

Further increasing the set of buyers leads to equilibrium configurations where the capacities are interior and the total number of products is close to the number of buyers. This may be the pattern with a unique equilibrium, or two closest asymmetric equilibria, or also with two *additional* asymmetric equilibria around the first two cases. It seems that the equilibrium share of capacities from the maximal possible slightly decreases, but not monotonically, and the limit is unclear (Table 3).

Starting with three sellers, we have the equilibrium with zero profits and maximal utilities for all sellers. However, there exists an interior profile that Pareto dominates the most competitive equilibrium (Table 4).

	0	1	2	3
0	(0; 0)	(0; 1)	(0; 2)	(0; 3)
1	(1; 0)	(0.6364; 0.6364)	(0.3642; 0.9922)	(0.1629; 0.9962)
2	(2; 0)	(0.9922; 0.3642)	(0.4231; 0.4231)	(0.1058; 0.264)
3	(3; 0)	(0.9962; 0.1629)	(0.264; 0.1058)	(0; 0)

Table 2: Payoff matrix for sellers game with capacities: $n_b = 3$, $n_s = 2$, $u = 1$, $c = 0$.

Table 4: Payoff matrix for sellers game with capacities: $n_b = 3$, $n_s = 3$, $u = 1$, $c = 0$.

0				
	0	**1**	**2**	**3**
0	(0; 0; 0)	(0; 1; 0)	(0; 2; 0)	(0; 3; 0)
1	(1; 0; 0)	(0.6364; 0.6364; 0)	(0.3642; 0.9922; 0)	(0.1629; 0.9962; 0)
2	(2; 0; 0)	(0.9922; 0.3642; 0)	(0.4231; 0.4231; 0)	(0.1058; 0.264; 0)
3	(3; 0; 0)	(0.9962; 0.1629; 0)	(0.264; 0.1058; 0)	(0; 0; 0)
1				
	0	**1**	**2**	**3**
0	(0; 0; 1)	(0; 0.6364; 0.6364)	(0; 0.9922; 0.3642)	(0; 0.9962; 0.1629)
1	(0.6364; 0; 0.6364)	(0.3284; 0.3284; 0.3284)	(0.1621; 0.565; 0.1621)	(0.0577; 0.5749; 0.0577)
2	(0.9922; 0; 0.3642)	(0.565; 0.1621; 0.1621)	(0.2615; 0.2615; 0.0545)	(0.0653; 0.1925; 0.008)
3	(0.9962; 0; 0.1629)	(0.5749; 0.0577; 0.0577)	(0.1925; 0.0653; 0.008)	(0; 0; 0)

Continued on next page

Table 4: Payoff matrix for sellers game with capacities: $n_b = 3$, $n_s = 3$, $u = 1$, $c = 0$. (Continued)

2				
	0	**1**	**2**	**3**
0	(0; 0; 2)	(0; 0.3642; 0.9922)	(0; 0.4231; 0.4231)	(0; 0.264; 0.1058)
1	(0.3642; 0; 0.9922)	(0.1621; 0.1621; 0.565)	(0.0545; 0.2615; 0.2615)	(0.008; 0.1925; 0.0653)
2	(0.4231; 0; 0.4231)	(0.2615; 0.0545; 0.2615)	(0.107; 0.107; 0.107)	(0.0153; 0.0705; 0.0153)
3	(0.264; 0; 0.1058)	(0.1925; 0.008; 0.0653)	(0.0705; 0.0153; 0.0153)	(0; 0; 0)
3				
	0	**1**	**2**	**3**
0	(0; 0; 3)	(0; 0.1629; 0.9962)	(0; 0.1058; 0.264)	(0; 0; 0)
1	(0.1629; 0; 0.9962)	(0.0577; 0.0577; 0.5749)	(0.008; 0.0653; 0.1925)	(0; 0; 0)
2	(0.1058; 0; 0.264)	(0.0653; 0.008; 0.1925)	(0.0153; 0.0153; 0.0705)	(0; 0; 0)
3	(0; 0; 0)	(0; 0; 0)	(0; 0; 0)	(0; 0; 0)

Coming back to the framework of the prisoners' dilemma, most symmetric equilibria are Pareto dominated by some profile with lower capacities. This opens the possibility of collusion among sellers, especially in the long run interaction, when they strategically restrict the supply.

5 Conclusion

This paper presents a comprehensive analysis of the influence of seller capacity on competition in a finite market. It addresses the optimal capacity and states the controversial influence of capacity growth on a given seller. The analysis contributes into the literature about the existence of equilibria in directed search models and enriches the algorithm of solving the buyers' subgame. Summing up, the firm size associated with its capacity is a fine tuning of the model, and the choice of small versus large is more complicated than that between 1 and 2 product per firm.

Starting with homogeneous products, buyers who are symmetric in all aspects, and sellers who are symmetric in costs, we discover a novel dimension of product differentiation. Different capacities produce different availability, which in turn biases buyers' choice, redistributes the demand, and diverges prices. This falsifies the dilemma between the price and quantity competition as two distinct ways to explain the market. Instead of this, one can see a deeper analogy of price-quantity competition with price-location competition, but also including the limitations stemming from friction.

