NATIONAL RESEARCH UNIVERSITY
HIGHER SCHOOL OF ECONOMICS

Andrey G. Maksimov, Marina S. Telezhkina

SIMILARITY OF STRUCTURAL CHANGES: THE CASE OF UNIVERSITY ENROLLMENT RATES

BASIC RESEARCH PROGRAM

WORKING PAPERS

SERIES: ECONOMICS

WP BRP 271/EC/2024

Andrey G. Maksimov[1], *Marina S. Telezhkina*[2]

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The paper examines similarity of models with structural changes among heterogeneous panel data units. We propose applying a cosine metric to compare angles between vectors of weighted coefficients as a measure of closeness of economic models. Testing whether the cosine metric value is zero against nonzero, positive, and negative alternatives enriches traditional testing results. The latter merely indicate that models are different since the vectors of coefficients could not be treated as equal. We suggest interpreting nonzero values of a cosine metric as evidence of similarities in the factor structure. This means that similar factors are significant; majority of them affect the dependent variable either in the same direction or opposite. Applying the methodology to study dynamics of university enrollment rates in various countries, the paper provides evidence for the existence of similarities in the factors driving university enrollment rate dynamics. It identifies sustainable cluster divisions and applies the cosine metric to different groups of countries. Notably, evidence is provided that post-communist countries are more similar in the factor structure of the dynamics of university enrollment rates to developed countries than to other developing countries. Increasing access to the internet among population strongly positively contributes to explanation of dynamics of higher education enrollment rates in almost all countries in the 1990s to the beginning of the 2000s.

JEL Classification: C12, C23 I25.

Keywords: panel data, clusterization, structural changes, cosine metric, developed countries, developing countries, post-communist countries.

[1] National Research University Higher School of Economics. Professor of the Chair of Economic Theory and Econometrics; E-mail: amaksimov@hse.ru
[2] National Research University Higher School of Economics. Doctoral candidate in Economics; E-mail: m.s.telezhkina@gmail.com

Introduction

At the turn of the century, society engaged in fundamental transformations in production processes, in economies and in international relations that, among others, caused changes in dynamics of many other macroeconomic parameters in countries. Technological changes, growing access to the internet and globalization are among the main drivers of changes in dynamics of many macroeconomic parameters that could be the results of structural breaks in economic models, namely shifts in factor structures. Analysis of heterogeneity of models with structural breaks could substantially contribute to understanding and modelling institutional mechanisms.

Okui and Wang [Okui and Wang, 2021] present one of the recent papers delivering an approach to account for heterogeneity among sample units in estimation of panel data models. In particular, they offer a clusterization algorithm with instantaneous estimation of number and points of structural breaks. A test presented in the current paper is aimed at enriching the results of estimation of models with structural breaks among heterogenous panel data units: to estimate similarity of models although they are different by definition.

The novelty of the research is that the test for similarity of models is formulated in terms of a cosine metric calculated for values of coefficients in models. This allows to speak not only about direct equality of vectors of coefficients as in traditional tests, but rather to verify their closeness to each other by measuring angle between vectors that enhances interpretation of the results. In particular, if the cosine metric is equal to zero, then the models are different by the factor structure. If a value of the cosine metric is positive, then the models are close by the factors` contributions to the dynamics of the dependent variable. The closer the value of the metric is to unity, the more similar the models are both in terms of significance and values of the coefficients. Conversely, a negative value of the cosine metric implies that the same factors in the models have opposite influence on the dependent variable. Thus, verifying a hypothesis of the cosine metric being equal to zero, one attempts to gather some information from heterogeneity: while traditional tests check for strict equality of vectors of coefficients and state that the models are different, the cosine metric test could verify whether the models have some similarities in their factor structures, or they are totally different. The results could be helpful in terms of further theoretical modelling of dynamic of a parameter and structural changes. If the cosine metric is positive, then one could describe the dynamics of the dependent variable in both countries within the same mechanisms. In cases of null-equality or negativity of the cosine metric one needs different theoretical specifications to describe the dynamics of the dependent variable in countries.

To illustrate the approach of testing for similarity of models with structural changes, we apply it to the analysis of dynamics of university enrollment rates in countries. Along with a fast rise in computer technologies and the growth of international relations, at the end of the 20th and the beginning of the 21st centuries dynamics of parameters of higher education systems also changed significantly worldwide. The authors [Schofer and Meyer, 2005] draw attention to similarities in dynamics of higher education enrollment rates in countries: growth of the parameter accelerates almost in the same period in the majority of countries; pace of the growth after acceleration is quite similar. The scholars suggest that there are common factors behind acceleration of growth rate of university enrollment in countries that make greater contribution in dynamics of the parameter than local factors. Assuming that changes in dynamics of university enrollment rates are the result of a structural changes in models of the parameter, we reformulate the idea of Schofer and Meyer [Schofer and Meyer, 2005] into a hypothesis of similar structural changes in dynamics of the parameter in countries. In other words, if common factors drive the acceleration of university enrollment rates, then, strictly speaking, we expect simultaneous increase or decrease in the impact of the factors on dynamics of the parameter after the structural change compared to their impact before the structural change. Given lags and peculiarities of institutional structures, it can be difficult to verify simultaneous changes in roles of factors. Thus, we suggest that if we observe similarities in factor structures and the impact of factors on dynamics of the dependent variable, then this provides a weak argument in favor of common reasons behind higher education expansion. It is important to note that we could be able to conclude there are similar reasons behind higher education expansion only if a factor structure of the parameter does not experience huge transformations and institutional structure of system of higher education remains mainly unchanged. We attempt to apply traditional regression analysis methods as well as the cosine metric to verify either common shifts in values of coefficients or even similarities in models in general.

The objectives of the research are to develop a methodology for testing for similarity of structural changes in panel data models while accounting for heterogeneity in panel data units; analyze structural changes in dynamics of university enrollment rates.

The shocks in the world economy since 2020 have resulted in structural changes in markets, including labor market. This contributes to the relevance of the current research, which suggests a methodology to analyze similarity of econometric models. Results of such analysis could be further suggestive for the purpose of economic modelling of mechanisms in countries.

The methodology of the current research involves several steps, in particular, estimation of heterogeneous structural breaks and testing for similarity of models and structural changes. Given that, the paper is organized as follows: literature on methods of structural break points` estimation in panel data is observed in section 1; the methodological part in section 2 proposes the testing procedure; in section 3 the application of the proposed methodology to the analysis of dynamics of higher education enrollment rates is described.

1. Literature Review

Use of panel data allows one to increase the range of data at hand and obtain consistent estimates. Nevertheless, an issue of heterogeneity among panel data units arises. The most traditional fixed and random effects specifications allow only for constant country or time specific effects assuming homogeneity of slope coefficients which is often not the case. To account for heterogeneity in influence of factors on a dependent variable the authors of [Pesaran and Smith, 1995] propose mean-group estimates (MG). The method assumes estimation of coefficients for each panel data unit separately and then taking an average of coefficient estimates.

Recently, the idea of clustering units as a method to account for panel units` heterogeneity receives growing attention. In the group fixed effects method (GFE) Bonhomme and Manresa [Bonhomme and Manresa, 2015] propose to put into one group units that get lower residual sum of squares with a given vector of coefficients` estimates. The authors offer to repeat the procedure till the moment it converges to a sustainable distribution of panel data units across groups.

The main focus of the current research is on processes exhibiting structural changes. Despite clear understanding of the fact that a moment of a structural break cannot be exactly the same across panel data units, a significant number of researchers focus on developing techniques for estimating a moment of a common structural break for all units of a panel, trying to exploit the benefits of panel data in terms of consistency of estimates. Wachter and Tzavalis [Wachter and Tzavalis, 2012] develop the idea to choose an estimate of a structural break that maximizes the difference between the values of GMM objective functions before and after a structural break. Baltagi, Kao, Liu [Baltagi et al., 2015] propose to define a moment of structural change, that minimizes the sum of RSS of the models estimated on subsamples before and after a moment of structural break. Qian and Su [Qian and Su, 2016] approach the issue with the use of the lasso shrinkage dimension algorithm. The authors introduce penalized least squares method (PLS) to apply to the model in differences with the dummy-variables accounting for structural changes in coefficients and penalty for growing differences in values of coefficient induced by a structural change. If there are endogeneity in a model under consideration, then the authors recommend applying penalized GMM, where traditional GMM objective function is enriched with the penalty term for growing difference in values of coefficients due to a structural change.

A number of researchers attempt to account for heterogeneity in moments of structural breaks. In the paper [Okui and Wang, 2021] the methodology of structural breakpoint estimation AGFL (adaptive group fused lasso) [Qian and Su, 2016] is combined with the clustering procedure GFE of Bonhomme and Manresa [Bonhomme and Manresa, 2015]. Liao [Liao, 2008] makes an assumption that moments of structural breaks in a model among different units of a panel are derived from one distribution. The authors propose the use of Bayesian procedure to estimate moments of structural breaks.

Since methodology of Okui and Wang [Okui and Wang, 2021] ensures obtaining simultaneously consistent clustering division, consistent estimates of number and points of structural breaks in each of the clusters as well as consistent coefficient estimates the further research exploits the methodology, so that it makes sense to describe the methodology in more details.

