HIGHER SCHOOL OF ECONOMICS
NATIONAL RESEARCH UNIVERSITY

Claudia M. Landeo, Maxim Nikitin, Sergei Izmalkov

OPTIMAL CIVIL JUSTICE DESIGN

Working Paper WP9/2024/02
Series WP9
Research of economics and finance

Moscow
2024

Editor of the Series WP9
"Research of economics and finance"
Maxim Nikitin

Claudia M. Landeo, Maxim Nikitin, Sergei Izmalkov.

Optimal Civil Justice Design [Electronic resource]: Working paper WP9/2024/02/ Claudia M. Landeo, Maxim Nikitin, Sergei Izmalkov; National Research University Higher School of Economics. – Electronic text data (700 Kb). – Moscow: HSE Publishing House, 2024. – (Series WP9 "Research of economics and finance"). – 128 p.

We study the optimal civil justice design in an environment with two-sided private information and two players, a victim and an injurer. Using a mechanism design approach, we identify the properties that must be satisfied by an optimal civil justice system to ensure access to justice and maximal compensation to the victims at the minimum expected cost of producing evidence, and characterize the optimal cost-allocation rule. Our main findings are as follows. First, full revelation of private information requires the production of evidence in just a subset of legal cases. Second, the American rule where each party pays his own cost of producing evidence arises endogenously as the socially optimal cost-allocation rule only under certain conditions. Third, a tort reform that implements the proposed mechanism in real-world settings is feasible.

JEL Categories: C72, D86, K10, L23.

Keywords: Civil Justice; Evidence; Tort Reform; American Rule; English Rule; Mechanism Design; Two-Sided Private Information; Access to Justice; Victim's Compensation; American Civil Justice System.

Claudia M. Landeo[*] – University of Alberta, Department of Economics; Henry Marshall Tory Building 7-25, Edmonton, AB T6G 2H4, Canada; landeo@ualberta.ca

Maxim Nikitin – International College of Economics and Finance, HSE University; 11 Pokrovskiy Boulevard, Moscow, Russia; mnikitin@hse.ru

Sergei Izmalkov – New Economic School; 3 Nobel St., Skolkovo, Moscow, Russia; sizmalkov@nes.ru

[*]We thank Emiliano Catonini, Tatiana Mayskaya, Peio Zuazo-Garin and Jack Ochs for insightful discussions and comments. We are grateful for suggestions from participants at the 2024 Stony Brook Center for Game Theory Festival and the HSE University Microeconomic Theory Workshop.

1 Introduction

The U.S. tort system provides $172 billion in gross compensation to the plaintiffs each year. A significant portion of this compensation represents lawyers' fees and other costs of producing evidence (Engstrom, 2014).[1] Although the production of evidence is a core component of any civil justice system and the cost of producing evidence influences the actual compensation received by the victims, previous theoretical work on civil litigation abstracts from the analysis of the optimal production of evidence.[2] Our paper contributes to the law and economics literature by presenting the first application of mechanism design to the optimal design of the civil justice system.[3] The fundamental goals of the civil justice system and the optimal production of evidence are considered in our design. We demonstrate that the optimal civil justice mechanism minimizes the social welfare loss associated with an accident by providing access to justice to the victims and maximal compensation to the victims confronting liable injurers at the minimum expected

[1] In civil litigation, the cost of producing evidence encompasses the litigation costs and the lawyers' fees. Litigation costs include the costs associated with the production of information by each party and the legal discovery process (i.e., the formal process of exchanging information between parties about the evidence and witnesses that will be presented at trial). They also include the costs associated with the additional production of information at trial. Litigation costs represent approximately $5.2 billion (Engstrom, 2014). Lawyers' fees represent approximately one-third of the plaintiff's gross or net compensation.

[2] For seminal work on litigation, see Shavell (1982), Png (1983), Bebchuk (1984), Reinganum and Wilde (1986), Schweizer (1989) and Spier (1992, 1994). For seminal work on care taking and litigation, see Png (1987) and Landeo et al. (2007). For seminal work on third-party funding of litigation, see Daughety and Reinganum (2014) and Landeo and Nikitin (2018).

[3] More specifically, previous literature has applied mechanism-design tools to study specific civil justice institutions such as the English and American rule for allocation of litigation costs. In contrast, our paper studies the optimal design of the whole civil justice system.

cost of producing evidence. In contrast to previous work, our comprehensive approach allows us to characterize optimal cost-allocation rules under an optimal production of evidence. We demonstrate that a tort reform that implements the proposed mechanism in real-world settings is feasible.

We study the design of the optimal civil justice system in an environment characterized by two-sided private information and two players, a victim and an injurer. Without loss of generality, we focus on direct-revelation civil justice mechanisms where the victim and the injurer truthfully report their types (Myerson, 1981, 1979). We identify the properties that must be satisfied by an optimal civil justice system to ensure access to justice and maximal compensation to the victims at the minimum expected cost of producing evidence. Our analysis demonstrates that the cost of producing evidence, the probability of confronting liable injurers, and the society's concern regarding the victim's compensation determine the optimal civil justice system.[4] We show that, when the optimal production of evidence is implemented, full revelation of private information is achieved by investigating just a subset of legal cases. Our findings suggest that the cost-allocation rule applied in the American civil justice system, where each party pays their own cost of producing evidence, is not always socially optimal. The English rule, where the liable injurer or the victim confronting a non-liable injurer pays all the cost of producing evidence, is the optimal cost-allocation rule when the likelihood of liable injurers is moderate and the society's concern about restoring the victim's welfare is sufficiently high. In contrast to inefficient real-world civil litigation procedures, under the optimal mechanism, all victims get access to justice, the victims who confront liable injurers get maximal compensation, and evidence is produced at the minimum expected social cost by investigating just a subset of the legal cases.

The civil justice system is modeled as follows. We assumes that a neg-

[4]The probability of confronting liable injurers will henceforth be referred to as "probability of liable injurers"

ative event (an accident) caused by the injurer generated damages to the victim, and that the injurer might be liable or legally responsible for these damages and hence, might be required to pay compensation to the victim. The damage level, which is private information, represents the victim's type. Our setting allows for a continuum of damage levels. The injurer's liability, which is also private information, represents the injurer's type, non-liable and liable types. The types are independently distributed, and the distributions are commonly known.

A welfare-maximizing direct-revelation civil justice mechanism with a truth-telling equilibrium can be described as follows. A victim initiates the civil procedure by deciding to participate in the mechanism and the injurer is compelled to participate.[5] Both parties are required to report their types. The mechanism produces evidence, perfectly reveals the types, and ends with transfers from the liable injurers to the victims.

The social planner's civil justice design problem consists of a mathematical optimization problem. The objective function is represented by the social welfare loss associated with an accident, which includes the expected harm from an accident, the expected cost of producing evidence, the expected infringement of the victim's right of access to justice or right to fully participate in the legal system (Landeo and Nikitin, 2018), and the expected infringement of the right of the victim confronting a liable injurer to be fully compensated. The players' constraints ensure the victim's participation in the mechanisms and the parties' truthful report of types.

We begin our analysis with a benchmark model where the cost of producing evidence is paid by the social planner. This simplifying assumption allows us to introduce the main component of our methodological approach. We characterize the optimal civil justice system by adopting a two-step approach. In the first step, we identify the interim probabilities of investigation

[5]These features of the mechanism are aligned with the voluntary participation of the plaintiff and the compulsory participation of the defendant in civil litigation.

that induce the victim to truthfully reveal his damage type. We show that the cost of producing evidence and the probability of liable injurers determine the interim probabilities of investigation. In the second step, we verify whether the interim probabilities of investigation also induce the liable injurer to truthfully report her liability type, and apply adjustments to the interim probabilities of investigation. We show that the optimal production of evidence involves the investigation of just a subset of legal cases. If the probability of liable injurers is sufficiently low, irrespective of the victim's reported damages, then only legal cases where the injurer reports to be non-liable might be investigated. If the probability of liable injurers is sufficiently high, then legal cases where the injurer reports to be non-liable and the victim reports relatively low damages and legal cases where the injurer reports to be liable and the victim reports relatively high damages might be investigated.

We then demonstrate that a tort reform that implements the optimal mechanism in real-world settings is feasible. The proposed tort reform consists of adding a first stage, an "Information-Revelation Stage" to the current civil litigation procedures. We model civil litigation using a game-theoretic setting with two-sided incomplete information and three Bayesian risk-neutral players, a social planner, a plaintiff (victim) and a defendant (injurer). In the first stage, the Information-Revelation Stage, the social planner chooses a direct-revelation mechanism involving incentives to the plaintiff to file a lawsuit (initiate the mechanism) and incentives to the plaintiff and defendant to truthfully report their types. The social planner then informs the reported types to the plaintiff and the defendant, and a standard two-stage litigation game starts. The plaintiff makes a take-it-or-leave-it settlement offer to the defendant, and the defendant decides whether to accept the offer. In case of acceptance, the legal case is settled out of court; otherwise, a costly trial occurs. We show that a direct-revelation mechanism with a truth-telling equilibrium exists in this environment, and that this mechanism is the same as the optimal mechanism of the benchmark model. In equilibrium, perfect revelation of private information is achieved by producing evidence on just a subset of legal cases. All legal cases are settled out of court and hence, the likelihood of trial and the corresponding litigation costs are minimized.

Next, we study the optimal civil justice design with endogenous allocation of the cost of producing evidence between the victim and the injurer. The assumption that the social planner pays the cost of producing evidence is relaxed. In contrast to the previous literature, the design of cost-allocation rules takes into account the optimal production of evidence. We show that our main qualitative findings regarding the design of the optimal production of evidence also hold in this environment. New insight regarding the application of the two most common cost-allocation institutions, the English and the American rules, are derived. Under the English rule, the liable injurer or the victim confronting a non-liable injurer pays all the cost of producing evidence. Under the American rule, which is applied in the American civil justice system, each party pays her own cost of producing evidence. We show that the American rule is not always socially optimal. When the likelihood of liable injurers is moderate and the society's concern regarding the victim's compensation is sufficiently high, the English rule is the socially-optimal rule. Our analysis underscores the robustness of our benchmark model findings regarding the optimal production of evidence, and the tractability of our framework to study more complex civil justice systems.

Important policy implications are derived from our paper. Our analysis demonstrates that the optimal civil justice mechanism shares some of the features present in the American civil litigation system but also underscores other relevant factors for the design of optimal civil justice institutions. In contrast to the American civil justice system where the production of evidence occurs in all the legal cases that are filed, we show that, in the optimal civil justice system, the truthful revelation of private information is achieved by devoting society's resources to the investigation of only a subset of legal cases. When the production of evidence is optimal, the social planner achieves the core goal of providing access to justice and maximal compensation to the victims at the minimum cost of producing evidence. The extension to our benchmark model provides additional policy implications. When the allocation of the cost of producing evidence is endogenous, we demonstrate that the optimal cost allocation does not always correspond to the cost-allocation rule applied in the American civil justice system.

The closest to our work is the very small theoretical literature on the design of civil litigation institutions using a mechanism-design approach.[6] Schweizer (1989) studies a game-theoretic model of litigation with two-sided incomplete information. Mechanism-design tools are also used to explore whether the inefficient equilibrium under which litigation is not avoided with certainty is due to the take-it-or-leave-it bargaining procedure adopted in the game-theoretic model. The framework assumes that the litigants' outside option is to proceed to trial where the private information is perfectly revealed and the English rule for cost allocation is applied. The findings suggest that there are more important factors that preclude efficiency, and that the efficient outcome does not always exist. Spier (1994) analyzes the effects of Rule 68 for cost allocation between a plaintiff and a defendant on the likelihood of out-of-court settlement using a game-theoretic model of one-sided incomplete information.[7] Mechanism design is then applied to a two-sided private information setting to study the optimal cost-allocation rule that maximizes the probability of settlement. The framework assumes that the litigants' outside option is to go to trial, where the private information is perfectly revealed and the American rule for cost-allocation is applied. The results

[6] For brevity, this section summarizes only the papers that are closely related to our work.

[7] Under Rule 68, cost allocation depends on the pretrial settlement offers. When a litigant rejects an out-of-court settlement offer and gets a less favorable judgment at trial, she must compensate the other litigant for certain costs incurred after the offer was made.

suggest that the optimal rule for the allocation of the pretrial litigation costs resembles Rule 68.

Klement and Neeman (2005) investigate the optimal cost-allocation rule that minimizes the rate of litigation subject to maintaining deterrence using one-sided private-information framework and a mechanism-design approach. This paper also assumes that the litigants' outside option is to proceed to trial and that the defendant's private information is perfectly revealed at trial. Their findings suggest that the optimal cost-allocation mechanism involves instruments that resemble the English rule. Demougin and Fluet (2006) use a mechanism-design approach to study the optimal standard of proof rule for determining negligence that maximizes deterrence, and find that "the more-likely-than-not" rule induces maximal care-taking incentives for potential injurers.[8]

These papers differ significantly from our work. Schweizer's (1989) application of mechanism design is limited to exploring a specific outcome associated with his game-theoretic environment. The focus of Spier (1994) and Klement and Neeman (2005) is on the design of cost-allocation rules that maximize the likelihood of out-of-court settlement. Demougin and Fluet's (2006) paper is centered on the design of optimal standard of evidence rules. In contrast to our work, the design of civil justice institutions in these papers does not consider the optimal production of evidence. Importantly, these previous applications of mechanism design abstract from the fundamental goals of the civil justice system of providing access to justice and full compensation to the victims confronting liable injurers.

Another strand of this literature is related to the application of mechanism design to the design of criminal law institutions. Silva (2019) studies truth-telling mechanisms for groups of suspects where only one is guilty, and

[8]Under the "more-likely-than-not" rule, a defendant is considered liable when the court determines that it is "more likely than not" that she did not fulfill the social standard of care. A degree of certainty above 50% is generally applied.

finds that the optimal mechanism involves leniency for confession before investigation. Siegel and Strulovici (2023) investigate optimal deterrence with direct-revelation mechanisms for harmful acts committed by single injurers. They study two deterrence settings, and demonstrate that the optimal mechanisms resemble criminal law institutions such as plea bargaining and binary verdicts. None of these papers study optimal civil justice institutions.

Our work is also connected to the theoretical literature on the economic analysis of civil litigation (Shavell 1982, Bebchuk 1984, Reinganum and Wilde 1986, Png 1987, Spier 1992, Landeo et al. 2007). Applied economic models of pretrial settlement bargaining have been developed to explain the sources of negotiation breakdown in civil litigation and to propose reforms aimed at improving the efficiency of civil litigation institutions. Primarily game theory has influenced these studies. A limitation of this approach is that the effectiveness of a specific litigation institution might be affected by the features of the bargaining model. In particular, the properties of the litigation institution are sensitive to the timing of the game. Importantly, the design of the litigation institutions does not take into account the optimal production of evidence. We extend this literature by providing the first general application of mechanism design to the study of optimal civil justice institutions under an optimal production of evidence.

The rest of the article is organized as follows. Section 2 presents the main assumptions and notation used in the benchmark model, and describes the social planner's civil justice design problem. Section 3 presents the analysis of the benchmark model, shows that the goals of the civil justice system can be achieved by producing evidence in just a subset of legal cases, and provides a real-world application. Section 4 studies the optimal civil justice design under endogenous allocation of the cost of producing evidence between the injurer and the victim, and shows that the cost-allocation rule applied in the American civil justice system is not always socially optimal. Section 5 provides concluding remarks. Formal proofs are presented in the Appendices.

2 Civil Justice System

This section introduces the stylized civil justice system studied in this paper. We presents the basic notation, describe the direct-revelation civil justice mechanism with a truth-telling equilibrium, and discusses the social planner's civil justice design problem.

2.1 Basic Notation

Our framework involves two-sided private information and two risk-neutral players, a victim V and an injurer I. A negative event (an accident) caused by the injurer generated damages A to the victim, and that the injurer might be liable or legally responsible for these damages and hence, might be required to pay compensation to the victim. Damages $A \in [0, \bar{A}]$ represent the victim's type, where $A = 0$ and $A = \bar{A}$ refer to the lowest and highest damages experienced by the victim, respectively. A is distributed with probability density function $g(A)$ and cumulative distribution function $G(A)$, and $g(A) > 0 \; \forall A \in [0, \bar{A}]$. Liability $L \in \{0, 1\}$ represents the injurer's type, where $L = 0$ denotes a non-liable type and $L = 1$ denotes a liable type. The probability of liable injurers is $p \in (0, 1)$. The victim's and the injurer's types are private information and the distributions of types are commonly known. The victim's and injurer's types are stochastically independent.[9]

The social planner's goal is to ensure the victim's access to justice or participation in the civil justice system (Landeo and Nikitin, 2018) and maximal compensation to the victims confronting liable injurers.[10] The social planner's adjudication of compensation to the victim is based on the injurer's liability type. We denote this criterion as the *social planner's adjudication criterion*. For any realization of $A \in [0, \bar{A}]$, if the injurer is liable ($L = 1$),

[9]Our findings also hold in environments with non-independent types that do not allow the social planner to use one player's information to learn about the other player's type.

[10]See Section 2.3.2 for details about the social welfare loss function.

then the victim should be compensated with transfer A; otherwise, no compensation should be granted.[11] The victim and the injurer have limited financial resources. We denote the victim's and the injurer's financial resources as $W^V > 0$ and $W^I > \bar{A}$.[12]

The social planner's problem is derived from the fact that she does not know the victim's and the injurer's types. The optimal civil justice design should allow the social planner to accomplish her goal by revealing the parties' types at the minimum social cost of producing evidence.

2.2 Civil Justice Mechanisms

Without loss of generality, we focus our attention on a special class of civil justice mechanisms, the direct-revelation civil justice mechanisms with a truth-telling equilibrium (Myerson, 1981, 1979). This mechanism can be described as follows. The victim initiates the civil procedure by deciding to participate in the mechanism and the injurer is compelled to participate.[13] If the victim decides to participate, both parties are required to simultaneously and confidentially report their types, (r^V, r^I) to the social planner, where $r^V \in [0, \bar{A}]$ and $r^I \in \{0, 1\}$ denote the victim's and the injurer's reports, respectively.

Although the players' types are private information, the social planner can *produce evidence* at a cost to investigate the reported types. We assume that the technology allows the social planner to perfectly learn the types. The costly technology for the production of evidence is described by a pair

[11] In other words, we implicitly assume that the social planner uses an exogenous social standard of care for liability determination.

[12] The last assumption indicates that although the injurer has limited financial resources, his financial resources suffice to compensate a victim of type $A \in [0, \bar{A}]$, i.e., he is not judgment proof.

[13] These features of the mechanism are aligned with the voluntary participation of the plaintiff and the compulsory participation of the defendant in civil litigation.

of cost functions. We denote $C_0(A)$ as the cost of producing evidence when the injurer's type is non-liable and the victim's type is A, and $C_1(A)$ as the cost of producing evidence when the injurer's type is liable and the victim's type is A.[14] We assume that $C_i(0) > 0$ $(i = 0, 1)$, and $\forall A \in [0, \bar{A}]$, $\frac{\partial C_i(A)}{\partial A} > 0$ $(i = 0, 1)$.[15] We allow for $C_0(A) \geq C_1(A)$ and $C_0(A) < C_1(A)$. We denote $q(r^V, r^I)$ as the probability of investigation.

We assume that the social planner can also impose a fine $f^i \in [0, \bar{f}^i]$ when party i's report r^i is found to be untruthful $(i = I, V)$. Given that the victim and the injurer have limited financial resources, $\bar{f}^i \leq W^i$ $(i = V, I)$. The transfers and fines that result from the production of evidence are as follows. First, when the reports are not investigated, the injurer transfers r^V to the victim if $r^I = 1$. Second, when the reports are investigated and found to be truthful ($r^V = A$ and $r^I = L$), the injurer transfers r^V to the victim if $r^I = 1$. Third, when the reports are investigated and the injurer's report is found to be untruthful ($r^I \neq L$), the injurer pays fine $f^I \in [0, \bar{f}^I]$ to the social planner, and the social planner transfers r^V to the victim if $L = 1$.[16] Fourth, when the reports are investigated and the victim's report is found to be untruthful ($r^V \neq A$), the victim does not receive any transfers and pays fine $f^V \in [0, \bar{f}^V]$ to the social planner.[17] Fifth, when the reports are

[14]In Section 3, the benchmark model, we assume that the cost of producing evidence is paid by the social planner. In Section 4, we endogenize the allocation of the cost of producing evidence between the injurer and the victim.

[15]The victim has an incentive to untruthfully report higher damages to get higher compensation. As a result of the higher investigation complexity when damages are higher, stronger investigation efforts and higher costs might be required.

[16]For mathematical tractability, the rule consists of just imposing a fine to the untruthful injurer. We show that in equilibrium, the fine is the maximal, $\bar{f}^I$. Given that the injurer is financially constrained, $\bar{f}^I = W^I$. Alternatively, we could define the maximal fine as $W^I - A$, charge this fine to the liable injurer and require the liable injurer to pay the compensation A. Notice that the two rules produce the same incentive to the injurer in terms of expected loss, and hence, are equivalent.

[17]The component of the rule consisting on not providing any compensation to the victim

investigated and the victim's and injurer's reports are found to be untruthful ($r^V \neq A$ and $r^I \neq L$), the victim does not receive any transfers, and the victim and the injurer pay fines $f^V \in [0, \bar{f}^V]$ and $f^I \in [0, \bar{f}^I]$ to the social planner.

Two types of constraints must be imposed to ensure participation in the mechanism and truthful reports: individual-rationality constraints and incentive-compatibility constraints. To simplify the notation, we denote the probability of investigation $q(r^V, r^I)$ when the injurer reports to be non-liable and the victim reports to be type A as $q_0(A) \in [0, 1]$, and the probability of investigation $q(r^V, r^I)$ when the injurer reports to be liable and the victim reports to be type A as $q_1(A) \in [0, 1]$.[18] First, a victim's individual-rationality constraint is required to ensure that the victim will be willing to participate in the civil justice mechanism. Second, victim's and injurer's incentive-compatibility constraints are required to ensure that the players will be willing to truthfully report their types.[19]

In sum, in the class of direct-revelation civil justice mechanisms with a truth-telling equilibrium, the victim initiates the civil procedure by deciding to participate in the mechanism, the injurer is compelled to participate, and both parties are required to report their types to the social planner. The mechanism produces evidence at a cost, perfectly reveals the types, and ends with transfers from the liable injurers to the victims.

allows the social planner to strengthen the incentives to truthful revelation of the victim's types without increasing the expected social cost.

[18]For brevity, the lemmas are included in Appendix A.

[19]In other words, truthful reports must be a Bayesian Nash Equilibrium in the civil justice game. See Section 2.3.1 for details about the players' constraints.

2.3 Social Planner's Civil Justice Design Problem

The social planner's problem of the design of the civil justice mechanism consists of a mathematical optimization problem. The social planner should choose the direct-revelation civil justice mechanisms with a truth-telling equilibrium that minimizes the social welfare loss function SWL associated with an accident. Given that the class of direct-revelation mechanisms with a truth-telling equilibrium requires the satisfaction of constraints (1) and (2), the social planner's optimization problem should be expressed in terms of the actual types.

The social welfare loss function SWL consists of several components. First, it includes the social welfare loss associated with the expected harm from an accident $H = \mathbb{E}[A] = \int_0^{\bar{A}} Ag(A)dA$. Second, it includes the social welfare loss associated with the expected cost of producing evidence (i.e., expected cost of investigation) $\mathbb{E}[C(A)] =$ $\int_0^{\bar{A}} [pq_1(A)C_1(A) + (1-p)q_0(A)C_0(A)]\, g(A)dA$. Third, it includes the social welfare loss associated with the expected infringement of the victim's right of access to justice $\theta\mathbb{E}[\eta(A)]$, where $\mathbb{E}[\eta(A)] = \int_0^{\bar{A}} \eta(A)g(A)dA$ represents the mass of victims that do not trigger the mechanism, i.e., do not get "Access to Justice," and $\theta \geq 0$ represents the society's concern regarding the victims' right of access to justice (Landeo and Nikitin, 2018).[20] Fourth, it includes the social loss associated with the expected infringement of the right of the victim confronting liable injurers to be fully compensated $\Lambda\mathbb{E}[\xi(A)]$, where $\mathbb{E}[\xi(A)] \geq 0$ represents the magnitude of undercompensation of the victim confronting liable injurers across victim's types, and $\Lambda \geq 0$ represents the society's concern regarding restoring the welfare of victims confronting liable

[20] In our environment, participation in the mechanism represents "Access to Justice" for victims. In the context of civil litigation, "access to justice" refers to the victim's ability to fully participate in the legal system (by filing a lawsuit agains the injurer, and if necessary, by bringing the case to trial). See Landeo and Nikitin (2018) for a formal definition of access to justice.

injurers to the level they experienced before the accident.

Definition 1. *The social welfare loss function SWL is defined as follows.*

$$SWL = H + \mathbb{E}[C(A)] + \theta\mathbb{E}[\eta(A)] + \Lambda\mathbb{E}[\xi(A)] =$$

$$= \int_0^{\bar{A}} Ag(d)dA + \int_0^{\bar{A}} [pq_1(A)C_1(A) + (1-p)q_0(A)C_0(A)]\, g(A)dA +$$

$$+\theta \int_0^{\bar{A}} \eta(A)g(A)dA + \Lambda \int_0^{\bar{A}} \xi(A)g(A)dA.$$

The social planner's problem is to minimize the SWL by choosing the optimal probabilities of investigation, $q_0(A)$ and $q_1(A)$, subject to the victim's and injurer's constraints. iIntuitively, the social planner's goal is to provide access to justice to the victim and restore the welfare of the victim confronting liable injurers to the level she experienced before the accident by inducing the victim and the injurer to truthfully reveal their private information at the minimum cost of producing evidence.

3 Benchmark Model

We begin our analysis by assuming that the social planner pays the cost of producing evidence. This simplified assumption allows us to introduce the main component of our methodological approach. (In Section 4, we study the optimal civil justice with endogenous allocation of the cost of producing evidence between the victim and the injurer, and characterizing the optimal cost-allocation rule.)

3.1 Players' Constraints

The set of players's constraint first includes the victim's individual-rationality (participation) constraint, $pA \geq 0 \ \forall A \in [0, \bar{A}]$, which is trivially satisfied $\forall A \in [0, \bar{A}]$. Hence, the victim has always an incentive to participate and the mechanism is always triggered.[21] Second, it includes the victim's incentive-compatibility constraint. Lemma 1 in Appendix A shows that it suffices to consider the victim's incentive-compatibility constraint for the lowest damage type, $A = 0$: the victim's expected payoff when she truthfully reports her type should be greater than or equal to her expected payoff when she untruthfully reports a higher type: $0 \geq p(1-q_1(A'))A' - [pq_1(A') + (1-p)q_0(A')]\bar{f}^V$ $\forall A' \in [0, \bar{A}]$. This constraint should hold $\forall A' \in [0, \bar{A}]$. Hence, the victim's incentive-compatibility constraint is equal to: $\forall A \in [0, \bar{A}]$,

$$[pq_1(A) + (1-p)q_0(A)]\bar{f}^V \geq p(1 - q_1(A))A. \tag{1}$$

Third, it includes the the incentive-compatibility constraints for the liable and non-liable injurers: the injurer's expected loss when he truthfully reports his type should be lower than or equal to his expected loss when she untruthfully reports his type. Lemma 2 shows that the liable injurer's incentive-compatibility constraint is equal to:

$$\int_0^{\bar{A}} Ag(A)dA \leq \bar{f}^I \int_0^{\bar{A}} q_0(A)g(A)dA. \tag{2}$$

Finally, the non-liable injurer's incentive compatibility constraint is equal to:

$$0 \geq -\int_0^{\bar{A}} \left[(1 - q_1(A))A + q_1(A)f^I\right] g(A)dA,$$

which is trivially satisfied $\forall A \in [0, \bar{A}]$ and $\forall f^I \in [0, \bar{f}^I]$. Hence, the non-liable injurer will always truthfully report his type. As demonstrated in Appendix A (Lemmas 1 and 2), the application of maximal fines $\bar{f}^V$ and $\bar{f}^I$ allows the social planner to save resources on investigation. Given that the victim and the injurer have limited financial resources, $\bar{f}^V = W^V$ and $\bar{f}^I = W^I$.[22]

[21] Remember that, when the victim triggers the mechanism, the injurer is compelled to participate. Hence, an injurer's individual-rationality constraint is not required.

[22] Remember that the injurer and the victim are financially constrained, and that $W^I > \bar{A}$ and $W^V > 0$, by assumption. If $\bar{f}^V$ and $\bar{f}^I$ could go to $+\infty$, the incentive-compatibility

3.2 Social Planner's Civil Justice Design Problem

Note first that the term H is exogenous. Second, given that the victim's individual-rationality constraint is trivially satisfied for $A \in [0, \bar{A}]$, the victim always has an incentive to trigger the mechanism, i.e., the victim always fully participates in the legal system. As a consequence, the victim always gets access to justice and hence, $\mathbb{E}[\eta(A)] = 0$. Third, given that the social planner pays the cost of producing evidence, the victims confronting liable injurers are fully compensated and their welfare is restored and hence, $E[\xi(A)] = 0$.[23]

Hence, the social planner's problem is reduced to minimize the expected cost of producing evidence $\mathbb{E}[C(A)]$:

$$\min_{q_0(A), q_1(A)} \left\{ \int_0^{\bar{A}} \left[p q_1(A) C_1(A) + (1-p) q_0(A) C_0(A) \right] g(A) dA \right\}$$

subject to the victim's incentive-compatibility constraint (1), $[pq_1(A) + (1 - p)q_0(A)]\bar{f}^V \geq p(1 - q_1(A))A \ \forall A \in [0, \bar{A}]$, the liable injurer's incentive-compatibility constraint (2), $\int_0^{\bar{A}} A g(A) dA \leq \bar{f}^I \int_0^{\bar{A}} q_0(A) g(A) dA$, and the feasibility constraints for the probabilities of investigation, $0 \leq q_i(A) \leq 1$ $(i = 0, 1)$.

We adopt a two-step approach to characterize the optimal probabilities of investigation. In the first step, we characterize the interim probabilities of investigation $q_0(A)$ and $q_1(A)$ that satisfy the victim's incentive-compatibility

constraints (1) and (2) would be satisfied for $q_0(A)$ and $q_1(A)$ approaching zero. In other words, when the victim and the injurer have unlimited financial resources, the truthful revelation of types is achieved at zero social cost of producing evidence.

[23]Formally, $\mathbb{E}[\xi(A)] = p \int_0^{\bar{A}} \alpha_1 q_1(A) C_1(A) g(A) dA$, where α_1 represents the victim's share of the cost of producing evidence when the injurer reports to be liable. Given that the social planner pays the cost of producing evidence, $\alpha_1 = 0$. In Section 6, we study the optimal allocation of the cost of producing evidence. We use an extension of our benchmark model consisting on endogenizing the allocation of the cost of producing evidence. In this setting, the victim might share the cost of producing evidence, $\alpha_1 \geq 0$. As a result, the victim might not be fully compensated and hence, $\mathbb{E}[\xi(A)] \geq 0$.

constraint (1) and the feasibility constraints for the probability of investigation. In the second step, we verify whether the interim probability of investigation $q_0(A)$ also satisfies the liable injurer's incentive-compatibility constraint (2). If not, adjustments to the interim probabilities of investigation $q_0(A)$ and $q_1(A)$ are implemented.[24] The next sections outline the main steps in the characterization of the optimal production of evidence. Formal proofs and description of the numerical example are presented in Appendices A and B.

3.3 Interim Probabilities of Investigation: Step 1

We characterize the interim probabilities of investigation $q_0(A)$ and $q_1(A)$ that satisfy the victim's incentive-compatibility constraint (1) and feasibility constraints for the probabilities of investigation, $0 \leq q_i(A) \leq 1$ $(i = 0, 1)$.[25]

3.3.1 Analysis of Type-A Victim

The social planner's problem involves optimization across victim's types. Given that the victim's incentive-compatibility constraint is specified for a particular type A, it is appropriate to start the analysis with the social planner's optimization problem for an individual type-A victim.

Denote the expected cost of producing evidence associated with a type-A victim as $\mathbb{E}[C] = pq_1(A)C_1(A) + (1-p)q_0(A)C_0(A)$. The social planner's problem is:

$$\min_{q_0(A), q_1(A)} \{pq_1(A)C_1(A) + (1-p)q_0(A)C_0(A)\}$$

[24] Mathematically, the social planner's problem consists of a linear programming problem, i.e., the objective function and the constraints are linear (Vohra, 2011). The order of the analysis of linear constraints does not affect the solution of the problem.