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Appendix

Proof of the Lemma 1.

$$\zeta(x;n_b,h) = \sum_{s=0}^{n_b-1}\left(C^s_{n_b-1}\times x^s\times(1-x)^{n_b-1-s}\right)\times\min\left(\frac{h}{s+1},1\right) =$$

$$= \sum_{s=0}^{h-1} C^s_{n_b-1}\times x^s\times(1-x)^{n_b-1-s} + \sum_{s=h}^{n_b-1} C^s_{n_b-1}\times x^s\times(1-x)^{n_b-1-s}\times\frac{h}{s+1} \quad (21)$$

We also know that

$$\sum_{s=0}^{n_b-1} C^s_{n_b-1}\times x^s\times(1-x)^{n_b-1-s} = 1 \quad (22)$$

as sum of all probabilities and

$$\sum_{s=0}^{n_b-1} C^s_{n_b-1}\times x^s\times(1-x)^{n_b-1-s}\times\frac{1}{s+1} = \zeta(x;n_b,1) = \frac{1-(1-x)^{n_b}}{n_b\times x}, \quad (23)$$

as it was demonstrated in equation 3.

Using that we can rewrite $\zeta(x;n_b,h)$ as

$$\zeta(x;n_b,h) = \sum_{s=0}^{h-1} C^s_{n_b-1}\times x^s\times(1-x)^{n_b-1-s} +$$
$$+ h\times\left[\frac{1-(1-x)^{n_b}}{n_b\times x} - \sum_{s=0}^{h-1} C^s_{n_b-1}\times x^s\times(1-x)^{n_b-1-s}\times\frac{1}{s+1}\right] \quad (24)$$

The first summation term of the expression above (denote it $\omega_1(x;n_b,h)$) is equal to $\mathbb{P}(X_1 \leqslant h-1)$ where X_1 is a random variable equal to the number of successes in the n_b-1 Bernoulli experiments with the probability of success x by definition.

Let consider the second summation term.

$$\omega_2(x;n_b,h) = \sum_{s=0}^{h-1} C^s_{n_b-1}\times x^s\times(1-x)^{n_b-1-s}\times\frac{1}{s+1} \quad (25)$$

After taking a derivative and making some math we can get

$$\frac{\partial\omega_2(x;n_b,h)}{\partial x} = \frac{1}{x(1-x)}\left[\omega_1(x;n_b,h)-\omega_2(x;n_b,h)\right] - \frac{(n_b-1)\times\omega_2(x;n_b,h)}{1-x} =$$
$$= \frac{1}{1-x}\times$$
$$\times\left[\frac{1}{x}\left(1-h\times C^h_{n_b-1}\times B_x\left(h,n_b-h\right)-\omega_2(x;n_b,h)\right)-(n_b-1)\times\omega_2(x;n_b,h)\right], \quad (26)$$

where $B_x\left(h,n_b-h\right)$ is an incomplete beta function.

Combining this differential equation with one of the border conditions $\omega_2(0;n_b,h) = 1$ and $\omega_2(1;n_b,h)=0$, we can solve Cauchy problem and finally get

$$\omega_2(x;n_b,h) = \frac{1}{n_b}\times[C^h_{n_b-1}\times x^{h-1}\times(1-x)^{n_b-h} +$$
$$+\frac{1}{x}\times(\omega_1(x;n_b,h)-(1-x)^{n_b})]. \quad (27)$$

Putting all together and simplifying,

$$\begin{aligned}
\zeta(x;n_b,h) = \omega_1(x;n_b,h) + h \times \left[\frac{1-(1-x)^{n_b}}{n_b \times x}\right] - \\
- h \times \frac{1}{n_b} \times \left[C_{n_b-1}^{h} \times x^{h-1} \times (1-x)^{n_b-h} + \frac{1}{x} \times \left(\omega_1(x;n_b,h) - (1-x)^{n_b}\right)\right] = \\
= \omega_1(x;n_b,h) + \frac{h}{n_b \times x} - \frac{h}{n_b \times x} \times \omega_1(x;n_b,h) - \frac{h}{n_b} \times C_{n_b-1}^{h} \times x^{h-1} \times (1-x)^{n_b-h} = \\
= \omega_1(x;n_b,h) + \frac{h}{n_b \times x} \times \left[1 - \omega_1(x;n_b,h) - C_{n_b-1}^{h} \times x^{h} \times (1-x)^{n_b-h}\right] = \\
= \omega_1(x;n_b,h) + \frac{h}{n_b \times x} \times I_x\left(h+1, n_b-h\right),
\end{aligned} \tag{28}$$

where $I_x\left(h+1, n_b-h\right)$ is a regularized incomplete beta function.

Finally, by definition $I_x\left(h+1, n_b-h\right)$ is equal to $\mathbb{P}(X_2 > h)$, where X_2 is a random variable equal to the number of successes in the n_b Bernoulli experiments with the probability of success x.