The authors [Okui and Wang, 2021] consider a panel data $\{\{y_{it}, x_{it}\}_{t=1}^{T}\}_{i=1}^{N}$ where y_{it} is a dependent variable, x_{it} is a $k \times 1$ vector of regressors, t stays for time, i – for an observational unit. It is assumed that there is a grouped structure among observational units with G groups. The set of groups is defined as $\mathbb{G} = \{1, \dots G\}$; $g_i \in \mathbb{G}$ indicates the group membership of unit i. A data generating process for units belonging to one of the groups is described by an equation $y_{it} = x_{it}'\beta_{g_i,t} + \varepsilon_{it}$, $i = 1, \dots N, t = 1, \dots T$ where ε_{it} is an error term with zero mean, so that all units in the same group share the same time-varying coefficient $\beta_{g,t}, g \in \mathbb{G}$ [Okui and Wang, 2021]. Let β be a vector aggregating all values of coefficients for all groups $\beta = \left(\beta_{1,1}', \dots \beta_{1,T}', \beta_{2,1}' \dots \beta_{2,T}', \beta_{G,1}' \dots \beta_{G,T}'\right)$. Let $\mathcal{B} \subset \mathbb{R}^k$ is the parameter space for each $\beta_{g,t}$. The parameter space for β is $\mathcal{B}^{GT}$. Let γ be the vector of g_is, such that $\gamma = \{g_1, \dots g_N\}$; $\mathbb{G}^N$is the parameter space for γ. For each group there are m_g structural breaks in coefficients happening in break dates $(T_{g,1}, \dots T_{g,m_g})$ [Okui and Wang, 2021]. Coefficients` and points` of structural break estimates are obtained simultaneously as a result of the following optimization problem [Okui and Wang, 2021]:

$$\left(\hat{\beta}, \hat{\gamma}\right) = \underset{(\beta,\gamma)\in\mathcal{B}^{GT}\times\mathbb{G}^N}{\operatorname{argmin}} \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it} - x_{it}'\beta_{g_i,t})^2 + \lambda\sum_{g\in\mathbb{G}}\sum_{t=2}^{T}\dot{\omega}_{g,t}\left\|\beta_{g,t} - \beta_{g,t-1}\right\|, \quad (1)$$

where $\gamma = (g_1, \dots, g_N)$ is group identificator, $\mathbb{G}^N$ is parameter space for γ, λ is tuning parameter, $\dot{\omega}_{g,t}$ is data-driven weight $\dot{\omega}_{g,t} = \left\|\dot{\beta}_{g,t} - \dot{\beta}_{g,t-1}\right\|^{-\kappa}$, $\dot{\beta}$ is a preliminary estimate, κ is a user-specified constant[3], $\|\cdot\|$ is L1 norm such that for a vector $z = (z_1 \dots z_n)$, $\|z\| = \sum_{i=1}^{n} |z_i|$. Starting vector for the iterative procedure can be found as a solution of the following task [Okui and Wang, 2021]:

$$(\dot{\beta}, \dot{\gamma}) = \underset{(\beta,\gamma)\in\mathbb{B}^{GT}\times\mathbb{G}^N}{\operatorname{argmin}} \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it} - x_{it}'\beta_{g_i,t})^2, (2).$$

The parameter λ is chosen minimizing the information criterion proposed in [Qian and Su, 2016] $IC(\lambda) = \frac{1}{NT}\sum_{j=1}^{m_\lambda+1}\sum_{t=T_{j-1}+1}^{T_j}\sum_{i=1}^{N}(y_{it} - x_{it}'\hat{\beta}_{g_i,j})^2 + \rho_{NT}k(m_\lambda + 1)$, where m_λ is the number of breaks associated with the parameter λ, j is a number of time periods given m_λ structural breaks, T_j is the end of each of the j time periods, $\hat{\beta}_{g_i,j}$ is the vector of coefficients` estimates for each g and j, ρ_{NT} is the penalty term on the amount of breaks, $\rho_{NT} = c * \, ln(NT)/\sqrt{NT}$ with $c = 0.05$, following [Qian and Su, 2016].

The consistent number of groups is chosen by minimization the BIC proposed by [Bonhomme and Manresa, 2015] $BIC(G) = \frac{1}{NT}\sum_{j=1}^{m+1}\sum_{t=T_{j-1}+1}^{T_j}\sum_{i=1}^{N}\left(y_{it} - x_{it}'\hat{\beta}_{g_i,j}\right)^2 + \hat{\sigma}^2 * \frac{n_p(G)+N}{NT} * \ln(NT)$, where $\hat{\sigma}^2$ is a scaling parameter, the estimate of the variance of ε_{it}, $n_p(G)$ – total number of estimated coefficients.

The estimation method of [Okui and Wang, 2021] allows to obtain consistent cluster division and consistent and asymptotically normal coefficient estimates with the rate of convergence $\sqrt{N_K, I_{K,j}}$, where N_K is the number of units in a cluster and $I_{K,j}$ is a number of time periods between breaks.

The procedure has some limitations. Firstly, the speed of convergence depends heavily on consistency of preliminary estimates $(\dot{\beta}, \dot{\gamma})$, which are available when initial values are random [Bonhomme and Manresa, 2015]. The authors of [Okui and Wang, 2021] note that in order to obtain consistent preliminary estimates of $(\dot{\beta}, \dot{\gamma})$, it is recommended to perform additional iterative procedure, trialing a range of initial values until achieving consistent grouping. Such preliminary estimates $(\dot{\beta}, \dot{\gamma})$ guarantee right weights $\dot{\omega}_{g,t}$ and ensures fast convergence of the procedure of grouping and structural breakpoints estimation. The authors [Okui and Wang, 2021]

[3] Examining performance of the estimator on simulated data, the authors [Okui and Wang, 2021] take the value of κ equal to two, following the adaptive Lasso literature.

also point out the problem of choice of number of clusters: the more the clusters, the fewer breaks in each cluster are found, that could worsen model fit. It also seems reasonable to stress the existence of the “curse of dimensionality” problem influencing performance of the procedure: the more regressors are considered, the more orthogonal vector of coefficients could appear to be, the higher the probability of empty clusters.

The current research complements the literature addressing whether models with structural breaks are close to each other. This approach allows to uncover similarities in different models. In particular, we combine procedures for clustering units of a panel and break detection with testing for similarity of vectors of coefficients and their changes in models for different clusters. Euclidean distance is among the most widely used metrics. It calculates the straight-line distance between two points in n-dimensional space and is sensitive to outliers. Cosine similarity metric accounts for differences in direction of vectors, measuring angle between two vectors. The latter seems to be promising in the context of similarity of models with structural changes as it allows to consider different models and still find commonalities particularly in factor structure. If angle between vectors of coefficients is acute, then the models can be considered quite similar with the factors influencing the dependent variable in the same direction. If angle is straight, then the coefficients in the models are opposite in sign. In other words, the factor structure is similar, but the factors` contribution in the dynamics of the dependent variable is opposite in sign. If vectors of coefficients are orthogonal, then the models are different. Interpretation of values of the cosine metric from the point of view of models` similarity in econometric models is discussed in more detail in the next section of the paper.

Since distribution of cosine metric is unknown in majority of cases, then testing procedures, based on cosine metric, require the use of bootstrap.

Panel data bootstrap procedures are largely borrowed from time-series bootstrap algorithms and differentiate them by maintenance of the assumption of cross-sectional independence, by accounting for incidental parameter bias [Goncalves and Kaffo, 2015]. The most recent paper of Kapetanios [Kapetanios, 2008] illustrates the asymptotic validity of cross-sectional bootstrap for the large N, fixed T case, assuming cross sectional independence, disclosing its application both in cases of parametric and nonparametric bootstrap. Goncalves and Kaffo also remain within the assumption of cross-sectional independence, but they try to account for incidental parameter bias [Goncalves and Kaffo, 2015]. The latter occurs in case of small-T panels and assumes that the estimated variation of the estimate of fixed-effect parameter is equal to $E(\hat{\sigma}^2_{\widehat{\alpha}_i}) = \sigma^2_{\widehat{\alpha}_i}\frac{T-1}{T}$ and is biased. Recursive-design and fixed-design residual wild bootstrap procedures are proposed in [Goncalves and Kaffo, 2015] as allowing to obtain consistent parameter estimates, given incidental parameter bias. The procedures are parametric, so that both assume resampling of residuals, multiplying residuals on a random number $e_{it}^* = e_{it} * \eta_{it}$, $\eta_{it} \sim iid[0,1]$, and then resampling the dependent variable so that $y_{it}^* = \hat{\alpha}_i + \hat{\theta} y_{it-1}^* + e_{it}^*$ [Goncalves and Kaffo, 2015]. Then the estimates of the parameters according to the recursive-design and fixed-design procedures can be found with formulas (3) and (4), where $\bar{y}_i \equiv \frac{1}{T}\sum_{t=1}^{T} y_{it}$, $\bar{y}_i^* \equiv \frac{1}{T}\sum_{t=1}^{T} y_{it}^*$, $\bar{y}_{i-} \equiv \frac{1}{T}\sum_{t=1}^{T} y_{it-1}$, $\bar{y}_{i-}^* = \frac{1}{T}\sum_{t=1}^{T} y_{it-1}^*$, y_{it}^* is a vector of values of the dependent variable in a bootstrap sample [Goncalves and Kaffo, 2015]

$$\widehat{\theta^*}_{rd} = \left(\frac{1}{NT}\sum_{i=1}^{n}\sum_{t=1}^{T}(y_{it-1}^* - \bar{y}_{i-}^*)^2\right)^{-1} \frac{1}{NT}\sum_{i=1}^{n}\sum_{t=1}^{T}(y_{it-1}^* - \bar{y}_{i-}^*)(y_{it}^* - \bar{y}_i^*), \quad (3)$$

$$\widehat{\theta^*}_{fd} = \left(\frac{1}{NT}\sum_{i=1}^{n}\sum_{t=1}^{T}(y_{it-1} - \bar{y}_{i-})^2\right)^{-1} \frac{1}{NT}\sum_{i=1}^{n}\sum_{t=1}^{T}(y_{it-1} - \bar{y}_{i-})(y_{it}^* - \bar{y}_i^*), \quad (4)$$

In the paper [Goncalves, 2011] the author describes moving block bootstrap procedure that allows for different forms of cross-sectional dependence as well as time-series dependence. The parameters estimates` consistency depends on the level of cross-sectional dependence, but by simulation the author shows that quite high levels of cross-sectional dependence still allow to obtain consistent estimates. The method assumes resampling of the $N \times l$ blocks, where N is the number of panel data units in initial dataset, l is the length of a block in time dimension. Blocks are overlapping, beginning in each moment of time, so that the last block starts from $(T - l + 1)$, where T – is the time length of initial dataset. Thus, considering observations of a variable Y for T periods for n units, in notations in table 1, the blocks $B_{1,l}$ ... $B_{T-l+1,l}$ are resampled. One of the limitations of the procedure is that it does not describe, how to choose the length of a block.