[25] For brevity, the main text of the paper only includes the main propositions. The additional propositions, lemmas and claims are included in Appendix A.

subject to the victim's incentive-compatibility constraint (1) and the feasibility constraints $0 \leq q_i(A) \leq 1$ $(i = 0, 1)$. Claim 1 in Appendix A shows that the victim's incentive-compatibility constraint holds as an equality. Hence, the victim's incentive compatibility constraint (1) becomes:

$$(p\bar{f}^V + pA)q_1(A) + (1-p)\bar{f}^V q_0(A) = pA. \tag{3}$$

Solving for $q_0(A)$:

$$q_0(A) = -\frac{p(\bar{f}^V + A)}{(1-p)\bar{f}^V} q_1(A) + \frac{pA}{(1-p)\bar{f}^V}.$$

Therefore,

$$\mathbb{E}[C] = \frac{1}{\bar{f}^V}\{q_1[pC_1(A)\bar{f}^V - pC_0(A)(\bar{f}^V + A)] + pAC_0(A)\}$$

and

$$\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} = \frac{[pC_1(A)\bar{f}^V - pC_0(A)(\bar{f}^V + A)]}{\bar{f}^V}.$$

When $\frac{[pC_1(A)\bar{f}^V - pC_0(A)(\bar{f}^V + A)]}{\bar{f}^V} < 0$, which is equivalent to $A > \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} < 0$; when $\frac{[pC_1(A)\bar{f}^V - pC_0(A)(\bar{f}^V + A)]}{\bar{f}^V} = 0$, which is equivalent to $A = \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} = 0$; and, when $\frac{[pC_1(A)\bar{f}^V - pC_0(A)(\bar{f}^V + A)]}{\bar{f}^V} > 0$, which is equivalent to $A < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} > 0$.

The interim probabilities of investigation for type-A victim should also satisfy the feasibility constraints $0 \leq q_i(A) \leq 1$ $(i = 0, 1)$. Suppose that $A > \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ and therefore, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} < 0$. The interim $q_1(A)$ is a corner solution with $q_1(A)$ taking the maximal value constrained by the feasibility constraint $q_1(A) \leq 1$ and the victim's incentive-compatibility constraint holding as an equality, equation (3). Solving for $q_1(A)$, we get $q_1(A) = \big(\frac{1}{pA + p\bar{f}^V}\big)[pA - (1-p)\bar{f}^V q_0(A)]$. $q_1(A)$ takes the maximal value when $q_0(A) = 0$: $q_1(A) = \frac{A}{A + \bar{f}^V}$. Hence, the interim probabilities of investigation for a victim of type A are $q_0(A) = 0$ and $q_1(A) = \frac{A}{A + \bar{f}^V} < 1$.

Suppose that $A = \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ and therefore, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} = 0$. Claim 2 shows that, when $A = \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$, the interim probabilities of investigation $q_0(A)$ and $q_1(A)$ involve infinitely many values, $q_0(A) \in \big[0, \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V}\big]$

and $q_1(A) \in \left[0, \frac{A}{A+\bar{f}^V}\right]$.[26] We consider here the minimum value for $q_0(A)$ and the corresponding $q_1(A)$ such that constraint (3) and the feasibility constraints are satisfied. Hence, the interim probabilities of investigation for a victim of type A are $q_0(A) = 0$ and $q_1(A) = \frac{A}{A+\bar{f}^V} < 1$.

Suppose now that $A < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$ and therefore, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} > 0$. The interim $q_1(A)$ is a corner solution with $q_1(A)$ taking the minimal value constrained by the feasibility constraint $q_1(A) \geq 0$ and equation (3). We need to check whether $q_1(A) = 0$ satisfies the victim's incentive-compatibility constraint. Evaluating equation (3) at $q_1(A) = 0$ and solving for $q_0(A)$ yields $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$. We also need to verify whether $q_0(A)$ satisfies the feasibility constraints. Define $A^0(p) \equiv \left(\frac{1-p}{p}\right)\bar{f}^V$. Claim 4 verifies that $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1$ only when $A \leq \left(\frac{1-p}{p}\right)\bar{f}^V = A^0(p)$. Hence, two mutually-exclusive cases are possible. If $A \leq A^0(p)$, then the interim probabilities of investigation are $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$. If $A > A^0(p)$, then $q_0(A) = 1$. The minimal feasible $q_1(A)$ is the one that corresponds to $q_0(A) = 1$. Evaluating equation (3) at $q_0(A) = 1$ and solving for $q_1(A)$ yields $q_1(A) = 1 - \frac{\bar{f}^V}{p(\bar{f}^V+A)}$. Hence, the interim probabilities of investigation are $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(\bar{f}^V+A)}$. The feasibility constraints $0 \leq q_1(A) \leq 1$ hold $\forall A \in [0, \bar{A}]$ and $\forall p \in (0, 1)$.

Proposition 1 characterizes the interim probabilities of investigation.

Proposition 1. *Suppose $p \in (0,1)$. The interim probabilities of investigation for a victim of type A are as follows. (1) If $A \geq \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$, then $q_0(A) = 0$ and $q_1(A) = \frac{A}{\bar{f}^V+A}$. (2) If $A < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$ and $A \leq \left(\frac{1-p}{p}\right)\bar{f}^V = A^0(p)$, then $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$. (3) If $A < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$ and $A > \left(\frac{1-p}{p}\right)\bar{f}^V = A^0(p)$, then $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(\bar{f}^V+A)}$.*

[26] Claim 2 also shows that the optimal mechanism is not unique for every p-value when p is sufficiently high.

3.3.2 Analysis Across Victim's Types

We now characterize the interim probabilities of investigation across victim's types. If $C_0(A) \geq C_1(A)\ \forall A \in [0, \bar{A}]$, then

$$A \geq \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V \tag{4}$$

is satisfied $\forall A \in [0, \bar{A}]$. Hence, by Proposition 1, the social planner incentivizes the victim only through $q_1(A)$ across victim's types. We denote this case as Environment 1. If $C_0(A) < C_1(A)\ \forall A \in [0, \bar{A}]$ and $\bar{A} < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ $\forall A \in [0, \bar{A}]$, then condition (4) is never satisfied. Hence, by Proposition 1, the social planner incentivizes the victim through $q_0(A)$ across victim's types. We denote this case as Environment 2.[27]

Consider first Environment 1. Suppose $C_0(A) \geq C_1(A)\ \forall A \in [0, \bar{A}]$. Therefore, condition $A \geq \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ is satisfied $\forall A \in [0, \bar{A}]$. By Proposition 1, the interim probabilities of investigation are: $q_0(A) = 0$ and $q_1(A) = \frac{A}{\bar{f}^V + A} < 1\ \forall A \in [0, \bar{A}]$. The interim probabilities of investigation hold $\forall$ $p \in (0, 1)$. Consider now Environment 2. Suppose $C_0(A) < C_1(A)\ \forall A \in [0, \bar{A}]$ and $\bar{A} < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$. Therefore, condition $A \geq \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ is never satisfied. By Proposition 1, if $A \leq \big(\frac{1-p}{p}\big)\bar{f}^V = A^0(p)$, then the interim probabilities of investigation are $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} \leq 1$ and $q_1(A) = 0$; if $A > \big(\frac{1-p}{p}\big)\bar{f}^V = A^0(p)$, then the interim probabilities of investigation are $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(A + \bar{f}^V)} < 1$.

Define $p^0 \equiv \frac{\bar{f}^V}{\bar{f}^V + \bar{A}}$. By Claim 5, if $p \leq p^0$, then $\bar{A} \leq A^0(p)$. Therefore, every $A \in [0, \bar{A}]$ is lower than or equal to $A^0(p)$. If $p > p^0$, then $\bar{A} > A^0(p)$. Therefore, $A \in [0, \bar{A}]$ can be greater than or lower than $A^0(p)$. Hence,

[27] Claim 3 shows that there are multiple cases. The relationship among $\bar{A}$, $\big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ and $\big(\frac{1-p}{p}\big)\bar{f}^V$ determines multiple combinations of cases and hence, multiple states of the world or environments. We study Case 2(b)i. here. This case occurs when $C_0(A) < C_1(A)$ $\forall A \in [0, \bar{A}]$, $\bar{A} < \min\big\{\big(\frac{1-p}{p}\big)\bar{f}^V, \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V\big\}$ and $\big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V \leq \big(\frac{1-p}{p}\big)\bar{f}^V$. Formal analysis of the other environments is available from the authors upon request.

two mutually-exclusive p-segments are possible. p-Segment 1: If $p \in [0, p^0]$, then the interim probabilities of investigation are $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1$ and $q_1(A) = 0 \ \forall A \in [0, \bar{A}]$. p-Segment 2: If $p \in (p^0, 1]$, then the interim probabilities of investigation are as follows. If $A \leq A^0(p)$, then $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1$ and $q_1(A) = 0$. If $A > A^0(p)$, then $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(A+\bar{f}^V)} > 0$. Claims 4 and 6 verify that the feasibility constraint $q_0(A) \leq 1$ is satisfied. It simple to show that the feasibility constraints $q_0(A) \geq 0$ and $0 \leq q_1(A) \leq 1$ are also satisfied.

Proposition 2 characterizes the interim probabilities of verification across victim's types for Environments 1 and 2. It also shows that Environment 2 encompasses two p-segments, p-Segment 1 and p-Segment 2.

Proposition 2. *The interim probabilities of investigation for Environments 1 and 2 across victim's types are as follows.*

1. *Environment 1: If $C_0(A) \geq C_1(A) \ \forall A \in [0, \bar{A}]$, then the interim probabilities of investigation are: $q_0(A) = 0$ and $q_1(A) = \frac{A}{\bar{f}^V + A} < 1$ $\forall A \in [0, \bar{A}]$ and $\forall p \in (0, 1)$.*

2. *Environment 2: If $C_0(A) < C_1(A) \ \forall A \in [0, \bar{A}]$ and $\bar{A} < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V \ \forall A \in [0, \bar{A}]$, then the interim probabilities of investigation are as follows. (a) p-Segment 1: If $p \in (0, p^0]$, then the interim probabilities of investigation are $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0 \ \forall A \in [0, \bar{A}]$. (b) p-Segment 2: If $p \in (p^0, 1)$, then the interim probabilities of investigation are $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0 \ \forall A \in [0, A^0(p)]$, and $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(A+\bar{f}^V)} \ \forall A \in (A^0(p), \bar{A}]$.*

Intuitively, to incentivize the victim to truthfully reveal her type while economizing on investigation costs, the social planner should take into account the cost of producing evidence, the likelihood of liable injurers and the victim's damages.

3.4 Optimal Production of Evidence: Step 2

This section verifies whether the interim probabilities of investigation also satisfy the liable injurer's incentive-compatibility constraint (2). If not, adjustments to the interim probabilities of investigation are implemented. The probabilities of investigation that satisfy constraints (1) and (2) are denoted as optimal probabilities of investigation and represent the optimal production of evidence.

As discussed in the Appendix, there are two possible procedures to adjust the interim probabilities of investigation, Procedures 1 and 2. Procedure 1 consists of an increase in $q_0(A)$ and a decrease in $q_1(A)$ until the liable injurer's incentive-compatibility constraint (2) is satisfied as an equality while keeping the victim's incentive-compatibility constraint (1) satisfied as an equality. Procedure 2 consists of an increase in $q_0(A)$ without decreasing $q_1(A)$ until the liable injurer's incentive-compatibility constraint is satisfied as an equality while keeping the victim's incentive-compatibility constraint (1) satisfied. As stated in Corollary 1, Procedures 1 and 2 can be applied to Environment 1, and only Procedure 2 can be applied in Environment 2. Propositions 3 and 4 show that the application of adjustment procedures should start at the lowest A, and that Procedure 1 is more efficient that Procedure 2 and hence, when both procedures can be applied, Procedure 1 should be applied first.

For brevity, only the analysis of Environment 1 will be presented here. (The analysis for Environment 2 is presented in the Appendix.) Next, we identify the p-segments and identify the procedures that should be applied. Our findings suggest that there are three states of the world in each Environment involving low, moderate and high probabilities of liable injurers p. We then implement the adjustment procedures, and characterize the optimal production of evidence for each state of the world. Our analysis demonstrates that complete revelation of private information is achieved by focusing the society's investigation efforts on just a subset of legal cases.[28]

Remember that Environment 1 occurs when $C_0(A) \geq C_1(A)\ \forall A \in (0, \bar{A}]$. We characterize the optimal probabilities of investigation. We first verify whether the liable injurer's incentive-compatibility constraint (2) is satisfied:

$$\int_0^{\bar{A}} Ag(A)dA \leq \bar{f}^I \int_0^{\bar{A}} q_0(A)g(A)dA. \tag{2}$$

As Proposition 2 states, the interim probabilities of investigation are: $\forall p \in (0,1)$ and $\forall A \in [0, \bar{A}]$, $q_0(A) = 0$ and $q_1(A) = \frac{A}{f^V + A}$. Given that $q_0(A) = 0$ $\forall A \in [0, \bar{A}]$, constraint (2) is never satisfied. Hence, the application of an adjustment procedure to increase $q_0(A)$ is required. Given that $q_1(A) > 0$ $\forall A \in [0, \bar{A}]$, Procedure 1, the most efficient procedure, can be implemented across A-values. Procedure 1 starts from the lowest values of A. The social planner increases $q_0(A)$ and reduces $q_1(A)$ while keeping the victim's incentive-compatibility constraint (1) satisfied as an equality. The procedure continues until the liable injurer's incentive-compatibility constraint (2) is satisfied as an equality. If the liable injurer's incentive-compatibility constraint is still not satisfied after exhausting the implementation of Procedure 1, then Procedure 2 should be implemented.

Claim 4 in Appendix A shows that, $\forall p \in (0,1)$, $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{f^V} \leq 1 \iff A \leq A^0(A)$. By Claim 5, $\bar{A} > A^0(p) \iff p > p^0$. By Claim 6, $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{f^V} \leq 1\ \forall A \in [0, \bar{A}]$ if $p \leq p^0$. Therefore, to satisfy the feasibility constraint $q_0(A) \leq 1$, Procedure 1 can be exhausted $\forall A \in [0, \bar{A}]$ only when $p \leq p^0$. The next corollary summarizes this result.

Corollary 3. *When $p \leq p^0$, the application of Procedure 1 can be exhausted $\forall A \in [0, \bar{A}]$. When $p > p^0$, the implementation of Procedure 1 can be exhausted only $\forall A \in [0, A^0(p)]$.*

[28] For brevity, the main text of the paper only includes the propositions. The lemmas and claims are included in Appendix A.

3.4.1 p-Segments

This section shows that the optimal production of evidence depends on the probability of liable injurers p. Specifically, it demonstrates that there are three p-segments in Environment 1 that differ in the application of the adjustment procedures, and hence, in the optimal probabilities of investigation: p-Segment 1 where $p \in (0, \tilde{p}]$, p-Segment 2.1 where $p \in (\tilde{p}, \bar{p}]$ and p-Segment 2.2 where $p \in (\bar{p}, 1)$. Proposition 5 identifies a sufficient condition for the existence of these three p-segments in Environment 1. Define $\bar{g} = \max_A\{g(A)\}$ and $\underline{g} = \min_A\{g(A)\}$.

Proposition 5. *Suppose $C_0(A) \geq C_1(A)$ $\forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)$ and $p \in (0, 1)$. If $\bar{g} < 2\underline{g}$, then there are three mutually-exclusive p-segments: p-Segment 1 where $p \in (0, \tilde{p}]$, p-Segment 2.1 where $p \in (\tilde{p}, \bar{p}]$, and p-Segment 2.2 where $p \in (\bar{p}, 1)$.*

Next, we provide an intuitive discussion. Formal analysis is presented in Appendix A.

p-Segments 1 and 2

We first characterize $\tilde{p}$ and demonstrate that $p \in (0, 1)$ is divided into two main segments: p-Segment 1 where $p \in (0, \tilde{p}]$ and p-Segment 2 where $p \in (\tilde{p}, 1)$.

The interim probabilities of investigation are: $\forall p \in (0, 1)$ and $\forall A \in [0, \bar{A}]$, $q_0(A) = 0$ and $q_1(A) = \frac{A}{f^V + A}$. Consider an injurer of type A and apply Procedure 1. The maximal increase in $q_0(A)$ while reducing $q_1(A)$ such that the victim's incentive-compatibility constraint (1) is satisfied as an equality corresponds to $q_1(A) = 0$. Using equality (3), the corresponding $q_0(A)$ is $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{f^V}$.

Define $\tilde{p}$ as the p-value such that, after exhausting the application of Procedure 1 across A-values, i.e., after reducing $q_1(A)$ to zero $\forall A \in [0, \bar{A}]$, the liable injurer's incentive-compatibility constraint evaluated at the adjusted interim probabilities of verification is satisfied as an equality:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{\bar{A}} \Big(\frac{\tilde{p}}{1-\tilde{p}}\Big)\frac{A}{\bar{f}^V} g(A)dA, \tag{5}$$

which can be rewritten as:

$$\int_0^{\bar{A}} Ag(A)dA = \frac{\bar{f}^{\,I}}{\bar{f}^V}\Big(\frac{\tilde{p}}{1-\tilde{p}}\Big)\int_0^{\bar{A}} Ag(A)dA.$$

Hence,

$$\tilde{p} = \frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I},$$

where $0 < \frac{\bar{f}^V}{\bar{f}^V+\bar{f}^I} < 1$. Proposition 5 shows that $\tilde{p} < p^0$. Claim 6 shows that the feasibility constraint $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} \leq 1$ holds $\forall p \in (0, \tilde{p}]$ and $\forall A \in [0, \bar{A}]$.

p-Segments 2.1 and 2.2

We now characterize $\bar{p}$ and demonstrate that p-Segment 2, $p \in (\tilde{p}, 1)$, is divided into two segments: p-Segment 2.1 where $p \in (\tilde{p}, \bar{p}]$ and p-Segment 2.2 where $p \in (\bar{p}, 1)$.

Suppose $p \in (\tilde{p}, 1)$. By Claim 6, when $p \in (\tilde{p}, p^0]$, after exhausting Procedure 1 $\forall A \in [0, \bar{A}]$, the feasibility constraint for $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} \leq 1$ holds $\forall A \in [0, \bar{A}]$. By Claim 4, when $p \in (p^0, 1)$, after exhausting the application of Procedure 1, the feasibility constraint $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} \leq 1$ only holds when $A \leq A^0(p)$. Hence, Procedure 1 can be exhausted only $\forall A \in [0, A^0(p)]$.

Define $\bar{p}$ as the p-value such that, after exhausting the application of Procedure 1 for $A \in [0, A^0(\bar{p})]$, the liable injurer's incentive-compatibility constraint evaluated at the adjusted interim probabilities of investigation is satisfied as an equality, where $A^0(\bar{p}) = \big(\frac{1-\bar{p}}{\bar{p}}\big)\bar{f}^V$:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I\Big[\int_0^{A^0(\bar{p})}\Big(\frac{\bar{p}}{1-\bar{p}}\Big)\frac{A}{\bar{f}^V} g(A)dA + \int_{A^0(\bar{p})}^{\bar{A}} 0g(A)dA\Big]. \tag{6}$$

Lemma 5 shows that there exists a unique $A^0(\bar{p})$ such that $0 < A^0(\bar{p}) < \bar{A}$, and there exists a unique $\bar{p}$ such that $p^0 < \bar{p} < 1 \ \forall A \in [0, \bar{A}]$. The next corollary summarizes the order relationship of $\tilde{p}$, $\bar{p}$ and p^0.

Corollary 4. $0 < \tilde{p} < p^0 < \bar{p} < 1 \ \forall A \in [0, \bar{A}]$.

3.4.2 Optimal Probabilities of Investigation

p-Segment 1

Suppose $p \in (0, \tilde{p}]$. We show that Procedures 1 and 2 should be applied. Consider first the application of Procedure 1. Given that $\tilde{p} < p^0$, the feasibility constraint of $q_0(A) \leq 1$ still holds when the implementation of Procedure 1 is exhausted $\forall A \in [0, \bar{A}]$. After exhausting the application of Procedure 1 across $A \in [0, \bar{A}]$, the adjusted interim probabilities of verification are $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} < 1$ and $q_1(A) = 0$. At the adjusted interim probabilities of verification,

$$\int_0^{\bar{A}} Ag(A)dA > \bar{f}^I \int_0^{\bar{A}} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA,$$

by the definition of $\tilde{p}$ and $\frac{\partial\left(\frac{p}{1-p}\right)}{\partial p} > 0$. Therefore, Procedure 2 should be also application. Remember that Procedure 2 consists of increasing $q_0(A)$ without reducing $q_1(A)$ until the liable injurer's incentive-compatibility constraint is satisfied as an equality. By the feasibility constraint for $q_0(A)$, the highest value for $q_0(A)$ is 1. Starting at the lowest values of A, the social planner increases $q_0(A)$ to 1 for $A \in [0, A^1(p)]$, where $A^1(p)$ is the A-threshold such that the liable injurer's incentive-compatibility constraint holds as an equality:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \left[\int_0^{A^1(p)} 1g(A)dA + \int_{A^1(p)}^{\bar{A}} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA\right]. \quad (8)$$

Lemma 4 verifies that there exists a unique $A^1(p)$ and that $A^1(p) < \bar{A}$. Lemma 4 also verifies that there exists a unique $A^1(\tilde{p})$ and that $A^1(\tilde{p}) = 0$.

Therefore, when $p = \tilde{p}$,

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{\bar{A}} \Big(\frac{p}{1-p}\Big)\frac{A}{\bar{f}^V} g(A)dA,$$

which is aligned with the definition of $\tilde{p}$.

Hence, the optimal production of evidence in p-Segment 1 involves the following optimal probabilities of investigation. For $A \in [0, A^1(p)]$, $q_0(A) = 1$ and $q_1(A) = 0$. For $A \in (A^1(p), \bar{A}]$, $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} < 1$ and $q_1(A) = 0$, where $A^1(p)$ is determined implicitly by the liable injurer's incentive compatibility constraint (2) written as an equality, $\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \big[\int_0^{A^1(p)} g(A)dA + \int_{A^1(p)}^{\bar{A}} \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} g(A)dA \big]$. The optimal social welfare loss for p-Segment 1 is:

$$SWL^1 = H + \mathbb{E}[C(A)]^1 + \theta\mathbb{E}[\eta(A)] + \Lambda E[\xi(A)] =$$

$$= \int_0^{\bar{A}} Ag(A)dA + (1-p)\times$$

$$\times\Bigg[\int_0^{A^1(p)} C_0(A)g(A)dA + \int_{A^1(p)}^{\bar{A}} \bigg(\frac{p}{1-p}\bigg)\frac{A}{\bar{f}^V} C_0(A)g(A)dA\Bigg] + 0 + 0.$$

p-Segment 2.1

Suppose $p \in (\tilde{p}, \bar{p}]$, where $\tilde{p} < p^0 < \bar{p}$. Two cases are possible. Suppose first that $p \in (\tilde{p}, p^0]$. By Claim 5, if $p \leq p^0$, then $\bar{A} \leq A^0(p)$. Therefore, the feasibility constraint $q_0(A) \leq 1$ holds when the implementation of Procedure 1 is exhausted $\forall A \in [0, \bar{A}]$. We show that only Procedure 1 should be applied and the application of Procedure 1 should not be exhausted $\forall A \in [0, \bar{A}]$. Given that $p > \tilde{p}$, by the definition of $\tilde{p}$ and by $\frac{\partial(\frac{p}{1-p})}{\partial p} > 0$, after exhausting the application of Procedure 1 $\forall A \in [0, \bar{A}]$, the liable injurer's incentive-compatibility constraint is:

$$\int_0^{\bar{A}} Ag(A)dA < \bar{f}^I \int_0^{\bar{A}} \Big(\frac{p}{1-p}\Big)\frac{A}{\bar{f}^V} g(A)dA.$$

Therefore, Procedure 1 should be exhausted only for $A \in [0, A^{2.1}(p)]$, where $A^{2.1}(p)$ corresponds to A-threshold such that the liable injurer's incentive-compatibility constraint holds as an equality:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \left[\int_0^{A^{2.1}(p)} \left(\frac{p}{1-p} \right) \frac{A}{\bar{f}^V} g(A)dA + \int_{A^{2.1}(p)}^{\bar{A}} 0g(A)dA \right]. \quad (9)$$

Lemma 5 verifies that there exists a unique $A^{2.1}(p)$ and that $0 < A^{2.1}(p) < \bar{A}$.

Suppose now that $p \in (p^0, \bar{p}]$. By Claim 5, if $p > p^0$, then $\bar{A} > A^0(p)$. Therefore, the feasibility constraint for $q_0(A)$ holds when the implementation of Procedure 1 is exhausted only for $A \in [0, A^0(p)]$. Lemma 5 also verifies that there exists a unique $A^{2.1}(p) \leq A^0(p)$ and that $0 < A^{2.1}(p) < \bar{A}$.

Hence, the optimal production of evidence in p-Segment 2.1. involves the following optimal probabilities of investigations. For $A \in [0, A^{2.1}(p)]$, $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$. For $A \in (A^{2.1}(p), \bar{A}]$, $q_0(A) = 0$ and $q_1(A) = \frac{A}{\bar{f}^V + A} < 1$, where $A^{2.1}(p)$ is determined implicitly by the liable injurer's incentive compatibility constraint (2) written as an equality, $\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{A^{2.1}(p)} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA$. The optimal social welfare loss for p-Segment 2.1 is:

$$SWL^{2.1} = H + \mathbb{E}[C(A)]^{2.1} + \theta\mathbb{E}[\eta(A)] + \Lambda E[\xi(A)] =$$

$$= \int_0^{\bar{A}} Ag(A)dA + (1-p)\int_0^{A^{2.1}(p)} \left(\frac{p}{1-p} \right) \frac{A}{\bar{f}^V} C_0(A)g(A)dA +$$

$$+p\int_{A^{2.1}(p)}^{\bar{A}} \left(\frac{A}{\bar{f}^V + A} \right) C_1(A)g(A)dA + 0 + 0.$$

p-Segment 2.2

Suppose $p \in (\bar{p}, 1)$. Given that $\bar{p} > p^0$, $\bar{A} > A^0(p)$, by Claim 5. Therefore, the feasibility constraint for $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1$ holds when the application of Procedure 1 is exhausted only for $A \in [0, A^0(p)]$. Lemma 6 shows that, after exhausting the application of Procedure 1 for $A \in [0, A^0]$, the liable injurer's incentive-compatibility constraint is still not satisfied:

$$\int_0^{\bar{A}} Ag(A)dA > \bar{f}^I \Big[\int_0^{A^0(p)} \left(\frac{p}{1-p}\right) \frac{A}{\bar{f}^V} g(A)dA + \int_{A^0(p)}^{\bar{A}} 0g(A)dA \Big].$$

Therefore, the social planner should apply Procedure 1 by increasing $q_0(A)$ to 1 and reducing $q_1(A)$ to $1 - \frac{\bar{f}^V}{p(\bar{f}^V + A)}$ to $A > A^0(p)$. Lemma 6 also shows that this adjustments should be applied $\forall A \in (A^0(p), A^{2.2}(p)]$, where $A^{2.2}(p)$ corresponds to the A-threshold such that the liable injurer's incentive-compatibility constraint is satisfied as an equality:

$$\int_0^{\bar{A}} Ag(A)dA =$$

$$= \bar{f}^I \Big[\int_0^{A^0(p)} \left(\frac{p}{1-p}\right) \frac{A}{\bar{f}^V} g(A)dA + \int_{A^0(p)}^{A^{2.2}(p)} 1g(A)dA + \int_{A^{2.2}(p)}^{\bar{A}} 0g(A)dA \Big]. \tag{10}$$

Finally, Lemma 6 verifies that there exists a unique $A^{2.2}(p)$ such that $A^0(p) < A^{2.2}(p) < \bar{A}$.

Hence, the optimal production of evidence in p-Segment 2.2. involves the following optimal probabilities investigation. For $A \in [0, A^0(p)]$, $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$. For $A \in (A^0(p), A^{2.2}(p)]$, $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(\bar{f}^V+A)}$. For $A \in (A_{2.2}(p), \bar{A}]$, $q_0(A) = 0$ and $q_1(A) = \frac{A}{A+\bar{f}.^V}$, where $A^{2.2}(p)$ is determined implicitly by the liable injurer's incentive-compatibility constraint (2) written as an equality, $\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \big[\int_0^{A^0(p)} \left(\frac{p}{1-p}\right) \frac{A}{\bar{f}^V} g(A)dA + \int_{A^0(p)}^{A^{2.2}(p)} g(A)dA \big]$ and $A^0(p) = \left(\frac{1-p}{p}\right)\bar{f}^V$. The optimal social welfare loss for p-Segment 2.2 is:

$$SWL^{2.2} = H + \mathbb{E}[C(A)]^{2.2} + \theta\mathbb{E}[\eta(A)] + \Lambda E[\xi(A)] =$$

$$= \int_0^{\bar{A}} Ag(A)dA + (1-p)\times$$

$$\times \Big[\int_0^{A^0(p)} \left(\frac{p}{1-p}\right) \frac{A}{\bar{f}^V} C_0(A)g(A)dA + \int_{A^0(p)}^{A^{2.2}(p)} C_0(A)g(A)dA \Big] +$$

Table 1: Optimal Production of Evidence – Environment 1

p-Segment	A-Segment	Optimal $q_0(A)$	Optimal $q_1(A)$
p-Segment 1	$A \in [0, A^1(p)]$	1	0
$p \in (0, \tilde{p}]$	$A \in (A^1(p), \bar{A}]$	$\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$	0
p-Segment 2.1	$A \in [0, A^{2.1}(p)]$	$\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$	0
$p \in (\tilde{p}, \bar{p}]$	$A \in (A^{2.1}(p), \bar{A}]$	0	$\frac{A}{\bar{f}^V + A}$
p-Segment 2.2	$A \in [0, A^0(p)]$	$\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$	0
$p \in (\bar{p}, 1)$	$A \in (A^0(p), A^{2.2}(p)]$	1	$1 - \frac{\bar{f}^V}{p(\bar{f}^V + A)}$
	$A \in (A^{2.2}(p), \bar{A}]$	0	$\frac{A}{\bar{f}^V + A}$

$$+p\left\{ \int_{A^0(p)}^{A^{2.2}(p)} \left[1 - \frac{\bar{f}^V}{p(\bar{f}^V + A)}\right] C_1(A)g(A)dA + \right.$$
$$\left. + \int_{A^{2.2}(p)}^{\bar{A}} \left(\frac{A}{\bar{f}^V + A}\right) C_1(A)g(A)dA \right\} + 0 + 0.$$

Table 1 summarizes the optimal production of evidence in Environment 1. In contrast to the American civil justice system where evidence is produced in every filed case, and hence society's resources are diverted from productive activities to the costly production of evidence in every legal case, our analysis suggests that the optimal production of evidence involves just a subset of legal cases.

When the probability of liable injurers is sufficiently low ($p \leq \tilde{p}$), the victim's gains from misreporting are low. Therefore, the production of evidence just in legal cases where the injurer reports to be non-liable suffices to incentivize the victim and the liable injurer to truthfully report their types. When probability of liable injurers is sufficiently high ($p > \tilde{p}$), the victim's gains from misreporting are high, and these gains increase with the reported damages. If the victim untruthfully reports relatively low damages, then the gains from misreporting are lower. Therefore, the production of evidence just in legal cases where the injurer reports to be non-liable suffices to incentivize the victim and the liable injurer to truthfully report their types.

If the victim untruthfully reports relatively high damages, the gains from misreporting are higher. Given that the victim receives compensation only from a liable injurer only, the production of evidence just in the legal cases where the injurer reports to be liable suffices to incentivize the victim to truthfully report her type. In addition, given that the legal cases where the injurer reports to be non-liable are investigated if the victim reports low damages, the resulting aggregate probability of investigation also suffices to incentivize the liable injurer to truthfully report his type because the injurer does not know the victim's type.

3.5 An Illustration: Uniform Distribution of Damages

A simple example using a uniform distribution of damages illustrates the results for the benchmark model. We focus on Environment 1, p-Segment 2.1 where $p \in (\tilde{p}, \bar{p}]$.[29] Appendix B presents formal analysis of the model with a uniform distribution of damages and discusses the numerical example.