Q.E.D. $\square$

Derivation of expression for $\frac{\partial \tilde{\gamma}_j^s(p_j,\mathbf{p}_{-j})}{\partial p_j}$ *(20).*

$$\begin{cases}
L_i = (u-p_i) \times \zeta(\tilde{\gamma}_i^s; h_i) - (u-p_{i+1}) \times \zeta(\tilde{\gamma}_{i+1}^s; h_{i+1}) = 0, \;\; \forall i \in \{1,\ldots,n_s-1\} \\
L_{n_s} = \sum_{i=1}^{n_s} \tilde{\gamma}_i^s - 1 = 0
\end{cases} \tag{29}$$

$$\frac{\partial L}{\partial \tilde{\gamma}^s} \times \frac{\partial \tilde{\gamma}^s}{\partial p} = -\frac{\partial L}{\partial p} \iff \frac{\partial \tilde{\gamma}^s}{\partial p} = -\left(\frac{\partial L}{\partial \tilde{\gamma}^s}\right)^{-1} \times \frac{\partial L}{\partial p} \tag{30}$$

$$\frac{\partial L}{\partial \tilde{\gamma}^s} = \left(\begin{array}{ccccc|c}
\beta_1 & -\beta_2 & 0 & \cdots & 0 & 0 \\
0 & \beta_2 & -\beta_3 & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\
0 & \cdots & \cdots & \cdots & \beta_{n_s-1} & -\beta_{n_s} \\
\hline
1 & \cdots & \cdots & \cdots & 1 & 1
\end{array}\right) = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right), \tag{31}$$

where $\beta_i = (u-p_i) \times \zeta'\left(\tilde{\gamma}_i^s; h_i\right)$

$$\begin{aligned}
\left(\frac{\partial L}{\partial \tilde{\gamma}^s}\right)^{-1} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \\
\begin{pmatrix} A^{-1} + A^{-1}B\left(D - CA^{-1}B\right)^{-1}CA^{-1} & -A^{-1}B\left(D-CA^{-1}B\right)^{-1} \\ -\left(D-CA^{-1}B\right)^{-1}CA^{-1} & \left(D-CA^{-1}B\right)^{-1} \end{pmatrix}^{-1}
\end{aligned} \tag{32}$$

$$A^{-1} = \begin{pmatrix} \frac{1}{\beta_1} & \frac{1}{\beta_1} & \cdots & \frac{1}{\beta_1} & \frac{1}{\beta_1} & \frac{1}{\beta_1} \\ 0 & \frac{1}{\beta_2} & \cdots & \frac{1}{\beta_2} & \frac{1}{\beta_2} & \frac{1}{\beta_2} \\ \vdots & \ddots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & \frac{1}{\beta_{n_s-2}} & \frac{1}{\beta_{n_s-2}} \\ 0 & \cdots & \cdots & 0 & 0 & \frac{1}{\beta_{n_s-1}} \end{pmatrix} \tag{33}$$

$$\begin{aligned}\left(D - CA^{-1}B\right)^{-1} &= \left(1 - \begin{pmatrix} 1 & \cdots & 1 \end{pmatrix} \times A^{-1} \times \begin{pmatrix} 0 \\ \vdots \\ 0 \\ -\beta_{n_s} \end{pmatrix}\right)^{-1} = \\ &= \left(1 + \beta_{n_s} \times \sum_{i=1}^{n_s-1} \frac{1}{\beta_i}\right)^{-1} = \frac{1}{1 + \beta_{n_s} \times \sum_{i=1}^{n_s-1} \frac{1}{\beta_i}} = (*)\end{aligned} \tag{34}$$

$$CA^{-1} = \begin{pmatrix} \frac{1}{\beta_1} & \sum_{i=1}^{2} \frac{1}{\beta_i} & \cdots & \sum_{i=1}^{n_s-1} \frac{1}{\beta_i} \end{pmatrix} \tag{35}$$

$$A^{-1}B = -\begin{pmatrix} \frac{\beta_{n_s}}{\beta_1} \\ \vdots \\ \frac{\beta_{n_s}}{\beta_{n_s-1}} \end{pmatrix} \tag{36}$$

$$A^{-1}BC = -\beta_{n_s} \times \begin{pmatrix} \frac{1}{\beta_1} & \cdots & \frac{1}{\beta_1} \\ \vdots & \cdots & \vdots \\ \frac{1}{\beta_{n_s-1}} & \cdots & \frac{1}{\beta_{n_s-1}} \end{pmatrix} \tag{37}$$

$$A^{-1}BCA^{-1} = -\beta_{n_s} \times \begin{pmatrix} \frac{1}{\beta_1} \times \frac{1}{\beta_1} & \frac{1}{\beta_1} \times \sum_{i=1}^{2} \frac{1}{\beta_i} & \cdots & \frac{1}{\beta_1} \times \sum_{i=1}^{n_s-1} \frac{1}{\beta_i} \\ \vdots & \cdots & \cdots & \vdots \\ \frac{1}{\beta_{n_s-1}} \times \frac{1}{\beta_1} & \frac{1}{\beta_{n_s-1}} \times \sum_{i=1}^{2} \frac{1}{\beta_i} & \cdots & \frac{1}{\beta_{n_s-1}} \times \sum_{i=1}^{n_s-1} \frac{1}{\beta_i} \end{pmatrix} \tag{38}$$