Tab.1 Moving blocks bootstrap

$B_{1,l}$:	$Z_{1n} = (Y_{11} \dots Y_{1n})$	$Z_{2n} = (Y_{21} \dots Y_{2n})$	…	$Z_{ln} = (Y_{l1} \dots Y_{ln})$
	…	…	…	…
$B_{t,l}$:	Z_{tn}	$Z_{t+1\,n}$	…	$Z_{t+l\,n}$
$B_{t+1,l}$:	$Z_{t+1\,n}$	$Z_{t+2\,n}$	…	$Z_{t+l+1\,n}$
	…	…	…	…
$B_{T-l+1,l}$:	$Z_{T-l+1\,n}$	$Z_{T-l+2\,n}$	…	Z_{Tn}

Having observed a range of structural break point estimation algorithms, heterogenous panel data estimation procedures, the use of the methodology of [Okui and Wang, 2021] seems to be justified. It allows to obtain simultaneously consistent cluster division, consistent estimates of number of points of structural breaks and consistent coefficients` estimates. In order to perform the test for the cosine metric for vectors of coefficients` estimates moving block bootstrap procedure is chosen. The choice is guided by its applicability to a wide range of cases, including datasets with cross-sectional and time dependence. The validity of cosine metric for bootstrap is proved in the next section; the formulated hypothesis and testing procedure are also described there.

2. Methodology

Assume that there is a true cluster structure consisting of G clusters, so that $\mathbb{G} = \{1, \dots G\}$ is a set of clusters. Thus, a data generating process is unique for all units of a cluster $g \in \mathbb{G}$ and has one structural break:

$$y_{it} = (1 - d_{it})x_{it}'\beta_g^{pre} + d_{it}x_{it}'\beta_g^{post} + \varepsilon_{it}, \text{(5)}$$

where x_{it} is a vector $(k \times 1)$ of values of regressors for a panel unit i at the moment t, β_g^{pre}, $\beta_g^{post}$are $(k \times 1)$ vectors of values of coefficients before and after the structural change respectively for a cluster to which belongs a panel unit i, d_{it} is a binary variable catching a point of the structural change T_g, so that it takes a value of 1 if $t > T_g$. Let us consider a cosine metric calculated with the formula $Q = \frac{A'B}{\|A\| * \|B\|}$ where A, B are vectors, $A'B$ is their scalar multiplication and $\|A\|, \|B\|$ are their Euclidean norms. One can imagine applying it to analyze similarity of structural changes in models for clusters. This could allow to gather additional information about clusters even if traditional empirical tests and procedures say that the models are different. It still could be the case that structural changes involve shifts in influence of factors on the dependent variable in similar directions. Calculation a cosine metric for relative values of coefficients after and before a structural change allows to evaluate similarity of direction of shifts in factor structures. Cosine metric calculated for values of relative coefficients is equal (7):

$$Q = \frac{r_p' r_q}{\|r_p\| * \|r_q\|}, \text{(7)}$$

where r_p, r_q are vectors consisting of related values of coefficients after and before a structural change for clusters $p, q \in \mathbb{G}$, $r_p' = \left(\frac{\beta_{p1}^{post}}{\beta_{p1}^{pre}} \dots \frac{\beta_{pk}^{post}}{\beta_{pk}^{pre}}\right)$ $r_q' = \left(\frac{\beta_{q1}^{post}}{\beta_{q1}^{pre}} \dots \frac{\beta_{qk}^{post}}{\beta_{qk}^{pre}}\right)$. The cosine metric is defined if all coefficients before the structural change and at least one of the coefficients after the structural change in each of the models are nonzero. The cosine metric is ***zero*** if each coefficient after a structural change is zero in at least one of the models. This means that the structural transformation is such that some factors no longer contribute into dynamics of the variable and these factors are different in clusters. In other words, units of clusters face different structural changes. The cosine metric is also ***zero*** if there are some coefficients that change their sign as a result of a structural change *in both models* (for a given coefficient $\xi', \xi' = \{1, \dots k\} \frac{\beta_{p\xi'}^{post}}{\beta_{p\xi'}^{pre}} * \frac{\beta_{q\xi'}^{post}}{\beta_{q\xi'}^{pre}} > 0$) as well as some coefficients that change their impact *only in one* of the models (for a given coefficient $\xi', \xi' = \{1, \dots k\} \frac{\beta_{p\xi'}^{post}}{\beta_{p\xi'}^{pre}} * \frac{\beta_{q\xi'}^{post}}{\beta_{q\xi'}^{pre}} < 0$) and these changes compensate each other. This means that, generally, structural changes happen differently in models but the final conclusion should be based rather on economic theory and traditional approaches to interpretation of the results. The cosine metric is ***positive*** if either majority of terms are positive or terms that makes largest contribution into the value of the scalar multiplication of vectors in the numerator of (7): for a given coefficient $\xi', \xi' = \{1, \dots k\}$ $\frac{\beta_{p\xi'}^{post}}{\beta_{p\xi'}^{pre}} * \frac{\beta_{q\xi'}^{post}}{\beta_{q\xi'}^{pre}} > 0$. As it is seen from table 2, there are cases, which allows one to conclude about similar structural changes in models as well as opposite cases. This means that positiveness of the metric in (7) leads to overestimation of similarity of structural changes.

Tab. 2. Interpretation of positive cosine metric values cases in (7)

Signs of cosine metric factors	Signs of coefficients	Interpretation of structural changes in models for groups p and q
$\frac{\beta_{p\xi'}^{post}}{\beta_{p\xi'}^{pre}} > 0 \; and \; \frac{\beta_{q\xi'}^{post}}{\beta_{q\xi'}^{pre}} > 0$	$\beta_{p\xi'}^{pre} > 0;\ \beta_{p\xi'}^{post} > 0;$ $\beta_{q\xi'}^{pre} > 0; \beta_{q\xi'}^{post} > 0$	Similar
	$\beta_{p\xi'}^{pre} < 0;\ \beta_{p\xi'}^{post} < 0;$ $\beta_{q\xi'}^{pre} < 0; \beta_{q\xi'}^{post} < 0$	Similar
	$\beta_{p\xi'}^{pre} > 0; \beta_{p\xi'}^{post} > 0;$ $\beta_{q\xi'}^{pre} < 0; \beta_{q\xi'}^{post} < 0$	Different
	$\beta_{p\xi'}^{pre} < 0; \beta_{p\xi'}^{post} < 0;$	Different

	$\beta_{q\xi'}^{pre} > 0; \beta_{q\xi'}^{post} > 0$	
$\frac{\beta_{p\xi'}^{post}}{\beta_{p\xi'}^{pre}} < 0\ and\ \frac{\beta_{q\xi'}^{post}}{\beta_{q\xi'}^{pre}} < 0$	$\beta_{p\xi'}^{pre} > 0; \beta_{p\xi'}^{post} < 0;$ $\beta_{q\xi'}^{pre} > 0; \beta_{q\xi'}^{post} < 0$	Similar
	$\beta_{p\xi'}^{pre} < 0; \beta_{p\xi'}^{post} > 0;$ $\beta_{q\xi'}^{pre} < 0; \beta_{q\xi'}^{post} > 0$	Similar
	$\beta_{p\xi'}^{pre} > 0; \beta_{p\xi'}^{post} < 0;$ $\beta_{q\xi'}^{pre} < 0; \beta_{q\xi'}^{post} > 0$	Different
	$\beta_{p\xi'}^{pre} < 0; \beta_{p\xi'}^{post} > 0;$ $\beta_{q\xi'}^{pre} > 0; \beta_{q\xi'}^{post} < 0$	Different

The cosine metric is ***negative*** if each of the coefficients change its sign as a result of a structural change *only in one* of the models (Tab. 3). Nevertheless, the negative cosine metric also overestimates difference of structural changes: if it turns that coefficients after a structural change are of the same sign, then it seems reasonable to suggest that there are similar structural changes in models.

Tab. 3. Interpretation of negative cosine metric values cases in (7)

Signs of cosine metric factors	Signs of coefficients	Interpretation of structural changes in models for groups p and q
$\frac{\beta_{p\xi'}^{post}}{\beta_{p\xi'}^{pre}} > 0\ and\ \frac{\beta_{q\xi'}^{post}}{\beta_{q\xi'}^{pre}} < 0$	$\beta_{p\xi'}^{pre} > 0;\ \beta_{p\xi'}^{post} > 0;$ $\beta_{q\xi'}^{pre} < 0; \beta_{q\xi'}^{post} > 0$	Similar
	$\beta_{p\xi'}^{pre} < 0;\ \beta_{p\xi'}^{post} < 0;$ $\beta_{q\xi'}^{pre} > 0; \beta_{q\xi'}^{post} < 0$	Similar
	$\beta_{p\xi'}^{pre} > 0;\ \beta_{p\xi'}^{post} > 0;$ $\beta_{q\xi'}^{pre} > 0; \beta_{q\xi'}^{post} < 0$	Different
	$\beta_{p\xi'}^{pre} < 0;\ \beta_{p\xi'}^{post} < 0;$ $\beta_{q\xi'}^{pre} < 0; \beta_{q\xi'}^{post} > 0$	Different