Suppose that the victim's damage types A are uniformly distributed on the interval $[0, \bar{A}]$, where $g(A) = \frac{1}{\bar{A}}\ \forall A \in [0, \bar{A}]$, $G(A) = \frac{A}{\bar{A}}$, and $\int_0^{\bar{A}} Ag(A)dA = \frac{\bar{A}}{2}$. The relevant threshold $A^{2.1}(p)$ and $\bar{p}$ can be explicitly defined: $A^{2.1}(p) = \bar{A}\sqrt{\frac{(1-p)\bar{f}^V}{p\bar{f}^I}}$ and, $\bar{p} = \frac{\bar{f}^V \bar{f}^I}{\bar{f}^V \bar{f}^I + \bar{A}^2}$. Suppose that the cost of producing evidence functions are $C_0(A) = C_0 + c_0 A$ and $C_1(A) = C_1 + c_1 A$, where $C_i > 0$ and $c_i > 0$ $(i = 0, 1)$ are constants. The set of exogenous parameters is: $\{C_0, C_1, c_0, c_1, \bar{f}^V, \bar{f}^I, \bar{A}, p\} = \{1528, 690, 0.3, 0.01, 1800, 3600, 1200, 0.45\}$. The condition for Environment 1 becomes $C_0(A) = 1528 + 0.3A \geq 690 + 0.01A = C_1(A)\ \forall A \in [0, 1200]$.[30]

Table 3 summarizes our results. Intuitively, in a state of the world where the cost of producing evidence in legal cases where the injurer reports to be non-liable and the victim reports damages $A \in [0, 1200]$ is greater than or

[29] See Appendix B for an example of Environment 2, p-Segment 2 where $p \in (p^0, 1]$.
[30] After simplification, $-838 < 0.29A\ \forall A \in [0, 1200]$.

Table 3: Numerical Example – Optimal Production of Evidence

Environment	p-Segment	A-Segment	Optimal $q_0(A)^a$	Optimal $q_1(A)^a$
Environment 1	$(0.33, 0.82]$	$[0, 938]$	$q_0(A) = 0.213$	$q_1(A) = 0$
(p-Segment 2.1)		$(938, 1200]$	$q_0(A) = 0$	$q_1(A) = 0.373$

Note: [a]For each A-segment, $q_0(A)$ and $q_1(A)$ are evaluated at the average A-value; $p = 0.45$ is used.

equal to the cost of producing evidence in legal cases where the injurer reports to be liable and the victim reports damages $A \in [0, 1200]$, Environment 1 emerges. Evidence might be produced only in legal cases where the injurer reports to be non-liable and the victim reports sufficiently low damages ($A \leq 938$). Evidence might be also produced in legal cases where the injurer reports to be liable if the victim reports sufficiently high damages ($A > 938$). The expected harm from an accident, equal to the expected victim's damages, is $H = \frac{1200}{2} = 600$. The optimal expected cost of producing evidence for p-Segment 2.1 is $\mathbb{E}[C(A)]^{2.1} = 343$, the optimal expected cost from the infringement of the victim's right of access to justice is $\theta\mathbb{E}[\eta(A)] = 0$ and the optimal expected cost from the infringement of the right of the victims confronting liable injurers to be fully compensated is $\Lambda\mathbb{E}[\xi(A)] = 0$. Hence, $SWL^{2.1} = 943$.

Our findings regarding the optimal production of evidence suggest that, under the optimal direct-revelation mechanism, the victims always participate in the mechanism, $\mathbb{E}[\eta(A)] = 0$, the victims and the injurers truthfully reveal their types, the victims confronting liable injurers are fully compensated, $\mathbb{E}[\xi(A)] = 0$, and the expected cost of producing evidence $\mathbb{E}[C(A)]$ is minimal. In other words, the social planner achieves the goal of providing access to justice to the victims and restoring the welfare of the victims confronting liable injurers at the minimum expected cost of producing evidence. It is worth noting that, in contrast to the inefficient production of evidence in real-life settings, the truthful revelation of private information requires the production of evidence only in a subset of the legal cases.

3.6 Application in Real-World Settings: Tort Reform

This section demonstrates that the proposed optimal mechanism has real-world applications and significant policy implications. Consider a tort reform consisting of adding a first stage, an "Information-Revelation Stage" to the current civil litigation procedures. We model this environment by adding this first stage to a standard model of litigation.

Consider the following game with two-sided incomplete information and three Bayesian risk-neutral players, a social planner, a plaintiff (victim) and a defendant (injurer). The sequence of moves is as follows. Nature moves first and determines the players' types, given the probabilities distributions described before, and privately informs the type to each player. The types are independently distributed, and the distributions are commonly known. Then, the Information-Revelation Stage starts. In this stage, the social planner chooses a direct-revelation mechanism involving incentives to the (potential) plaintiff to file a lawsuit (initiate the mechanism by deciding to participate) and incentives to the plaintiff and the defendant to truthfully report their types, i.e., the social planner chooses a direct-revelation mechanism with a truth-telling equilibrium. As in the benchmark model, the social planner produces evidence to verify the reports with probabilities $q_0(A)$ and $q_1(A)$. $C_0(A)$ and $C_1(A)$ denote the cost of producing evidence. We assume that $C_0(A) \geq C_1(A)$,[31] and that the social planner pays the cost of producing evidence.[32] The social planner imposes fine $f^i \in [0, \bar{f}^i]$ $(i = P, D)$ in case of misreporting.

The social planner then informs the reported types to the plaintiff and

[31] This environment corresponds to Environment 1 of the benchmark model.

[32] Our qualitative findings also hold when the plaintiff and the defendant pay the cost of producing evidence.

the defendant, and a standard two-stage litigation game starts. The plaintiff makes a take-it-or-leave-it settlement offer S to the defendant, and after observing the offer, the defendant decides whether to accept or reject it. In case of acceptance, the legal case is settled out of court; otherwise, a costly trial occurs. We assume that each party pays her own litigation costs, $K^i > 0$ $(i = P, D)$, and that $\bar{f}^P \leq W^P - K^P$ and $\bar{f}^D \leq W^D - (\bar{A} + K^D)$.[33] Compensation to the plaintiff is granted only when the defendant is liable.

Next, we show that a direct-revelation mechanism with a truth-telling equilibrium exists, and that this mechanism is the same as the optimal direct-revelation mechanism of the benchmark model.

3.6.1 Litigation Game

Consider the equilibrium strategies of a litigation game of complete information. At trial, the plaintiff of type A gets compensation A only when the defendant is liable. Given that each party pays their own litigation cost, K^V and K^I, the plaintiff gets payoff $A - K^V$ when the defendant is liable, and $-K^V$ when the defendant is non-liable; and, the defendant gets payoff $-(A+K^I)$ when he is liable, and, $-K^I$ when he is non-liable. Consider now the pretrial bargaining negotiations. In equilibrium, the plaintiff of type A makes pretrial settlement offer $S = A + K^I$ to the liable defendant, an offer $S = K^I$ to the non-liable defendant; and, both types of defendants accept the offer. Hence, out-of-court settlement is achieved with certainty.

3.6.2 Direct-Revelation Mechanism with Truth-Telling Equilibrium

We show now that the social planner's problem, objective function and constraints, is the same as the benchmark model. As a result, in equilibrium,

[33] These assumptions ensure that the defendant is not judgement proof, and the plaintiff has sufficient financial resources to pay her own litigation costs.

the social planner chooses a direct-revelation mechanism with a truth-telling equilibrium that is the same as the optimal mechanism of the benchmark model.

Note first that, as in the benchmark model, the goal of the social planner in this game-theoretic environment is to minimize the social welfare loss associated with an accident by providing access to justice to the victims and maximal compensation to the victims confronting liable injurers at the minimum expected cost of producing evidence.

Second, we show that the relevant constraints are the same as in the benchmark model. The plaintiff's individual rationality (participation) constraint is $\forall A \in [0, \bar{A}]$, $p(A + K^I) \geq 0$, which is trivially satisfied $\forall A \in [0, \bar{A}]$. The plaintiff's incentive-compatibility constraint is as follows. Given that the plaintiff of type zero will have the strongest gain from misreporting, it suffices to evaluate her incentives. When the plaintiff truthfully reveals her type, her expected payoff is K^I. When she misreport her type, her expected payoff is $[p(1 - q_1(A))A] - \{[pq_1(A) + (1 - p)q_0(A)]\bar{f}^V\} + K^I$. Hence, the plaintiff's incentive-compatibility constraint is $[p(1 - q_1(A))A] - \{[pq_1(A) + (1 - p)q_0(A)]\bar{f}^V\} + K^I \leq K^I$. After simplification,

$$[pq_1(A) + (1 - p)q_0(A)]\bar{f}^V \geq p(1 - q_1(A))A, \qquad (1)$$

which is the same as in the benchmark model. The liable defendant's incentive-compatibility constraint is as follows. When the liable defendant truthfully reports his type, his expected loss is $\int_0^{\bar{A}} Ag(A)dA + K^I$. When he misreports his type, his expected loss is $\bar{f}^I \int_0^{\bar{A}} q_0(A)g(A)dA + K^I$. Hence, the liable defendant's incentive-compatibility constraint is $\int_0^{\bar{A}} Ag(A)dA + K^I \leq \bar{f}^I \int_0^{\bar{A}} q_0(A)g(A)dA + K^I$. After simplification,

$$\int_0^{\bar{A}} Ag(A)dA \leq \bar{f}^I \int_0^{\bar{A}} q_0(A)g(A)dA, \qquad (2)$$

which is the same as in the benchmark model. Finally, the non-liable defendant's incentive-compatibility constraint is $-K^I \geq$

$\geq -\left\{ \int_0^{\bar{A}} \left[(1-q_1(A))A + q_1(A)f^I\right] g(A)dA + K^I \right\}$. After simplification, $0 \geq -\int_0^{\bar{A}} \left[(1-q_1(A))A + q_1(A)f^I\right] g(A)dA$, which is the same as in the benchmark model, and is trivially satisfied $\forall A \in [0, \bar{A}]$ and $\forall f^I \in [0, \bar{f}^I]$. Therefore, as in the benchmark model, the relevant constraints are constraint (1) and (2).

Hence, the social planner's problem consists of choosing the probabilities of investigation $q_0(A)$ and $q_1(A)$ that minimize the expected cost of producing evidence subject to the victim's incentive-compatibility constraint (1) and the liable injurer's incentive-compatibility constraint (2). Given that the social planner's problem is the same as in the benchmark model, the equilibrium probabilities of investigation $q_0(A)$ and $q_1(A)$ correspond to the optimal mechanism of the benchmark model.

Our previous analysis demonstrates that the optimal mechanism of the benchmark model has real-world applications and hence, significant policy implications. Under the proposed tort reform, perfect revelation of private information is achieved by producing evidence on just a subset of legal cases. In equilibrium, all cases are settled out of court and hence, the likelihood of trial and the corresponding litigation cost are minimized.

4 Optimal Civil Justice Design with Endogenous Cost Allocation

We extend our benchmark model by endogenizing the allocation of the cost of producing evidence between the victim and the injurer. In contrast to the previous literature on civil justice, we characterize the optimal cost-allocation rule under an optimal production of evidence. Our analysis demonstrates that the key insights of the benchmark model regarding the optimal production of evidence extend to this setting. We also derive important new insights regarding the role of the cost of producing evidence and the society's concern about restoring the victim's welfare on the optimal cost allocation. Our findings suggest that the cost-allocation rule applied in the American civil justice system, where each party pays his own cost of producing evidence, is not always the socially-optimal cost-allocation rule.

Our benchmark framework assumes that the social planner pays the cost of producing evidence. We assume now that the victim and the injurer might pay a share of the cost of producing evidence. Denote α_i such that $0 \leq \alpha_i \leq 1$ as the victim's share of $C_i(A)$ $(i = 0, 1)$. We also assume that $W^I > \max\{C_0(\bar{A}), \bar{A} + C_1(\bar{A})\}$ and $W^V > \max\{C_0(\bar{A}), C_1(\bar{A})\}$.[34] All the other assumptions of the benchmark model hold here.

The next sections outline the main steps in the characterization of the optimal production of evidence $q_i(A)$ and the optimal allocation of the cost of producing evidence α_i $(i = 0, 1)$.[35] The proof of the main Proposition (Proposition 12) is presented in Appendix A. Formal analysis of the optimal mechanism and additional proofs are included in Appendix C. Description of the numerical example is presented in Appendix D.[36]

4.1 Players' Constraints

As in the benchmark model, the victim's incentive-compatibility constraint, $\forall A, A' \in [0, \bar{A}]$,

$$p(A - \alpha_1 q_1(A) C_1(A)) - (1-p)\alpha_0 q_0(A) C_0(A) \geq$$

[34] The first assumption indicates that although the injurer has limited financial resources, his financial resources suffice to pay the cost of evidence production and to compensate the victim, i.e., he is not judgment proof. The second assumption indicates that although the victim has limited financial resources, her financial resources suffice to pay the evidence production cost, i.e., she is not judgment proof.

[35] Although $q_i(A, \alpha_0, \alpha_1)$ $(i = 0, 1)$, to simplify notation, we use $q_i(A)$ $(i = 0, 1)$ across sections.

[36] For brevity, the main text of the paper only includes the main proposition.

$$\geq p(1-q_1(A'))A' - [pq_1(A') + (1-p)q_0(A')]\bar{f}^V$$

and the liable injurer's incentive-compatibility constraint

$$\int_0^{\bar{A}} Ag(A)dA + (1-\alpha_1)\int_0^{\bar{A}} q_1(A)C_1(A)g(A)dA \leq \bar{f}^I \int_0^{\bar{A}} q_0(A)g(A)dA$$

are not trivially satisfied. In contrast to the benchmark model, the victim's individual rationality constraint, $\forall A \in [0, \bar{A}]$,

$$pA - [p\alpha_1 q_1(A)C_1(A) + (1-p)\alpha_0 q_0(A)C_0(A)] \geq 0$$

and the non-liable injurer's incentive-compatibility constraint

$$(1-\alpha_0)\int_0^{\bar{A}} q_0(A)C_0(A)g(A)dA \leq \int_0^{\bar{A}} [(1-q_1(A))A + q_1(A)\bar{f}^I]g(A)dA$$

are also not trivially satisfied.

4.2 Social Planner's Civil Justice Design Problem

Given that now the victim might share the cost of producing evidence, $\mathbb{E}[\xi(A)] = p\int_0^{\bar{A}} \alpha_1 q_1(A)C_1(A)g(A)dA \geq 0$. It represents the expected infringement of the right of the victims confronting liable injurers to be fully compensated or the victims' expected undercompensation.

The social planner's problem is to minimize the SWL function by choosing the optimal production of evidence $q_i(A)$ and the optimal allocation of the cost of producing evidence α_i $(i = 0, 1)$, subject to the victim's incentive-compatibility constraint, the victim's individual-rationality constraint, the liable injurer's incentive-compatibility constraint, the non-liable injurer's incentive-compatibility constraint, and the feasibility constraints $0 \leq q_i(A) \leq 1$ and $0 \leq \alpha_i \leq 1$ $(i = 0, 1)$.

Note first, the term H is exogenous. Second, given that the victims confronting liable injurers might pay a share of the cost of producing evidence, they might not be fully compensated and their welfare might not be totally restored, i.e., $\mathbb{E}[\xi(A)] \geq 0$. Third, given that the victim's individual rationality constraint is not trivially satisfied, some victims might decide not to participate, i.e., $\mathbb{E}[\eta(A)] \geq 0$. Therefore, the social planner's problem is:

$$\min_{q_0(A,\alpha_0,\alpha_1),q_1(A,\alpha_0,\alpha_1),\alpha_0,\alpha_1} \left\{ \int_0^{\bar{A}} [pq_1(A)C_1(A) + (1-p)q_0(A)C_0(A)]\, g(A)dA + \right.$$

$$\left. +\theta \int_0^{\bar{A}} \eta(A)g(A)dA + \Lambda p \int_0^{\bar{A}} \alpha_1 q_1(A)C_1(A)g(A)dA \right\}$$

subject to the victim's incentive-compatibility constraint, the victim's individual-rationality constraint, the liable injurer's incentive-compatibility constraint, the non-liable injurer's incentive-compatibility constraint, and the feasibility constraints $0 \leq q_i(A) \leq 1$ and $0 \leq \alpha_i \leq 1$ $(i = 0, 1)$. Importantly, in the optimal mechanism, by ensuring that the victim's individual-rationality constraint is satisfied, the victims will fully participate in the legal system and get access to justice and $\eta(A)$ will be zero $\forall A \in [0, \bar{A}]$. By also ensuring that the victim's share of the cost of producing evidence α_1 is optimal, maximal compensation to the victims confronting liable injurers will be ensured in the optimal mechanism. Hence, the main goal of the civil justice system of access to justice and maximal compensation to the victims confronting liable injurers is achieved at the minimum cost of producing evidence.

The methodology proposed in the benchmark model can be extended to accommodate this more complex framework. The characterization of the optimal production of evidence $q_0(A)$ and $q_1(A)$ and the optimal allocation of the cost of producing evidence α_0 and α_1 now requires a five-step procedure. In the first four steps, presented in Appendix C, we characterize the interim probabilities of investigation taking the shares of the cost of producing evidence as given but including the feasibility constraints $0 \leq \alpha_i \leq 1$ $(i = 0, 1)$. In the fifth step, presented in the next section, we use the interim probabilities of investigation to characterize the optimal probabilities of investigation and the optimal shares of the cost of producing evidence.[37] Our findings suggest that, in addition to the cost of producing evidence and the probability of liable injurers, the optimal mechanism also depends on the society's concern about restoring the victim's welfare Λ.

4.3 Optimal Cost Allocation and Optimal Production of Evidence

We characterize the optimal cost allocation and the optimal production of evidence. Our focus is on Environment 1, p-Segment 2.1.[38] We first characterize the optimal cost allocation, i.e., the optimal α_i $(i = 0, 1)$. We then characterize the optimal production of evidence by evaluating $A^{2.1}(p, \alpha_1)$ at the optimal α_1. The next corollary summarizes the interim probabilities of investigation obtained in Steps 1–4, and underscores that only α_1 affects the interim probabilities of investigation.

Corollary 6. *Suppose* $C_0(A) \geq C_1(A)(1 + \Lambda)$ $\forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)(1 + \Lambda)$ *and* $p \in (\tilde{p}, \bar{p}]$. *The interim probabilities of investigation are as follows. For* $A \in [0, A^{2.1}(p, \alpha_1)]$, $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ *and* $q_1(A) = 0$. *For* $A \in (A^{2.1}(p, \alpha_1), \bar{A}]$, $q_0(A) = 0$ *and* $q_1(A) = \frac{A}{\bar{f}^V + A} < 1$. $A^{2.1}(p, \alpha_1)$ *is determined implicitly by the liable injurer's incentive compatibility constraint written as an equality,* $\int_0^{\bar{A}} Ag(A)dA + (1 - \alpha_1)\int_{A^{2.1}(p,\alpha_1)}^{\bar{A}} \frac{A}{\bar{f}^V + A} C_1(A)g(A)dA = \bar{f}^I \int_0^{A^{2.1}(p,\alpha_1)} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA$.

Given that the victim's individual-rationality constraint is satisfied at the interim probabilities of investigation $\forall \alpha_i$ $(i = 0, 1)$, $\theta\mathbb{E}[\eta(A)] = 0$. Therefore, the function $\mathbb{E}[C(A)] + \theta\mathbb{E}[\eta(A)] + \Lambda\mathbb{E}[\xi(A)]$, evaluated at the interim

[37] Given that this is a linear-programming problem, the order on the analysis of constraints does not affect the characterization of the optimal instruments.

[38] The analysis of the optimal mechanisms for the other two p-segments of Environment 1 and for Environment 2 is available from the authors upon request.

probabilities of investigation becomes:

$$\mathbb{E}[C(A)] + \Lambda\mathbb{E}[\xi(A)] =$$

$$= \left[p \int_{A^{2.1}(p,\alpha_1)}^{\bar{A}} \left(\frac{A}{\bar{f}^V + A}\right) C_1(A) g(A) dA + \right.$$

$$\left. +(1-p) \int_0^{A^{2.1}(p,\alpha_1)} \left(\frac{p}{1-p}\right) \frac{A}{\bar{f}^V} C_0(A) g(A) dA \right] +$$

$$+p\Lambda \int_{A^{2.1}(p,\alpha_1)}^{\bar{A}} \alpha_1 \left(\frac{A}{\bar{f}^V + A}\right) C_1(A) g(A) dA,$$

where $A^{2.1}(p, \alpha_1)$ is implicitly defined by the liable injurer's incentive-compatibility constraint holding as an equality:

$$\int_0^{\bar{A}} A g(A) dA + (1-\alpha_1) \int_{A^{2.1}(p,\alpha_1)}^{\bar{A}} \left(\frac{A}{\bar{f}^V + A}\right) C_1(A) g(A) dA =$$

$$= \bar{f}^I \int_0^{A^{2.1}(p,\alpha_1)} \left(\frac{p}{1-p}\right) \frac{A}{\bar{f}^V} g(A) dA.$$

After simplification,

$$\mathbb{E}[C(A)] + \Lambda\mathbb{E}[\xi(A)] =$$

$$= p(1 + \Lambda\alpha_1) \int_{A^{2.1}(p,\alpha_1)}^{\bar{A}} \left(\frac{A}{\bar{f}^V + A}\right) C_1(A) g(A) dA +$$

$$+p \int_0^{A^{2.1}(p,\alpha_1)} \frac{A}{\bar{f}^V} C_0(A) g(A) dA.$$

Given that the $\mathbb{E}[C(A)] + \Lambda\mathbb{E}[\xi(A)]$ does not depend on α_0, any $\alpha_0 \in [0, 1]$ is optimal. Therefore, the social planner's problem is reduced to:

$$\min_{\alpha_1 \in [0,1]} \{\mathbb{E}[C(A)] + \Lambda\mathbb{E}[\xi(A)]\}.$$

The characterization of the optimal α_1 involves several steps. We outline the main steps here. The proof of Proposition 13 in Appendix A provides formal analysis.

Although $A^{2.1}$ is a function of p and α_1, i.e., $A^{2.1}(p, \alpha_1)$, to simplify the notation, we use $A^{2.1}$ in (most parts of) the analysis, and denote $\mathbb{E}[C(A)] + \Lambda\mathbb{E}[\xi(A)]$ as S.

$$\frac{\partial S}{\partial \alpha_1} =$$

$$= p \int_{A^{2.1}}^{\bar{A}} \frac{A}{A + \bar{f}^V} C_1(A) g(A) dA \times$$

$$\times \left[\frac{-\frac{A^{2.1}}{\bar{f}^V} C_0(A^{2.1}) + \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1}) + \Lambda\Big(\frac{A^{2.1}}{\bar{f}^V}\frac{p}{1-p}\bar{f}^I + \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})\Big)}{\frac{A^{2.1}}{\bar{f}^V}\frac{p}{1-p}\bar{f}^I + (1-\alpha_1)\frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})} \right].$$

Proposition 13 shows that the sign of $\frac{\partial S}{\partial \alpha_1}$ is ambiguous and depends on Λ. In other words, any sufficient conditions that solve this ambiguity should include Λ.

We use the function $\Lambda^0(\alpha_1)$, defined as the Λ-function that results from equating $\frac{\partial S}{\partial \alpha_1}$ to zero,

$$\Lambda^0(\alpha_1) = \frac{\frac{A^{2.1}}{\bar{f}^V} C_0(A^{2.1}) - \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})}{\frac{A^{2.1}}{\bar{f}^V}\frac{p}{1-p}\bar{f}^I + \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})},$$

to find sufficient conditions on Λ such that the ambiguity of the sign of $\frac{\partial S}{\partial \alpha_1}$ is solved. Next, we need to evaluate the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1}$:

$$\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} = \frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} \frac{\partial A^{2.1}}{\partial \alpha_1},$$

where $\frac{\partial A^{2.1}}{\partial \alpha_1} < 0$. Proposition 12 shows that the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}}$ is ambiguous, and therefore, the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1}$ is also ambiguous. Hence, additional sufficient conditions are required. We show that when $\frac{\partial C_1(A)}{\partial A} > \mu \ \forall A \in [0, \bar{A}]$, $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} < 0$ and hence, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} > 0$; when $\frac{\partial C_1(A)}{\partial A} < \mu \ \forall A \in [0, \bar{A}]$, $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} > 0$ and hence, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} < 0$; and, when $\frac{\partial C_1(A)}{\partial A} = \mu \ \forall A \in [0, \bar{A}]$, $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} = 0$ and hence, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} = 0$.[39] To complete the analysis, we show that $\frac{\partial \frac{\partial S}{\partial \alpha_1}}{\partial \Lambda} > 0$, and define the Λ-thresholds $\bar{\Lambda}^0 \equiv \max_{\alpha_1} \Lambda^0(\alpha_1)$ and $\underline{\Lambda}^0 \equiv \min_{\alpha_1} \Lambda^0(\alpha_1)$.

[39] μ represents a threshold for $\frac{\partial C_1(A)}{\partial A}$, and is formally characterized in Proposition 12.

We can now characterize the three main mutually-exclusive Λ-segments, Cases 1–3, and the optimal α_1 for each case. (1) Case 1: If $\Lambda < \underline{\Lambda}^0$, then $\Lambda < \Lambda^0(\alpha_1)\ \forall \alpha_1 \in [0,1]$, by the definition of $\underline{\Lambda}^0$. Therefore, $\frac{\partial S}{\partial \alpha_1} < 0$, by the definition of $\Lambda^0(\alpha_1)$ and $\frac{\partial \frac{\partial S}{\partial \alpha_1}}{\partial \Lambda} > 0$. Hence, the optimal $\alpha_1 = 1$. (2) Case 2: If $\Lambda > \bar{\Lambda}^0$, then $\Lambda > \Lambda^0(\alpha_1)\ \forall \alpha_1 \in [0,1]$, by the definition of $\bar{\Lambda}^0$. Therefore, $\frac{\partial S}{\partial \alpha_1} > 0$, by the definition of $\Lambda^0(\alpha_1)$ and $\frac{\partial \frac{\partial S}{\partial \alpha_1}}{\partial \Lambda} > 0$. Hence, the optimal $\alpha_1 = 0$. (3) Case 3: If $\Lambda \in [\underline{\Lambda}^0, \bar{\Lambda}^0]$, then $\Lambda < \Lambda^0(\alpha_1)$ for some $\alpha_1 \in [0,1]$ and/or $\Lambda > \Lambda^0(\alpha_1)$ for some $\alpha_1 \in [0,1]$, by the ambiguity of the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1}$ due to the ambiguity of the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}}$. Hence, we need to use the additional sufficient conditions on μ to solve the ambiguity of the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1}$. We characterize three mutually-exclusive cases: Case 3(a) where $\frac{\partial C_1(A)}{\partial A} > \mu$ and hence, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} > 0$; Case 3(b) where $\frac{\partial C_1(A)}{\partial A} < \mu$ and hence, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} < 0$; and, Case 3(c) where $\frac{\partial C_1(A)}{\partial A} = \mu$, and hence, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} = 0$. Consider, for instance, Case 3(b). Proposition 12 shows that, when $\Lambda \in (\underline{\Lambda}^0, \bar{\Lambda}^0)$, there exists an optimal $\alpha_1 \in (0,1)$ for each $\Lambda \in (\underline{\Lambda}^0, \bar{\Lambda}^0)$, i.e., $\alpha_1(\Lambda) \in (0,1) = \arg\min_{\alpha_1 \in [0,1]}\{S\}$.[40] Proposition 12 summarizes the optimal cost allocation.[41]

Proposition 12. *Suppose* $C_0(A) \geq C_1(A)(1+\Lambda)\ \forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)(1+\Lambda)$ *and* $p \in (\tilde{p}, \bar{p}]$. *The optimal cost allocation is as follows.*

1. *If* $\Lambda < \underline{\Lambda}^0$, *then any* $\alpha_0 \in [0,1]$ *and* $\alpha_1 = 1$ *are optimal.*

2. *If* $\Lambda > \bar{\Lambda}^0$, *then any* $\alpha_0 \in [0,1]$ *and* $\alpha_1 = 0$ *are optimal.*

3. *If* $\Lambda \in [\underline{\Lambda}^0, \bar{\Lambda}^0]$, *then three cases are possible.*

   (a) *Suppose* $\frac{\partial C_1(A)}{\partial A} > \mu\ \forall A \in [0, \bar{A}]$.

      i. *If* $\Lambda = \underline{\Lambda}^0$, *then any* $\alpha_0 \in [0,1]$ *and* $\alpha_1 = 1$ *are optimal.*

[40] This case corresponds to 3(b)iii in Proposition 12.

[41] $\hat{\Lambda}$ represents a threshold for Λ, and is formally characterized in Proposition 12.

ii. If $\Lambda = \bar{\Lambda}^0$*, then any* $\alpha_0 \in [0,1]$ *and* $\alpha_1 = 0$ *are optimal.*

iii. If $\Lambda \in (\underline{\Lambda}^0, \bar{\Lambda}^0)$*, then two cases are possible.*

A. If $\Lambda \in (\underline{\Lambda}^0, \hat{\Lambda})$*, then any* $\alpha_0 \in [0,1]$ *and* $\alpha_1 = 1$ *are optimal.*

B. If $\Lambda \in [\hat{\Lambda}, \bar{\Lambda}^0)$*, then any* $\alpha_0 \in [0,1]$ *and* $\alpha_1 = 0$ *are optimal.*

(b) Suppose $\frac{\partial C_1(A)}{\partial A} < \mu \ \forall A \in [0, \bar{A}]$.

i. If $\Lambda = \underline{\Lambda}^0$*, then any* $\alpha_0 \in [0,1]$ *and* $\alpha_1 = 1$ *are optimal.*

ii. If $\Lambda = \bar{\Lambda}^0$*, then any* $\alpha_0 \in [0,1]$ *and* $\alpha_1 = 0$ *are optimal.*

iii. If $\Lambda \in (\underline{\Lambda}^0, \bar{\Lambda}^0)$*, then any* $\alpha_0 \in [0,1]$ *and* $\alpha_1(\Lambda) \in (0,1)$ *are optimal.*

(c) Suppose $\frac{\partial C_1(A)}{\partial A} = \mu \ \forall A \in [0, \bar{A}]$.

i. If $\Lambda = \underline{\Lambda}^0$*, then any* $\alpha_0 \in [0,1]$ *and* $\alpha_1 = 1$ *are optimal.*

ii. If $\Lambda = \bar{\Lambda}^0$*, then any* $\alpha_0 \in [0,1]$ *and* $\alpha_1 = 0$ *are optimal.*

iii. If $\Lambda \in (\underline{\Lambda}^0, \bar{\Lambda}^0)$*, then any* $\alpha_0 \in [0,1]$ *and any* $\alpha_1 \in [0,1]$ *are optimal.*

Given that neither $\mathbb{E}[C(A)]$ nor $\Lambda\mathbb{E}[\xi(A)]$ depend on α_0, any $\alpha_0 \in [0,1]$ is optimal across cases. Consider now the optimal α_1. By Claim 11, $\frac{\partial \mathbb{E}[C(A)]}{\partial \alpha_1} < 0$ and $\frac{\partial \Lambda \mathbb{E}[\xi(A)])}{\partial \alpha_1} \geq 0$. When $\Lambda < \underline{\Lambda}^0$ (Case 1), the effect of α_1 on $\mathbb{E}[\xi(A)]$ less than offsets the effect of α_1 on $\mathbb{E}[C(A)]$.[42] Hence, the overall effect of α_1 on S is negative and the optimal $\alpha_1 = 1$. intuitively, when the society's concern about restoring the welfare of a victim confronting a liable injurer is sufficiently low, social welfare loss is minimized by allocating the cost of producing evidence only to the victim, only to the injurer, or to both the victim and the injurer when the injurer is non-liable (any $\alpha_0 \in [0,1]$

[42]Hence, $\frac{\partial S}{\partial \alpha_1} < 0$.

is optimal), and by allocating the cost of producing evidence only to the victim when the injurer is liable ($\alpha_1 = 1$). Hence, neither the American rule where each party pays her own cost of producing evidence ($\alpha_i \in (0,1)$, $i = 0, 1$) nor the English rule where the victim pays all the cost of producing evidence when the injurer is non-liable and the injurer pays all the cost of producing evidence when he is liable ($\alpha_0 = 1$ and $\alpha_1 = 0$) are the optimal cost-allocation rules. We denote the optimal rule as "Alternative rule."[43]

When $\Lambda > \bar{\Lambda}^0$ (Case 2), the effect of α_1 on $\mathbb{E}[\xi(A)]$ more than offsets the effect of α_1 on $\mathbb{E}[C(A)]$.[44] Hence, the overall effect of α_1 on S is positive and the optimal $\alpha_1 = 0$. Intuitively, when the society's concern about restoring the the welfare of a victim confronting a liable injurer is sufficiently high, social welfare loss is minimized by allocating the cost of producing evidence only to the victim, only to the injurer, or to both the victim and the injurer when the injurer is non-liable (any $\alpha_0 \in [0, 1]$ is optimal), and by allocating the cost of producing evidence only to the injurer when the injurer is liable ($\alpha_1 = 0$). Given that any $\alpha_0 \in [0, 1]$ is optimal, the social planner can allocate the cost of producing evidence only to the victim when the injurer is not liable ($\alpha_0 = 1$). The optimal cost-allocation rule involving $\alpha_0 = 1$ and $\alpha_1 = 0$ resembles the English rule.[45] When $\Lambda \in [\underline{\Lambda}^0, \bar{\Lambda}^0]$ (Case 3), the effect of α_1 on $\mathbb{E}[\xi(A)]$ less than offsets, more than offsets or is equal to the effect of α_1 on $\mathbb{E}[C(A)]$.[46] Hence, the overall effect of α_1 on S is negative, positive or equal to zero, and the optimal $\alpha_1 = 1$ or $\alpha_1 = 0$ or $\alpha_1 \in (0, 1)$. For an illustration of the American rule as the optimal cost-allocation rule in our framework, consider Case 3(b)iii where $\Lambda \in (\underline{\Lambda}^0, \bar{\Lambda}^0)$ and $\frac{\partial C_1(A)}{\partial A} < \mu$. Proposition 12 shows that, for each $\Lambda \in (\underline{\Lambda}^0, \bar{\Lambda}^0)$, the effect of α_1 on $\mathbb{E}[\xi(A)]$ is lower (greater) than the effect of α_1 on $\mathbb{E}[C(A)]$ when α_1 is lower (greater)

[43] Similar findings apply to Cases 3(a)i, 3(a)iii.A, 3(b)i and 3(c)i of Proposition 12.