$$\left(\frac{\partial L}{\partial \tilde{\gamma}^s}\right)^{-1} = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}, \tag{39}$$

where

$$Q_{11} =$$
$$\begin{pmatrix} \frac{1}{\beta_1}\left[1 - \frac{\beta_{n_s}}{\beta_1}(*)\right] & \frac{1}{\beta_1}\left[1 - \sum_{i=1}^{2} \frac{\beta_{n_s}}{\beta_i}(*)\right] & \cdots & \frac{1}{\beta_1}\left[1 - \sum_{i=1}^{n_s-1} \frac{\beta_{n_s}}{\beta_i}(*)\right] \\ \frac{1}{\beta_2}\left[0 - \frac{\beta_{n_s}}{\beta_1}(*)\right] & \frac{1}{\beta_2}\left[1 - \sum_{i=1}^{2} \frac{\beta_{n_s}}{\beta_i}(*)\right] & \cdots & \frac{1}{\beta_2}\left[1 - \sum_{i=1}^{n_s-1} \frac{\beta_{n_s}}{\beta_i}(*)\right] \\ \vdots & \ddots & \ddots & \vdots \\ \frac{1}{\beta_{n_s-1}}\left[0 - \frac{\beta_{n_s}}{\beta_1}(*)\right] & \frac{1}{\beta_{n_s-1}}\left[0 - \sum_{i=1}^{2} \frac{\beta_{n_s}}{\beta_i}(*)\right] & \cdots & \frac{1}{\beta_{n_s-1}}\left[1 - \sum_{i=1}^{n_s-1} \frac{\beta_{n_s}}{\beta_i}(*)\right] \end{pmatrix} \tag{40}$$

$$Q_{12} = \begin{pmatrix} \frac{\beta_{n_s}}{\beta_1} \\ \vdots \\ \frac{\beta_{n_s}}{\beta_{n_s-1}} \end{pmatrix} \times (*) \tag{41}$$

$$Q_{21} = -\begin{pmatrix} \frac{1}{\beta_1} & \sum_{i=1}^{2} \frac{1}{\beta_i} & \cdots & \sum_{i=1}^{n_s-1} \frac{1}{\beta_i} \end{pmatrix} \times (*) \tag{42}$$

$$Q_{22} = (*) \tag{43}$$

$$\frac{\partial L}{\partial p} = \\ -\begin{pmatrix} \zeta(\tilde{\gamma}_1^s; h_1) & -\zeta(\tilde{\gamma}_2^s; h_2) & 0 & \cdots & 0 & 0 \\ 0 & \zeta(\tilde{\gamma}_2^s; , h_2) & -\zeta(\tilde{\gamma}_3^s; , h_3) & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \zeta(\tilde{\gamma}_{n_s-1}^s; , h_{n_s-1}) & -\zeta(\tilde{\gamma}_{n_s}^s; , h_{n_s}) \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \tag{44}$$

$$\frac{\partial \tilde{\gamma}^s}{\partial p} = \\ \begin{pmatrix} \frac{\zeta\left(\tilde{\gamma}_1^s;h_1\right)}{\beta_1}\left[1-\frac{\beta_{n_s}}{\beta_1}(*)\right] & \frac{\zeta\left(\tilde{\gamma}_2^s;h_2\right)}{\beta_1}\left[-\frac{\beta_{n_s}}{\beta_2}(*)\right] & \cdots & \frac{\zeta\left(\tilde{\gamma}_{n_s}^s;h_{n_s}\right)}{\beta_1}\left[1-\sum_{i=1}^{n_s-1}\frac{\beta_{n_s}}{\beta_i}(*)\right] \\ \frac{\zeta\left(\tilde{\gamma}_1^s;h_1\right)}{\beta_2}\left[-\frac{\beta_{n_s}}{\beta_1}(*)\right] & \frac{\zeta\left(\tilde{\gamma}_2^s;h_2\right)}{\beta_2}\left[1-\frac{\beta_{n_s}}{\beta_2}(*)\right] & \cdots & \frac{\zeta\left(\tilde{\gamma}_{n_s}^s;h_{n_s}\right)}{\beta_2}\left[1-\sum_{i=1}^{n_s-1}\frac{\beta_{n_s}}{\beta_i}(*)\right] \\ \vdots & \ddots & \ddots & \vdots \\ -\frac{\zeta\left(\tilde{\gamma}_1^s;h_1\right)}{\beta_1}(*) & -\frac{\zeta\left(\tilde{\gamma}_2^s;h_2\right)}{\beta_2}(*) & \cdots & \frac{\zeta\left(\tilde{\gamma}_{n_s}^s;h_{n_s}\right)}{\beta_{n_s}}\left[-\sum_{i=1}^{n_s-1}\frac{\beta_{n_s}}{\beta_i}(*)\right] \end{pmatrix} \tag{45}$$