$\frac{\beta_{p\xi'}^{post}}{\beta_{p\xi'}^{pre}} < 0 \; and \; \frac{\beta_{q\xi'}^{post}}{\beta_{q\xi'}^{pre}} > 0$	$\beta_{p\xi'}^{pre} < 0; \beta_{p\xi'}^{post} > 0;$ $\beta_{q\xi'}^{pre} > 0; \beta_{q\xi'}^{post} > 0$	Similar
	$\beta_{p\xi'}^{pre} > 0; \beta_{p\xi'}^{post} < 0;$ $\beta_{q\xi'}^{pre} < 0; \beta_{q\xi'}^{post} < 0$	Similar
	$\beta_{p\xi'}^{pre} > 0; \beta_{p\xi'}^{post} < 0;$ $\beta_{q\xi'}^{pre} > 0; \beta_{q\xi'}^{post} > 0$	Different
	$\beta_{p\xi'}^{pre} < 0; \beta_{p\xi'}^{post} > 0;$ $\beta_{q\xi'}^{pre} < 0; \beta_{q\xi'}^{post} < 0$	Different

In general, zero values of the cosine metric (7) allow one to conclude that models for groups face different structural changes. Nevertheless, non-zero values of the cosine metric (7) lack a precise and insightful interpretation, that is why it seems reasonable to introduce additional metric not for vectors of relative values but for vectors of coefficients themselves (8). On the one hand, it accounts for opposite signs of coefficients in models for groups more correctly, decreasing the value of the cosine metric so that the models are treated to be less similar. On the other hand, the cosine metric (8) can be considered as weak evidence of similarity of structural changes: it does not compare changes themselves but each of the coefficients. The metric is calculated as follows:

$$Q = \frac{\beta_p' \beta_q}{\|\beta_p\| * \|\beta_q\|}, \; (8)$$

where β_p, β_q are $(2k \times 1)$ vectors of coefficients before and after a structural change for clusters p and q, $\beta_g' = \left(\beta_g^{pre'} \; \beta_g^{post'}\right), \; g = (p, q), \; g \in \mathbb{G}$. The cosine metric is defined if at least one coefficient in each model is not equal to zero. The cosine metric is equal to ***zero*** if each of the coefficients is equal to zero at least in one of the models for one of the clusters. That means that there are differences in factor structures in models. If there are some coefficients of the same sign in two models and some coefficients of opposite sign, then it could also turn out that the value of the cosine metric is zero. Factors, which coefficients are of the same sign in both models, increase the value of the cosine metric. If they are factors that make the largest contribution in the value of the nominator of the cosine metric or there are majority of factors with similar by sign coefficients, the cosine metric is ***positive***.

We continue further considering both approaches to analyze similarity of structural changes as being complementary: the one based on the cosine metric for vectors of relative coefficients as well as another measuring cosine of the angle between vectors of coefficients themselves.

To account for significance of coefficients when estimating cosine metrics, it makes sense to apply weights to the estimates of coefficients. Comparing structural changes in models an estimate of value of the cosine metric is found according to (9):

$$\hat{Q} = \frac{\hat{r}_p'\hat{r}_q}{\|\hat{r}_p\| * \|\hat{r}_q\|}, \quad (9)$$

where $\hat{r}_p, \hat{r}_q$ are vectors of relation of weighted coefficient estimates after and before the structural break. The weights are different for coefficients before and after a structural change. The former multiplies logarithm of absolute value of t-statistic of the respective coefficient before a structural change and sign of the coefficient`s estimate: $w_{g\xi}^{pre} = \ln\left(\left|t_{g\xi}^{pre}\right| + 1\right) * sign\ (\hat{\beta}_{g\xi}^{pre}), \xi \in [1, k]$, $g = (p, q),\ g \in \mathbb{G}, t_{g\xi}^{pre}$ is the value of t-statistic of the test for null-equality of the respective coefficient. The weight for a coefficient after a structural change is calculated as follows: $w_{g\xi}^{post} = \ln\left(\left|t_{g\xi}^{post}\right| + 1\right), \xi \in [1, k],\ g = (p, q),\ g \in \mathbb{G}, t_{g\xi}^{post}$ is the value of t-statistic of the test for null-equality of the respective coefficient. Both weights are higher for significant coefficients. Accounting additionally for a sign of a coefficient before a structural change, on the one hand, allows to ignore it and account only for a sign of a coefficient after a structural change as it provides more information about a nature of a structural change. On the other hand, ignoring sign of a coefficient before a structural change, one excludes the situations which overestimate similarity of models with a positive cosine metric value (Tab. 2).

The value of the cosine metric for vectors of coefficients in general is calculated as in (10):

$$\hat{Q} = \frac{\widehat{\beta_p^w}' \widehat{\beta_q^w}}{\|\widehat{\beta_p^w}\| * \|\widehat{\beta_q^w}\|}, \quad (10)$$

where $\widehat{\beta_p^w}, \widehat{\beta_q^w}$ are vectors of weighted coefficient estimates for clusters p and q, each element of a matrix is equal to a value of a coefficient multiplied by its weight $\hat{\beta}_{g\xi} * w_{g\xi},\ w_{g\xi} = \ln\left(\left|t_{g\xi}\right| + 1\right),\ \xi \in [1, k], g = (p, q), g\ \in \mathbb{G},\ \ t_{g\xi}$ is a value of t-statistic of the respective coefficient. As it is already noted for the cosine metric (8), the value of the cosine metric (10) is also positive if the majority of coefficients are of the same sign, especially those that are significant and make greater contribution into values of the dependent variable. One could suggest interpreting these cases as arguments in favor of similarities in models.

One could imagine a test for a hypothesis of a cosine metric being equal to zero against different alternatives: a value of a cosine metric being nonzero, positive or negative. A hypothesis of null equality of a cosine metric could be applied either to compare structural changes or models in general. In the first case, the result allows one to conclude about similarity of ways in which changes are transmitted to dynamics of dependent variables in different clusters. If a value of the cosine metric is close to zero, one considers structural changes in the models for clusters to be different. Economically speaking, the changes are associated with either different factors or different shifts in the influence of the factors. The closer a value of the cosine metric is to unity, the more similar structural changes are between the observed units. This means that changes are associated with similar shifts in the influence of the factors on the dependent variable. If a value of the cosine metric is close to (-1), one would rather speak about different structural changes, probably associated with similar factors, but their influence on the dependent variable changes in the opposite way. If a cosine metric is calculated for vectors of coefficients of two models, then when it is equal to zero, one could say that the models of the parameter are different by factor structure. A positive value of the cosine metric would suggest a positive correlation between vectors of coefficients, in other words, factors influence the dependent variable almost similarly. By a negative value of the cosine metric, one would suppose a negative correlation between vectors of coefficients, which could mean that the same factors impact the dependent variable oppositely. Nevertheless, this last result could also be insightful as it indicates that there are the same factors that contribute significantly to the dynamics of the dependent variable, which could occur in countries with similar level of economic and social development, political stability etc.

Theoretical distributions of the cosine metrics (9) and (10) depend on properties of data, model estimation method. That substantially complicates definition of the theoretical distribution. Given that it makes sense to apply a bootstrap method to estimate standard deviation and test statistics for the estimates of value of the cosine metrics (9) and (10). It seems reasonable to assume that a cosine metric $Q(A, B)$ meets the general requirements of bootstrap procedure [Efron, 1979]: the following facts allow to conclude about regularity of the metric (6):

- definiteness: $\forall A, B \in \mathbb{R}^L, \|A\| > 0, \|B\| > 0, \exists\ Q(A, B) \in\ \mathbb{R}$
- scale consistency: $\forall \alpha, \beta \neq 0, Q(\alpha A, \beta B) = Q(A, B)$
- smoothness: $\forall l \in [1, L], \forall p \in \mathbb{N}, \exists\ \frac{\partial^p Q}{\partial (A_l)^p}(A, B), \frac{\partial^p Q}{\partial (B_l)^p}(A, B)$

As it concerns distribution of values of a cosine metric in its application to regression coefficients` estimates, then the values of $\hat{Q}$ are identically distributed if the coefficients` estimates are obtained within the same model specification and by the same estimation method. If bootstrap subsamples are chosen independently, then the values of $\hat{Q}$ are independently distributed.

The following procedure is to be followed to conduct a null-equality test estimating the cosine metrics as in (9) and (10):

1) obtain consistent cluster division, break points estimates and coefficients` estimates with the use of the method by [Okui and Wang, 2021]; calculate values of $\hat{Q}$ for the pairs of clusters;

2) on subsamples before and after the structural break perform the moving block bootstrap [Goncalves, 2011] for pairs of values of the dependent variable and the regressors for clusters; obtain bootstrap estimates of the cosine metric Q^*; estimate standard deviation of the estimate $\hat{Q}$ as equal to the bootstrap estimate of standard deviation σ^* - standard deviation of the values of Q^*;

3) calculate a value of a test statistic on the observed data $\hat{S} = \frac{\hat{Q}}{\sigma^*}$

4) get a bootstrap distribution of values of the statistic for the test of null equality of the cosine metric in two steps

a) transform data such that the null hypothesis is correct for it: for the pair of vectors, for which a cosine metric is calculated, make such transformation of one element of a one of the vectors, so that the vectors are orthogonal[4]. Forecast values of the dependent variable for observations of one of the clusters with the new vector of the coefficients`.

b) on subsamples before and after the structural break perform the moving block bootstrap [Goncalves, 2011] for pairs of forecasted values of the dependent variable and the regressors for one of the clusters and the dependent variable and the regressors for another cluster. Obtain bootstrap coefficients` estimates, bootstrap estimates of the cosine metric θ^*, bootstrap distribution of the statistic and its quantiles.

[4] A change in value of a coefficient changes also a value of its t-statistic and a weight in the formula of the cosine metric. But if choose a coefficient that makes the greatest impact into value of a cosine metric and, among them, has the highest p-value, then small change in the value of this coefficient will, on the one hand, allow to achieve zero value of a cosine metric, on the other hand, will incur such a small change in the weight of the coefficient, that is equal to logarithm of t-statistics, that could be ignored. However, this approach could not guarantee a perfect zero-equality of the cosine metric calculated for the transformed data, that is why one could notice a bit of asymmetry in bootstrap distribution of the cosine metrics. In line with the abovementioned idea, one could choose a coefficient equal to an average fixed effect to be changed in order to achieve zero equality of the cosine metric. An average fixed effect coefficient turns out to be always significant, so that small changes in the value do not substantially alter its p-value and its weight used in the estimate of the cosine metric.

c) conclude about the hypothesis of the cosine metric being equal to zero by comparing the estimated value of the statistic calculated on the second step and the quantiles of its distribution obtained on the step 4b.