[44] Hence, $\frac{\partial S}{\partial \alpha_1} > 0$.

[45] Similar findings apply to Cases 3(a)ii, 3(a)iii.B, 3(b)ii, and 3(c)ii of Proposition 12.

[46] Hence, $\frac{\partial S}{\partial \alpha_1} < 0$, $\frac{\partial S}{\partial \alpha_1} > 0$ or $\frac{\partial S}{\partial \alpha_1} = 0$.

than the optimal α_1. Hence, the optimal $\alpha_1(\Lambda) \in (0,1)$. Given that any $\alpha_0 \in [0,1]$ is optimal, the social planner can choose any $\alpha_0 \in (0,1)$. The optimal cost-allocation rule involving $\alpha_i \in (0,1)$ $(i = 0,1)$ resembles the American rule.[47]

Important policy implications are derived. Our findings suggest that the cost of producing evidence and the society's concern about fully restoring the victim's welfare are determinant in the characterization of the optimal cost-allocation rule. In contrast to previous work on cost-allocation rules in civil litigation, we find that the English rule is not always the optimal cost-allocation rule. This result might be explained by the general features of our framework that allow us to study the design of optimal cost-allocation rules under an optimal production of evidence. Importantly, our results suggest that the cost-allocation rule applied in the American civil justice system, the American rule, is not always the socially-optimal rule. We provide conditions under which the American, the English rule or the Alternative rule might be socially optimal.[48]

4.4 An Illustration: Uniform Distribution of Damages

A simple example using a uniform distribution of damages illustrates the results for the model with endogenous cost allocation. We focus on Environment 1, p-Segment 2.1 where $p \in (\tilde{p}, \bar{p}]$. Appendix D presents formal analysis of the model with a uniform distribution of damages and discusses the numerical example.

As in Section 4.4, suppose that the victim's damage types A are uniformly distributed on the interval $[0, \bar{A}]$,[49] Suppose also that 41% of $C_0(A)$

[47] Similarly, in Case 3(c)iii of Proposition 12, the American rule might be optimal.

[48] As in the case of the benchmark model, a tort reform that implements the optimal mechanism in real-world settings is feasible. Formal analysis and proofs are available from the authors upon request.

[49] The relevant thresholds $A^{2.1}(p, \alpha_1)$ and $\bar{p}(\alpha_1)$ cannot be explicitly defined and hence,

Table 4: Numerical Example – Optimal Cost Allocation and Optimal Production of Evidence

	$\Lambda < \underline{\Lambda}^0$	$\Lambda \in [\underline{\Lambda}^0, \bar{\Lambda}^0]$	$\Lambda > \bar{\Lambda}^0$
Cost Allocation	$\alpha_0 = 0$	$\alpha_0 = 0.41$	$\alpha_0 = 1$
	$\alpha_1 = 1$	$\alpha_1 = 0.41$	$\alpha_1 = 0$
	(*Alternat. Rule*)	(*American Rule*)	(*English Rule*)
Prod. of Evidence[a]	$A \in [0, 938]$	$A \in [0, 962]$	$A \in [0, 976]$
	$q_0(A) = 0.213$	$q_0(A) = 0.218$	$q_0(A) = 0.222$
	$q_1(A) = 0$	$q_1(A) = 0$	$q_1(A) = 0$
	$A \in (938, 1200]$	$A \in (962, 1200]$	$A \in (976, 1200]$
	$q_0(A) = 0$	$q_0(A) = 0$	$q_0(A) = 0$
	$q_1(A) = 0.373$	$q_1(A) = 0.371$	$q_1(A) = 0.377$
Social Welfare Loss	959.94	962.14	960.06
H	600.00	600.00	600.00
$\mathbb{E}[C(A)]$	342.86	353.59	360.06
$\theta\mathbb{E}(\eta(A)]$	0	0	0
$\Lambda\mathbb{E}(\xi(A)]$	17.08	8.55	0

Note: [a]For each A-segment, $q_0(A)$ and $q_1(A)$ are evaluated at the average A-value and $p = 0.45$.

and $C_1(A)$ corresponds to the cost of producing evidence associated with the victim, and 59% of $C_0(A)$ and $C_1(A)$ corresponds to the cost of producing evidence associated with the injurer.[50] The cost of producing evidence functions and the set of exogenous parameters used in Section 5.3 also hold here. In addition, we use three Λ-values: $\Lambda \in \{0.30, 0.40, 0.50\}$. In this numerical example, $\frac{\partial C_1(A)}{\partial A} < \mu$ and therefore, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} < 0$. Hence, $\underline{\Lambda}^0 = \Lambda^0(\alpha = 1) = 0.396$ and $\bar{\Lambda}^0 = \Lambda^0(\alpha_1 = 0) = 0.402$. The expected harm from an accident, equal to the expected victim's damages, is $H = \frac{1200}{2} = 600$ across Λ-values. Table 4 summarizes the main results. Columns 2, 3 and 4 are constructed using $\Lambda \in \{0.30, 0.40, 0.50\}$.

Consider Column 2. The state of the world where $\Lambda = 0.30 < 0.396 = \underline{\Lambda}^0$ corresponds to Case 1 of Proposition 13 where the optimal $\alpha_0 \in [0, 1]$ and the optimal $\alpha_1 = 1$. Hence, the optimal cost-allocation rule is the Alternative rule.[51] Consider Column 3. The state of the world where $\Lambda = 0.40 \in (0.396, 0.402) = (\underline{\Lambda}, \bar{\Lambda}^0)$ corresponds to Case 3(b)iii of Proposition 12 where the optimal $\alpha_0 \in [0, 1]$ and the optimal $\alpha_1(\alpha_1) \in (0, 1)$. Given that $\Lambda = 0.40$, $\alpha_1(0.40) = 0.41$. By assumption, 41% of $C_i(A)$ corresponds to the cost of producing evidence associated with the victim ($i = 0, 1$). Hence, when the optimal $\alpha_0 = 0.41$ and the optimal $\alpha_1 = 0.41$, this cost-allocation rule corresponds to the American Rule. Consider Column 4. The state of the world where $\Lambda = 0.50 > 0.402 = \bar{\Lambda}^0$ corresponds to Case 2 of Proposition 12 where the optimal $\alpha_0 \in [0, 1]$ and the optimal $\alpha_1 = 0$. Hence, when the optimal $\alpha_0 = 1$ and the optimal $\alpha_1 = 0$, this cost-allocation rule

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they can be characterized only numerically. The MATLAB software is used to construct this numerical example.

[50] Remember that $C_i(A)$ refers to the cost of producing evidence when the injurer reports type i *and* the victim reports damages $A \in [0, \bar{A}]$ ($i = 0, 1$), i.e., the cost of producing evidence for *both* parties involved in a legal case.

[51] Given that $\alpha_1 = 1$, the term of the liable injurer's incentive-compatibility constraint that depends on α_1 cancels out. Hence, the optimal probabilities of investigation presented in Section 5.3 also hold here.

corresponds to the English Rule. It is worth notice that when Λ is sufficiently high, the welfare of the victims confronting liable injurers is fully restored, $\mathbb{E}[\xi(A)]^{2.1} = 0$.

In sum, although a framework involving an endogenous allocation of the cost of producing evidence obviously raises some new and interesting issues, the main insights derived from our benchmark model, the methodology, and the implication for the design of optimal civil justice institutions remain relevant. In contrast to the previous literature, the design of the cost allocation rule takes into account the optimal production of evidence. Our analysis demonstrates that the optimal mechanism shares some features present in the American civil litigation system but also underscores other relevant factors for the design of optimal civil justice institutions. Our results demonstrate the robustness of our previous findings and the tractability of our framework to study complex civil justice institutions.

5 Summary and Conclusions

This paper presents the first application of mechanism design to the design of the civil justice system. The fundamental goals of the civil justice system and the optimal production of evidence are considered in our design. The optimal civil justice mechanism minimizes the social welfare loss associated with an accident by providing access to justice to the victims and maximal compensation to the victims confronting liable injurers at the minimum expected cost of producing evidence. In contrast to previous work, our comprehensive approach allows us to characterize optimal cost-allocation rules under an optimal production of evidence. We demonstrate that a tort reform that implements the optimal mechanism in real-world settings is feasible. The proposed tort reform consists of adding an “Information-Revelation Stage” to the current civil litigation procedures. In equilibrium, perfect revelation of private information is achieved by producing evidence on just a subset of legal cases. Out-of-court settlement is achieved with certainty and hence, the likelihood of trial and the corresponding litigation costs are minimized.

Important policy implications are derived. We show that the optimal civil litigation mechanisms have features that parallel many of those in the American civil justice system but also underscores other relevant factors for the design of optimal civil justice institutions. In contrast to inefficient real-world civil litigation procedures, under the optimal civil justice mechanism, full revelation of private information is achieved by producing evidence in just a subset of legal cases. We show that the cost-allocation rule applied in the American civil justice system is not always socially optimal.

Additional relevant extension can be investigated. For instance, our framework can be extended to study the optimal civil justice design under an endogenous negligence rule (see Landeo and Nikitin, 2024). In this setting, the probability of liable injurers will be endogenous and determined by the social standard of care and the potential injurers' care-taking decisions (precaution) given their precaution-cost types. Our methodology is also applicable to study this more complex environment. In the optimal civil justice design, truthful revelation of private information is achieved by investigating just a subset of legal cases. Importantly, under the optimal negligence rule, only a subset of potential injurers decide to be negligent. These findings suggest that our results regarding the optimal production of evidence are robust, and the assumption regarding liability types is appropriate. This, and other extensions, remain fruitful areas for future research.

References

Bebchuk, L.A. 1984. "Litigation and Settlement under Imperfect Information." *RAND Journal of Economics,* pp. 404-415.

Daughety, A.F. and Reinganum, J.F. 2014. "The Effects of Third-Party Funding of Plaintiffs on Settlement." *American Economic Review,* pp. 2552-2566.

Demougin, D. and Fluet, C. 2006. "Preponderance of Evidence." *European Economic Review,* pp. 963-976.

Engstrom, Nora F. 2014. "Lawyers Lending: Costs and Consequences." *DePaul Law Review,* pp. 377-446.

Klement, A. and Neeman, Z. 2005. "Against Compromise: A Mechanism Design Approach." *Journal of Law and Economics*, 21: 285–314.

Landeo, C.M. and Nikitin, M. 2024. "Optimal Negligence Rule Design." Mimeo, University of Alberta and International College of Economics and Finance NRU – HSE.

Landeo, C.M. and Nikitin, M. 2018. "Financially-Constrained Lawyers: An Economic Theory of Legal Disputes." *Games and Economic Behavior,* pp. 625-647.

Landeo, C.M., Nikitin, M., and Baker, S. 2007. "Deterrence, Lawsuits and Litigation Outcomes under Court Errors." *Journal of Law, Economics and Organization*, pp. 57-97.

Myerson, R.B. 1981. "Optimal Auction Design." *Mathematics of Operations Research*, pp. 58-73.

Myerson, R.B. 1979. "Incentive Compatibility and the Bargaining Problem." *Econometrica*, pp. 61-73.

Png, I.P.L. 1987. "Litigation, Liability, and the Incentives for Care." *Journal of Public Economics,* pp. 61-85.

Png, I.P.L. 1983. "Strategic Behavior in Suits, Settlement, and Trial." *Bell Journal of Economics,* pp. 539-550.

Reinganum, J.F. and Wilde, L. 1986. "Settlement, Litigation, and the Allocation of Litigation Costs." *RAND Journal of Economics,* pp. 557-566.

Schweizer, U. 1989. "Litigation and Settlement under Two-Sided Incomplete Information." *Review of Economic Studies*, pp. 163–177.

Shavell, S. 1982. "Suit, Settlement, and Trial: A Theoretical Analysis under Alternative Methods for the Allocation of Legal Costs." *Journal of Legal Studies*, pp. 55-81.

Siegel, R. and Strulovici, B. 2023. "Judicial Mechanism Design." *American Economic Journal: Microeconomics*, pp. 243–270.

Silva, F. 2019. "If We Confess Our Sins." *International Economic Review*, pp. 1–24.

Spier, K.E. 1994. "Pretrial Bargaining and the Design of Fee-Shifting Rules." *RAND Journal of Economics*, pp. 197–214.

Spier, K.E. 1992. "The Dynamics of Pretrial Negotiations." *Review of Economic Studies*, pp. 93–108.

Vohra, R. 2011. "Mechanism Design: A Linear Programming Approach." *Econometric Society Monograph.* Cambridge University Press.

Appendix A

This Appendix presents the main proofs of propositions and lemmas, the technical aspects related to the adjustments to the interim probabilities of investigation, and the formal analysis of Environment 2. (Additional formal analysis is presented in the Supplementary Material, Appendices B, C, and D.)

Lemma 1. *Suppose* $p \in (0,1)$. *The victim's incentive-compatibility constraint is* $[pq_1(A) + (1-p)q_0(A)]\bar{f}^V \geq p(1-q_1(A))A$, $\forall A \in [0,\bar{A}]$.

Proof. By truthfully reporting her type, a victim with type $A \in [0,\bar{A}]$ gets pA. No victim has an incentive to report a lower type. When the victim reports a type $A' > A$ ($A' \in (0,\bar{A}]$), the victim gets A' instead of A only when the injurer is liable and investigation does not occur, and gets zero compensation and pays fine f^V when investigation occurs. The victim's expected payoff is equal to:

$$[p(1-q_1(A'))A' + (1-p)(1-q_0(A'))0] + [pq_1(A')(0) + (1-p)q_0(A')(0)]-$$

$$-[pq_1(A') + (1-p)q_0(A')]f^V =$$

$$= [p(1-q_1(A'))A'] - \{[pq_1(A') + (1-p)q_0(A')]f^V\},$$

where the term in brackets represents the expected gains from misreporting, and the term in curly brackets represents the expected loss from misreporting. Therefore, the incentive-compatibility constraint for a victim with type A is:

$$pA \geq [p(1-q_1(A'))A'] - \{[pq_1(A') + (1-p)q_0(A')]f^V\}.$$

By setting $f^V \in [0,\bar{f}^V]$ as high as possible, the social planner will spend less resources on verification. Given that the victim has limited financial resources, $\bar{f}^V = W^V$. Therefore,

$$pA \geq [p(1-q_1(A'))A'] - [pq_1(A') + (1-p)q_0(A')]\bar{f}^V.$$

This inequality can be rewritten as:

$$p(A'-A) \leq \{pq_1(A')A' + [pq_1(A') + (1-p)q_0(A')]\bar{f}^V\}.$$

If this constraint holds for a victim with the lowest type ($A = 0$), then it also holds for any $A \in (0,\bar{A}]$. Therefore, it suffices to consider the victim's incentive-compatibility constraint for the lowest damage type: For $A = 0$ and $A' \in (0,\bar{A}]$,

$$pA' \leq \{pq_1(A')A' + [pq_1(A') + (1-p)q_0(A')]\bar{f}^V\},$$

which can be written as:

$$0 \geq p(1 - q_1(A'))A' - [pq_1(A') + (1 - p)q_0(A')]f^V.$$

Rearranging terms:

$$[pq_1(A') + (1 - p)q_0(A')]f^V \geq p(1 - q_1(A'))A'.$$

This constraint should hold $\forall A' \in [0, \bar{A}]$. Hence, the victim's incentive-compatibility constraint can be expressed in terms of iA: $\forall A \in [0, \bar{A}]$,

$$[pq_1(A) + (1 - p)q_0(A)]\bar{f}^V \geq p(1 - q_1(A))A. \tag{1}$$

■

Lemma 2. *Suppose $p \in (0, 1)$. The liable injurer's incentive-compatibility constraint is $\int_0^{\bar{A}} Ag(A)dA \leq \bar{f}^I \int_0^{\bar{A}} q_0(A)g(A)dA$.*

Proof. When a liable injurer truthfully reports his type, the injurer pays $\mathbb{E}[A]$ and gets an expected payoff equal to:

$$-\mathbb{E}[A] = -\int_0^{\bar{A}} Ag(A)dA,$$

By pretending to be non-liable, a liable I gets an expected payoff equal to:

$$\mathbb{E}[(1 - q_0(A))(0)] - \mathbb{E}[q_0(A)f^I] = -f^I \int_0^{\bar{A}} q_0(A)g(A)dA.$$

Therefore, the liable injurer's incentive-compatibility constraint is:

$$-\int_0^{\bar{A}} Ag(A)dA \geq -f^I \int_0^{\bar{A}} q_0(A)g(A)dA,$$

which can be rewritten as:

$$\int_0^{\bar{A}} Ag(A)dA \leq f^I \int_0^{\bar{A}} q_0(A)g(A)dA.$$

By setting $f^I \in [0, \bar{f}^I]$ as high as possible, the social planner will economize on verification efforts. Given that the injurer has limited financial resources, $\bar{f}^I = W^I$. Hence, the liable injurer's incentive-compatibility constraint is:

$$\int_0^{\bar{A}} Ag(A)dA \leq \bar{f}^I \int_0^{\bar{A}} q_0(A)g(A)dA. \tag{2}$$

■

Claim 1. *Suppose $p \in (0,1)$. The victim's incentive-compatibility constraint for a victim of type $A \in [0, \bar{A}]$ holds as an equality at the interim probabilities of investigation $0 \leq q_i(A) \leq 1$ $(i = 0, 1)$.*

Proof. Suppose $p \in (0,1)$. We show by contradiction. The victim's incentive-compatibility constraint can be rewritten as $(p\bar{f}^V + pA)q_1(A) + (1-p)\bar{f}^V q_0(A) \geq pA$. Suppose that the constraint holds as a strict inequality, $(p\bar{f}^V + pA)q_1 + (1-p)\bar{f}^V q_0(A) > pA$, and that $q_0(A)$ and $q_1(A)$ are the interim probabilities of investigation, i.e., they minimize $\mathbb{E}[C] = pq_1(A)C_1(A) + (1-p)q_0(A)C_0(A)$. The constraint is not satisfied at $q_0(A) = 0$ and $q_1(A) = 0$. Hence, at least one $q_i(A) > 0$ $(i = 0, 1)$. First, suppose that $q_1(A) > 0$. It suffices to show that a reduction in $q_1(A)$ still satisfies the constraint. Define $\Psi \equiv (p\bar{f}^V + pA)q_1(A) + (1-p)\bar{f}^V q_0(A) - pA > 0$. Assume that $q_1(A)$ is reduced by $\frac{\Psi}{2(p\bar{f}^V + pA)}$. The constraint is now $\big(p\bar{f}^V + pA)(q_1(A) - \frac{\Psi}{2(p\bar{f}^V + pA)}\big) + (1-p)\bar{f}^V q_0(A) = pA + \Psi - \frac{\Psi}{2} = pA + \frac{\Psi}{2} > pA$. Hence, a reduction in $q_1(A)$ still satisfies the constraint but reduces the expected cost of producing evidence. Contradiction follows. Second, suppose $q_0(A) > 0$. It suffices to show that a reduction in $q_0(A)$ still satisfies the constraint. Assume that $q_0(A)$ is reduced by $\frac{\Psi}{2(1-p)\bar{f}^V}$. The constraint is now $\big(p\bar{f}^V + pA)q_1 + (1-p)\bar{f}^V(q_0(A) - \frac{\Psi}{2(1-p)\bar{f}^V}\big) = pA + \Psi - \frac{\Psi}{2} = pA + \frac{\Psi}{2} > pA$. Hence, a reduction in $q_0(A)$ still satisfies the constraint but reduces the expected cost of producing evidence. Contradiction follows. We conclude that the incentive-compatibility constraint for victim of type A holds as an equality. ■

Claim 2. *Suppose $A = \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ and $p \in (0,1)$. The optimal mechanism is not unique for every p-value when $p > \frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I}$.*

Proof.

1. We identify the set of interim probabilities of investigation. The maximal $q_0(A)$ and $q_1(A)$ such that (3) is satisfied are obtained by evaluating (3) at $q_1(A) = 0$ and $q_0(A) = 0$, respectively. The interim probabilities of investigation are $q_0(A) \in \big[0, \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V}\big]$ and $q_1(A) \in \big[0, \frac{A}{A + \bar{f}^V}\big]$.

2. We show that the optimal mechanism is not unique for every p when $p > \frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I}$.

We structure the continuum of $q_0(A)$ and $q_1(A)$as follows. We know that $q_1(A) \in [0, \frac{A}{A+\bar{f}^V}]$. Define $\psi \equiv \frac{q_1(A)}{\frac{A}{A+\bar{f}^V}}$ such that $\psi \in [0,1]$. Hence, $q_1(A) = \psi\frac{A}{A+\bar{f}^V}$, and any value of $\psi \in [0,1]$ corresponds to a value of $q_1(A)$ consistent with the set of interim $q_1(A)$.

Given constraint (3),

$$q_0(A) = \frac{pA}{(1-p)\bar{f}^V} - \frac{p(\bar{f}^V + A)}{(1-p)\bar{f}^V}\psi\frac{A}{A+\bar{f}^V} = \frac{pA}{(1-p)\bar{f}^V}(1-\psi)$$

When $\psi = 1$, the interim probabilities of investigation are the same as the ones in the case where $A \geq \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$. When $\psi = 0$, the interim probabilities of investigation are the same as the ones in the case where $A < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$. When $\psi \in (0,1)$, the interim probabilities of investigation correspond to the ones that satisfy (3) and the feasibility constraints.

It is simple to show that for $p \in (0, \tilde{p}]$, the optimal mechanism is unique for each p. Specifically, $q_0(A)$ and $q_1(A)$ are the same as the ones for the cases where $A \geq \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ and $A < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$. Take the case of $p = \frac{\bar{f}^V}{\bar{f}^V+\bar{f}^I}$. The optimal mechanism is unique for this p-value: $q_0(A) = \frac{pA}{(1-p)\bar{f}^V}$ and $q_1(A) = 0\ \forall A \in [0, \bar{A}]$. The liable injurer's incentive-compatibility constraint is satisfied as equality:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{\bar{A}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA.$$

Consider $p = \frac{\bar{f}^V}{\bar{f}^V+\bar{f}^I} + \varepsilon$, where $\varepsilon > 0$, a small number. If an adjustment $\forall A \in [0, \bar{A}]$, consisting of increasing $q_0(A)$ and decreasing $q_1(A)$ such that the liable injurer's incentive-compatibility constraint while keeping the victim's incentive-compatibility holding as an equality, is implemented, the liable injurer's incentive-compatibility constraint is:

$$\int_0^{\bar{A}} Ag(A)dA < \bar{f}^I \int_0^{\bar{A}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA$$

Hence, the adjustment should be implemented only for $A \in [0, A^*]$, where A^* is determined by the incentive-compatibility constraint written as an equality for $p = \frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I} + \epsilon$:

$$\int_0^{\bar{A}} Ag(A)dA =$$

$$= \bar{f}^I \left[\int_0^{A^*} \frac{pA}{(1-p)\bar{f}^V} g(A)dA + \int_{A^*}^{\bar{A}} \frac{pA}{(1-p)\bar{f}^V} (1-\psi) g(A)dA \right]$$

In the optimal mechanism, $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$ for $A \in [0, A^*]$; $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}(1-\psi)$ and $q_1(A) = \psi\frac{A}{A+\bar{f}^V}$ for $A \in (A^*, \bar{A}]$. Therefore, each ψ-value generates an optimal mechanism. Given that there are infinitely many ψ-values ($\psi \in [0,1]$), the optimal mechanism is not unique for $p = \frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I} + \epsilon$. A similar approach can be used to evaluate any $p \in \left(\frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I}, 1\right)$.

■

Claim 3. *Suppose $p \in (0,1)$. Across victim's types, the interim probabilities of investigation encompass multiple cases.*

Proof.

1. Suppose that $C_0(A) \geq C_1(A)$ $\forall A \in [0, \bar{A}]$. Therefore, $A \geq \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$ $\forall A \in [0, \bar{A}]$. Hence, by Proposition 1, the interim probabilities of investigation are: When $A \in [0, \bar{A}]$,

$$\begin{cases} q_0(A) = 0 \\ q_1(A) = \frac{A}{\bar{f}^V + A} < 1. \end{cases}$$

2. Suppose that $C_0(A) < C_1(A)$ $\forall A \in [0, \bar{A}]$. Therefore, $A \geq \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$ or $A < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$.

   (a) Suppose $A \geq \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$. Therefore, by Proposition 1, the interim probabilities of investigation are: When $A \in \left[\left(\frac{C_1(A)}{C_0(A)} - \right.\right.$

$1\Big)\bar{f}^V, \bar{A}\Big]$,

$$\begin{cases} q_0(A) = 0 \\ q_1(A) = \frac{A}{\bar{f}^V + A} < 1. \end{cases}$$

(b) Suppose $A < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$. Therefore, $\left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V \leq \left(\frac{1-p}{p}\right)\bar{f}^V$ or $\left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V > \left(\frac{1-p}{p}\right)\bar{f}^V$.

i. Suppose $\left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V \leq \left(\frac{1-p}{p}\right)\bar{f}^V$ and $A < \min\left\{\left(\frac{1-p}{p}\right)\bar{f}^V, \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V\right\}$. Therefore, by Proposition 1, the interim probabilities of investigation are: When $A \in \left[0, \min\left\{\left(\frac{1-p}{p}\right)\bar{f}^V, \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V\right\}\right)$,

$$\begin{cases} q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1 \\ q_1(A) = 0. \end{cases}$$

ii. Suppose $\left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V > \left(\frac{1-p}{p}\right)\bar{f}^V$ and $A \leq \left(\frac{1-p}{p}\right)\bar{f}^V$. Therefore, by Proposition 1, the interim probabilities of investigation are: When $A \in \left[0, \left(\frac{1-p}{p}\right)\bar{f}^V\right]$,

$$\begin{cases} q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1 \\ q_1(A) = 0. \end{cases}$$

iii. Suppose $\left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V > \left(\frac{1-p}{p}\right)\bar{f}^V$ and $A > \left(\frac{1-p}{p}\right)\bar{f}^V$. Therefore, by Proposition 1, the interim probabilities of investigation are: When $A \in \left(\left(\frac{1-p}{p}\right)\bar{f}^V, \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V\right)$,

$$\begin{cases} q_0(A) = 1 \\ q_1(A) = 1 - \frac{\bar{f}^V}{p(A + \bar{f}^V)} < 1. \end{cases}$$

■

Define $A^0(p) \equiv \left(\frac{1-p}{p}\right)\bar{f}^V$.

Claim 4. $\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1 \ \forall p \in (0,1) \iff A \leq A^0(p)$.

Proof. $\forall p \in (0,1)$, $\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1$ if $A \leq \left(\frac{1-p}{p}\right)\bar{f}^V = A^0(p)$. The same logic applies to the proof that $\forall p \in (0,1)$, $A \leq \left(\frac{1-p}{p}\right)\bar{f}^V = A^0(p)$ if $\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1$. ■

Proposition 1. *Suppose $p \in (0,1)$. The interim probabilities of investigation for a victim of type A are as follows.*

1. *If $A \geq \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$, then $q_0(A) = 0$ and $q_1(A) = \frac{A}{\bar{f}^V + A}$.*
2. *If $A < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$ and $A \leq \left(\frac{1-p}{p}\right)\bar{f}^V = A^0(p)$, then $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$.*
3. *If $A < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$ and $A > \left(\frac{1-p}{p}\right)\bar{f}^V = A^0(p)$, then $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(\bar{f}^V + A)}$.*

Proof. Let $\mathbb{E}[C] = pq_1(A)C_1(A) + (1-p)q_0(A)C_0(A)$ be the expected cost of producing evidence associated with a victim's of type A. The social planner's problem is as follows.

$$\min_{q_0(A), q_1(A)} \{pq_1(A)C_1(A) + (1-p)q_0(A)C_0(A)\}$$

subject to the victim's incentive-compatibility constraint (1) and the feasibility constraints $0 \leq q_0(A) \leq 1$ and $0 \leq q_1(A) \leq 1$. Claim 1 shows that the victim's incentive-compatibility constraint holds as an equality. Hence, the victim's incentive compatibility constraint (1) becomes:

$$(p\bar{f}^V + pA)q_1(A) + (1-p)\bar{f}^V q_0(A) = pA. \tag{3}$$

The optimization problem is now reduced to a minimization with an equality constraint. Solving for $q_0(A)$:

$$q_0(A) = -\frac{p(\bar{f}^V + A)}{(1-p)\bar{f}^V} q_1(A) + \frac{pA}{(1-p)\bar{f}^V}.$$

Substituting $q_0(A)$ in the objective function,

$$\mathbb{E}[C] = \frac{1}{\bar{f}^V}\{q_1[pC_1(A)\bar{f}^V - pC_0(A)(\bar{f}^V + A)] + pAC_0(A)\}.$$

Hence,

$$\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} = \frac{[pC_1(A)\bar{f}^V - pC_0(A)(\bar{f}^V + A)]}{\bar{f}^V}.$$

When $\frac{pC_1(A)\bar{f}^V - pC_0(A)(\bar{f}^V+A)}{\bar{f}^V} < 0$, which is equivalent to $A > \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} < 0$; when $\frac{pC_1(A)\bar{f}^V - pC_0(A)(\bar{f}^V+A)}{\bar{f}^V} = 0$, which is equivalent to $A = \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} = 0$; and, when $\frac{pC_1(A)\bar{f}^V - pC_0(A)(\bar{f}^V+A)}{\bar{f}^V} > 0$, which is equivalent to $A < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} > 0$.

We now characterize the interim probabilities of investigation for a victim of type A by incorporating the feasibility constraints $0 \leq q_0(A) \leq 1$ and $0 \leq q_1(A) \leq 1$ in the analysis. Suppose $A > \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} < 0$. The interim $q_1(A)$ is a corner solution with $q_1(A)$ taking the maximal value constrained by $q_1(A) \leq 1$ and equation (3). Solving for $q_1(A)$, we get $q_1(A) = \big(\frac{1}{pA + p\bar{f}^V}\big)[pA - (1-p)\bar{f}^V q_0(A)]$. Note that $q_1(A)$ takes the maximal value if $q_0(A) = 0$: $q_1(A) = \frac{A}{A+\bar{f}^V}$. Hence, the interim probabilities of investigation for a victim of type A are $q_0(A) = 0$ and $q_1(A) = \frac{A}{A+\bar{f}^V} < 1$.

Suppose that $A = \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ and therefore, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} = 0$. Claim 2 shows that, when $A = \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$, the interim probabilities of investigation $q_0(A)$ and $q_1(A)$ involve infinitely many values, $q_0(A) \in \big[0, \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V}\big]$ and $q_1(A) \in \big[0, \frac{A}{A+\bar{f}^V}\big]$. Claim 2 also shows that the optimal mechanism is not unique for p sufficiently high. We consider here the minimum value for $q_0(A)$ and the corresponding $q_1(A)$ such that constraint (3) and the feasibility constraints are satisfied. Hence, the interim probabilities of investigation for a victim of type A are $q_0(A) = 0$ and $q_1(A) = \frac{A}{A+\bar{f}^V} < 1$.