$$\frac{\partial \tilde{\gamma}_i^s}{\partial p_i} = \frac{\zeta\left(\tilde{\gamma}_i^s; h_i\right)}{(u-p_i)\times\zeta'\left(\tilde{\gamma}_i^s; h_i\right)} \times \\ \left[1 - \frac{1}{(u-p_i)\times\zeta'\left(\tilde{\gamma}_i^s; h_i\right)} \times \frac{(u-p_{n_s})\times\zeta'\left(\gamma_{n_s}; h_{n_s}\right)}{1+(u-p_{n_s})\times\zeta'\left(\gamma_{n_s}; h_{n_s}\right)\times\sum_{i=1}^{n_s-1}\frac{1}{(u-p_i)\times\zeta'\left(\tilde{\gamma}_i^s;h_i\right)}}\right] \tag{46}$$

$$\frac{\partial \tilde{\gamma}_i^s}{\partial p_i} = \zeta\left(\tilde{\gamma}_i^s; h_i\right) \times \frac{1}{(u-p_i)\times\zeta'\left(\tilde{\gamma}_i^s; h_i\right)} \times \\ \left[1 - \frac{1}{(u-p_i)\times\zeta'\left(\tilde{\gamma}_i^s; h_i\right)} \times \left(\frac{1}{\sum_{i=1}^{n_s}\frac{1}{(u-p_i)\times\zeta'\left(\tilde{\gamma}_i^s;h_i\right)}}\right)\right] \tag{47}$$

□

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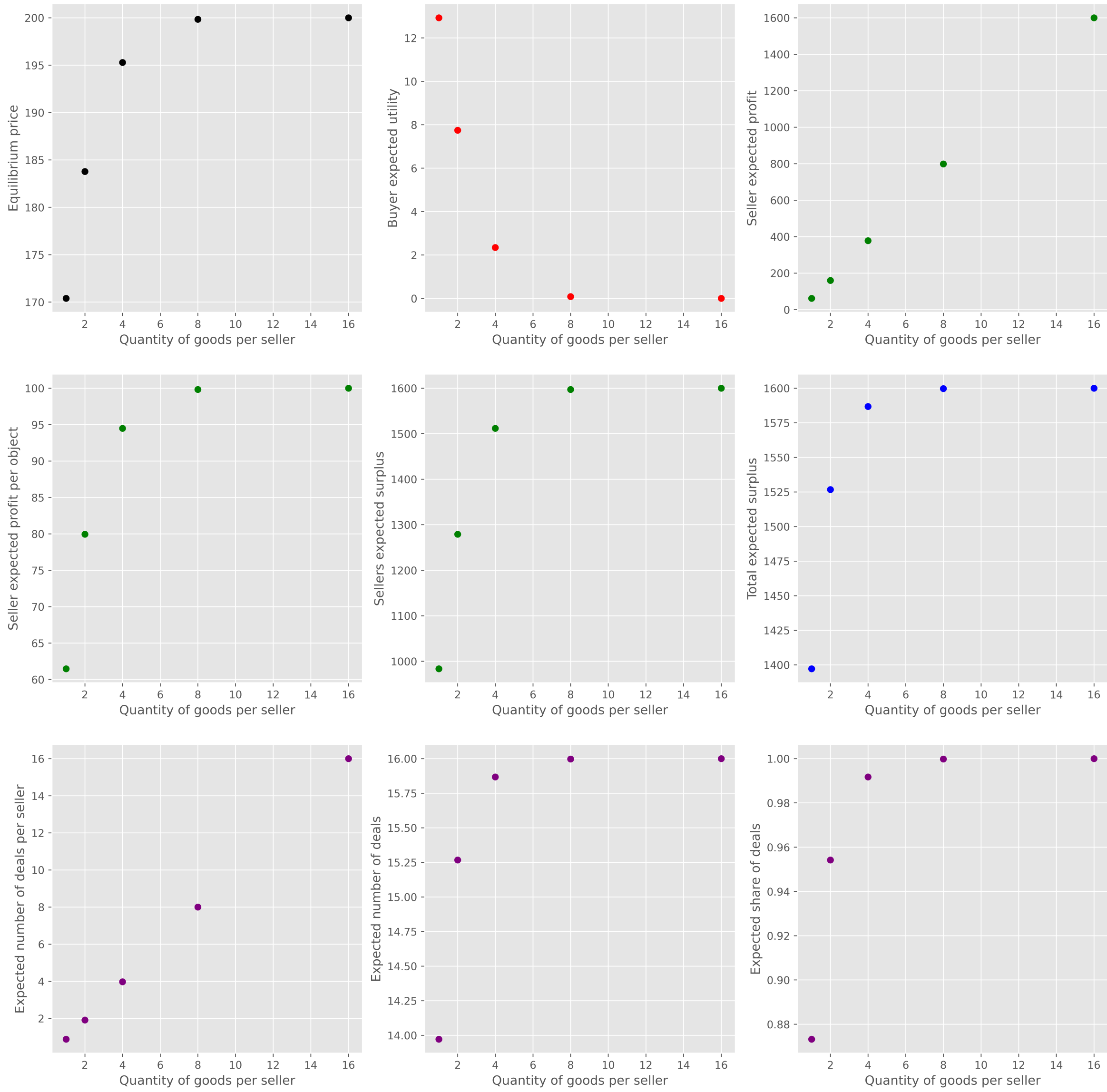

Figure 5: Equilibrium characteristics as a function of market concentration, under $n_b = 32$, $H = 16$, $c = 100$, $u = 200$.