Speaking about interpretation of the results it seems reasonable to point out some peculiarities. Firstly, one of the main assumptions of the methodology is that dynamics of a parameter under consideration is described by the same model specification before and after a structural break. In other words, it seems reasonable to consider structural changes within the same economic model. Conversely, if significant institutional transformations take place that are associated with changes in economic mechanisms, then these make the results of the analysis of similarity of structural changes with the use of the cosine metrics misinterpretable. Secondly, another assumption of the methodology is that there are structural breaks in dynamics of a parameter. It is important to verify both theoretically and empirically that it is reasonable. Thirdly, it is necessary to ensure that an econometric model is specified correctly. Inappropriate model specification could lead to inconsistent estimates that will confuse the results of the analysis of similarities of structural changes. It also is important to pay attention to multicollinearity, otherwise the results could be misleading. Fourthly, one of the limitations of the procedure is the curse of dimensionality problem: the higher the dimension of the vectors, the greater are the chances that they are orthogonal. One could use that approach considering groups of factors that could also be suggestive from the point of the analysis of similarities in reaction of economies on shocks.

To conclude, it is important to stress that the procedure is assumed to be applied to analysis of economic data, that allows us to narrow a number of possible special cases and corner solutions to be considered. Moreover, it makes sense to combine the described procedure with the traditional approach to interpretation of regression estimates that involves necessary alignment with economic theory. The next section presents results of application of the described procedure to analysis of similarity of structural changes in dynamics of university enrollment rates.

3. Empirical results

Countries differ significantly by institutional structure, educational policy and many other factors. For instance, there are systems with less government involvement and a greater role of market mechanisms in determining the cost of education and the number of educational placements, as seen in the United States and the United Kingdom. In contrast, other systems exhibit stronger government regulation, including requirements for educational programs and the availability of state-funded placements, as in Russia and certain European countries. Nevertheless, we would like to argue that there are still similarities between them, in particular, in the dynamics of enrollment rates both within and between the groups of countries.

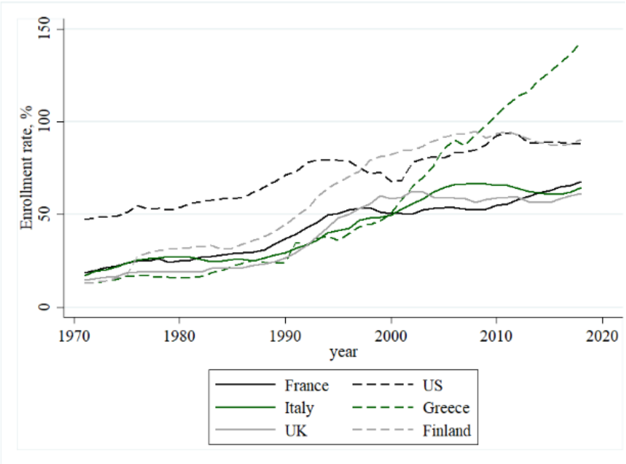

Fig. 1. University enrollment rate dynamics among the developed[5] countries

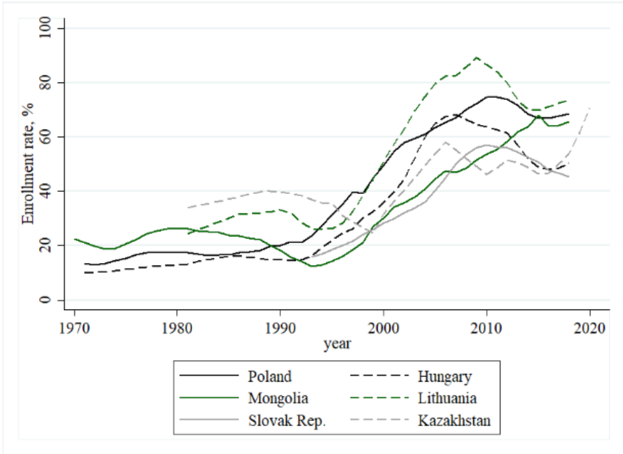

Fig. 2. University enrollment rate dynamics among the developing countries

As it is evident from figures 1, 2, almost all countries experience acceleration of growth rate of university enrollment at the end of the century, that begin to decelerate until 2010. It seems that in developed countries growth rate of university enrollment becomes higher earlier, at the end of the 1980s, whereas in developing countries it starts later, in the 1990s. In the 2000s, growth rate of enrollment among the developed countries becomes more moderate, in some of the developing countries it becomes even negative. Schofer and Meyer [Schofer and Meyer, 2005] pay attention to similarity in dynamics of the parameter among countries and come up with the hypothesis that there are common drivers of the changes in the dynamics, namely, technological changes and globalization. They increase demand for a highly qualified workforce in the 1990s, driving up demand for university degrees, when the pace of technological changes becomes more moderate and the trend towards globalization starts to change, growth rates of economies decrease, the pace of growth of demand for higher education also slows. Although we cannot check the hypothesis of common drivers of the acceleration of growth of university enrollment in the 1990s due to limitations of the data at hand, we verify the hypothesis indirectly through analysis of structural changes in the model in the 2000s. If the acceleration of growth of the parameter in the 1990s-2000s is associated with the same factors in countries, then in the period of deceleration of the

[5] We single out developed countries (developed economies) and developing ones (emerging market and developing economies) according to the level of economic development suggested by the IMF: "IMF World economic outlook. A long and difficult ascent", oct. 2020, pp. 121.

pace of growth of the parameter, the impact of those factors will naturally decrease. All in all, the above arguments support the relevance of examining similarities in models of university enrollment dynamics across countries.

Growth rate of university enrollment is influenced by both demand-side and supply-side determinants. The former include household income, demand for labor among university graduates and individuals with higher education, and relative wages of university graduates compared to individuals without higher education [Toutkoushian and Paulsen, 2016]. In countries with less government involvement in the education system, supply-side factors might include average cost of higher education, number of educational institutions, and level of competition among institutions offering different levels of service quality [Toutkoushian and Paulsen, 2016]. In countries where the government plays a substantial role in education policy, the supply of educational services depends largely on the level of government expenditures on higher education. This, in turn, is influenced by national income, domestic and foreign policy objectives, and the needs of industries for highly skilled professionals [Toutkoushian and Paulsen, 2016]. Technological advancements and market globalization in the late 20th century likely impacted the dynamics of youth access to higher education, resulting in accelerated growth rates in university enrollment across nearly all countries. It can be assumed that the effects of these shocks on higher education systems differ between countries with varying levels of government involvement. In countries with limited government involvement, the emergence of new technologies and globalized markets primarily impacts the demand for high-skilled labor [Acemoglu and Autor, 2011], increasing the relative wages of this group and, subsequently, the demand for higher education, which then prompts a supply response. In contrast, in countries where the state has greater involvement in the higher education system, technological advancements and market globalization likely affect not only demand but also the supply of educational services. Growing participation in international trade increased corporates` revenues, boosts tax revenues and government budgets, enabling higher public spending on higher education. Expanding access to the internet at the end of 1990s – beginning of 2000s could rise the share of people informed about higher education opportunities, enhance access to it through availability of online application procedures, and contribute to perception of popularity of higher education through social networks. Increasing internet coverage could also give access to information about vacancies and skills required, ensuring an enhanced view of skills required and possible working opportunities, which could contribute to growing demand for higher education. Moreover, rising internet coverage could also facilitate emergence of new jobs directly related to online work, such as web development, which could also require high skills and university education. All in all, the regressors in the model for the growth rate of university enrollment include the growth rate of internet access, growth rates in high-, middle-, and low-skilled employment, to capture the influence of technological development on the economy. Including GDP per capita in the list of regressors makes it possible to account for income as a factor of demand for higher education. At the same time, values of GDP per capita are correlated with values of government expenditures on higher education, which is a supply factor in countries with a high level of government participation in the higher education system. It seems reasonable to account for lags in decisions of economic agents. In particular, it can be assumed that the correlation between employment structure dynamics and the growth in university enrollment rates among youth manifests with a lag of two or more periods, due to delays in employers' responses to shifts in labor demand and in individuals' educational decision-making. Since both private and public investments in education are long-term, it is likely that university enrollment rates among youth correlate more strongly with lagged values rather than the current level of real GDP per capita.

Suppose that dynamics of growth rate of university enrollment from 1995 to 2020 is formed within a model with one structural change[6]. Assume that there are clusters of countries such that the dynamics of the parameter are formed within the same model in countries of the same cluster. The model specification is considered in the following form:

$$enr_{it} = \alpha_i + \beta_1 H_{it-2} + \beta_2 M_{it-2} + \beta_3 Int_{it-1} + \beta_4 Int_{it-2} + \beta_5 GDPpc_{it-1} + \beta_6 GDPpc_{it-2} + \gamma_0 D_t + \gamma_1 H_{it-2} D_t + \gamma_2 M_{it-2} D_t + \gamma_3 Int_{it-1} D_t + \gamma_4 Int_{it-2} D_t + \gamma_5 GDPpc_{it-1} D_t + \gamma_6 GDPpc_{it-2} D_t + \varepsilon_{it}, \quad (11)$$

where i – country identifier, t – time, enr – growth rate of university enrollment, H, M – growth rates of shares of high and middle skilled workers respectively, Int – growth rate of share of people with access to internet, $GDPpc$ – growth rate of GDP per capita, D – dummy variable of the time of structural break, =1 after a structural break, =0 otherwise.