Suppose now that $A < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$, $\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} > 0$. The interim $q_1(A)$ is a corner solution with $q_1(A)$ taking the minimal value constrained by $q_1(A) \geq 0$ and equation (3). We need to check whether $q_1 = 0$ satisfies the victim's incentive-compatibility constraint. Evaluating equation (3) at $q_1(A) = 0$ and solving for $q_0(A)$ yields $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V}$. We also need to verify whether $q_0(A)$ satisfies the feasibility constraints. By Claim 4, $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} \leq 1$ when $A \leq \big(\frac{1-p}{p}\big)\bar{f}^V = A^0(p)$. Hence, two mutually-exclusive cases are possible. If $A \leq \big(\frac{1-p}{p}\big)\bar{f}^V = A^0(p)$, then the interim probabilities of investigation are $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$. If $A > \left(\frac{1-p}{p}\right)\bar{f}^V = A^0(p)$, then the minimal feasible $q_1(A)$ is the one that corresponds to $q_0(A) = 1$. Evaluating equation (3) at $q_0(A) = 1$ and solving for $q_1(A)$ yields $q_1(A) = 1 - \frac{\bar{f}^V}{p(\bar{f}^V+A)}$. Hence, the interim probabilities of investigation are $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(\bar{f}^V+A)} < 1$. ■

Define $p^0 \equiv \frac{\bar{f}^V}{\bar{f}^V+\bar{A}}$.

Claim 5. $\bar{A} > A^0(p) \iff p > p^0$.

Proof. If $\bar{A} > \left(\frac{1-p}{p}\right)\bar{f}^V = A^0(p)$, solving for p, $p > \frac{\bar{f}^V}{\bar{f}^V+\bar{A}} = p^0$. The same logic applies to the proof that $p > p^0 \implies \bar{A} > A^0(p)$. ■

Claim 6. $\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1 \; \forall A \in [0, \bar{A}]$ *if* $p \leq p^0$.

Proof. Given that $\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ is increasing in A, $\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1 \; \forall A \in [0, \bar{A}]$ if this holds for the highest value of A. Hence, $\left(\frac{p}{1-p}\right)\frac{\bar{A}}{\bar{f}^V} \leq 1$ if $p \leq \frac{\bar{f}^V}{\bar{f}^V+\bar{A}} = p^0$. ■

Proposition 2. *The interim probabilities of investigation for Environments 1 and 2 across victim's types are as follows.*

1. *Environment 1: If $C_0(A) \geq C_1(A) \; \forall A \in [0, \bar{A}]$, then the interim probabilities of investigation are: $q_0(A) = 0$ and $q_1(A) = \frac{A}{\bar{f}^V+A} < 1$ $\forall A \in [0, \bar{A}]$ and $\forall p \in (0, 1)$.*
2. *Environment 2: If $C_0(A) < C_1(A) \; \forall A \in [0, \bar{A}]$ and $\bar{A} < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V \; \forall A \in [0, \bar{A}]$, then the interim probabilities of investigation are as follows.*
   - *(a) p-Segment 1: If $p \in (0, p^0]$, then the interim probabilities of investigation are $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0 \; \forall A \in [0, \bar{A}]$.*
   - *(b) p-Segment 2: If $p \in (p^0, 1)$, then the interim probabilities of investigation are $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0 \; \forall A \in [0, A^0(p)]$, and $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(A+\bar{f}^V)} \; \forall A \in (A^0(p), \bar{A}]$.*

Proof.

1. Suppose $C_0(A) \geq C_1(A)\ \forall A \in [0, \bar{A}]$. Therefore, $A \geq \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ is satisfied $\forall A \in [0, \bar{A}]$. By Proposition 1, the interim probabilities of investigation are: $q_0(A) = 0$ and $q_1(A) = \frac{A}{\bar{f}^V + A} < 1\ \forall A \in [0, \bar{A}]$. The interim probabilities of investigation hold $\forall\ p \in (0, 1)$.

2. Suppose $C_0(A) < C_1(A)\ \forall A \in [0, \bar{A}]$ and $\bar{A} < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$. Therefore, $A \geq \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ is never satisfied. By Proposition 1, if $A \leq \big(\frac{1-p}{p}\big)\bar{f}^V = A^0(p)$ or equivalently, if $p \leq \frac{\bar{f}^V}{\bar{f}^V + A}$, then the interim probabilities of investigation are $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} \leq 1$ by Claim 5, and $q_1(A) = 0$; if $A > \big(\frac{1-p}{p}\big)\bar{f}^V = A^0(p)$, or equivalently, if $p > \frac{\bar{f}^V}{\bar{f}^V + A}$, then the interim probabilities of investigation are $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(A + \bar{f}^V)} < 1$. When $p \leq p^0$, $\bar{A} \leq A^0(p)$ by Claim 4. Therefore, every $A \in [0, \bar{A}]$ is lower than or equal to A^0. When $p > p^0$, $\bar{A} > A^0(p)$ by Claim 4. Therefore, $A \in [0, \bar{A}]$ can be greater than or lower than A^0. Hence, two mutually-exclusive p-segments are possible:

   (a) p-Segment 1: If $p \in (0, p^0]$, the interim probabilities of investigation are $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} \leq 1$ by Claim 5, and $q_1(A) = 0$ $\forall A \in [0, \bar{A}]$.

   (b) p-Segment 2: If $p \in (p^0, 1)$: The interim probabilities of investigation are as follows.

      i. If $A \leq A^0(p)$, then $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} \leq 1$ by Claim 5, and $q_1(A) = 0$.

      ii. If $A > A^0(p)$, then $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(A + \bar{f}^V)} > 0$.

   The interim probabilities of investigation in Environment 2 hold $\forall A \in [0, \bar{A}]$ and $\forall\ p \in (0, 1)$.

■

Adjustments to the Interim Probabilities of Investigation

This section discusses technical aspects of the application of adjustments to the interim probabilities of investigation to satisfy the liable injurer's incentive-compatibility constraint (2), $\int_0^{\bar{A}} A g(A) dA \leq \bar{f}^I \int_0^{\bar{A}} q_0(A) g(A) dA$. First, we define the two possible adjustment procedures, and denote them as Procedures 1 and 2.

Definition 2. *Procedure 1 consists of an increase in* $q_0(A)$ *and a decrease in* $q_1(A)$ *until the liable injurer's incentive-compatibility constraint (2) is satisfied as an equality while keeping the victim's incentive-compatibility constraint (1) satisfied as an equality. Procedure 2 consists of an increase in* $q_0(A)$ *without decreasing* $q_1(A)$ *until the liable injurer's incentive-compatibility constraint (2) is satisfied as an equality while keeping the victim's incentive-compatibility constraint (1) satisfied.*

In Environment 1, the interim $q_0(A)$ is always zero. Hence, constraint (2) is never satisfied and an adjustment is required. Given that the interim $q_1(A) > 0$, Procedures 1 and 2 can be implemented. In Environment 2, even when $q_0(A) > 0$, f^I is limited to $\bar{f}^I$. Therefore, constraint (2) might not be satisfied and an adjustment might be required. When $p \in (0, p^0]$ and $A \in [0, \bar{A}]$, although the interim $q_0(A) < 1$, the interim $q_1(A) = 0$. Hence, only Procedure 2 can be implemented. When $p \in (p^0, 1]$, an adjustment can be applied only for relatively low A-values. Specifically, when $A \in (0, A^0(p)]$, even when the interim $q_0(A) < 1$, the interim $q_1(A) = 0$. Hence, only Procedure 2 can be implemented. When $A \in (A^0(p), \bar{A}]$, although the interim $q_1(A) > 0$, the interim $q_0(A) = 1$. Hence, neither Procedure 1 nor Procedure 2 can be implemented. The next corollary summarizes this result.

Corollary 1. *Procedures 1 and 2 can be applied in Environment 1. Only Procedure 2 can be applied in Environment 2.*

Second, we study the efficiency of the adjustment procedures. Consider a victim of type A. Remember that $\mathbb{E}[C] = pq_1(A)C_1(A) + (1-p)q_0(A)C_0(A)$ represents the expected cost of producing evidence associated with a victim of type A. Let $u^I_{0|1} = \bar{f}^I q_0(A)$ represent the liable injurer's expected loss for misreporting associated with a victim of type A, $\frac{\partial u^I_{0|1}}{\partial q_0(A)}$ represent the change in the liable injurer's expected loss for misreporting due to an increase in $q_0(A)$, and $\frac{\partial \mathbb{E}[C]}{\partial q_0(A)}$ represent the change in the expected cost of producing evidence due to an increase in q_0 while keeping the victim's incentive-compatibility constraint satisfied. Next, we define the efficiency of Procedure i, $\Omega_i(A)$ $(i = 1, 2)$.

Definition 3. *Consider a victim of type A. Efficiency of Procedure i is defined as* $\Omega_i(A) = \frac{\frac{\partial u^I_{0|1}}{\partial q_0(A)}}{\frac{\partial \mathbb{E}[C]}{\partial q_0(A)}}$.

Intuitively, a high $\Omega_i(A)$ indicates a high increase in $u^I_{0|1}$ and/or a low increase in $\mathbb{E}[C]$ due to an increase in $q_0(A)$. Hence, the procedure with the highest $\Omega_i(A)$ exhibits the highest level of efficiency in incentivizing the liable injurer through an increase in $q_0(A)$. Lemma 3 characterizes $\Omega_1(A)$ and $\Omega_2(A)$.

$$\Omega_1(A) = \frac{\bar{f}^I}{(1-p)C_0(A) - (1-p)\frac{C_1(A)}{\left(1+\frac{A}{\bar{f}^V}\right)}}.$$

$$\Omega_2(A) = \frac{\bar{f}^I}{(1-p)C_0(A)}.$$

Both $\Omega_1(A)$ and $\Omega_2(A)$ apply to Environment 1, and only $\Omega_2(A)$ applies to Environment 2. Lemma 3 shows that when $C_0(0) > C_1(0)$, $\Omega_1(A) > 0$ exists $\forall A \in [0, \bar{A}]$.

Lemma 3. *(1) Suppose $C_0(A) \geq C_1(A)$ $\forall A \in ([0, \bar{A}]$. If and only if $C_0(0) > C_1(0)$, $\Omega_1(A) > 0$ exists. (2) If and only if $C_0(0) > 0$, $\Omega_2(A) > 0$ exists.*

Proof. We first characterize Ω_1 and Ω_2. By Definition 4, $\Omega_i(A) = \frac{\frac{\partial u^I_{0|1}}{\partial q_0(A)}}{\frac{\partial \mathbb{E}[C]}{\partial q_0(A)}}$. $\mathbb{E}[C] = pq_1(A)C_1(A) + (1-p)q_0(A)C_0(A)$ and $\frac{\partial u^I_{0|1}}{\partial q_0(A)} = \bar{f}^I$. Consider Procedure 1.

$$\frac{\partial \mathbb{E}[C]}{\partial q_0(A)} = \frac{\partial \mathbb{E}[C]}{\partial q_0(A)} + \frac{\partial \mathbb{E}[C]}{\partial q_1(A)}\frac{\partial q_1(A)}{\partial q_0(A)}.$$

The direct effect of an increase in $q_0(A)$, DE, is

$$DE = \frac{\partial \mathbb{E}[C]}{\partial q_0(A)} = (1-p)C_0(A) > 0.$$

$$\frac{\partial \mathbb{E}[C]}{\partial q_1(A)} = pC_1(A).$$

Given equation (3), $(p\bar{f}^V + pA)q_1(A) + (1-p)\bar{f}^V q_0(A) = pA$,

$$\frac{\partial q_1(A)}{\partial q_0(A)} = -\frac{(1-p)}{p\left(1+\frac{A}{\bar{f}^V}\right)}.$$

Therefore, the indirect effect of an increase in $q_0(A)$, IE, is

$$IE = \frac{\partial \mathbb{E}[C]}{\partial q_1(A)}\frac{\partial q_1(A)}{\partial q_0(A)} = pC_1(A)\left[-\frac{(1-p)}{p\left(1+\frac{A}{\bar{f}^V}\right)}\right] < 0.$$

The overall effect of an increase in $q_0(A)$, $\frac{\partial \mathbb{E}[C]}{\partial q_0(A)}$, is:

$$\frac{\partial \mathbb{E}[C]}{\partial q_0(A)} = (1-p)C_0(A) + pC_1(A)\left[-\frac{(1-p)}{p\left(1+\frac{A}{\bar{f}^V}\right)}\right] = (1-p)\left[C_0(A) - \frac{C_1(A)}{\left(1+\frac{A}{\bar{f}^V}\right)}\right].$$

Hence,

$$\Omega_1(A) = \frac{\bar{f}^I}{(1-p)\left[C_0(A) - \frac{C_1(A)}{\left(1+\frac{A}{\bar{f}^V}\right)}\right]}.$$

Consider Procedure 2.

$$\frac{\partial \mathbb{E}[C]}{\partial q_0(A)} = (1-p)C_0(A) = DE.$$

Hence,

$$\Omega_2(A) = \frac{\bar{f}^I}{(1-p)C_0(A)}.$$

1. Suppose $C_0(A) \geq C_1(A)\ \forall A \in (0, \bar{A}]$ and $C_0(0) > C_1(0)$. We show that

$$\Omega_1(A) = \frac{\bar{f}^I}{(1-p)\left[C_0(A) - \frac{C_1(A)}{\left(1+\frac{A}{\bar{f}^V}\right)}\right]} > 0$$

exists $\forall A \in [0, \bar{A}]$.

(a) When $A > 0$,

$$0 < \left(1 + \frac{A}{\bar{f}^V}\right) = \frac{f^V}{\bar{f}^V + A} < 1.$$

Therefore,

$$C_0(A) > \frac{C_1(A)}{\left(1+\frac{A}{f^V}\right)},$$

by $C_0(A) \geq C_1(A)$ $\forall A \in (0, \bar{A}]$. Hence,

$$\Omega_1(A) = \frac{\bar{f}^I}{(1-p)\Big[C_0(A) - \frac{C_1(A)}{\left(1+\frac{A}{f^V}\right)}\Big]} > 0$$

exists.

(b) When $A = 0$, $\left(1 + \frac{A}{f^V}\right) = 1$. Hence,

$$\Omega_1(A) = \frac{\bar{f}^I}{(1-p)(C_0(0) - C_1(0))} > 0$$

exists if and only if $C_0(0) > C_1(0)$.

2. Suppose $C_0(0) > 0$. We show that $\Omega_2(A) > 0$ exists.

$$\Omega_2(A) = \frac{\bar{f}^I}{(1-p)C_0(A)} > 0$$

exists $\forall A \in [0, \bar{A}]$ if and only if $C_0(0) > 0$ and $\frac{\partial C_0(A)}{\partial A} > 0$ by assumption.

■

Intuition follows. In Procedure 1, the effect of an increase in $q_0(A)$ on $E[C]$ involves a positive direct effect through $q_0(A)$ and a negative indirect effect through $q_1(A)$. The direct effect more than offsets the indirect effect and hence, the overall effect is positive. Lemma 3 also shows that when $C_0(0) > 0$, $\Omega_2 > 0$ exists $\forall A \in [0, \bar{A}]$. Intuitively, in Procedure 2, the overall effect of an increase in $q_0(A)$ on $E[C]$ involves just a positive direct effect through $q_0(A)$.

Next, we compare the efficiency of both procedures for a victim of type A in Environment 1. Suppose $C_0(A) \geq C_1(A)$ $\forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)$ and $C_0(0) > 0$,

$$\Omega_1(A) = \frac{\bar{f}^I}{(1-p)C_0(A) - (1-p)\frac{C_1(A)}{\left(1+\frac{A}{f^V}\right)}} > \frac{\bar{f}^I}{(1-p)C_0(A)} = \Omega_2(A).$$

In words, for a victim of type A, Procedure 1 is the most efficient adjustment procedure.

Third, we provide formal analysis of the application of Procedures 1 and 2 across victim's types, and identify the efficiency-superior procedure in Environment 1.[52]

Proposition 3 identifies a sufficient condition under which the application of Procedure 1 should start at the lowest value of A, and shows that Procedure 2 should always start at the lowest value of A.[53]

Proposition 3. *(1) Suppose* $C_0(A) \geq C_1(A)\ \forall A \in (0, \bar{A}]$ *and* $C_0(0) > C_1(0)$. *If* $\frac{\partial \frac{C_1(A)}{C_0(A)}}{\partial A} \leq 0\ \forall A \in [0, \bar{A}]$, *then the implementation of Procedure 1 should start at the lowest value of A. (2) Suppose* $C_0(0) > 0$. *Tthe implementation of Procedure 2 should start at the lowest value of A.*

Proof.

1. Suppose $C_0(A) \geq C_1(A)\ \forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)$ and $\frac{\partial \frac{C_1(A)}{C_0(A)}}{\partial A} \leq 0$ $\forall A \in [0, \bar{A}]$. $\Omega_1(A)$ can be written as:

$$\Omega_1(A) = \frac{\bar{f}^I}{(1-p)C_0(A)\left[1 - \frac{C_1(A)}{C_0(A)\left(1+\frac{A}{\bar{f}^V}\right)}\right]}.$$

Denote $\left[1 - \frac{C_1(A)}{C_0(A)\left(1+\frac{A}{\bar{f}^V}\right)}\right]$ as $\Psi(A)$.

$$\frac{\partial \Omega_1(A)}{\partial A} =$$

$$= \frac{\bar{f}^I}{1-p}\left[\frac{1}{C_0(A)}\left(-\frac{1}{(\Psi(A))^2}\right)\frac{\partial \Psi(A)}{\partial A} + \frac{1}{\Psi(A)}\left(-\frac{1}{(C_0(A))^2}\right)\frac{\partial C_0(A)}{\partial A}\right],$$

where $\frac{\partial C_0(A)}{\partial A} > 0$, by assumption of the model, and

$$\frac{\partial \Psi(A)}{\partial A} = -\frac{\partial \frac{C_1(A)}{C_0(A)}}{\partial A}\frac{1}{\left(1+\frac{A}{\bar{f}^V}\right)} - \frac{C_1(A)}{C_0(A)}\left[-\frac{1}{\left(1+\frac{A}{\bar{f}^V}\right)^2}\frac{1}{\bar{f}^V}\right] > 0$$

[52]Remember that only Procedure 2 can be implemented in Environment 2.

[53]Proposition 3 shows that the sign of $\frac{\partial \Omega_1(A)}{\partial A}$ is ambiguous. Therefore, Procedure 1 could also start at the highest value of A. For consistency of the methodology across procedures, we decided to impose a condition such that $\frac{\partial \Omega_1(A)}{\partial A} < 0$.

because $\frac{\partial \frac{C_1(A)}{C_0(A)}}{\partial A} \leq 0$, by assumption. Hence, $\frac{\partial \Omega_1(A)}{\partial A} < 0$.

2. Suppose $C_0(0) > 0$.

$$\Omega_2(A) = \frac{\bar{f}^I}{(1-p)C_0(A)}.$$

$$\frac{\partial \Omega_2(A)}{\partial A} = -\frac{\bar{f}^I \frac{\partial C_0(A)}{\partial A}}{(1-p)(C_0(A))^2} < 0,$$

because $\frac{\partial C_0(A)}{\partial A} > 0$ by assumption of the model.

■

Intuition follows. In Environment 1, when $\frac{\partial \frac{C_1(A)}{C_0(A)}}{\partial A} \leq 0 \ \forall A \in [0, \bar{A}]$, $\Omega_1(A)$ increases when A decreases. Across environments, $\Omega_2(A)$ increases when A decreases. Hence, the social planner should start the application of Procedure i $(i = 1, 2)$ at the lowest levels of A where Procedure i is more efficient. The next remark summarizes this result.

Proposition 4 identifies a necessary and sufficient condition for the efficiency superiority of Procedure 1 across victim's types in Environment 1 where both procedures can be applied.

Proposition 4. *Suppose $C_0(A) \geq C_1(A) \ \forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)$, $C_0(0) > 0$ and $\frac{\partial \frac{C_1(A)}{C_0(A)}}{\partial A} \leq 0 \ \forall A \in [0, \bar{A}]$. If and only if $C_0(A) - C_0(0) < \frac{C_1(A)\bar{f}^V}{A+\bar{f}^V} \ \forall A \in [0, \bar{A}]$, Procedure 1 is more efficient than Procedure 2 across victim's types.*

Proof. Suppose $C_0(A) \geq C_1(A) \ \forall A \in [0, \bar{A}]$, $C_0(0) > C_1(0)$, $C_0(0) > 0$, $\frac{\partial \frac{C_1(A)}{C_0(A)}}{\partial A} \leq 0$ and $C_0(A) - C_0(0) < \frac{C_1(A)\bar{f}^V}{A+\bar{f}^V} \ \forall A \in [0, \bar{A}]$. Given that $\frac{\partial \Omega_2(A)}{\partial A} < 0$, it suffices to compare $\Omega_2(A)$ for $A = 0$ and $\Omega_1(A)$ for $A \in [0, \bar{A}]$.

$$\Omega_1(A) = \frac{\bar{f}^I}{(1-p)\Big[C_0(A) - \frac{C_1(A)\bar{f}^V}{A+\bar{f}^V}\Big]} > \frac{\bar{f}^I}{(1-p)C_0(0)} = \Omega_2(0)$$

if and only if $C_0(A) - C_0(0) < \frac{C_1(A)\bar{f}^V}{A+\bar{f}^V}$. ■

Intuition follows. In Environment 1, when $C_0(A) - C_0(0) < \frac{C_1(A)\bar{f}^V}{A+\bar{f}^V}$ for $A \in [0, \bar{A}]$, the efficiency of Procedure 1 for any $A \in [0, \bar{A}]$ is strictly higher than the highest level of efficiency of Procedure 2 (which occurs when $A = 0$). Hence, the social planner should exhaust the implementation of Procedure 1, i.e., should increase $q_0(A)$ and reduce $q_1(A)$ across victim's types, before applying Procedure 2. The next corollary summarizes this result.

Corollary 2. *In Environment 1 where Procedures 1 and 2 can be applied, Procedure 1 should be applied first.*

Proposition 5. *Suppose $C_0(A) \geq C_1(A)$ $\forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)$ and $p \in (0, 1)$. There are three p-segments: p-Segment 1 where $p \in (0, \tilde{p}]$, p-Segment 2.1 where $p \in (\tilde{p}, \bar{p}]$, and p-Segment 2.2 where $p \in (\bar{p}, 1)$.*

Proof. Suppose $C_0(A) \geq C_1(A)$ $\forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)$ and $p \in (0, 1)$.

1. We show that $p \in (0, 1)$ is divided into two main p-segments, denoted as p-Segment 1 and p-Segment 2. To accomplish this, we need to show that there exists a unique p-threshold, denoted as $\tilde{p}$ such that $0 < \tilde{p} < 1$.
   (a) In the main text of the paper, we define $\tilde{p}$ as the p-threshold such that, after exhausting Procedure 1 $\forall A \in [0, \bar{A}]$, the liable injurer's incentive-compatibility constraint is satisfied as an equality.
   $$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{\bar{A}} \left(\frac{\tilde{p}}{1-\tilde{p}}\right) \frac{A}{\bar{f}^V} g(A)dA.. \tag{5}$$
   (b) We need to show that $\tilde{p} < p^0$. Otherwise, Procedure 1 could not be exhausted $\forall A \in [0, \bar{A}]$. Equation (5) can be rewritten as:
   $$\int_0^{\bar{A}} Ag(A)dA = \frac{\bar{f}^{\;I}}{\bar{f}^V} \left(\frac{\tilde{p}}{1-\tilde{p}}\right) \int_0^{\bar{A}} Ag(A)dA.$$
   Hence,
   $$0 < \frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I} = \tilde{p}.$$
   Given that $\bar{f}^I > \bar{A}$ by assumption,
   $$\tilde{p} < \frac{\bar{f}^V}{\bar{f}^V + \bar{A}} = p^0,$$
   where $p^0 < 1$.

(c) Finally, we need to show the uniqueness of $\tilde{p}$. Given that $\frac{\partial(\frac{p}{1-p})}{\partial p}$ is strictly increasing in p, the threshold $\tilde{p}$ such that, after exhausting Procedure 1 $\forall A \in [0, \bar{A}]$, the liable injurer's incentive-compatibility constraint holds as an equality is unique.

Hence, $p \in (0, 1)$ is divided into two main p-segments, p-Segment 1 where $p \in (0, \tilde{p}]$ and p-Segment 2 where $p \in (\tilde{p}, 1)$.

2. We show that p-Segment 2 where $p \in (\tilde{p}, 1)$ is divided into two additional p-segments, p-Segment 2.1 where $p \in (\tilde{p}, \bar{p}]$ and p-Segment 2.2 where $p \in (\bar{p}, 1)$. To accomplish this, we need to show that there exists a unique p-threshold, denoted as $\bar{p}$, such that $\tilde{p} < \bar{p} < 1$.

(a) In the main text of the paper, we define $\bar{p}$ as the p-value such that, after exhausting the implementation of Procedure 1 for $A \in [0, A^0(\bar{p})]$, the liable injurer's incentive-compatibility constraint evaluated at the adjusted interim probabilities of investigation is satisfied as an equality:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \left[\int_0^{A^0(\bar{p})} \left(\frac{\bar{p}}{1-\bar{p}} \right) \frac{A}{\bar{f}^V} g(A)dA + \int_{A^0(\bar{p})}^{\bar{A}} 0g(A)dA \right], \tag{6}$$

where $A^0(\bar{p}) = \left(\frac{1-\bar{p}}{\bar{p}}\right)\bar{f}^V$. We show now that there exists a unique $\bar{p}$ such that $p^0 < \bar{p} < 1$. Therefore, p-Segment 2 is divided into two p-segments, denoted as p-Segment 2.1 and p-Segment 2.2. Given that $A^0(p)$ is a strictly decreasing function of p, there is a one-to-one correspondence between $A^0(p)$ and p. Therefore, we can show the existence and uniqueness of $\bar{p}$ by showing that there exists a unique $A^0(\bar{p}) = \left(\frac{1-\bar{p}}{\bar{p}}\right)\bar{f}^V < \bar{A}$.

Define $\Psi(x) \equiv \bar{f}^I \int_0^x \frac{A}{x} g(A)dA$ for $x \in (0, \bar{A}]$. First, we show that $\lim_{x\to 0} \Psi(x) = 0$. Given that $\frac{dg(A)}{dA} \geq 0$, $g(A) \leq g(x)$ $\forall A \leq x$. Therefore, $\forall A \leq x$:

$$\lim_{x\to 0} \Psi(x) = \lim_{x\to 0} \bar{f}^I \int_0^x \frac{A}{x} g(A)dA \leq \lim_{x\to 0} \bar{f}^I \int_0^x \frac{A}{x} g(x)dA =$$

$$= \lim_{x\to 0} \bar{f}^I \left(\int_0^x \frac{A}{x} dA \right) g(x) = \bar{f}^I \lim_{x\to 0} \frac{x^2}{2x} g(x) = 0.$$

Given that $\Psi(x) > 0, \forall x \in (0, \bar{A}]$,

$$\lim_{x \to 0} \Psi(x) = 0.$$

Hence, $\int_0^{\bar{A}} Ag(A)dA > \lim_{x\to 0} \Psi(x) = 0$. Second, $\Psi(\bar{A}) = \bar{f}^I \int_0^{\bar{A}} \frac{A}{\bar{A}} g(A)dA = \frac{\bar{f}^I}{\bar{A}} \int_0^{\bar{A}} Ag(A)dA$. Therefore, $\int_0^{\bar{A}} Ag(A)dA < \Psi(\bar{A})$. Third, we show that there exists a unique $x = A^0(\bar{p}) < \bar{A}$ such that $\int_0^{\bar{A}} Ag(A)dA = \Psi(x)$. We show that $\Psi(x)$ is a differentiable function on x and therefore a continuous function, and a strictly increasing function on x. By assumption $\frac{\partial g(A)}{\partial A} \geq 0$,

$$\frac{\partial \Psi(x)}{\partial x} = \bar{f}^I \left(- \int_0^x \frac{A}{x^2} g(A)dA + g(x) \right) \geq$$

$$\geq \bar{f}^I \left(- \int_0^x \frac{A}{x^2} g(x)dA + g(x) \right) =$$

$$= \bar{f}^I g(x) \left(1 - \frac{x^2}{2x^2} \right) > 0.$$

Therefore, there exists a unique value of $x = A^0(\bar{p}) < \bar{A}$ such that

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{A^0(\bar{p})} \frac{A}{A^0(\bar{p})} g(A)dA.$$

Hence, there exists a unique $\bar{p}$.

(b) We need to show that $\bar{p} > p^0$. Otherwise, Procedure 1 could be exhausted $\forall A \in [0, \bar{A}]$. $A^0(\bar{p}) = \frac{(1-\bar{p})\bar{f}^V}{\bar{p}}$. Therefore, $\bar{p} = \frac{\bar{f}^V}{\bar{f}^V + A^0(\bar{p})}$. $A^0(\bar{p}) < \bar{A}$, by Part 2(a) of this proof. Hence,

$$\bar{p} = \frac{\bar{f}^V}{\bar{f}^V + A^0(\bar{p})} > \frac{\bar{f}^V}{\bar{f}^V + \bar{A}} = p^0,$$

by Claim 4. Given that $\tilde{p} < p^0$ by Part 2(a) of this proof, $\bar{p} > \tilde{p}$ and hence, $\bar{p} > p^0$ by Part 1(b) of this proof.

(c) We show that $\bar{p} < 1$. $A^0(\bar{p}) = \frac{(1-\bar{p})\bar{f}^V}{\bar{p}}$. Hence, $\bar{p} = \frac{\bar{f}^V}{\bar{f}^V + A^0(\bar{p})} < 1$.

Hence, p-Segment 2 where $p \in (\tilde{p}, 1)$ is divided into two p-segments, p-Segment 2.1 where $p \in (\tilde{p}, \bar{p}]$ and p-Segment 2.2 where $p \in (\bar{p}, 1)$.

■

Proposition 6. *Suppose $C_0(A) < C_1(A)$ $\forall A \in [0, \bar{A}]$, $\bar{A} < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$ and $p \in (0,1)$. There are three p-segments: p-Segment 1.1 where $p \in (0, \tilde{p}]$, p-Segment 1.2 where $p \in (\tilde{p}, p^0]$ and p-Segment 2 where $p \in (p^0, 1)$.*

Proof. Suppose $C_0(A) < C_1(A)$ $\forall A \in [0, \bar{A}]$, $\bar{A} < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$ and $p \in (0,1)$.

1. We show that p-Segment 1 where $p \in (0, p^0]$ is divided into two p-segments, p-Segment 1.1 where $p \in (0, \tilde{p}]$ and p-Segment 1.2 where $p \in (\tilde{p}, p^0]$. To accomplish this, we need to show that there exists a unique p-threshold, denoted as $\check{p}$, such that $0 < \check{p} < p^0$.

   (a) In the main text of the paper, we define $\check{p}$ as the p-threshold such that the liable injurer's incentive-compatibility constraint, evaluated at the interim probabilities of verification, is satisfied as an equality.

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{\bar{A}} \left(\frac{\check{p}}{1-\check{p}}\right)\frac{A}{\bar{f}^V} g(A)dA. \tag{7}$$

   Equation (7) can be rewritten as:

$$\int_0^{\bar{A}} Ag(A)dA = \frac{\bar{f}^{\;I}}{\bar{f}^V}\left(\frac{\check{p}}{1-\check{p}}\right)\int_0^{\bar{A}} Ag(A)dA.$$

   Hence,

$$0 < \frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I} = \check{p}.$$

   Given that $\bar{f}^I > \bar{A}$ by assumption,

$$\check{p} < \frac{\bar{f}^V}{\bar{f}^V + \bar{A}} = p^0,$$

where $p^0 < 1$. Note that $\check{p} = \tilde{p}$. Part 2(c) of Proposition 5 also hold here and therefore, the p-threshold is unique.

Hence p-Segment 1 where $p \in (0, p^0]$ is divided into two main p-Segments, denoted as p-Segment 1.1 where $p \in (0, \tilde{p}]$ and p-Segment 1.2 where $p \in (\tilde{p}, p^0]$.

(b) We need to show that p-Segment 1.2 where $p \in (\tilde{p}, p^0]$ is not divided into additional p-segments. To accomplish this goal, we need to show that there does not exist a threshold $p^{'}$ where $\tilde{p} < p^{'} < p^0$ such that the liable injurer's incentive-compatibility constraint, evaluated at the interim probabilities of investigation or at the adjusted interim probabilities of investigation after applying Procedure 2, is satisfied as an equality.