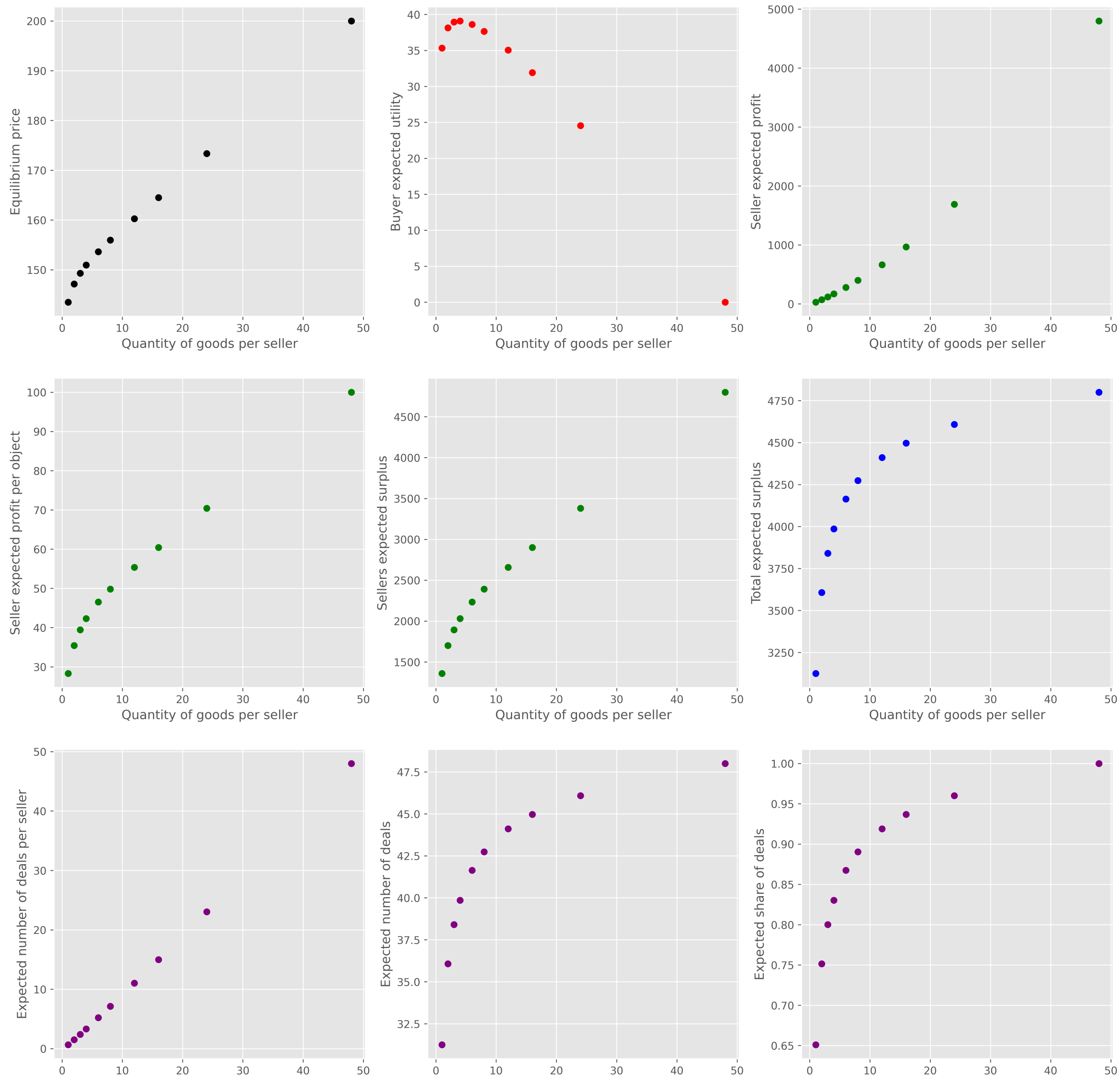

Figure 6: Equilibrium characteristics as a function of market concentration, under $n_b = 50$, $H = 48$, $c = 100$, $u = 200$.