The data of 91 countries for the period 1995-2020 from the databases of the World bank (https://data.worldbank.org) and the International Labor Organization (https://ilostat.ilo.org/) is

[6] One could imagine that there are more than one structural change in the dynamics of the parameter in some countries, for example, the first is connected directly with the dynamics of university enrollment rates and the second, connected with the global financial crisis in 2008-2009. First and foremost, we remain with the assumption of one structural break in the model of the parameter because that is required for the procedures exploited in the research: the procedure of Okui and Wang, 2021, that estimates moments of structural breaks and cluster division, requires that the dataset is large enough, otherwise, the procedure runs out of observations too often. The same is applied to the bootstrap procedure of Goncalves and Kaffo, 2015. That is why we limit the number of structural changes to be considered to one. Secondly, it also matters to notice that it could appear that there are no structural breaks. With the use of the IC applied within the procedure of Okui and Wang, 2021, we get evidence of one rather than no structural changes for all of the clusters. Nevertheless, we argue that the model for dynamic of university enrollment rate is minimally affected by the assumption of a single structural change. Educational system is relatively rigid, and changes in educational policy are infrequent. Moreover, the parameter offers some advantages over other macroeconomic parameters: its dynamic is only indirectly influenced by the financial crisis and migration waves, and these effects typically emerge only in the long term. Given the reasoning above, we choose to remain considering the dynamics of university enrollment rate to illustrate the procedure described in section 2.

used in the current research for the purpose of the analysis of similarity of models of university enrollment rate dynamics.

Following the procedure of [Okui and Wang, 2021], we have considered possible divisions of countries into 2-5 clusters and found that the best way to describe dynamics of university enrollment in countries is with three clusters. Having identified quite sustainable cluster division, we label the clusters as “higher HCI”, “lower HCI” and “least HCI” (Tab. 10 in Appendix). The first cluster consists of 38 countries, capturing nearly all economically developed nations. Most countries in this group have a Human Capital Index[7] (HCI) above 0.70 and maintain resilient educational systems that are largely insulated from economic shocks. The second cluster is made up of 21 developing countries, where HCI values mostly range from 0.5 to 0.7. In these countries, university enrollment rates among youth could be more susceptible to macroeconomic fluctuations. The third cluster, comprising 8 developing countries, has the lowest average HCI. Many of these nations experience political instability, leading to considerable fluctuations in youth enrollment rates, as they face numerous economic and political challenges.

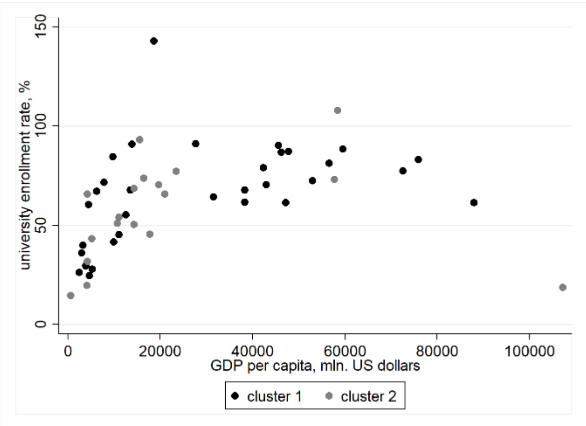

Fig. 3. Distribution of countries by clusters; values of university enrollment rate and GDP per capita in 2018

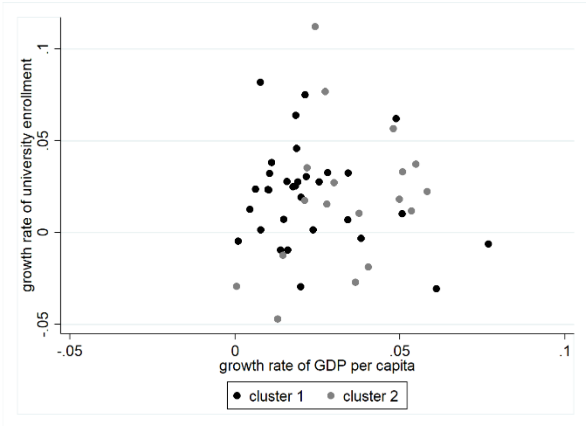

Fig. 4. Distribution of countries by clusters; growth rates of university enrollment and GDP per capita in 2018

The obtained distribution of countries by clusters seems to be justified. Firstly, almost all economically developed countries remain in one cluster (cluster 1 “higher HCI”). The same can be said about post-Soviet countries and post-communist countries of Eastern Europe (cluster 2 “lower HCI”). Secondly, as evident from fig. 3, countries with the highest level of GDP per capita are also in the same cluster. In fig. 4 one might notice that the grey cloud shifts towards right in

[7] We consider values of Human Capital Index in 2020. URL: https://ourworldindata.org/grapher/human-capital-index-in-2018?tab=table

comparison to the black cloud. Thus, the former illustrates countries with higher growth rates of GDP per capita, in particular developing ones, the latter mostly covers developed ones. The estimates of coefficients and moments of the structural breaks in the two clusters are presented in table 4 and are further used for the purpose of analyzing similarity of structural changes using the methodology proposed in the section 2.

Tab. 4. The coefficients`, break point estimates in the model (11) and cluster division obtained within the procedure [Okui and Wang, 2021]

	"Higher HCI"		"Lower HCI"		"Least HCI"	
	1995-2009	2010-2020	1995-2009	2010-2020	1995-2004	2004-2020
H_{t-2}	-0.12***	-0.24***	-0.08	0.10**	-	0.14
M_{t-2}	-0.15	-0.17**	0.007	0.08	-	1.67***
Int_{t-1}	0.02***	0.003	0.01	0.05**	-	0.09
Int_{t-2}	0.02***	-0.02	-0.01	0.04*	-	0.25***
$GDPpc_{t-1}$	-0.06	-0.005	-0.07	0.004**	-	0.01
$GDPpc_{t-2}$	-0.07	-0.002	0.12	0.0003	-	-0.008
$const$	0.01***	0.03***	0.06***	-0.01***	-	0.02*
$p-value(F)$	0.005	0.000	0.009	0.010	-	0.84
Number of observations	306	323	185	185	-	74
Number of countries	34	37	19	26	-	7
Wald test: H_0: no structural changes H_1: not H_0	H_1		H_1		-	
*** - 5%, ** - 10%, *- 15% level of significance						

The clustering procedure offered by Okui and Wang (2021) suggests a stable division into three clusters when the number of clusters is selected according to the BIC criterion. This provides evidence of underlying similarities in the models describing university enrollment rate dynamics among countries within each cluster. The estimated timing of structural breaks is the same for two of the three clusters, which include the majority of the countries. Generally, this finding contradicts both empirical evidence—such as the earlier start of higher education expansion in economically developed countries and the later expansion in developing ones (see Figs. 1, 2) - and the significant institutional differences across countries. At the same time, this result may reflect an overcrowding of the first cluster, labeled "higher HCI", which includes not only nearly all economically developed countries but also some developing ones. It may also be the case that university enrollment rate dynamics in certain countries are relatively insulated from the 2008–2009 economic crisis, which could have affected the estimated timing of structural breaks, particularly within the densely populated first cluster. It seems reasonable to draw attention to the third cluster named "least HCI" countries, where the estimated structural change in university enrollment dynamics occurs around 2004. The second period for this group of countries appears to coincide with political and economic stability, factors that boost demand for both labor and education. This likely explains the closer relationship between university enrollment dynamics and factors such as internet access and middle-skill employment trends in these countries. Notably, internet access expansion significantly influences university enrollment growth across all three clusters, albeit at different points in time. Differences in the timing of this effect may be attributed to the responsiveness of economic agents and the rigidity of social institutions. Furthermore, variations in the relationship between labor demand and higher education demand could account for the differences in coefficient signs for variables that track skill-level employment dynamics. In summary, identifying a stable cluster division does suggest meaningful similarities across countries. Once clusters are defined, similarities in the dynamics of higher education enrollment remain evident. However, the current division does not fully capture certain stylized facts about individual countries, possibly due to the small number of clusters. Nevertheless, it is worth noting that an alternative division into four clusters, which is the next-best option according to the BIC criterion [Okui and Wang, 2021], provides neither a clearer distinction among the countries within the "higher HCI" cluster nor substantially different estimates. Thus, it seems reasonable to consider alternative approaches to grouping countries to better highlight the similarities and differences in factors influencing university enrollment rates. Considering groups of developed and developing countries as suggested by the IMF (Tab. 11 in Appendix) the following estimates of coefficients and moments of structural changes are obtained (Tab. 5).

Tab. 5. The coefficients`, break point estimates in the model (11) obtained within the procedure [Baltagi et al., 2017] for developed and developing countries

	Developed		Developing	
	1995-2003	2004-2020	1995-2009	2010-2020
H_{t-2}	-0.06	0.18**	-0.12	-0.07
M_{t-2}	0.01	0.32***	-0.10	-0.07
Int_{t-1}	0.001	0.07***	0.01	0.02
Int_{t-2}	-0.003	0.03***	0.01**	-0.0005
$GDPpc_{t-1}$	-0.22	0.002	0.15	0.0005
$GDPpc_{t-2}$	0.27**	-0.003	-0.02	-0.0007
$const$	0.06***	0.01***	0.03***	0.02***
$p-value(F)$	0.000	0.001	0.000	0.000
Number of observations	168	448	191	260
Number of countries	30	30	27	40
Wald test: H_0: no structural changes H_1: not H_0	H_1		H_1	
*** - 5%, ** - 10%, *- 15% level of significance				

The results are rather controversial compared to those presented in table 4, which could point out diversity of countries in both groups of developed and developing countries and clusters. On the one hand, the results reflect the differences in moments of a structural change between the two groups of countries that matches previously mentioned empirical observations. On the other hand, the results demonstrate a common feature of the models of growth of university enrollment estimated for clusters and groups of countries by economic development. In particular, one could again notice positive and significant influence of expanding access to the internet on growth rate of university enrollment in both groups of countries with the value of the coefficient varying in the interval of 0.01-0.07. The results also highlight one possible distinctive feature of economically developed countries that consists in a tighter connection between employment dynamics and dynamics of university enrollment rates in these countries compared with economically developing countries. Nevertheless, the further results (Tab. 6) demonstrate that there is still much diversity in each of the groups of economically developed and developing countries: we single out countries with the most developed democracy institutions and other developed countries (Tab. 11 in Appendix), post-communist countries (Tab. 12 in Appendix) and other developing countries.