When $p \in (\tilde{p}, p^0]$, the liable injurer's incentive compatibility constraint, evaluated at the interim probabilities of investigation, is satisfied as a strict inequality:

$$\int_0^{\bar{A}} Ag(A)dA < \int_0^{\bar{A}} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA,$$

by the definition of $\tilde{p}$ and $\frac{\partial(\frac{p}{1-p})}{\partial p} > 0$. Therefore, there does not exist $\tilde{p} < p^{'} < p^0$ such that the liable injurer's incentive-compatibility constraint, evaluated at the interim probabilities of investigation or at the adjusted interim probabilities of investigation after applying Procedure 2, is satisfied as an equality.

Hence, p-Segment 1.2 where $p \in (\tilde{p}, p^0]$ is not divided into additional p-segments.

Hence, $p \in (0, p^0]$ is divided into two p-segments, p-Segment 1.1 where $p \in (0, \tilde{p}]$ and p-Segment 1.2 where $p \in (\tilde{p}, p^0]$.

2. We show that p-Segment 2 where $p \in (p^0, 1)$ is not divided into additional p-segments. To accomplish this goal, we need to show that there does not exist a threshold $p^{''}$ where $p^0 < p^{''} < 1$ such that the liable injurer's incentive-compatibility constraint, evaluated at the interim probabilities of investigation or at the adjusted interim probabilities of investigation after applying Procedure 2, is satisfied as an equality.

Suppose $p \in (0,1)$. By the definition of $\tilde{p}$,

$$\int_0^{\bar{A}} Ag(A)dA = \int_0^{A^0(p)} \left(\frac{\tilde{p}}{1-\tilde{p}}\right) \frac{A}{\bar{f}^V} g(A)dA + \int_{A^0(p)}^{\bar{A}} \left(\frac{\tilde{p}}{1-\tilde{p}}\right) \frac{A}{\bar{f}^V} g(A)dA,$$

where $\left(\frac{\tilde{p}}{1-\tilde{p}}\right)\frac{A}{\bar{f}^V} < 1 \ \forall A \in [0, \bar{A}]$, by $\tilde{p} < p^0$ and Claim 5.

Suppose $p \in (p^0, 1)$.

$$\int_0^{A^0(p)} \left(\frac{\tilde{p}}{1-\tilde{p}}\right) \frac{A}{\bar{f}^V} g(A)dA + \int_{A^0(p)}^{\bar{A}} \left(\frac{\tilde{p}}{1-\tilde{p}}\right) \frac{A}{\bar{f}^V} g(A)dA <$$

$$< \int_0^{A^0(p)} \left(\frac{p}{1-p}\right) \frac{A}{\bar{f}^V} g(A)dA + \int_{A^0(p)}^{\bar{A}} 1g(A)dA,$$

by $\frac{\partial(\frac{p}{1-p})}{\partial p} > 0$ and $\left(\frac{\tilde{p}}{1-\tilde{p}}\right)\frac{A}{\bar{f}^V} < 1 \ \forall A \in [0, \bar{A}]$.

As a result, when $p \in (p^0, 1)$, the liable injurer's incentive compatibility constraint, evaluated at the interim probabilities of investigation, is satisfied as a strict inequality:

$$\int_0^{\bar{A}} Ag(A)dA < \int_0^{A^0(p)} \left(\frac{p}{1-p}\right) \frac{A}{\bar{f}^V} g(A)dA + \int_{A^0(p)}^{\bar{A}} 1g(A)dA.$$

Therefore, there does not exist $p^0 < p^{''} < 1$ such that the liable injurer's incentive-compatibility constraint, evaluated at the interim probabilities of investigation or at the adjusted interim probabilities of investigation after applying Procedure 2, is satisfied as an equality.

Hence, p-Segment 2 where $p \in (p^0, 1)$ is not divided into additional p-segments.

■

Lemma 4. *Suppose* $C_0(A) \geq C_1(A) \ \forall A \in (0, \bar{A}]$*,* $C_0(0) > C_1(0)$ *and* $p \in (0, \tilde{p}]$*. There exists a unique* $0 \leq A^1(p) < \bar{A}$*.*

Proof. Suppose $C_0(A) \geq C_1(A) \ \forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)$ and $p \in (0, \tilde{p}]$. In the main text of the paper, we define $A^1(p)$ as the A-threshold such that, after exhausting Procedure 1 $\forall A \in [0, \bar{A}]$ and increasing $q_0(A)$

to 1 by applying Procedure 2 $\forall A \in [0, A^1(p)]$, the liable injurer's incentive-compatibility constraint holds as an equality:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \left[\int_0^{A^1(p)} 1g(A)dA + \int_{A^1(p)}^{\bar{A}} \left(\frac{p}{1-p}\right) \frac{A}{\bar{f}^V} g(A)dA \right]. \quad (8)$$

We show the existence and uniqueness of $A^1(p)$ and that $A^1(p) < \bar{A}$.

1. Suppose $p \in (0, \tilde{p}]$. We show that, after exhausting the application of Procedure 1 $\forall A \in [0, \bar{A}]$, the liable injurer's incentive-compatibility constraint is still not satisfied when $p \in (0, \tilde{p})$ and hence, Procedure 2 should be applied starting with the lowest A-value.

   Given that $p < p^0$, by Claim 4, $A < A^0(p)$ and therefore, Procedure 1 can be exhausted $\forall A \in [0, \bar{A}]$.

   (a) Suppose $p \in (0, \tilde{p})$. At the adjusted interim probabilities of verification,

   $$\int_0^{\bar{A}} Ag(A)dA > \bar{f}^I \int_0^{\bar{A}} \left(\frac{p}{1-p}\right) \frac{A}{\bar{f}^V} g(A)dA,$$

   by the definition of $\tilde{p}$ and $\frac{\partial\left(\frac{p}{1-p}\right)}{\partial p} > 0$.

   (b) Suppose $p = \tilde{p}$. At the adjusted interim probabilities of verification,

   $$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{\bar{A}} \left(\frac{p}{1-p}\right) \frac{A}{\bar{f}^V} g(A)dA,$$

   by the definition of $\tilde{p}$.

2. Suppose $p \in (0, \tilde{p}]$. We show that there exists a unique $A^1(p) < \bar{A}$ such that Procedure 2 is applied by increasing $q_0(A)$ to 1 only for $A \in [0, A^1(p)]$, and the liable injurer's incentive-compatibility constraint is satisfied as an equality.

Define $\psi(y)$ for $y \in [0, \bar{A}]$ as follows:

$$\psi(y) \equiv \bar{f}^I \left[\int_0^y g(A)dA + \int_y^{\bar{A}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA \right].$$

Note that

$$\psi(y) = \bar{f}^I \left[\int_0^{A^1(p)} g(A)dA + \int_{A^1(p)}^{\bar{A}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA \right]$$

when $y = A^1(p)$. We show $y = A^1(p) < \bar{A}$ exists and is unique.

(a) Suppose $p \in (0, \tilde{p})$. We show the existence and uniqueness of $A^1(p)$ such that $0 < A^1(p) < \bar{A}$.

$$\int_0^{\bar{A}} Ag(A)dA > \psi(0) = \bar{f}^I \int_0^{\bar{A}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA,$$

$$\int_0^{\bar{A}} Ag(A)dA < \psi(\bar{A}) = \bar{f}^I \int_0^{\bar{A}} g(A)dA = \bar{f}^I$$

and

$$\frac{\partial \psi(y)}{\partial y} = \bar{f}^I \left[g(y) - \frac{py}{(1-p)\bar{f}^V} g(y) \right] > 0.$$

$\psi(y)$ is differentiable and therefore is continuous. It is also strictly increasing in y. Hence, there exists a unique $y = A^1(p) \in (0, \bar{A})$ such that

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \left[\int_0^{A^1(p)} 1g(A)dA + \int_{A^1(p)}^{\bar{A}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA \right].$$

(b) Suppose $p = \tilde{p}$. We show the existence and uniqueness of $A^1(p)$ and that $A^1(p) = 0$. Evaluate $\psi(y)$ at $y = 0$.

$$\psi(0) = \bar{f}^I \left[\int_0^0 g(A)dA + \int_0^{\bar{A}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA \right].$$

By the definition of $\tilde{p}$, $\int_0^{\bar{A}} Ag(A)dA = \psi(0)$ when $p = \tilde{p}$. Therefore,

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \left[\int_0^0 1g(A)dA + \int_0^{\bar{A}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA \right].$$

Hence, $A^1(\tilde{p}) = 0$.

■

Lemma 5 *Suppose* $C_0(A) \geq C_1(A)$ $\forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)$ *and* $p \in (\tilde{p}, \bar{p}]$. *There exists a unique* $0 \leq A^{2.1}(p) < \bar{A}$.

Proof. Suppose $C_0(A) \geq C_1(A)$ $\forall A \in [0, \bar{A}]$ and $p \in (\tilde{p}, \bar{p}]$.

In the main text of the paper, we define $A^{2.1}(p)$ as the A-threshold such that, after exhausting Procedure 1 $\forall A \in [0, A^{2.1}(p)]$, the liable injurer's incentive-compatibility constraint holds as an equality:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \Big[\int_0^{A^{2.1}(p)} \frac{pA}{(1-p)\bar{f}^V} g(A)dA + \int_{A^{2.1}(p)}^{\bar{A}} 0g(A)dA \Big]. \quad (9)$$

We show the existence and uniqueness of $A^{2.1}(p)$ and that $0 \leq A^{2.1}(p) < \bar{A}$.

1. Suppose $p \in (\tilde{p}, p^0]$. By Claim 4, $A \leq A^0(p)$ and therefore, Procedure 1 can be exhausted $\forall A \in [0, \bar{A}]$.

   By the definition of $\tilde{p}$,

   $$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{\bar{A}} \frac{\tilde{p}A}{(1-\tilde{p})\bar{f}^V} g(A)dA$$

   Given that $p \in (\tilde{p}, p^0]$, $p > \tilde{p}$. By $\frac{\partial\left(\frac{p}{1-p}\right)}{\partial p} > 0$:

   $$\bar{f}^I \int_0^{\bar{A}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA > \bar{f}^I \int_0^{\bar{A}} \frac{\tilde{p}A}{(1-\tilde{p})\bar{f}^V} g(A)dA.$$

   The last inequality can be rewritten as:

   $$\frac{\bar{f}^I p}{(1-p)\bar{f}^V} \int_0^{\bar{A}} Ag(A)dA > \frac{\bar{f}^I \tilde{p}}{(1-\tilde{p})\bar{f}^V} \int_0^{\bar{A}} Ag(A)dA. \quad (1A)$$

Define $\Psi(z)$ for $z \in [0, \bar{A}]$.

$$\Psi(z) \equiv \int_0^z Ag(A)dA.$$

$\Psi(0) = 0$, $\Psi(\bar{A}) = \int_0^{\bar{A}} Ag(A)dA$.

$$\frac{\partial \Psi(z)}{\partial z} = zg(z) > 0.$$

$\Psi(z)$ is differentiable and therefore, $\Psi(z)$ is continuous. It is also strictly increasing in z. Therefore,

$$\frac{\bar{f}^I p}{(1-p)\bar{f}^V}\Psi(0) = 0 < \frac{\bar{f}^I \tilde{p}}{(1-\tilde{p})\bar{f}^V}\int_0^{\bar{A}} Ag(A)dA. \tag{2A}$$

Inequality (1A) can be expressed as:

$$\frac{\bar{f}^I p}{(1-p)\bar{f}^V}\Psi(\bar{A}) > \frac{\bar{f}^I \tilde{p}}{(1-\tilde{p})\bar{f}^V}\int_0^{\bar{A}} Ag(A)dA.$$

By (1A) and (2A), there exists a unique $z = A^{2.1}(p) \in (0, \bar{A})$ such that

$$\frac{\bar{f}^I p}{(1-p)\bar{f}^V}\Psi(A^{2.1}(p)) = \frac{\bar{f}^I \tilde{p}}{(1-\tilde{p})\bar{f}^V}\int_0^{\bar{A}} Ag(A)dA.$$

By the definition of $\Psi(z)$ and $z = A^{2.1}(p)$,

$$\frac{\bar{f}^I p}{(1-p)\bar{f}^V}\int_0^{A^{2.1}(p)} Ag(A)dA = \frac{\bar{f}^I \tilde{p}}{(1-\tilde{p})\bar{f}^V}\int_0^{\bar{A}} Ag(A)dA,$$

which can be written as:

$$\bar{f}^I \int_0^{A^{2.1}(p)} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}g(A)dA = \bar{f}^I \int_0^{\bar{A}} \left(\frac{\tilde{p}}{1-\tilde{p}}\right)\frac{A}{\bar{f}^V}g(A)dA.$$

Therefore, by the definition of $\tilde{p}$

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{A^{2.1}(p)} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}g(A)dA.$$

Hence, there exists a unique $0 < A^{2.1}(p) < \bar{A}$ such that

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \Big[\int_0^{A^{2.1}(p)} \frac{pA}{(1-p)\bar{f}^V} g(A)dA + \int_{A^{2.1}(p)}^{\bar{A}} 0g(A)dA \Big].$$

2. Suppose $p \in (p^0, \bar{p}]$. By Claim 4, $\bar{A} > A^0(p)$ and therefore, Procedure 1 can be exhausted only $\forall A \in [0, A^0(p)]$.

   We need to show that $A^{2.1}(p) \leq A^0(p)$ when $p \in (p^0, \bar{p}]$ and therefore, Procedure 1 can be exhausted $\forall A \in [0, A^{2.1}(p)]$. As a result, Part 1 of this proof also holds when $p \in (p^0, \bar{p}]$.

   To accomplish this goal, we first show that $A^{2.1}(p) < A^0(p)$ when $p \in (p^0, \bar{p})$. Second, we show that $A^{2.1}(p) = A^0(p)$ when $p = \bar{p}$.

   Define $m \equiv \frac{(1-p)\bar{f}^V}{p} = A^0(p)$ and $\psi(m)$ as:

$$\psi(m) \equiv \bar{f}^I \int_0^m \frac{A}{m} g(A)dA,$$

   where $\psi(m)$ is the right-hand side of the liable injurer's incentive compatibility constraint when Procedure 1 is exhausted $\forall A \in [0, A^0(p)]$.

   By assumption, $g(A)$ is non-decreasing in A. Therefore,

$$\frac{\partial \psi(m)}{\partial m} = \bar{f}^I \left[g(m) - \int_0^v \frac{A}{m^2} g(A)dA \right] \geq$$

$$\geq \bar{f}^I g(m) \left[1 - \frac{1}{m^2} \int_0^m Ag(A)dA \right] = \frac{\bar{f}^I g(m)}{2} > 0.$$

$$\frac{\partial \psi(m)}{\partial p} = \frac{\partial \psi(m)}{\partial m} \frac{\partial m}{\partial p} = -\frac{\bar{f}^I g(m)}{2} \frac{\bar{f}^V}{p^2} < 0.$$

   (a) Suppose $p \in (p^0, \bar{p})$. By Part 1 of this proof, when $p \in (\tilde{p}, p^0]$,

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{A^{2.1}(p)} \Big(\frac{p}{1-p} \Big) \frac{A}{\bar{f}^V} g(A)dA.$$

Given $\frac{\partial\psi(m)}{\partial p} < 0$, when $p \in (p^0, \bar{p})$,

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{A^{2.1}(p)} \Big(\frac{p}{1-p}\Big)\frac{A}{\bar{f}^V} g(A)dA <$$

$$< \bar{f}^I \int_0^{A^0(p)} \Big(\frac{p}{1-p}\Big)\frac{A}{\bar{f}^V} g(A)dA.$$

Hence, $A^{2.1}(p) < A^0(p)$ when $p \in (p^0, \bar{p})$.

(b) Consider $p = \bar{p}$. By the definition of $\bar{p}$,

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \Big[\int_0^{A^0(\bar{p})} \Big(\frac{\bar{p}}{1-\bar{p}}\Big)\frac{A}{\bar{f}^V} g(A)dA + \int_{A^0(\bar{p})}^{\bar{A}} 0g(A)dA\Big].$$

By the definition of $A^{2.1}(p)$, when $p = \bar{p}$,

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{A^{2.1}(\bar{p})} \Big(\frac{\bar{p}}{1-\bar{p}}\Big)\frac{A}{\bar{f}^V} g(A)dA + \int_{A^{2.1}(\bar{p})}^{\bar{A}} 0g(A)dA\Big].$$

Hence, $A^{2.1}(p) = A^0(p)$ when $p = \bar{p}$.

■

Lemma 6. *Suppose $C_0(A) \geq C_1(A)$ $\forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)$ and $p \in (\bar{p}, 1)$. There exists a unique $A^{2.2}(p)$ such that $A^0 < A^{2.2}(p) < \bar{A}$.*

Proof. Suppose $C_0(A) \geq C_1(A)$ $\forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)$ and $p \in (\bar{p}, 1)$. In the main text of the paper, we define $A^{2.2}(p)$ as the A-threshold such that, after exhausting Procedure 1 for $A \in [0, A^0(p)]$ and increasing $q_0(A)$ to 1 by applying Procedure 1 to $A \in (A^0(p), A^{2.2}(p)]$, the liable injurer's incentive-compatibility constraint holds as an equality:

$$\int_0^{\bar{A}} Ag(A)dA =$$

$$= \bar{f}^I \Big[\int_0^{\frac{(1-p)\bar{f}^V}{p}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA + \int_{\frac{(1-p)\bar{f}^V}{p}}^{A^{2.2}(p)} 1g(A)dA + \int_{A^{2.2}(p)}^{\bar{A}} 0g(A)dA\Big]. \tag{10}$$

We show existence and uniqueness of $A^{2.2}(p)$ and that $A^0 < A^{2.2}(p) < \bar{A}$, where $A^0 = \frac{(1-p)\bar{f}^V}{p}$.

1. Define v and $\Psi(v)$ for $p \in (0,1)$ and $v \in (0,\bar{A}]$, respectively, as follows.

$$v \equiv \frac{(1-p)\bar{f}^V}{p} = A^0(p).$$

$$\Psi(v) \equiv \bar{f}^I \Bigg[\int_0^v \frac{A}{v} g(A)dA + \int_v^{\bar{A}} g(A)dA \Bigg].$$

   (a) We show that, after exhausting the application of Procedure 1 $\forall A \in [0, A^0(p)]$, the liable injurer's incentive compatibility constraint is still not satisfied, and hence, Procedure 1 should be applied to $A > A^0(p)$.

   To accomplish this goal, we show that inequality (3A) holds:

$$\int_0^{\bar{A}} Ag(A)dA > \bar{f}^I \int_0^{\frac{(1-p)\bar{f}^V}{p}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA. \tag{3A}$$

   By the definition of $\bar{p}$:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{\frac{(1-\bar{p})\bar{f}^V}{\bar{p}}} \frac{\bar{p}A}{(1-\bar{p})\bar{f}^V} g(A)dA. \tag{6}$$

   We show that the right-hand-side of equation (6) is decreasing in p for $p \in (\bar{p}, 1)$.

$$\frac{\partial\Big(\bar{f}^I \int_0^{\frac{(1-p)\bar{f}^V}{p}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA\Big)}{\partial p} = \frac{\partial\Big(\bar{f}^I \int_0^v \frac{A}{v} g(A)dA\Big)}{\partial v}\frac{\partial v}{\partial p}.$$

   By assumption, $g(A)$ is non-decreasing in A. Therefore,

$$\frac{\partial\Big(\int_0^v \frac{A}{v} g(A)dA\Big)}{\partial v} = g(v) - \int_0^v \frac{A}{v^2} g(A)dA \geq$$

$$\geq g(v) - g(v)\int_0^v \frac{A}{v^2}dA = \frac{1}{2}g(v) > 0.$$

   By the definition of v,

$$\frac{\partial v}{\partial p} = -\frac{\bar{f}^V}{p^2} < 0.$$

Therefore,

$$\frac{\partial\Big(\bar{f}^I \int_0^{\frac{(1-p)\bar{f}^V}{p}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA\Big)}{\partial p} < 0.$$

Hence, when $p \in (\bar{p}, 1)$,

$$\int_0^{\bar{A}} Ag(A)dA > \bar{f}^I \int_0^{\frac{(1-p)\bar{f}^V}{p}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA.$$

(b) We show that, if $q_0(A)$ is increased to 1 by applying Procedure 1 to $A \in \big(\frac{(1-p)\bar{f}^V}{p}, \bar{A}\big]$, the liable injurer is satisfied as a strict inequality and hence, $q_0(A)$ should be increased to 1 by applying Procedure 1 to $A > \frac{(1-p)\bar{f}^V}{p}$ but $A < \bar{A}$.

To accomplish this goal, we show that inequality (4A) holds:

$$\int_0^{\bar{A}} Ag(A)dA <$$

$$< \bar{f}^I \Big[\int_0^{\frac{(1-p)\bar{f}^V}{p}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA + \int_{\frac{(1-p)\bar{f}^V}{p}}^{\bar{A}} 1g(A)dA \Big]. \quad (4A)$$

By the definitions of v and $\Psi(v)$, inequality (4A) can be expressed as:

$$\int_0^{\bar{A}} Ag(A)dA < \Psi(v).$$

Evaluating $\Psi(v)$ at $v = \bar{A}$:

$$\int_0^{\bar{A}} Ag(A)dA < \bar{f}^I \int_0^{\bar{A}} \frac{A}{\bar{A}} g(A)dA.$$

We show that $\Psi(v)$ is differentiable and therefore continuous, and that $\Psi(v)$ strictly decreasing in v:

$$\frac{\partial \Psi(v)}{\partial v} = \bar{f}^I \Big(-\int_0^v \frac{A}{v^2} g(A)dA + \frac{v}{v} g(v) - g(v) \Big) =$$

$$= -\bar{f}^I \int_0^v \frac{A}{v^2} g(A) dA < 0.$$

Therefore, $\forall v \in (0, \bar{A}]$

$$\int_0^{\bar{A}} Ag(A)dA < \bar{f}^I \left(\int_0^v \frac{A}{v} g(A)dA + \int_v^{\bar{A}} g(A)dA \right) = \Psi(v),$$

which can be written as:

$$\int_0^{\bar{A}} Ag(A)dA <$$

$$< \bar{f}^I \left[\int_0^{\frac{(1-p)\bar{f}^V}{p}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA + \int_{\frac{(1-p)\bar{f}^V}{p}}^{\bar{A}} g(A)dA \right].$$

2. We show that there exists a unique $u = A^{2.2}(p) \in \left(A^0(p), \bar{A}\right)$ such that

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \left[\int_0^{\frac{(1-p)\bar{f}^V}{p}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA + \int_{\frac{(1-p)\bar{f}^V}{p}}^{A^{2.2}(p)} g(A)dA \right].$$

Define $\psi(u)$ for $u \in \left[\frac{(1-p)\bar{f}^V}{p}, \bar{A}\right]$:

$$\psi(u) \equiv \bar{f}^I \int_{\frac{(1-p)\bar{f}^V}{p}}^{u} g(A)dA.$$

$\psi(\frac{(1-p)\bar{f}^V}{p}) = 0$ and $\psi(\bar{A}) = \bar{f}^V \int_{\frac{(1-p)\bar{f}^V}{p}}^{\bar{A}} g(A)dA.$

By assumption, $g(A) > 0$. Therefore,

$$\frac{\partial \psi(u)}{\partial u} = \bar{f}^I g(u) > 0.$$

$\psi(u)$ is differentiable and therefore continuous, and is strictly increasing in u.

Hence, there exists a unique $u = A^{2.2}(p) \in \big(A^0(p), \bar{A}\big)$ such that

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \Bigg[\int_0^{\frac{(1-p)\bar{f}^V}{p}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA + \Psi(A^{2.2}(p)) \Bigg],$$

which can be rewritten as:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \Bigg[\int_0^{\frac{(1-p)\bar{f}^V}{p}} \frac{pA}{(1-p)\bar{f}^V} g(A)dA + \int_{\frac{(1-p)\bar{f}^V}{p}}^{A^{2.2}(p)} g(A)dA \Bigg].$$

Environment 2: Optimal Production of Evidence – Step 2

Remember that Environment 2 occurs when $C_0(A) < C_1(A)\ \forall A \in [0, \bar{A}]$ and $\bar{A} < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$. We characterize the optimal probabilities of investigation.

Proposition 2 shows that there are two main p-segments in Environment 2, p-Segment 1 where $p \in (0, p^0]$ and p-Segment 2 where $p \in (p^0, 1)$. It also shows that the interim probabilities of investigation are as follows. p-Segment 1: $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} \leq 1$ and $q_1(A) = 0\ \forall A \in [0, \bar{A}]$. p-Segment 2: $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} \leq 1$ and $q_1(A) = 0$ for $A \in [0, A^0(p)]$, and $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(A+\bar{f}^V)}$ for $A \in (A^0(p), \bar{A}]$. $p^0 = \frac{\bar{f}^V}{\bar{f}^V + \bar{A}}$ and $A^0(p) = \big(\frac{1-p}{p}\big)\bar{f}^V$.

We first verify whether the liable injurer's incentive-compatibility constraint (2) is satisfied:

$$\int_0^{\bar{A}} Ag(A)dA \leq \bar{f}^I \int_0^{\bar{A}} q_0(A)g(A)dA. \tag{2}$$

Given that $q_0(A) > 0$ across p- and A-values but f^I is restricted to $\bar{f}^I$, constraint (2) might be satisfied only for sufficiently high p-values.[54] Hence, adjustments to the interim probabilities of investigation to increase $q_0(A)$ while keeping the victim's incentive compatibility constraint (1) satisfied might need to be implemented for low p-values. Finally note that, when

[54] Note that $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V} \leq 1$ for $p \in (0, p^0]$ across A-values and for $p \in (p^0, 1)$ and low A-values, and $q_0(A) = 1$ for $p \in (p^0, 1)$ and high A-values. Note also that $\frac{\partial(\frac{p}{1-p})}{\partial p} > 0$.

$p \in (0, p^0]$, $q_1(A) = 0\ \forall A \in [0, \bar{A}]$, and when $p \in (p^0, 1]$, $q_1(A) = 0\ \forall A \in [0, A^0(p)]$. Therefore, only Procedure 2 should be applied in those cases.[55]

Corollary 5. *In p-Segment 1 when $A \in [0, \bar{A}]$ and p-Segment 2 when $A \in [0, A^0(p)]$, only Procedure 2 can be implemented.*

5.0.1 Characterization of p-Segments

This section shows that the optimal production of evidence depends on the probability of liable injurers p. Proposition 6 demonstrates that there are three mutually-exclusive p-segments in Environment 2 that differ in the implementation of the adjustment procedures, and hence, in the optimal probabilities of investigation: p-Segment 1.1 where $p \in (0, \check{p}]$, p-Segment 1.2 where $p \in (\check{p}, p^0]$ and p-Segment 2 where $p \in (p^0, 1)$. It also shows that $\check{p} = \tilde{p}$.

Proposition 6. *Suppose $C_0(A) < C_1(A)\ \forall A \in [0, \bar{A}]$, $\bar{A} < \left(\frac{C_1(A)}{C_0(A)} - 1\right)\bar{f}^V$ and $p \in (0, 1)$. There are three p-segments: p-Segment 1.1 where $p \in (0, \tilde{p}]$, p-Segment 1.2 where $p \in (\tilde{p}, p^0]$ and p-Segment 2 where $p \in (p^0, 1)$.*

Next, we provide an intuitive discussion. Appendix A presents a formal proof.

Characterization of p̌: p-Segments 1.1 and 1.2

Suppose $p \in (0, p^0]$. This section characterizes $\check{p}$ and demonstrates that p-Segment 1 is divided into two segments: p-Segment 1.1 where $p \in (0, \check{p}]$ and p-Segment 1.2 where $p \in (\check{p}, p^0]$.

Define $\check{p}$ as the p-value such that the liable injurer's incentive-compatibility constraint evaluated at the interim probabilities of verification is satisfied as an equality:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{\bar{A}} \left(\frac{\check{p}}{1-\check{p}}\right)\frac{A}{\bar{f}^V} g(A)dA, \tag{7}$$

which can be rewritten as:

$$\int_0^{\bar{A}} Ag(A)dA = \frac{\bar{f}^{\,I}}{\bar{f}^V}\left(\frac{\check{p}}{1-\check{p}}\right)\int_0^{\bar{A}} Ag(A)dA.$$

[55] Remember that Procedure 2 consists of increasing $q_0(A)$ without reducing $q_1(A)$ until constraint (2) is satisfied as an equality while keeping constraint (1) satisfied.

Hence,

$$\check{p} = \frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I},$$

where $0 < \frac{\bar{f}^V}{\bar{f}^V+\bar{f}^I} < 1$. Finally note that

$$\check{p} = \frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I} = \tilde{p},$$

and $\tilde{p} < p^0$, by Proposition 5.

Hence, $\tilde{p}$ divides p-Segment 1 into two main segments p-Segments, p-Segment 1.1 where $p \in (0, \tilde{p}]$ and p-Segment 1.2 where $p \in (\tilde{p}, p^0]$. Proposition 6 verifies that p-Segment 1.2 and p-Segment 2 are not divided into additional p-segments.

5.0.2 Optimal Probabilities of Investigation

p-Segment 1.1

Suppose $p \in (0, \tilde{p}]$. The interim probabilities of investigation are $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1$ and $q_1(A) = 0 \; \forall A \in [0, \bar{A}]$. In case of requiring adjustments, only Procedure 2 could be applied. At the interim probabilities of verification:

$$\int_0^{\bar{A}} Ag(A)dA > \bar{f}^I \int_0^{\bar{A}} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA,$$

by the definition of $\tilde{p}$ and $\frac{\partial\left(\frac{p}{1-p}\right)}{\partial p} > 0$. Therefore, Procedure 2 should be implemented. Starting at the lowest value of A, the social planner should increase $q_0(A)$ to 1 without reducing $q_1(A)$ only for $A \in [0, A^{1.1}(p)]$, where $A^{1.1}(p)$ is the A-threshold such that the liable injurer's incentive-compatibility constraint holds as an equality:

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \Big[\int_0^{A^{1.1}(p)} 1g(A)dA + \int_{A^{1.1}(p)}^{\bar{A}} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA\Big].$$

Hence, $A^{1.1}(p) = A^1(p)$. Lemma 4 verifies that there exists a unique $A^1(p)$ and that $A^1(p) < \bar{A}$. Lemma 4 also verifies that there exists a unique $A^1(\tilde{p})$ and that $A^1(\tilde{p}) = 0$. Therefore, when $p = \tilde{p}$,

$$\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \int_0^{\bar{A}} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA,$$

which is aligned with the definition of $\tilde{p}$.

Hence, the optimal production of evidence for p-Segment 1.1 involves the following optimal probabilities of investigation: $q_0(A) = 1$ and $q_1(A) = 0$ for $A \in [0, A^1(p)]$, and $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$ for $A \in (A^1(p), \bar{A}]$, where $\tilde{p} = \frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I}$ and $A^1(p)$ is implicitly defined using the liable injurer's incentive-compatibility constraint holding as an equality, $\int_0^{\bar{A}} Ag(A)dA = \bar{f}^I \left[\int_0^{A^1(p)} g(A)dA + \int_{A^1(p)}^{\bar{A}} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA \right]$. The optimal social welfare loss function for p-Segment 1.1 is:

$$SWL^{1.1} = H + \mathbb{E}[C(A)]^{1.1} + \theta\mathbb{E}[\eta(A)] + \Lambda E[\xi(A)] =$$

$$= \int_0^{\bar{A}} Ag(A)dA +$$

$$+(1-p)\left[\int_0^{A^1(p)} C_0(A)g(A)dA + \int_{A^1(p)}^{\bar{A}} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} C_0(A)g(A)dA\right] + 0 + 0.$$

p-Segment 1.2

Suppose $p \in (\tilde{p}, p^0]$. The interim probabilities of investigation are $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} \leq 1$ and $q_1(A) = 0 \ \forall A \in [0, \bar{A}]$. In case of requiring adjustments, only Procedure 2 could be applied.

Given that $p > \tilde{p}$, at the interim probabilities of verification:

$$\int_0^{\bar{A}} Ag(A)dA < \bar{f}^I \int_0^{\bar{A}} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA$$

by the definition of $\tilde{p}$ and $\frac{\partial\left(\frac{p}{1-p}\right)}{\partial p} > 0$. Therefore, the implementation of Procedure 2 is not required. At the optimal probabilities of investigation, the liable injurer's incentive-compatibility constraint is satisfied as a strict inequality.