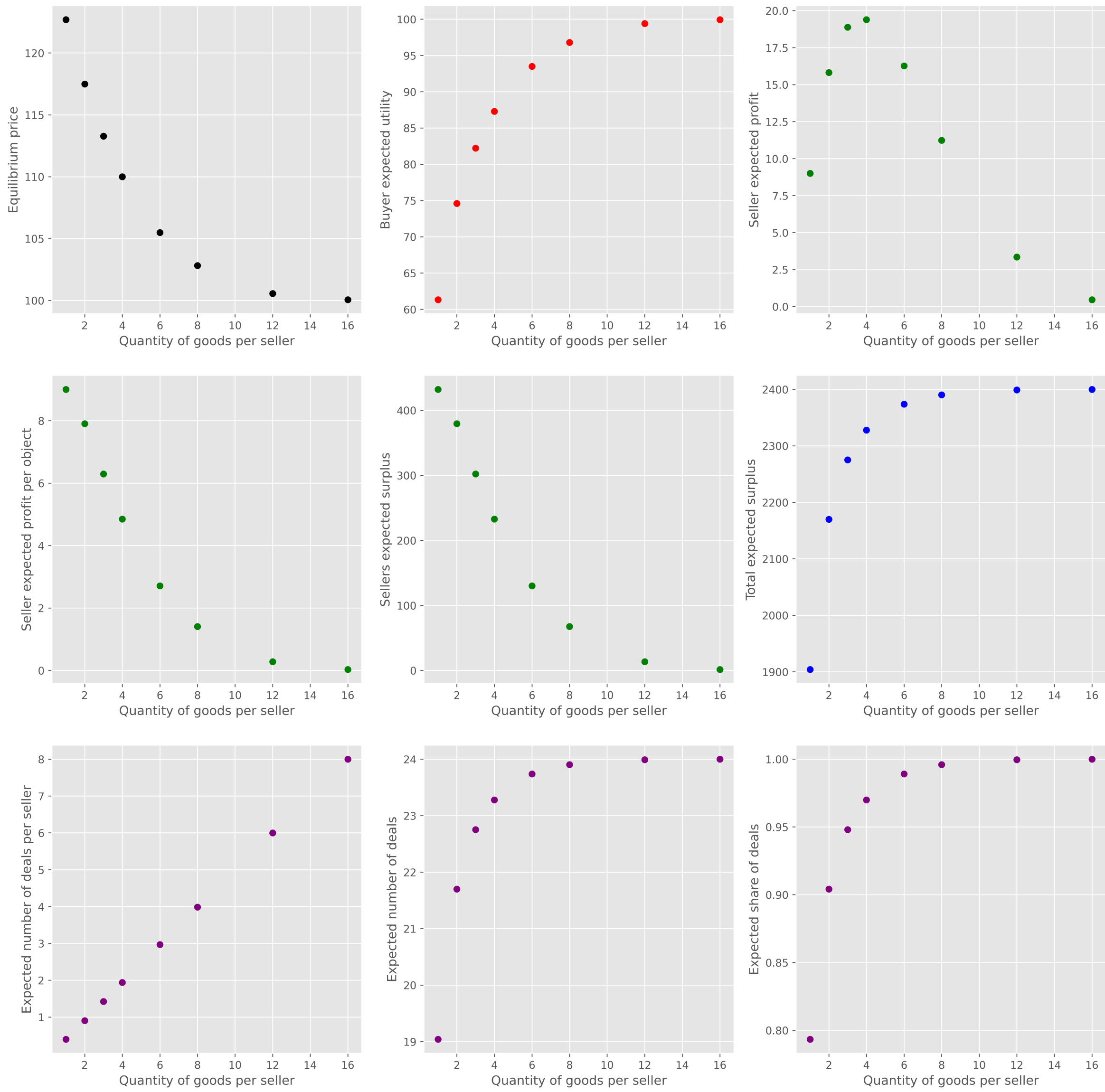

Figure 7: Equilibrium characteristics as a function of market concentration, under $n_b = 24$, $H = 48$, $c = 100$, $u = 200$.