Tab. 6. The coefficients`, break point estimates in the model (11) obtained within the procedure [Baltagi et al., 2017] for countries with the most developed democracies and developing countries

	Democracies		Other developed	
	1995-2005	2006-2020	1995-2003	2004-2020
H_{t-2}	0.07	0.20	-0.11	0.22**
M_{t-2}	0.09	-0.002	0.04	0.60***
Int_{t-1}	0.01**	-0.08	-0.01	0.11***
Int_{t-2}	-0.001	-0.07	-0.002	0.03**
$GDPpc_{t-1}$	-0.69***	0.002	0.03	0.002
$GDPpc_{t-2}$	0.53**	0.002	0.23	-0.005
$const$	0.04***	0.01***	0.07***	0.01***
$p-value(F)$	0.000	0.017	0.000	0.001
Number of observations	122	202	70	222
Number of countries	17	16	13	14
Wald test: H_0: no structural changes H_1: not H_0	H_1		H_1	

*** - 5%, ** - 10%, *- 15% level of significance

According to the estimates of moments of structural breaks the most developed democracies face structural changes in dynamics of university enrollment later than other developed countries, that could probably occur because of more stability of society and institutions in the first group of countries. One could again observe a positive correlation between growth of internet coverage and growth of university enrollment, although observed in different time periods in the groups of countries. Along with economic growth in almost all countries worldwide in the beginning of the 2000s dynamics of university enrollment shows stronger correlation with dynamics of GDP per capita in countries with strongest democracy institutions than in other developed countries. The difference could possibly be explained by more or less stability of labor market situation. Given much stability of it in countries with stronger democracy institutions, economic growth stimulates demand for higher education through increasing availability of grants, stipendiums, rising governmental support of higher education. Among other developed countries economic growth results in rising investments and increasing demand for labor, that leads to growth of demand for education. The sign of the correlation between growth rate of university enrollment and growth rate of GDP per capita is estimated to be negative with the first lag of the factor variable and positive with the second lag of the factor variable in the countries with the most developed democracy institutions. This could be explained by the fact that acceleration of growth of GDP per capita leads to higher inflation, so that higher education becomes unaffordable for some groups of people. When the wages and wage expectations adjust to a new level of inflation, then getting higher education becomes again a profitable strategy. Paying attention to correlation between dynamics of employment by skill levels and dynamics of enrollment in other developed countries, one would notice the greatest estimate for the variable of growth rate of middle skill employment. This is also the case when the regression is estimated for all developed countries (Tab. 5). The possible explanation for this could be that middle skill employment is considered as a potential employment for students that opt for working during studies or potential employment in case of dropout.

Tab. 7. The coefficients`, break point estimates in the model (11) obtained within the procedure [Baltagi et al., 2017] for post-communist and other developing countries

	Post-communist		Other developing	
	1995-2010	2011-2020	1995-2005	2006-2020
H_{t-2}	-0.18*	-0.33***	-0.28**	-0.01
M_{t-2}	-0.03	-0.10	-0.38*	0.10

Int_{t-1}	0.02***	0.01	0.006	0.01
Int_{t-2}	0.02***	0.05	0.007	0.005
$GDPpc_{t-1}$	0.29***	0.001	-0.25	-0.0004
$GDPpc_{t-2}$	-0.22*	-0.002	0.16	0.003
$const$	0.03***	0.01***	0.04***	0.03***
$p-value(F)$	0.000	0.000	0.000	0.18
Number of observations	176	164	54	199
Number of countries	18	19	14	28
Wald test: H_0: no structural changes H_1: not H_0	H_1		H_1	
*** - 5%, ** - 10%, *- 15% level of significance				

Having singled out post-communist countries among economically developing countries we also observe heterogeneity within the group (Tab. 7). The estimate of a structural breakpoint appears to be later for post-communist countries than for the rest of developing countries. It could be explained with the change in political regimes or beginning of period of political stability and economic prosperity: in all considered post-communist countries the regime had changed by 1993, whereas in most of the rest developing countries it came to changes in regime and political stability only in 2000s – this is caught by the estimate of the structural breakpoint. Regarding post-communist countries, the estimate of point of a structural break is in 2010. This is in line with the economic theory that states that after transformations in political regimes and economic systems at the end of the 90s, the beginning of the 2000s was the time of formation of institutions and markets accompanied by fast economic growth. This grows was probably interrupted by the world financial crisis in 2008, followed by structural changes in model of growth of university enrollment. One might notice that fast economic growth was one of the drivers of high growth rates of university enrollment in post-communist countries in the 2000s, as well as growing access to the internet. Economic growth means higher households` income, making higher education more affordable. Financial support of education from government is also traditionally greater in periods of economic growth, which also contributes to greater availability of higher education, more educational places, higher quality of education etc. Negative coefficient for growth rate of

GDP per capita could be related to the influence of inflation, which makes getting higher education either financial unavailable or financially irrational for some groups of applicants or students. Negative relationship between growth rate of high skill employment and university enrollment rates could be explained by an oversupply of employees with university education, erosion of higher education standards, low higher education wage premiums.

Tab. 8. Comparison of *models* for growth rate of university enrollment between the groups of countries with the use of cosine metric for vectors of coefficient estimates

	Clusters[8]	Developed and developing	Democracies and other developed	Post-communist and developed	Democracies and post-communist	Other developed and post-communist	Other developing and post-communist
$\hat{Q}$	-0.44	-0.37	0.13	-0.79	-0.73	-0.29	-0.09
$\hat{\sigma}(\hat{Q})$	0.17	0.32	0.30	0.33	0.28	0.25	0.18
St.	-2.59	-1.15	0.43	-2.39	-2.60	-1.16	-0.5
$q_{5\%}$	-1.53	-2.10	-0.86	-0.82	-0.70	-1.38	-0.65
$q_{95\%}$	1.92	1.21	2.05	2.03	1.50	1.6	2.50
$H_0: Q = 0$ $H_1: Q < 0$ $H_1: Q > 0$	$H_0: Q = 0$	$H_0: Q = 0$	$H_0: Q = 0$	$H_1: Q < 0$	$H_1: Q < 0$	$H_0: Q = 0$	$H_0: Q = 0$
Wald test: H_0: models are equal H_1: not H_0	H_1	H_1	H_1	H_1	H_1	H_1	H_1

Having the metric calculated for models in general, which implies measuring the angle between vectors of weighted coefficients, we obtain the results presented in table 8. First and foremost, one could notice that the results of the cosine metric test can substantially enrich the results obtained within the traditional Wald test for equality of models. As long as the Wald test

[8] The cosine metric is calculated for clusters of “higher HCI” and “lower HCI” countries (Tab. 10)

states that the models of growth rate of university enrollment are different between all groups under consideration, the cosine metric test can indicate whether the vectors of coefficients in the models are either orthogonal or negative, or positive. The former means that values of the dependent variable form under the influence of different lists of factors. When value of the cosine metric is either positive or negative, it could suggest that the factor structure has similarities, but impact of most of factors on the dependent variable is either similar or opposite in sign. The results suggest that post-communist countries have much more similarities with developed countries than with other developing countries. In particular, dynamics of university enrollment is tightly connected with economic dynamics both in developed countries and post-communist. In particular, economic growth in the 2000s leads to an increase of governmental support of higher education, rising availability of grants and stipendiums, so that dynamics of GDP per capita turns out to be the factor with the strongest contribution into dynamics of university enrolment both in countries with the strongest democracy institutions and post-communist countries. Moreover, both in post-communist and developed countries dynamics of shares of high and middle skilled employment correlates with dynamics of university enrollment but with different signs. The test for null-equality of the cosine metric provides evidence for a difference between post-communist and other developing countries: while economic factors define dynamics of university enrollment in the former, political stability and strength of social institutions probably play the most important role in the latter. Moreover, the results in table 8 highlight differences between economically developed countries, in particular between countries with the most developed democracy institutions and other developed countries as discussed earlier (Tab. 6).

One could also imagine checking not only similarity of vectors of coefficients in models but also similarity of structural changes by applying the cosine metric to vectors of values of relative weighted coefficients before and after a structural change. This could allow one to get an estimate of similarity of structural changes given significancy of coefficients. Applying this method to models of dynamics of university enrollment, comparison of relative changes in values of coefficients suggests that structural changes are mainly connected with different factors (Tab. 9).