Hence, the optimal production of evidence for p-Segment 1.2 involves the following optimal probabilities of investigation: $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0 \ \forall A \in [0, \bar{A}]$, where $p^0 = \frac{\bar{f}^V}{\bar{f}^V + \bar{A}}$. The optimal social welfare loss for p-Segment 1.2 is:

$$SWL^{1.2} = H + \mathbb{E}[C(A)]^{1.2} + +\theta\mathbb{E}[\eta(A)] + \Lambda E[\xi(A)] =$$

$$= \int_0^{\bar{A}} Ag(A)dA + (1-p)\int_0^{\bar{A}} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} C_0(A)g(A)dA + 0 + 0.$$

p-Segment 2

Suppose $p \in (p^0, 1)$. The interim probabilities of investigation are: $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} < 1$ and $q_1(A) = 0$ for $A \in [0, A^0(p)]$, and $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(A+\bar{f}^V)}$ for $A \in (A^0(p), \bar{A}]$, where $p^0 = \frac{\bar{f}^V}{\bar{f}^V + \bar{A}}$ and $A^0(p) = \left(\frac{1-p}{p}\right)\bar{f}^V$. In case of requiring adjustments, only Procedure 2 could be applied for $A \in [0, A^0(p)]$.

Given that $p > \tilde{p}$, at the interim probabilities of verification:

$$\int_0^{\bar{A}} Ag(A)dA < \int_0^{A^0(p)} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA + \int_{A^0(p)}^{\bar{A}} 1g(A)dA,$$

by the definition of $\tilde{p}$ and $\frac{\partial\left(\frac{p}{1-p}\right)}{\partial p} > 0$. Therefore, the implementation of Procedure 2 is not required. At the optimal probabilities of investigation, the liable injurer's incentive-compatibility constraint is satisfied as a strict inequality.

Hence, the optimal production of evidence for p-Segment 2 involves the following optimal probabilities of investigation: $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$ for $A \in [0, A^0(p)]$, and $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(A+\bar{f}^V)}$ for $A \in (A^0(p), \bar{A}]$, where $A^0(p) = \left(\frac{1-p}{p}\right)\bar{f}^V$ and $p^0 = \frac{\bar{f}^V}{\bar{f}^V + \bar{A}}$. The optimal social welfare loss for p-Segment 2 is:

$$SWL^2 = H + \mathbb{E}[C(A)]^2 + \theta\mathbb{E}[\eta(A)] + \Lambda E[\xi(A)] =$$

$$= \int_0^{\bar{A}} Ag(A)dA + (1-p)\left[\int_0^{A^0(p)} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} C_0(A)g(A)dA + \right.$$

$$\left. + \int_{A^0(p)}^{\bar{A}} C_0(A)g(A)dA\right] + p\int_{A^0}^{\bar{A}} \left[1 - \frac{\bar{f}^V}{p(\bar{f}^V + A)}\right] C_1(A)g(A)dA + 0 + 0.$$

Table 2 summarizes the optimal production of evidence for each p-segment in Environment 2. Similar to Environment 1, our analysis suggests that the optimal production of evidence involves just a subset of legal cases.

When the probability of liable injurers is sufficiently low ($p \leq p^0$), the victim's gains from misreporting are low. Therefore, the production of evidence just in legal cases where the injurer reports to be non-liable suffices to incentivize the victim and the liable injurer to truthfully report their types. When the probability of liable injurers is sufficiently high ($p > p^0$), the victim's gains from misreporting are high and these gains increase with the

Table 2: Optimal Production of Evidence – Environment 2

p-Segment	A-Segment[a]	Optimal $q_0(A)$	Optimal $q_1(A)$
p-Segment 1.1	$A \in [0, A^1(p)]$	1	0
$p \in (0, \tilde{p}]$	$A \in (A^1(p), \bar{A}]$	$\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$	0
p-Segment 1.2	$A \in [0, \bar{A}]$	$\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$	0
$p \in (\tilde{p}, p^0]$			
p-Segment 2	$A \in [0, A^0(p)]$	$\left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$	0
$p \in (p^0, 1)$	$A \in (A^0(p), \bar{A}]$	1	$1 - \frac{\bar{f}^V}{p(\bar{f}^V + A)}$

Note: [a] $A^{1.1}(p) = A^1(p)\ \forall p \in [0, \tilde{p}]$.

reported damages. If the victim untruthfully reports relatively low damages, the gains from misreporting are lower. Therefore, the production of evidence just in legal cases where the injurer reports to be non-liable suffices to incentivize the victim and the liable injurer to truthfully report their types. If the victim untruthfully reports relatively high damages, the gains from misreporting are higher. Therefore, the production of evidence in the legal cases where the injurer reports to be non-liable and legal cases where the injurer reports to be liable is required to incentivize the victim to truthfully report her type.

Proof of Proposition 12. Although $A^{2.1}$ is a function of p and α_1, i.e., $A^{2.1}(p, \alpha_1)$, to simplify the notation, we use $A^{2.1}$ in (most parts of) the proof. To simplify the notation, we also denote $\mathbb{E}[C(A)] + \Lambda\mathbb{E}[\xi(A)]$ as S. Finally, we denote $\frac{\partial C_i(A)}{\partial A}$ as $C_i'(A)$ $(i = 0, 1)$.

Consider first the characterization of the optimal α_0 first. The S function does not depend on α_0. Hence, across cases, any $\alpha_0 \in [0, 1]$ is optimal.

Consider now the characterization of the optimal α_1. The proof consists of several steps.

1. Evaluate the sign of $\frac{\partial S}{\partial \alpha_1}$. By the Chain Rule,

$$\frac{\partial S}{\partial \alpha_1} =$$

$$= \left[\frac{A^{2.1}}{\bar{f}^V} pC_0(A^{2.1})g(A^{2.1}) - p(1 + \Lambda\alpha_1)\left(\frac{A^{2.1}}{A^{2.1} + \bar{f}^V}\right)C_1(A^{2.1})g(A^{2.1})\right] \times$$

$$\times \frac{\partial A^{2.1}}{\partial \alpha_1} + \int_{A^{2.1}}^{\bar{A}} p\Lambda\Big(\frac{A}{A+\bar{f}^V}\Big) C_1(A) g(A) dA,$$

where

$$\frac{\partial A^{2.1}}{\partial \alpha_1} = -\frac{\int_{A^{2.1}}^{\bar{A}} \frac{A}{A+\bar{f}^V} C_1(A) g(A) dA}{\Big[\frac{A^{2.1}}{\bar{f}^V}\frac{p}{1-p}\bar{f}^I + (1-\alpha_1)\frac{A^{2.1}}{\hat{A}^1+\bar{f}^V} C_1(A^{2.1})\Big] g(A^{2.1})}.$$

$\frac{\partial S}{\partial \alpha_1}$ can be rewritten as:

$$\frac{\partial S}{\partial \alpha_1} = p \int_{A^{2.1}}^{\bar{A}} \frac{A}{A+\bar{f}^V} C_1(A) g(A) dA \times$$

$$\times \left[-\frac{\frac{A^{2.1}}{\bar{f}^V} C_0(A^{2.1}) - (1+\Lambda\alpha_1)\frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})}{\frac{A^{2.1}}{\bar{f}^V}\frac{p}{1-p}\bar{f}^I + (1-\alpha_1)\frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})} + \Lambda \right] =$$

$$= p \int_{A^{2.1}}^{\bar{A}} \frac{A}{A+\bar{f}^V} C_1(A) g(A) dA$$

$$\Bigg[\frac{-\Big(\frac{A^{2.1}}{\bar{f}^V} C_0(A^{2.1}) - \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})\Big)}{\frac{A^{2.1}}{\bar{f}^V}\frac{p}{1-p}\bar{f}^I + (1-\alpha_1)\frac{A^{2.1}}{\hat{A}^1+\bar{f}^V} C_1(A^{2.1})} +$$

$$+ \frac{\Lambda\Big(\frac{A^{2.1}}{\bar{f}^V}\frac{p}{1-p}\bar{f}^I + \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})\Big)}{\frac{A^{2.1}}{\bar{f}^V}\frac{p}{1-p}\bar{f}^I + (1-\alpha_1)\frac{A^{2.1}}{\hat{A}^1+\bar{f}^V} C_1(A^{2.1})}\Bigg].$$

Consider the numerator of the expression in brackets. Analyze the sign of the first term in parenthesis:

$$\Big(\frac{A^{2.1}}{\bar{f}^V} C_0(A^{2.1}) - \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})\Big) > \frac{A^{2.1}}{\bar{f}^V}(C_0(A^{2.1}) - C_1(A^{2.1})) >$$

$$> \frac{A^{2.1}}{\bar{f}^V}(C_0(A^{2.1}) - (1+\Lambda) C_1(A^{2.1})) > 0.$$

Therefore, the term $-\Big(\frac{A^{2.1}}{\bar{f}^V} C_0(A^{2.1}) - \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})\Big) < 0$. This term can be larger or smaller than the positive second term of the numerator, $\Lambda\Big(\frac{A^{2.1}}{\bar{f}^V}\frac{p}{1-p}\bar{f}^I + \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})\Big)$, by absolute value. Hence, the sign of $\frac{\partial S}{\partial \alpha_1}$ is ambiguous, and depends on Λ. In other words, any sufficient conditions that solve this ambiguity should include Λ.

2. Define $\Lambda^0(\alpha_1)$ as the Λ-function that results from equating $\frac{\partial S}{\partial \alpha_1}$ to zero. $\Lambda^0(\alpha_1)$ will be used to find sufficient conditions and characterize α_1 in Step 6 of this proof.

   Equate $\frac{\partial S}{\partial \alpha_1}$ to zero, solve for Λ and denote this function of α_1 as $\Lambda(\alpha_1)$:

$$\Lambda^0(\alpha_1) = \frac{\frac{A^{2.1}}{\bar{f}^V} C_0(A^{2.1}) - \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})}{\frac{A^{2.1}}{\bar{f}^V}\frac{p}{1-p}\bar{f}^I + \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})}.$$

3. Evaluate the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1}$. $\Lambda^0(\alpha_1)$ depends on α_1 through $A^{2.1}(p,\alpha_1)$ only. By the Chain Rule,

$$\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} = \frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} \frac{\partial A^{2.1}}{\partial \alpha_1}.$$

$$\frac{\partial A^{2.1}}{\partial \alpha_1} = -\frac{\int_{A^{2.1}}^{\bar{A}} \frac{A}{A+\bar{f}^V} C_1(A) g(A) dA}{\left[\frac{A^{2.1}}{\bar{f}^V}\left(\frac{p}{1-p}\right)\bar{f}^I + (1-\alpha_1)\frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})\right] g(A^{2.1})} < 0.$$

   Evaluate now the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}}$. $\Lambda^0(\alpha_1)$ can be rewritten as:

$$\Lambda^0(\alpha_1) = \frac{(A^{2.1}+\bar{f}^V)C_0(A^{2.1}) - \bar{f}^V C_1(A^{2.1})}{(A^{2.1}+\bar{f}^V)\left(\frac{p}{1-p}\right)\bar{f}^I + \bar{f}^V C_1(A^{2.1})}.$$

   Define $\Psi \equiv \frac{p}{1-p}\bar{f}^I$.

$$\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} =$$

$$= \frac{[C_0(A^{2.1}) + (A^{2.1}+\bar{f}^V)C_0'(A^{2.1}) - \bar{f}^V][(A^{2.1}+\bar{f}^V)\Psi + \bar{f}^V C_1(A^{2.1})]}{[(A^{2.1}+\bar{f}^V)\Psi + \bar{f}^V C_1(A^{2.1})]^2} -$$

$$- \frac{[\Psi + \bar{f}^V C_1'(A^{2.1})][(A^{2.1}+\bar{f}^V)C_0(A^{2.1}) - \bar{f}^V C_1(A^{2.1})]}{[(A^{2.1}+\bar{f}^V)\Psi + \bar{f}^V C_1(A^{2.1})]^2}.$$

   This expression can be rewritten as:

$$\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} =$$

$$= \frac{C_1(A^{2.1})\bar{f}^V(C_0(A^{2.1}) + \Psi) + (A^{2.1} + \bar{f}^V)^2 C_0'(A^{2.1})\Psi}{[(A^{2.1} + \bar{f}^V)\Psi + \bar{f}^V C_1(A^{2.1})]^2} +$$

$$+ \frac{(A^{2.1} + \bar{f}^V)\bar{f}^V C_0'(A^{2.1})C_1(A^{2.1})}{[(A^{2.1} + \bar{f}^V)\Psi + \bar{f}^V C_1(A^{2.1})]^2} -$$

$$- \frac{[(A^{2.1} + \bar{f}^V)\bar{f}^V C_1'(A^{2.1})(C_0(A^{2.1}) + \Psi)]}{[(A^{2.1} + \bar{f}^V)\Psi + \bar{f}^V C_1(A^{2.1})]^2}.$$

Terms in the first two lines are positive and the term in brackets (third line) is negative. Therefore, the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}}$ is ambiguous. Hence, the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1}$ is ambiguous.

Solve the ambiguity of the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1}$ by findings sufficient conditions on $C^{'}(A)$. Take the numerator of $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}}$, equate it to zero, and solve for $C_1'(A^{2.1})$. Therefore, when

$$C_1'(A^{2.1}) =$$

$$\frac{C_1(A^{2.1})\bar{f}^V(C_0(A^{2.1}) + \frac{p}{1-p}\bar{f}^I)}{(A^{2.1} + \bar{f}^V)\bar{f}^V(C_0(A^{2.1}) + \frac{p}{1-p}\bar{f}^I)} +$$

$$+ \frac{(A^{2.1} + \bar{f}^V)^2 C_0^{'}(A^{2.1})\frac{p}{1-p}\bar{f}^I + (A^{2.1} + \bar{f}^V)\bar{f}^V C_0^{'}(A^{2.1})C_1(A^{2.1})}{(A^{2.1} + \bar{f}^V)\bar{f}^V(C_0(A^{2.1}) + \frac{p}{1-p}\bar{f}^I)},$$

$\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} = 0$. Define μ as

$$\mu = \frac{C_1(A)\bar{f}^V(C_0(A) + \frac{p}{1-p}\bar{f}^I)}{(A + \bar{f}^V)\bar{f}^V(C_0(A) + \frac{p}{1-p}\bar{f}^I)} +$$

$$+ \frac{(A + \bar{f}^V)^2 C_0'(A)\frac{p}{1-p}\bar{f}^I + (A + \bar{f}^V)\bar{f}^V C_0'(A)C_1(A)}{(A + \bar{f}^V)\bar{f}^V(C_0(A) + \frac{p}{1-p}\bar{f}^I)}.$$

When $\frac{\partial C_1(A)}{\partial A} > \mu \ \forall A \in [0, \bar{A}]$, the numerator of $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}}$ is negative. Therefore, $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} < 0$ and hence, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} > 0$.

When $\frac{\partial C_1(A)}{\partial A} < \mu \; \forall A \in [0, \bar{A}]$, the numerator of $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}}$ is positive. Therefore, $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} > 0$ and hence, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} < 0$.

When $\frac{\partial C_1(A)}{\partial A} = \mu \; \forall A \in [0, \bar{A}]$, the numerator of $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}}$ is equal to zero. Therefore, $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} = 0$ and hence, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} = 0$.

4. Define $\bar{\Lambda}^0 \equiv \max_{\alpha_1} \Lambda^0(\alpha_1)$ and $\underline{\Lambda}^0 \equiv \min_{\alpha_1} \Lambda^0(\alpha_1)$.

5. Evaluate the sign of $\frac{\partial \frac{\partial S}{\partial \alpha_1}}{\partial \Lambda}$.

$$\frac{\partial \frac{\partial S}{\partial \alpha_1}}{\partial \Lambda} = p \int_{A^{2.1}}^{\bar{A}} \frac{A}{A + \bar{f}^V} C_1(A) g(A) dA \times$$
$$\times \left[\frac{(A^{2.1} + \bar{f}^V) \frac{p}{1-p} \bar{f}^I + \bar{f}^V C_1(A^{2.1})}{(A^{2.1} + \bar{f}^V) \frac{p}{1-p} \bar{f}^I + (1 - \alpha_1) \bar{f}^V C_1(A^{2.1})} \right] > 0.$$

6. Find sufficient conditions and characterize the optimal α_1. Steps 3–5 of this proof are used to characterize three mutually-exclusive Λ-segments, Cases 1–3.[56]

Case 1: If $\Lambda < \underline{\Lambda}^0$, then $\Lambda < \Lambda^0(\alpha_1) \; \forall \alpha_1 \in [0, 1]$, by the definition of $\underline{\Lambda}^0$. Therefore, $\frac{\partial S}{\partial \alpha_1} < 0$, by the definition of $\Lambda^0(\alpha_1)$ and $\frac{\partial \frac{\partial S}{\partial \alpha_1}}{\partial \Lambda} > 0$. Hence, the optimal $\alpha_1 = 1$.

Case 2: If $\Lambda > \bar{\Lambda}^0$, then $\Lambda > \Lambda^0(\alpha_1) \; \forall \alpha_1 \in [0, 1]$, by the definition of $\bar{\Lambda}^0$. Therefore, $\frac{\partial S}{\partial \alpha_1} > 0$, by the definition of $\Lambda^0(\alpha_1)$ and $\frac{\partial \frac{\partial S}{\partial \alpha_1}}{\partial \Lambda} > 0$. Hence, the optimal $\alpha_1 = 0$.

Case 3: If $\Lambda \in [\underline{\Lambda}^0, \bar{\Lambda}^0]$, then $\Lambda < \Lambda^0(\alpha_1)$ for some $\alpha_1 \in [0, 1]$ and/or $\Lambda > \Lambda^0(\alpha_1)$ for some $\alpha_1 \in [0, 1]$, by the ambiguity of the sign of

[56] In Cases 1 and 2, we do not use the result about the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1}$. Hence, additional conditions to solve the ambiguity of the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1}$ are not required. Additional conditions are only required in Case 3.

$\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1}$ due to the ambiguity of the sign of $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}}$ (Step 3 of this proof). Hence, additional sufficient conditions are required.

Next, we characterize three mutually-exclusive cases such that $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1}$ is lower than, equal to or greater than zero, Cases 3(a), 3(b) and 3(c). We use the conditions on μ derived in Step 3 of this proof.

(a) If $\frac{\partial C_1(A)}{\partial A} > \mu \, \forall A \in [0, \bar{A}]$, then, $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} < 0$. Therefore, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} > 0$.

   i. If $\Lambda = \underline{\Lambda}^0$, then $\Lambda < \Lambda^0(\alpha_1)$ for $\alpha_1 \in (0, 1]$ and $\Lambda = \Lambda^0(\alpha_1)$ for $\alpha_1 = 0$, by the definition of $\underline{\Lambda}^0$. Therefore, $\frac{\partial S}{\partial \alpha_1} < 0$ for $\alpha_1 \in (0, 1]$ and $\frac{\partial S}{\partial \alpha_1} = 0$ for $\alpha_1 = 0$, by the definition of $\Lambda^0(\alpha_1)$. Hence for $\Lambda = \underline{\Lambda}$, $\arg\min_{\alpha_1 \in [0,1]}\{S\} = 1$.

   ii. If $\Lambda = \bar{\Lambda}^0$, then $\Lambda > \Lambda^0(\alpha_1)$ for $\alpha_1 \in [0, 1)$ and $\Lambda = \Lambda^0(\alpha_1)$ for $\alpha_1 = 1$, by the definition of $\bar{\Lambda}^0$. Therefore, $\frac{\partial S}{\partial \alpha_1} > 0$ for $\alpha_1 \in [0, 1)$ and $\frac{\partial S}{\partial \alpha_1} = 0$ for $\alpha_1 = 1$, by the definition of $\Lambda^0(\alpha_1)$. Hence for $\Lambda = \bar{\Lambda}, \arg\min_{\alpha_1 \in [0,1]}\{S\} = 0$.

   iii. If $\Lambda \in (\underline{\Lambda}^0, \bar{\Lambda}^0)$, then $\Lambda < \Lambda^0(\alpha_1)$ for some $\alpha_1 \in [0, 1]$ and $\Lambda > \Lambda^0(\alpha_1)$ some other $\alpha_1 \in [0, 1]$, by the definitions of $\underline{\Lambda}^0$ and $\bar{\Lambda}^0$. Therefore, $\frac{\partial S}{\partial \alpha_1} < 0$ for some $\alpha_1 \in (0, 1]$ and $\frac{\partial S}{\partial \alpha_1} > 0$ for some other $\alpha_1 \in [0, 1]$, by the definition of $\Lambda^0(\alpha_1)$. Given that S is a continuous function of $\alpha_1 \in [0, 1]$, by the Extreme Value Theorem, an optimal α_1 exists. Next, we evaluate whether there exists corner and/or interior optimal α_1.

   We start by showing that $\alpha_1 \in (0, 1)$ is not an optimal α_1. By the continuity of $\Lambda^0(\alpha_1)$ and $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} > 0$, there exists a unique $\alpha_1^{'} \in (0, 1)$ such that $\Lambda = \Lambda^0(\alpha_1^{'})$. Given that $\Lambda^0(\alpha_1)$ is strictly increasing in α_1, when $\alpha_1 \in (0, \alpha_1^{'})$, $\Lambda > \Lambda^0(\alpha_1)$ and therefore, $\frac{\partial S}{\partial \alpha_1} > 0$; and, when $\alpha_1 \in (\alpha_1^{'}, 1)$, $\Lambda < \Lambda^0$ and therefore, $\frac{\partial S}{\partial \alpha_1} < 0$. As a result, $\alpha_1^{'} \in (0, 1) = \arg\max_{\alpha_1 \in [0,1]}\{S\}$. Hence, an optimal corner $\alpha_1 = 0$ and/or $\alpha_1 = 1$ must exist.

Next, we find additional sufficient conditions and characterize the corner optimal α_1.

$$S(\alpha_1, \Lambda) = \int_0^{\bar{A}} Ag(d)dA +$$

$$+p(1+\Lambda\alpha_1)\int_{A^{2.1}(p,\alpha_1)}^{\bar{A}} \left(\frac{A}{A+\bar{f}^V}\right) C_1(A)g(A)dA +$$

$$+p\int_0^{A^{2.1}(p,\alpha_1)} \frac{A}{\bar{f}^V} C_0(A)g(A)dA.$$

Compare S at optimal $\alpha_1 = 1$ and optimal $\alpha_1 = 0$. Optimal $\alpha_1 = 1$ occurs when $\Lambda = \underline{\Lambda}^0$, by point (i).

$$S(\alpha_1 = 1, \Lambda = \underline{\Lambda}) < S(\alpha_1 = 0, \Lambda = \underline{\Lambda}).$$

Optimal $\alpha_1 = 0$ occurs when $\Lambda = \bar{\Lambda}^0$, by point (i).

$$S(\alpha_1 = 0, \Lambda = \bar{\Lambda}) < S(\alpha_1 = 1, \Lambda = \bar{\Lambda}).$$

Hence, either $\alpha_1 = 0$ or $\alpha_1 = 1$ can be optimal under certain conditions on Λ.

We now characterize the sufficient conditions on Λ under which $\alpha_1 = 0$ is optimal and $\alpha_1 = 1$ is optimal. First, analyze the relationship between Λ and S for $\alpha_1 = 0$ and $\alpha_1 = 1$. Note that Λ does not affect $A^{2.1}(p, \alpha_1)$.

$$\frac{\partial S(\alpha_1 = 1, \Lambda)}{\partial \Lambda} = p\int_{A^{2.1}(p,1)}^{\bar{A}} \frac{A}{A+\bar{f}^V} C_1(A)g(A)dA > 0$$

and

$$\frac{\partial S(\alpha_1 = 0, \Lambda)}{\partial \Lambda} = 0,$$

i.e., $S(\alpha_1 = 1, \Lambda)$ is strictly increasing in Λ and $S(\alpha_1 = 0, \Lambda)$ is constant. Therefore, the two curves can cross in just one point. Hence, by the continuity of $S(\alpha_1 = 0, \Lambda)$ and $S(\alpha_1 = 1, \Lambda)$ with respect to Λ, there exists a unique $\hat{\Lambda}$ such that $S(\alpha_1 = 1, \Lambda) \leq S(\alpha_1 = 0, \Lambda)$ for $\Lambda \in (\underline{\Lambda}, \hat{\Lambda}]$, and $S(\alpha_1 = 1, \Lambda) > S(\alpha_1 = 0, \Lambda)$ for $\Lambda \in (\hat{\Lambda}, \bar{\Lambda})$. Hence, $S(\alpha_1 = 1, \Lambda)$ and $S(\alpha_1 = 0, \Lambda)$ intersect at point $\Lambda = \hat{\Lambda}$.

Characterize $\hat{\Lambda}$. At $\Lambda = \hat{\Lambda}$, $S(\alpha_1 = 0, \hat{\Lambda}) = S(\alpha_1 = 1, \hat{\Lambda})$,

$$\int_{A^{2.1}(p,0)}^{\bar{A}} \frac{A}{A + \bar{f}^V} C_1(A)g(A)dA + \int_0^{A^{2.1}(p,0)} \frac{A}{\bar{f}^V} C_0(A)g(A)dA =$$

$$= (1 + \hat{\Lambda}) \int_{A^{2.1}(p,1)}^{\bar{A}} \frac{A}{A + \bar{f}^V} C_1(A)g(A)dA +$$

$$+ \int_0^{A^{2.1}(p,1)} \frac{A}{\bar{f}^V} C_0(A)g(A)dA.$$

Solving for $\hat{\Lambda}$,

$$\hat{\Lambda} =$$

$$= \frac{\int_{A^{2.1}(p,0)}^{\bar{A}} \frac{A}{A+\bar{f}^V} C_1(A)g(A)dA + \int_0^{A^{2.1}(p,0)} \frac{A}{\bar{f}^V} C_0(A)g(A)dA}{\int_{A^{2.1}(p,1)}^{\bar{A}} \frac{A}{A+\bar{f}^V} C_1(A)g(A)dA} -$$

$$- \frac{\int_0^{A^{2.1}(p,1)} \frac{A}{\bar{f}^V} C_0(A)g(A)dA}{\int_{A^{2.1}(p,1)}^{\bar{A}} \frac{A}{A+\bar{f}^V} C_1(A)g(A)dA} - 1.$$

Hence, the optimal α_1 are as follows.

A. If $\Lambda \in (\underline{\Lambda}, \hat{\Lambda}]$, then the optimal $\alpha_1 = 1$.

B. If $\Lambda \in (\hat{\Lambda}, \bar{\Lambda})$, then the optimal $\alpha_1 = 0$.

(b) If $\frac{\partial C_1(A)}{\partial A} < \mu \, \forall A \in [0, \bar{A}]$, then, $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} > 0$. Therefore, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} < 0$.

i. If $\Lambda = \underline{\Lambda}^0$, then $\Lambda < \Lambda^0(\alpha_1)$ for $\alpha_1 \in [0, 1)$ and $\Lambda = \Lambda^0(\alpha_1)$ for $\alpha_1 = 1$, by the definition of $\underline{\Lambda}^0$. Therefore, $\frac{\partial S}{\partial \alpha_1} < 0$ for $\alpha_1 \in [0, 1)$ and $\frac{\partial S}{\partial \alpha_1} = 0$ for $\alpha_1 = 1$, by the definition of $\Lambda^0(\alpha_1)$. Hence for $\Lambda = \underline{\Lambda}^0$, $\arg\min_{\alpha_1 \in [0,1]}\{S\} = 1$.

ii. If $\Lambda = \bar{\Lambda}^0$, then $\Lambda > \Lambda^0(\alpha_1)$ for $\alpha_1 \in (0,1]$ and $\Lambda = \Lambda^0(\alpha_1)$ for $\alpha_1 = 0$, by the definition of $\bar{\Lambda}^0$. Therefore, $\frac{\partial S}{\partial \alpha_1} > 0$ for $\alpha_1 \in (0,1]$ and $\frac{\partial S}{\partial \alpha_1} = 0$ for $\alpha_1 = 0$, by the definition of $\Lambda^0(\alpha_1)$. Hence for $\Lambda = \bar{\Lambda}^0, \arg\min_{\alpha_1 \in [0,1]}\{S\} = 0$.

iii. If $\Lambda \in (\underline{\Lambda}, \bar{\Lambda})$, then $\Lambda < \Lambda^0(\alpha_1)$ for some $\alpha_1 \in [0,1]$ and $\Lambda > \Lambda^0(\alpha_1)$ some other $\alpha_1 \in [0,1]$, by the definitions of $\underline{\Lambda}^0$ and $\bar{\Lambda}^0$. Therefore, $\frac{\partial S}{\partial \alpha_1} < 0$ for some $\alpha_1 \in (0,1]$ and $\frac{\partial S}{\partial \alpha_1} > 0$ for some other $\alpha_1 \in [0,1]$, by the definition of $\Lambda^0(\alpha_1)$. Given that S is a continuous function of $\alpha_1 \in [0,1]$, by the Extreme Value Theorem, an optimal α_1 exists. Next, we evaluate whether there exists corner and/or interior optimal α_1.

We show that there exists an interior $\alpha_1 \in (0,1)$ for each $\Lambda \in (\underline{\Lambda}^0, \bar{\Lambda}^0)$. By the continuity of $\Lambda^0(\alpha_1)$ and $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} < 0$, there exists a unique $\alpha_1^{'} \in (0,1)$ such that $\Lambda = \Lambda^0(\alpha_1^{'})$. At $\alpha_1 = \alpha_1^{'}$, $\frac{\partial S}{\partial \alpha_1} = 0$ by the definition of $\Lambda^0(\alpha_1)$. When $\alpha_1 \in (0, \alpha_1^{'})$, $\Lambda < \Lambda^0(\alpha_1)$, by $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} < 0$. Therefore, $\frac{\partial S}{\partial \alpha_1} < 0$. When $\alpha_1 \in (\alpha_1^{'}, 1)$, $\Lambda > \Lambda^0$, by $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} < 0$. Therefore, $\frac{\partial S}{\partial \alpha_1} > 0$. Hence, $\alpha_1^{'} \in (0,1) = \arg\min_{\alpha_1 \in [0,1]}\{S\}$.

Given that $\frac{\partial S}{\partial \alpha_1} < 0$ for $\alpha_1 \in (0, \alpha_1^{'})$ and $\frac{\partial S}{\partial \alpha_1} > 0$ for $\alpha_1 \in (\alpha_1^{'}, 1)$, a corner optimal α_1, zero or 1, does not exist.

(c) If $\frac{\partial C_1(A)}{\partial A} = \mu \; \forall A \in [0, \bar{A}]$, then $\frac{\partial \Lambda^0(\alpha_1)}{\partial A^{2.1}} = 0$. Therefore, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} = 0$. Hence, $\underline{\Lambda}^0(\alpha_1) = \bar{\Lambda}^0(\alpha_1) = \Lambda^0$.

i. If $\Lambda < \Lambda^0$, then $\frac{\partial S}{\partial \alpha_1} < 0 \; \forall \alpha_1 \in [0,1]$, by the definition of $\Lambda^0(\alpha_1)$. Hence, the optimal $\alpha_1 = 1$.

ii. If $\Lambda > \Lambda^0$, then $\frac{\partial S}{\partial \alpha_1} > 0 \; \forall \alpha_1 \in [0,1]$, by the definition of $\Lambda^0(\alpha_1)$. Hence, the optimal $\alpha_1 = 0$

iii. If $\Lambda = \Lambda^0$, then $\frac{\partial S}{\partial \alpha_1} = 0 \; \forall \alpha_1 \in [0,1]$, by the definition of $\Lambda^0(\alpha_1)$. Hence, any $\alpha_1 \in [0,1]$ is optimal.

Appendix B. Benchmark Model – Uniform Distribution Model and Numerical Example

This Appendix presents the model with a uniform distribution of damages (Section B.1) and the numerical example (Section B.2) for the benchmark model. We focus on Environment 1, p-Segment 2.1 where $p \in (\tilde{p}, \bar{p}]$, and Environment 2, p-Segment 2 where $p \in (p^0, 1]$.

B.1 Model with a Uniform Distribution of Damages

Assume that A is uniformly distributed over $A \in [0, \bar{A}]$, where $g(A) = \frac{1}{\bar{A}}$ $\forall A \in [0, \bar{A}]$, $G(A) = \frac{A}{\bar{A}}$, and $\int_0^{\bar{A}} Ag(A)dA = \frac{\bar{A}}{2}$.

Thresholds $\bar{\mathbf{p}}$ and $\mathbf{A^{2.1}}$.– $\bar{p}$ and $A^{2.1}(p)$ can be explicitly defined.