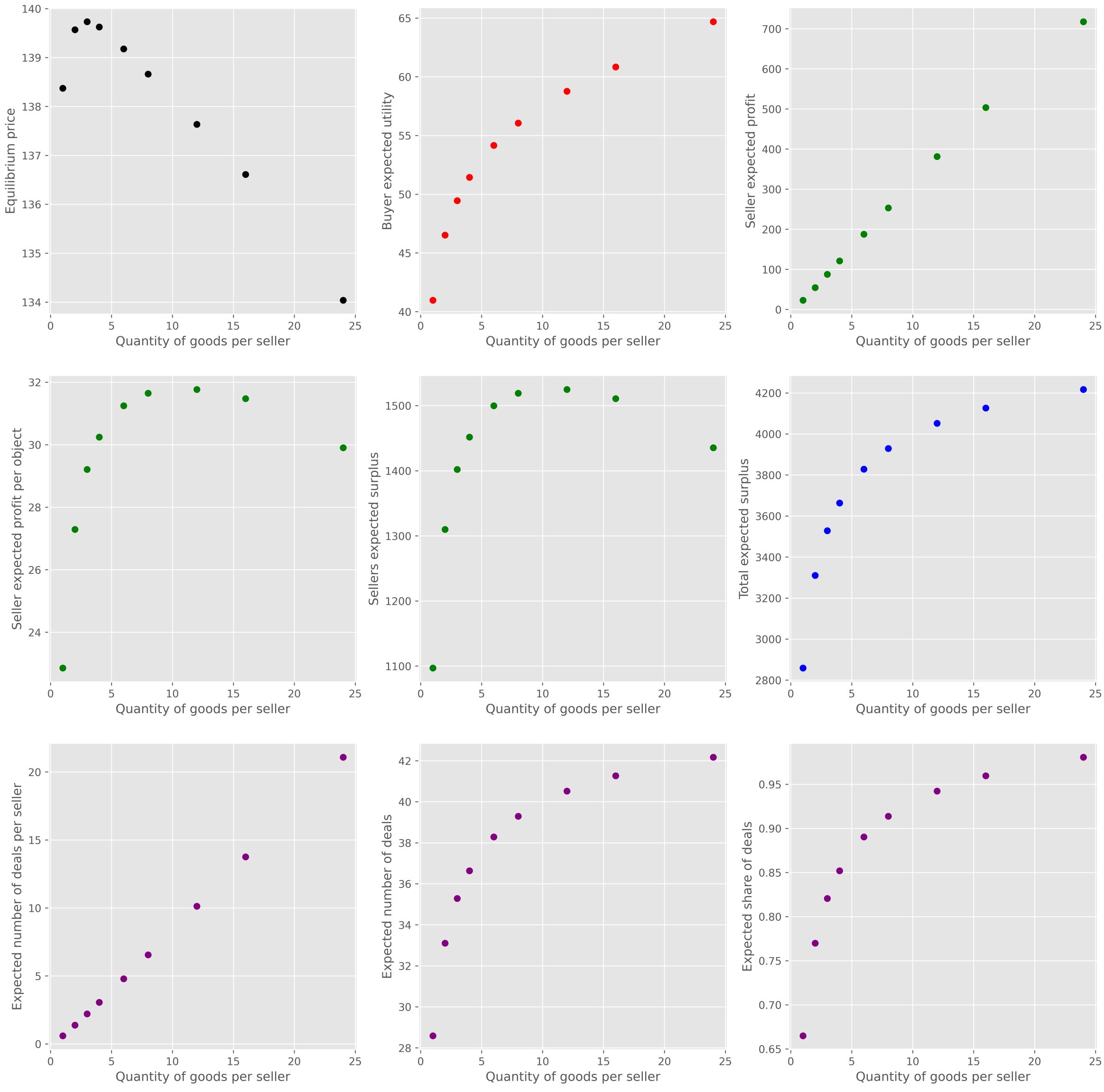

Figure 8: Equilibrium characteristics as a function of market concentration, under $n_b = 43$, $H = 48$, $c = 100$, $u = 200$.

Number of buyers	**Pure strategy equilibrium profiles**	**Number of pure strategy equilibrium**
2	(2; 1), (1; 2)	2
3	(3; 1), (2; 2), (1; 3)	3
4	(2; 2)	1
5	(3; 3)	1
6	(3; 3)	1
7	(4; 3), (3; 4)	2
8	(5; 3), (4; 4), (3; 5)	3
9	(5; 4), (4; 5)	2
10	(6; 4), (5; 5), (4; 6)	3
11	(6; 5), (5; 6)	2
12	(6; 6)	1
13	(6; 6)	1
14	(7; 6), (6; 7)	2
15	(8; 6), (7; 7), (6; 8)	3
16	(8; 7), (7; 8)	2
17	(9; 7), (8; 8), (7; 9)	3
18	(10; 7), (9; 8), (8; 9), (7; 10)	4
19	(10; 8), (8; 10)	2
20	(10; 9), (9; 10)	2
25	(13; 10), (12; 11), (11; 12), (10; 13)	4
30	(14; 14)	1
50	(23; 23)	1
100	(47; 44), (46; 45), (45; 46), (44; 47)	4

Table 3: Equilibrium profiles in sellers capacity game as a function of n_b, under $n_s = 2$, $u = 1$, $c = 0$.

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0	(0; 0)	(0; 1)	(0; 2)	(0; 3)	(0; 4)	(0; 5)	(0; 6)	(0; 7)
1	(1; 0)	(0.9641; 0.9641)	(0.9103; 1.8796)	(0.83; 2.706)	(0.7222; 3.3882)	(0.5917; 3.8538)	(0.4559; 4.0341)	(0.3619 3.9785)
2	(2; 0)	(1.8796; 0.9103)	(1.7172; 1.7172)	(1.497; 2.3568)	(1.2296; 2.7643)	(0.9425; 2.8902)	(0.6916; 2.7643)	(0.5647; 2.6062)
3	(3; 0)	(2.706; 0.83)	(2.3568; 1.497)	(1.9294; 1.9294)	(1.4612; 2.0815)	(1.0158; 1.9621)	(0.6841; 1.6973)	(0.5527; 1.5402)
4	(4; 0)	(3.3882; 0.7222)	(2.7643; 1.2296)	(2.0815; 1.4612)	(1.4128; 1.4128)	(0.853; 1.1582)	(0.4949; 0.8692)	(0.3753; 0.7412)
5	(5; 0)	(3.8538; 0.5917)	(2.8902; 0.9425)	(1.9621; 1.0158)	(1.1582; 0.853)	(0.5728; 0.5728)	(0.252; 0.3353)	(0.1536; 0.2408)
6	(6; 0)	(4.0341; 0.4559)	(2.7643; 0.6916)	(1.6973; 0.6841)	(0.8692; 0.4949)	(0.3353; 0.252)	(0.0925; 0.0925)	(0.026; 0.0356)
7	(7; 0)	(3.9785; 0.3619)	(2.6062; 0.5647)	(1.5402; 0.5527)	(0.7412; 0.3753)	(0.2408; 0.1536)	(0.0356; 0.026)	(0; 0)

Table 5: Payoff matrix for sellers game with capacities: $n_b = 7$, $n_s = 2$, $u = 1$, $c = 0$.