Tab. 9. Comparison of *structural changes* in models for growth rate of university enrollment with the use of cosine metric for relation of coefficients before and after a structural change

	Clusters [9]	Developed and developing	Democracies and other developed	Post-communist and developed	Post-communist and democracies	Post-communist and other developed	Post-communist and other developing
$\hat{Q}$	-0.07	-0.28	-0.28	-0.48	-0.09	-0.43	-0.02
$\hat{\sigma}(\hat{Q})$	0.35	0.41	0.44	0.38	0.49	0.43	0.29
St.	-0.20	-0.68	-0.64	-1.26	-0.18	-1.00	-0.07
$q_{5\%}$	-2.15	-1.43	-1.87	-1.92	-1.54	-1.60	-1.89
$q_{95\%}$	1.86	1.82	1.74	1.68	1.65	1.67	1.83
$H_0: Q = 0$ $H_1: Q < 0$ $H_1: Q > 0$	$H_0: Q = 0$	$H_0: Q = 0$	$H_0: Q = 0$	$H_0: Q = 0$	$H_0: Q = 0$	$H_0: Q = 0$	$H_0: Q = 0$
Wald test: H_0: models are equal H_1: not H_0	H_1	H_1	H_1	H_1	H_1	H_1	H_1

Upon closer consideration, one could notice that structural changes in almost all of the models concern the influence of dynamics of the internet coverage. In democracies and post-communist countries, enhancing access to the internet contributes to dynamics of university enrollment in the first period and makes no contribution in the next period. In other developed countries, the situation is opposite, which means no substantial contribution to dynamics of the dependent variable in the first period and significant contribution in the second. It is important to notice that the rate of growth of access to the internet in other developed and post-communist countries is extremely high in the period before a structural break (0.86 and 0.78 in the groups respectively). One could assume that this enhancement had much more postponed effect than the two-period lag that is accounted for in the model. The internet promotes education through

[9] The cosine metric is calculated for clusters of “higher HCI” and “lower HCI” countries (Tab. 10)

university web pages, university communities etc., and extends access to information about vacancies, professions etc. contributing a lot to growth rate of university enrollment in the second period. At the same time, despite high growth rates of internet coverage in the first period in post-communist countries, we do not observe its significant influence on dynamics of university enrollment rates in the second period in this group of countries. This could suggest that impact of the internet coverage on dynamics of university enrollment is mediated by employment dynamics. It seems that in other developed countries, rates of enrollment are more tightly connected with dynamics of employment than in post-communist countries. Then growth of internet coverage in the first group of countries significantly increases employment rates and enhances a better match between skills and jobs, which contributes to an increase in wage premiums, eventually leading to rising demand for higher education. In post-communist countries, increase in internet coverage also leads to a rise in job advertisements on the internet but may be due to a slower workflow, it does not contribute to a better match between skills and job and does not significantly adjust skill premiums. That is why the effect on university enrollment rates is not as great. As expected, dynamics of access to the internet plays no significant role in dynamics of university enrollment in other developing countries.

Thus, having singled out the groups of countries to be much different from each other by definition – by clusterization, by economic development, by institutional structure, by historical background, we have ended up finding with some similarities. It seems interesting to find that, as factor structure of dynamics of university enrollment is concerned, post-communist countries have more similarities with economically developed countries, in particular countries with strongest democracy institutions, than with other developing countries. One of the reasons could be that post-communist countries considered in this research entered a period of political stability and economic growth in the 2000s, having also much business relations with developed countries and being involved into many agreements with developed countries. Despite the fact that structural changes in dynamics of university enrollment rate in the 2000s are mainly different, growth of public access to the internet is one of the common factors that contributes positively to dynamics of university enrollment rates.

Conclusion

The research suggests a methodology for approaching a hypothesis about similarity of dynamic panel data models with structural changes. As it concerns models` similarity, the traditional tests offer an opportunity to check the hypothesis about equality of coefficients` vectors. We argue that in the case of rejection of the hypothesis, one could gather more information by trying to get deeper into the nature of heterogeneity by applying the test using the cosine metric. Then, it could turn out that despite the models being quite different, there is still much in common that could significantly enrich the theory.

Assuming a cluster structure among panel data units and structural changes in dynamics of a considered parameter, we propose to compare weighted vectors of coefficients on the basis of the cosine metric, testing the hypothesis of its equality to zero against different alternatives. If there is no reason to reject the null hypothesis, values of the dependent variable are formed within different models. If the cosine metric is positive, the factor structures of the dependent variables have similarities and the factors` contributions for most of the coefficients are similar in sign. Since distribution of values of the cosine metric depends on many parameters and is challenging to verify theoretically, it seems reasonable to apply a bootstrap to the hypothesis testing. In the case of panel data, one can use the moving block bootstrap procedure [Goncalves, 2011], since it is applicable in cases of cross-sectional and time dependence.

The methodology of testing for similarity of structural changes is applied to the issue of similarity of drivers of university enrollment rate dynamics in countries. It is suggested in the literature [Schofer and Meyer, 2005] that technological development and globalization are the common factors for growth of university enrollment rates in both groups of countries at the end of the century and the beginning of the 2000s. Technological development drives up the comparative price of high skills and, consequently, demand for higher education. This period is also characterized by a high speed of computerization in offices, households, and at schools, as well as growing access to the internet. These tighten the world and enrich information flow, also between employers and potential employees, universities and applicants. Among others, it influences relative wages and raises wage premiums for high skills [Karz and Murphy, 1992], [Taber, 2001], contributing to the growing demand for higher education. Globalization enhances international competition, which also gives rise to demand for high skills, their comparative price and demand for higher education. Both technological development and globalization also stimulate aggregate supply and economic growth, which means also higher households` income and higher demand for higher education. In order to verify the hypothesis about similar drivers of higher education expansion, we single out homogeneous clusters. We first interpret the results in general, highlighting similarities and differences, and then enrich the results with the cosine metric tests for vectors or weighted coefficients and vectors of relative weighted coefficients before and after a structural change.

Within the procedure of [Okui and Wang, 2021], we identify three clusters of countries, homogeneous by the model of dynamics of university enrollment rates: so-called “higher HCI”, “lower HCI” and “least HCI” countries. The procedure from [Okui and Wang, 2021] allows to obtain the estimates of points of a structural break in 2010 in the clusters of “higher HCI” and “lower HCI” countries and in 2005 among the “least HCI” countries. The fact that a sustainable cluster division is obtained and the clusters and structural break points are mainly theoretically justified suggests that, generally, there are similarities in dynamics of university enrollment rates. Having obtained the coefficients` estimates before and after a structural break in the clusters, we single out a growth rate of access to the internet as a common factor with a significantly positive impact on values of the dependent variables. The cosine metric test identifies significant differences in factor structure of the dependent variable in the clusters.

In order to shed more light on the nature of similarities and diversity between countries in dynamics of university enrollment, we divide countries by level of economic development, strength of democracy institutions and historical aspect. This allows us to highlight heterogeneity among countries with the strongest democracy institutions and other economically developed countries related to the role of dynamics of employment shares and dynamics of GDP per capita. It seems that in countries with the strongest democracy institutions, demand for labor demonstrates more stability and employment seems to be more secured by labor unions and other institutions. In contrast to other economically developed countries, in these countries dynamics of university enrollment is not significantly related to growth of relative employment with high or middle skills. When developing countries are considered with exception of post-communist countries, peculiarities of the factor structure are also observed: there are almost no factors demonstrating significant relationships with dynamics of university enrollment rates. This means political factors, quality of institutions or any other not considered factors play much greater role in dynamics of the parameter under consideration than factors of labor market, access to the internet or general economic growth. The test based on the cosine metric provides evidence that the factor structure of dynamics of university enrollment in post-communist countries is much closer to that of countries with the strongest democracy institutions than to other economically developing countries. That seems quite justified, given the high level of coordination, agreements etc. between post-communist countries and countries with the strongest democracy institutions.

To conclude, we would like to note that when it concerns heterogeneity, it could turn out that there is more or less of it between countries. Even if the objects differ, the proposed test assesses the extent of their divergence, helping to determine whether the models are orthogonal or exhibit some degree of similarity. Unlike traditional tests that only highlight differences, this approach allows for a deeper examination, potentially uncovering meaningful similarities that enhance the results and conclusions.

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Appendix

Tab.10. Cluster division obtained within the procedured of Okui and Wang [Okui and Wang, 2021]

Cluster 1 “Higher HCI” (38)		
Argentina	Georgia	Panama
Austria	Germany	Peru
Azerbaijan	Honduras	Philippines
Belgium	Ireland	Portugal
Bulgaria	Jamaica	Russian Federation
Canada	Latvia	Serbia
Chile	Malaysia	Spain
Costa Rica	Mali	Sweden
Croatia	Mexico	Switzerland
Denmark	Moldova	United Kingdom
Estonia	Morocco	United States
Finland	Netherlands	Vietnam
France	Norway	
Cluster 2 “Lower HCI” (26)		
Australia	Italy	Slovak Republic
Belize	Kazakhstan	Slovenia
Brazil	Lithuania	Sri Lanka

El Salvador	Luxembourg	Thailand
Greece	Mongolia	Togo
Hungary	North Macedonia	Tunisia
Iceland	Paraguay	Ukraine
Indonesia	Poland	Uruguay
Israel	Romania	
Cluster 3 “Least HCI” (8)		
Albania	Namibia	
Cyprus	Pakistan	
Malta	Rwanda	
Mauritius		

Tab. 11. The list of economically developed countries

Contries with the strongest democracy institutions[10] (17)		
Austria	Ireland	United States
Australia	Spain	Finland
United Kingdom	Canada	France
Germany	Luxemburg	Switzerland
Denmark	Netherlands	Sweden
Iceland	Norway	
Other economically developed countries (16)		
Albania	Italy	Korea Rep.
Belgium	Cyprus	Slovak Republic
Greece	Latvia	Slovenia
Israel	Lithuania	Croatia

[10] Countries that constitute the upper half of the rating of countries by the values of the democracy index in 2020 calculated by The Economist Intelligence Group. URL: Democracy index, 2019 (ourworldindata.org)

	Malta	Czech Republic
	Portugal	Estonia

Tab. 12. The list of post-communist countries by level of economic development[11]

Economically developed (6)		
Latvia	Slovak Republic	Czech Republic
Lithuania	Slovenia	Estonia

Developing economies (15)		
Azerbaijan	Kazakhstan	Serbia
Albania	Moldova	North Macedonia
Bolgaria	Poland	Tajikistan
Hungary	Russian Federation	Ukraina
Georgia	Romania	Croatia

Andrey G. Maksimov
National Research University Higher School of Economics (Nizhny Novgorod, Russia). Professor of the Chair of Economic Theory and Econometrics;
E-mail: amaksimov@hse.ru,

Marina S. Telezhkina
National Research University Higher School of Economics (Nizhny Novgorod, Russia). Former senior lecturer of the Chair of Economic Theory and Econometrics;
E-mail: m.s.telezhkina@gmail.com

[11] There are only post-Soviet countries and East European countries which data is considered in the empirical part of the current research. Other post-communist countries are not considered because of the lack of data for the observed parameters.