Consider $\bar{p}$. When $p = \bar{p}$, the liable injurer's incentive-compatibility constraint is:

$$\int_0^{\bar{A}} A\frac{1}{\bar{A}}dA = \bar{f}^I \int_0^{\frac{(1-\bar{p})\bar{f}^V}{\bar{p}}} \frac{\bar{p}A}{(1-\bar{p})\bar{f}^V}\frac{1}{\bar{A}}dA.$$

Let $\Psi = \frac{(1-\bar{p})\bar{f}^V}{\bar{p}}$. The liable injurer's incentive-compatibility constraint can be written as:

$$\frac{\bar{A}}{2} = \bar{f}^I \frac{1}{\Psi}\frac{(\Psi)^2}{2}\frac{1}{\bar{A}}.$$

Solving for Ψ:

$$\Psi = \frac{\bar{A}^2}{\bar{f}^I} = \frac{(1-\bar{p})\bar{f}^V}{\bar{p}}.$$

Hence,

$$\bar{p} = \frac{\bar{f}^V\bar{f}^I}{\bar{f}^V\bar{f}^I + \bar{A}^2}.$$

Consider $A^{2.1}(p)$. Suppose $p \in (\tilde{p}, \bar{p}]$. The liable injurer's incentive-compatibility constraint is:

$$\int_0^{\bar{A}} A\frac{1}{\bar{A}}dA = \bar{f}^I \int_0^{A^{2.1}(p)} \frac{pA}{(1-p)\bar{f}^V}\frac{1}{\bar{A}}dA,$$

which can be written as:

$$\frac{\bar{A}}{2} = \bar{f}^I \frac{p}{(1-p)\bar{f}^V} \frac{(A^{2.1}(p))^2}{2} \frac{1}{\bar{A}}.$$

Hence,

$$A^{2.1}(p) = \bar{A}\sqrt{\frac{(1-p)\bar{f}^V}{p\bar{f}^I}}.$$

Social Welfare Loss Function for Environment 1.– Given the optimal mechanisms and the assumption regarding the distribution of damages, the social welfare loss function is as follows.

$$SWL^{2.1} = H(A) + (1-p)\int_0^{A^{2.1}(p)} \Big(\frac{p}{1-p}\Big)\frac{A}{\bar{f}^V} C_0(A)g(A)dA+$$

$$+p\int_{A^{2.1}(p)}^{\bar{A}} \Big(\frac{A}{A+\bar{f}^V}\Big) C_1(A)g(A)dA =$$

$$= \frac{\bar{A}}{2} + (1-p)\int_0^{A^{2.1}(p)} \Big(\frac{p}{1-p}\Big)\frac{A}{\bar{f}^V}\frac{C_0(A)}{\bar{A}}dA + p\int_{A^{2.1}(p)}^{\bar{A}} \Big(\frac{A}{A+\bar{f}^V}\Big)\frac{C_1(A)}{\bar{A}}dA.$$

Social Welfare Loss Function for Environment 2.– Given the optimal mechanisms and the assumption regarding the distribution of damages, the social welfare loss function is as follows.

$$SWL^2 = H(A) + (1-p)\int_0^{A^0} \frac{pA}{(1-p)\bar{f}^V} C_0(A)g(A)dA + (1-p)\int_{A^0}^{\bar{A}} C_0(A)g(A)dA+$$

B.2 Numerical Example

The model with a uniform distribution of damages model is used to construct this numerical example.

Environment 1

Assume $C_0(A) = C_0 + c_0 A$ and $C_1(A) = C_1 + c_1 A$. The set of exogenous parameters is:

$$\{C_0, C_1, c_0, c_1, \bar{f}^V, \bar{f}^I, \bar{A}, p\} = \{1528, 690, 0.3, 0.01, 1800, 3600, 1200, 0.45\}.$$

Given that $\bar{A} = 1200$, $H = \frac{\bar{A}}{2} = 600$.

The model conditions are satisfied under this numerical example.

1. The condition for Environment 1, $C_0(A) \geq C_1(A)\ \forall A \in [0, \bar{A}]$, becomes $1528 + 0.3A \geq 690 + 0.01A$. After simplification, $0.29A > -838\ \forall A \in [0, 1200]$.
2. The condition for Lemma 3 and first condition for Propositions 3 and 4, $C_0(0) > C_1(0)$, becomes $1528 > 690$.
3. The second condition for Proposition 3, $\frac{\partial \frac{C_1(A)}{C_0(A)}}{\partial A} \leq 0\ \forall A \in [0, \bar{A}]$, becomes

$$\frac{\partial \frac{C_1 + c_1 A}{C_0 + c_0 A}}{\partial A} = \frac{c_1 C_0 - c_0 C_1}{(C_0 + c_0 A)^2} = \frac{0.01(1528) - 0.3(690)}{(1528 + 0.3A)^2} =$$

$$= \frac{-191.72}{(1528 + 0.3A)^2} < 0$$

$\forall A \in [0, 1200]$.

4. The third condition for Proposition 3, $\frac{\partial C_0(A)}{\partial A} > 0\ \forall A \in [0, \bar{A}]$, becomes $c_0 = 0.3 > 0\ \forall A \in [0, 1200]$.
5. The second condition for Proposition 4 is $C_0(A) - C_0(0) < \frac{C_1(A)\bar{f}^V}{A + \bar{f}^V}$ $\forall A \in [0, \bar{A}]$. The left-hand side of the inequality is: $C_0(A) - C_0(0) = c_0 A = 0.3A$. The right-hand side of the inequality is $\frac{(690+0.01A)(1800)}{A+1800}$. Evaluate the left-hand side at the highest A-value and the numerator of the right-hand side at the lowest A-value and the denominator of the right-hand side at the highest A-value. After simplification, $360 < 414 = \frac{(690)(1800)}{3000}\ \forall A \in [0, 1200]$.

For an illustration, the next sections focus on p-Segment 2.1.

Social Welfare Loss Functions.– Given the optimal mechanisms and functional forms for $C_0(A)$ and $C_1(A)$, the social welfare loss function is as follows.

$$SWL^{2.1} = \frac{\bar{A}}{2} + (1-p)\int_0^{A^{2.1}(p)} \Big(\frac{p}{1-p}\Big)\frac{A}{\bar{f}^V}\frac{C_0 + c_0 A}{\bar{A}}dA +$$

$$+p\int_{A^{2.1}(p)}^{\bar{A}} \Big(\frac{A}{A+\bar{f}^V}\Big)\frac{C_1 + c_1 A}{\bar{A}}dA =$$

$$\frac{\bar{A}}{2} + \frac{p}{\bar{f}^V\bar{A}}\Big[C_0\frac{(A^{2.1})^2}{2} + c_0\frac{(A^{2.1})^3}{3}\Big] +$$

$$\frac{p}{\bar{A}}\Big[c_1\big(\frac{\bar{A}^2}{2} - \frac{(A^{2.1}(p))^2}{2}\big) + (C_1 - c_1\bar{f}^V)(\bar{A} - A^{2.1}(p)) -$$

$$-(C_1 - c_1\bar{f}^V)\bar{f}^V[\log(\bar{A}+\bar{f}^V) - \log(A^{2.1}(p)+\bar{f}^V)]\Big],$$

where $A^{2.1}(p) = \bar{A}\sqrt{\frac{(1-p)\bar{f}^V}{p\bar{f}^I}}$. Using the set of exogenous parameters, $SWL^{2.1}$ is computed.

Numerical Example.– The relevant p-thresholds are $\tilde{p} = \frac{1800}{1800+3600} = 0.33$ and $\bar{p} = \frac{(1800)(3600)}{(1800)(3600)+1200^2} = 0.82$. Therefore, $p \in (0.33, 0.82]$. In this example, $p = 0.45$ and therefore, $A^{2.1}(p) = 938$. Hence, the A-segments are $A \in [0, 938]$ and $A \in (938, 1200]$. The optimal probabilities of investigation for each A-segment, evaluated at the average A-values, $A = 469$ and $A = 1069$, are: $q_0(A) = \big(\frac{0.45}{1-0.45}\big)\frac{469}{1800} = 0.213$ and $q_1(A) = 0$; and, $q_0(A) = 0$ and $q_1(A) = \frac{1069}{1800+1069} = 0.373$. Intuitively, evidence might be produced only in legal cases where the injurer reports to be non-liable and the victim reports sufficiently low damages ($A \leq 938$). Evidence might be also produced in legal cases where the injurer reports to be liable if the victim reports sufficiently high damages ($A > 938$). The expected harm from an accident, equal to the expected victim's damages, is $H = \frac{1200}{2} = 600$. The optimal expected cost of producing evidence for p-Segment 2.1 is $\mathbb{E}[C(A)]^{2.1} = 343$, the optimal expected cost from the infringement of the victim's right of access to justice is $\theta\mathbb{E}[\eta(A)] = 0$ and the optimal expected cost from the infringement of the right of the victims confronting liable injurers to be fully compensated is $\Lambda\mathbb{E}[\xi(A)] = 0$. Hence, the social welfare loss for p-Segment 2.1 is $SWL^{2.1} = H + \mathbb{E}[C(A)]^{2.1} + \theta\mathbb{E}[\eta(A)] + \Lambda\mathbb{E}[\xi(A)] = 600 + 343 + 0 + 0 = 943$.

Environment 2

Assume that $C_0(A) = C_0 + c_0 A$ and $C_1(A) = C_1 + c_1 A$. The set of exogenous parameters is:

$$\{C_0, C_1, c_0, c_1, \bar{f}^V, \bar{f}^I, \bar{A}\} = \{500, 1200, 0.15, 0.03, 1800, 3600, 1200\}.$$

Given that $\bar{A} = 1200$, $H = \frac{\bar{A}}{2} = 600$.

The model conditions are satisfied under this numerical example.

1. The first condition for Environment 2, $C_0(A) < C_1(A)\ \forall A \in [0, \bar{A}]$, becomes $500 + 0.15A < 1200 + 0.3A$. After simplification, $-700 < 0.15A$, which holds $\forall A \in [0, 1200]$.

2. The second condition for Environment 2, $\bar{A} < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$, becomes $1200 < \big(\frac{1200+0.3A}{500+0.15A} - 1\big)(1800)$. Evaluate the numerator of the fraction in the right-hand side of the inequality at the lowest A-value and the denominator of the fraction in the right-hand side of the inequality at the highest A-value: $1200 < 1376.47 = \big(\frac{1200}{680} - 1\big)(1800)\ \forall A \in [0, 1200]$.

3. The condition for Lemma 3 and first condition for Proposition 3, $C_0(0) > 0$, becomes $500 > 0$.

4. The second condition for Proposition 3, $\frac{\partial C_0(A)}{\partial A} > 0\ \forall A \in [0, \bar{A}]$, becomes $0.15 > 0\ \forall A \in [0, 1200]$.

For an illustration, we focus on p-Segment 2 in the next two sections.

Social Welfare Loss Functions.– Given the optimal mechanisms and the functional forms for $C_0(A)$ and $C_1(A)$, the social welfare loss function is as follows.

$$SWL^2 = \frac{\bar{A}}{2} + (1-p)\int_0^{A^0} \frac{pA}{(1-p)\bar{f}^V} \frac{C_0 + c_0 A}{\bar{A}} dA + (1-p)\int_{A^0}^{\bar{A}} \frac{C_0 + c_0 A}{\bar{A}} dA +$$

Table B1: Numerical Example – Optimal Production of Evidence

Environment	p-Segment	A-Segment	Optimal $q_0(A)^a$	Optimal $q_1(A)^a$
Environment 2	$(0.60, 1]$	$[0, 735]$	$q_0(A) = 0.501$	$q_1(A) = 0$
(p-Segment 2)		$(735, 1200]$	$q_0(A) = 1$	$q_1(A) = 0.084$

Note: [a]For each A-segment, $q_0(A)$ and $q_1(A)$ are evaluated at the average A-value; $p = 0.71$ is used.

$$+p\int_{A^0}^{\bar{A}}\left[1-\frac{\bar{f}^V}{p(A+\bar{f}^V)}\right]\left(\frac{C_1+c_1A}{\bar{A}}\right)dA =$$

$$=\frac{\bar{A}}{2}+\frac{p}{\bar{f}^V\bar{A}}\left[C_0\frac{(A^0)^2}{2}+c_0\frac{(A^0)^3}{3}\right]+\frac{1-p}{\bar{A}}\left[C_0(\bar{A}-A^0)+c_0\left(\frac{\bar{A}^2}{2}-\frac{(A^0)^2}{2}\right)\right]+$$

$$+\frac{p}{\bar{A}}\left[C_1(\bar{A}-A^0)+c_1\left(\frac{\bar{A}^2}{2}-\frac{(A^0)^2}{2}\right)\right]-$$

$$-\frac{\bar{f}^V}{\bar{A}}\{c_1(\bar{A}-A^0)+(C_1-c_1\bar{f}^V)[\log(\bar{A}+\bar{f}^V)-\log(A^0+\bar{f}^V)]\}.$$

Using the set of exogenous parameters, SWL^2 is computed.

Numerical Example.– Table B1 summarizes our results. The relevant p-threshold is $p^0 = \frac{1800}{1800+1200} = 0.60$. Therefore, $p \in (0.60, 1]$. In this example, $p = 0.71$ and therefore, $A^0(p) = 735$. Hence, the A-segments are $A \in [0, 735]$ and $A \in (735, 1200]$. The expected harm from an accident, equal to the expected victim's damages, is $H = \frac{1200}{2} = 600$. The optimal probabilities of investigation for each A-segment, evaluated at the average A-values, $A = 368$ and $A = 968$ are: $q_0(A) = \left(\frac{0.71}{1-0.71}\right)\frac{368}{1800} = 0.501$ and $q_1(A) = 0$ and, $q_0(A) = 1$ and $q_1(A) = 1 - \frac{1800}{0.71(1800+968)} = 0.084$. Intuitively, evidence might be produced only in legal cases where the injurer reports to be non-liable and the victim reports sufficiently low damages ($A \leq 735$). Evidence might be also produced in legal cases where the injurer reports to be non-liable and liable if the victim reports sufficiently high damages ($A > 735$). The expected harm from an accident, equal to the expected victim's damages, is $H = \frac{1200}{2} = 600$. The optimal expected cost of producing evidence for p-Segment 2 is $\mathbb{E}[C(A)]^2 = 157$, the optimal expected cost from the infringement of the victim's right of access to justice is $\theta\mathbb{E}[\eta(A)] = 0$ and the optimal expected cost from the infringement of the right of the victims confronting liable injurers to be fully compensated is $\Lambda\mathbb{E}[\xi(A)] = 0$. Hence, the social welfare loss for p-Segment 2 is $SWL^2 = H + \mathbb{E}[C(A)]^2 + \theta\mathbb{E}[\eta(A)] + \Lambda\mathbb{E}[\xi(A)] = 600 + 157 + 0 + 0 = 757$.

Appendix C: Model with Endogenous Cost Allocation

This appendix presents the formal analysis of the model with endogenous cost allocation. It also includes the proofs of propositions (except for Proposition 12), lemmas and claims. The model setup and intuitive discussion are presented in the main text of the paper, Section 6. The proof of Proposition 12 (main proposition) is included in Appendix A.

As stated in Section 4, we assume that the victim and the injurer might pay a share of the cost of producing evidence. Therefore, the transfers and fines that result from the production of evidence are as follows. First, when reports are not investigated, the injurer transfers r^V to the victim if $r^I = 1$. Second, when the reports are investigated and found to be truthful ($r^V = A$ and $r^I = L$), the injurer transfers r^V to the victim if $r^I = 1$ and pays $(1-\alpha_i)$ of the cost of producing evidence $C_i(A)$ $(i = 0, 1)$. The victim pays α_i of the cost of producing evidence $C_i(A)$ $(i = 0, 1)$. Third, when the reports are investigated and the injurer's report is found to be untruthful ($r^I \neq L$), the injurer pays fine $f^I \in [0, \bar{f}^I]$ to the social planner, and the social planner transfers r^V to the victim if $L = 1$ and pays $(1-\alpha_i)$ of the cost of producing evidence $C_i(A)$ $(i = 0, 1)$. The victim pays α_i of the cost of producing evidence $C_i(A)$ $(i = 0, 1)$. Fourth, when the reports are investigated and the victim's report is found untruthful, the victim does not receive any transfers and pays fine $f^V \in [0, \bar{f}^V]$ to the social planner, and the social planner pays α_i of the cost of producing evidence $C_i(A)$ $(i = 0, 1)$. The injurer pays $(1 - \alpha_i)$ of the cost of producing evidence $C_i(A)$ $(i = 0, 1)$. Fifth, when the reports are investigated and the victim's and injurer's reports are found untruthful ($r^V \neq A$ and $r^I \neq L$): The victim does not receive any transfers, the victim and the injurer pay fines $f^V \in [0, \bar{f}^V]$ and $f^I \in [0, \bar{f}^I]$ to the social planner, and the social planner pays the cost of producing evidence $C_i(A)$ $(i = 0, 1)$.[57]

Players' Constraints

Victim's Individual-Rationality Constraint.– By participating in the mechanism and truthfully reporting her type, a victim with type A gets compensation A when the injurer is liable and pays an expected share of the cost of verification equal to $[p\alpha_1 q_1(A)C_1(A) + (1-p)\alpha_0 q_0(A)C_0(A)]$.

[57]The social planner pays α_i and $(1-\alpha_1)$ of the cost of producing evidence $C_i(A)$ $(i = 0, 1)$.

Her expected payoff is $pA - [p\alpha_1 q_1(A)C_1(A) + (1-p)\alpha_0 q_0(A)C_0(A)]$. The victim gets zero compensation if she decides not to trigger the mechanism. Therefore, the victim's individual-rationality constraint is: $\forall A \in [0, \bar{A}]$,

$$pA - [p\alpha_1 q_1(A)C_1(A) + (1-p)\alpha_0 q_0(A)C_0(A)] \geq 0.$$

Rearranging terms:

$$p(A - \alpha_1 q_1(A)C_1(A)) - (1-p)\alpha_0 q_0(A)C_0(A) \geq 0.$$

In contrast to the benchmark model, this constraint is not trivially satisfied.

Victim's Incentive-Compatibility Constraint.– By truthfully reporting her type, the victim with type A gets compensation $p(A - \alpha_1 q_1(A)C_1(A)) - (1-p)\alpha_0 q_0(A)C_0(A)$. Clearly, no victim has an incentive to report a lower type. When the victim reports a (weakly) higher type, $A' \in [0, \bar{A}]$, she gets a compensation $A' \geq A$ when investigation does not occur, and gets no compensation and pays fine f^V (which includes her share of the cost of producing evidence α_i) when investigation occurs. Her expected payoff is $p(1 - q_1(A'))A' - [pq_1(A') + (1-p)q_0(A')]f^V$. Therefore, the victim's incentive-compatibility constraint is: $\forall A, A' \in [0, \bar{A}]$,

$$p(A - \alpha_1 q_1(A)C_1(A)) - (1-p)\alpha_0 q_0(A)C_0(A) \geq$$

$$\geq p(1 - q_1(A')A' - [pq_1(A') + (1-p)q_0(A')]f^V.$$

Note that, by setting the fine $f^V \in [0, \bar{f}^V]$ as high as possible, the social planner will spend less resources on investigation. Given that the victim is financially constrained, $\bar{f}^V = W^V$. Therefore, the victim's incentive-compatibility constraint is: $\forall A, A' \in [0, \bar{A}]$,

$$p(A - \alpha_1 q_1(A)C_1(A)) - (1-p)\alpha_0 q_0(A)C_0(A) \geq$$

$$\geq p(1 - q_1(A'))A' - [pq_1(A') + (1-p)q_0(A')]\bar{f}^V.$$

Liable Injurer's Incentive-Compatibility Constraint.– When the liable injurer truthfully reports his type, $r_L = 1$, he pays expected damages $\mathbb{E}[A]$ and a share $(1 - \alpha_1)$ of the cost of producing evidence $C_1(A)$ when investigation occurs. His expected payoff $u_{1|1}$ is $u^I_{1|1} = -[\mathbb{E}[A] + (1-\alpha_1)\mathbb{E}[q_1(A)C_1(A)]] = -\big[\int_0^{\bar{A}} Ag(A)dA + (1-\alpha_1)\int_0^{\bar{A}} q_1(A)C_1(A)g(A)dA\big]$. By pretending to be non-liable, the liable injurer pays zero compensation when investigation does not occur, and pays fine f^I when investigation occurs. His expected payoff $u^I_{0|1}$ is $u^I_{0|1} = -[\mathbb{E}[(1 - q_0(A))0] - f^I\mathbb{E}[q_0(A)] =$

$-f^I \int_0^{\bar{A}}(q_0(A)g(A)dA$. By setting $f^I \in [0, \bar{f}^I]$ as high as possible, the social planner economizes on investigation efforts. Hence, the liable injurer's incentive-compatibility constraint:

$$\int_0^{\bar{A}} Ag(A)dA + (1-\alpha_1)\int_0^{\bar{A}} q_1(A)C_1(A)g(A)dA \leq \bar{f}^I \int_0^{\bar{A}} q_0(A)g(A)dA.$$

Given that the injurer has limited financial resources, $\bar{f}^I = W^I$.

Non-Liable Injurer's Incentive-Compatibility Constraint.– When the non-liable injurer truthfully reports his type, $r_L = 0$, he pays zero compensation and pays a share $(1-\alpha_0)$ of the cost of producing evidence $C_0(A)$ when investigation occurs. His expected payoff $u^I_{0|0}$ is $u^I_{0|0} = -(1-\alpha_0)\mathbb{E}[q_0(A)C_0(A)] = -(1-\alpha_0)\int_0^{\bar{A}} q_0(A)C_0(A)g(A)dA$. When the non-liable injurer reports to be liable, $r_L = 1$, he pays compensation only when investigation does not occur, and pays a fine f^I when investigation occurs. His expected payoff $u^I_{1|0}$ is $u^I_{1|0} = -[\mathbb{E}[(1-q_1(A))A] - f^I\mathbb{E}[q_1(A)] = -\int_0^{\bar{A}} \left\{(1-q_1(A))A + q_1(A)f^I\right\} g(A)dA$. By setting $f^I \in [0, \bar{f}^I]$ as high as possible, the social planner economizes on investigation efforts. Hence, the non-liable injurer's incentive-compatibility constraint:

$$(1-\alpha_0)\int_0^{\bar{A}} q_0(A)C_0(A)g(A)dA \leq \int_0^{\bar{A}} \left\{(1-q_1(A))A + q_1(A)\bar{f}^I\right\} g(A)dA.$$

In contrast to the benchmark model, this constraint is not trivially satisfied.

The next claim shows the relationship between victim's incentive-compatibility constraint in the current model and the victim's incentive-compatibility constraint in the benchmark model and the victim's individual-rationality constraint in the model with endogenous cost allocation. This result simplifies the procedure used to characterize the optimal mechanism.

Claims 7. *The victim's incentive-compatibility constraint in the current model (for given α_0 and α_1) holds when the victim's incentive-compatibility constraint in the benchmark model holds and the victim's individual-rationality constraint in the current model holds.*

Proof. Remember that the victim's incentive-compatibility constraint in the benchmark model is: $\forall A \in [0, \bar{A}]$,

$$[pq_1(A) + (1-p)q_0(A)]\bar{f}^V \geq [p(1-q_1(A)]A.$$

This constraint can be rewritten as:

$$[p(1-q_1(A)]A - [pq_1(A) + (1-p)q_0(A)]\bar{f}^V \leq 0.$$

The victim's individual-rationality constraint in the current model is: $\forall A \in [0, \bar{A}]$,

$$p(A - \alpha_1 q_1(A)C_1(A)) - (1-p)\alpha_0 q_0(A)C_0(A) \geq 0.$$

Combining the last two constraints: $\forall A, A' \in [0, \bar{A}]$

$$p(A - \alpha_1 q_1(A)C_1(A)) - (1-p)\alpha_0 q_0(A)C_0(A) \geq 0 \geq$$
$$\geq [p(1-q_1(A)]A - [pq_1(A) + (1-p)q_0(A)]\bar{f}^V$$

which implies the victim's incentive-compatibility constraint in the current model is: $\forall A, A' \in [0, \bar{A}]$,

$$p(A - \alpha_1 q_1(A)C_1(A)) - (1-p)\alpha_0 q_0(A)C_0(A) \geq$$
$$\geq p(1-q_1(A'))A' - [pq_1(A') + (1-p)q_0(A')]\bar{f}^V,$$

■

The next claim demonstrates that that the liable injurer's and non-liable injurer's incentive-compatibility constraints cannot be simultaneously violated. Importantly, this result holds for any set $\{\alpha_0, \alpha_1, q_0(A), q_1(A)\}$ that is not optimal ($q_i(A) \in [0,1]$ and $\alpha_i \in [0,1]$. $i = 0, 1$). Hence, it suffices to focus on the liable injurer's incentive-compatibility constraint. This result simplifies the procedure used to characterize the optimal mechanism.

Claim 8. *(1) If the the liable injurer's incentive-compatibility constraint is violated or satisfied as an equality, then the non-liable injurer's incentive-compatibility constraint is satisfied. (2) If the non-liable injurer's incentive-compatibility constraint is violated or satisfied as an equality, then the liable injurer's incentive-compatibility constraint is satisfied.*

Proof. Consider Part (1). Suppose the liable injurer's incentive-compatibility constraint is not satisfied or is satisfied as equality:

$$\int_0^{\bar{A}} Ag(A)dA + (1-\alpha_1)\int_0^{\bar{A}} q_1(A)C_1(A)g(A)dA \geq \int_0^{\bar{A}} q_0(A)\bar{f}^I g(A)dA.$$

Then,

$$\int_0^{\bar{A}} Ag(A)dA + \int_0^{\bar{A}} q_1(A)(\bar{f}^I - A)g(A)dA \geq$$

$$\geq \int_0^{\bar{A}} Ag(A)dA + (1-\alpha_1)\int_0^{\bar{A}} q_1(A)C_1(A)g(A)dA \geq$$

$$\int_0^{\bar{A}} q_0(A)\bar{f}^I g(A)dA \geq (1-\alpha_0)\int_0^{\bar{A}} q_0(A)C_0(A)g(A)dA.$$

Hence,

$$\int_0^{\bar{A}} Ag(A)dA + \int_0^{\bar{A}} q_1(A)(\bar{f}^I - A)g(A)dA \geq (1-\alpha_0)\int_0^{\bar{A}} q_0(A)C_0(A)g(A)dA.$$

Hence, the non-liable injurer's incentive-compatibility constraint is satisfied. Part (2) is verified using a similar approach. ■

Social Planner's Civil Justice Problem

As discussed in the main text of the paper, the only two relevant components of the social welfare function for the characterization of the optimal cost allocation and the optimal production of evidence are $\mathbb{E}[C(A)]$ and $\Lambda\mathbb{E}[\xi(A)]$. The social planner's problem is:

$$\min_{q_0(A,\alpha_0,\alpha_1), q_1(A,\alpha_0,\alpha_1), \alpha_0, \alpha_1} \{\mathbb{E}[C(A)] + \Lambda\mathbb{E}[\xi(A)]\}.$$

We adopt a five-step procedure to characterize the optimal probabilities of investigation and the optimal share of the cost of producing evidence. The first four steps characterize the interim probabilities of verification. We take α_1 and α_0 as given but include the feasibility constraints $0 \leq \alpha_i \leq 1$ $(i = 0, 1)$ in the analysis. In the last step, we use the interim probabilities of verification to characterize the optimal α_0 and α_1.

In Step 1, we characterize the interim $q_0(A)$ and $q_1(A)$ that satisfy the victim's incentive-compatibility constraint of the benchmark model (1), and the feasibility constraints for $q_i(A)$ and α_i $(i = 0, 1)$. By Claim 6, the victim's incentive-compatibility constraint of the current model holds when the victim's incentive-compatibility constraint of the benchmark model and the victim's individual-rationality constraint of the current model hold. Therefore, Step 1 corresponds to the first part of verification that the victim's incentive-compatibility constraint of the current model holds.

In Step 2, we evaluate whether the interim probabilities of investigation satisfy the liable injurer's incentive-compatibility constraint. By Claim 7, if the liable injurer's incentive-compatibility constraint is violated or satisfied as an equality, then the non-liable injurer's incentive-compatibility constraint is also satisfied. If the liable injurer's incentive-compatibility constraint is not satisfied under the interim probabilities of investigation characterized in Step 1, adjustments to satisfy this constraint as an equality are implemented. The non-liable injurer's incentive compatibility constraint will be still satisfied under the adjusted interim probabilities of investigation, by Claim 7. If the liable injurer's incentive-compatibility constraint is satisfied under the interim probabilities of investigation characterized in Step 1 no further adjustments are required. Two mutually-exclusive cases occur. First, if the injurer's incentive-compatibility constraint is satisfied as an equality, then the non-liable injurer is also satisfied and hence, no further adjustments are required. Second, if the injurer's incentive-compatibility constraint is satisfied as an inequality, then we need to verify whether the non-liable injurer is also satisfied. If not, further adjustments to satisfy the non-liable injurer's incentive-compatibility constraint as an equality should be implemented. By Claim 7, the liable injurer's incentive-compatibility constraint will be still satisfied under the adjusted interim probabilities of investigation.

In Step 3, we evaluate whether the interim probabilities of investigation characterized in Step 2 satisfy the victim's individual-rationality constraint. If not, adjustments are implemented. Therefore, given Claim 6, Step 3 also corresponds to second part of verification that the victim's incentive-compatibility constraint of the current model holds.

In Step 4, we verify whether the interim probabilities of investigation still satisfy the victim's and liable injurer's incentive-compatibility constraints. If not, adjustments are implemented. We show that the optimal probabilities of investigation of the benchmark model correspond to the interim probabilities of investigation of this model. In contrast to the benchmark model, the threshold $A^{2.1}(p, \alpha_1)$ now depends on α_1.

In Step 5, we characterize the optimal cost allocation and optimal production of evidence. We first characterize the optimal cost allocation, α_0 and α_1 by evaluating $\mathbb{E}[C(A)] + \Lambda\mathbb{E}[\xi(A)]$ at the interim probabilities of investigation, and minimizing this function with respect to α_0 and α_1. We then characterize the optimal production of evidence by evaluating the threshold $A^{2.1}(p, \alpha_1)$ of the interim probabilities of investigation at the optimal α_1.

Step 1: Analysis of Victim's Incentive-Compatibility Constraint (Part 1)

Claim 9. *Suppose $p \in (0,1)$. Part 1 of the victim's incentive-compatibility constraint for a victim of type $A \in [0, \bar{A}]$, $[pq_1(A) + (1-p)q_0(A)]\bar{f}^V \geq [p(1-q_1(A)]A$ holds as an equality at the interim probabilities of investigation $0 \leq q_i(A) \leq 1$ $(i = 0, 1)$.*

Proof. The proof follows the same logic than the proof of Claim 1 of the benchmark model. ■

Claim 10. *Suppose $p \in (0,1)$. Suppose $A = \big(\frac{C_1(A)(1+\Lambda)}{C_0(A)} - 1\big)\bar{f}^V$ or $A = \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$. The optimal mechanism is not unique for $p > \frac{\bar{f}^V}{\bar{f}^V + \bar{f}^I}$.*

Proof. The proof follows the same logic than the proof of Claim 2 of the benchmark model. ■

Following the benchmark model, we define $p^0 \equiv \frac{\bar{f}^V}{\bar{f}^V + A}$ and $A^0(p) \equiv \big(\frac{1-p}{p}\big)\bar{f}^V$. Claims 3–5 of the benchmark model also hold here.

Proposition 7. *Suppose $p \in (0,1)$. The interim probabilities of investigation for a victim of type A are as follows.*

1. *If $A \geq \big(\frac{C_1(A)(1+\Lambda)}{C_0(A)} - 1\big)\bar{f}^V$, then $q_0(A) = 0$ and $q_1(A) = \frac{A}{\bar{f}^V + A}$.*
2. *If $A < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ and $A \leq \big(\frac{1-p}{p}\big)\bar{f}^V$, then $q_0(A) = \big(\frac{p}{1-p}\big)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$.*
3. *If $A < \big(\frac{C_1(A)}{-}1\big)\bar{f}^V$ and $A > \big(\frac{1-p}{p}\big)\bar{f}^V$, then $q_0(A) = 1$ and $q_1(A) = 1 - \frac{\bar{f}^V}{p(\bar{f}^V + A)}$.*

Proof. The proof follows the same logic than the proof of Proposition 1 of the benchmark model. ■

Proposition 8. *The interim probabilities of investigation for Environments 1 and 2 across victim's types are as follows.*

1. *Environment 1: If $C_0(A) \geq C_1(A)(1+\Lambda)$, then the interim probabilities of investigation are: $q_0(A) = 0$ and $q_1(A) = \frac{A}{\bar{f}^V + A} < 1$ $\forall A \in [0, \bar{A}]$ $\forall p \in (0,1)$.*
2. *Environment 2: If $C_0(A) < C_1(A)$ $\forall A \in [0, \bar{A}]$ and $\bar{A} < \big(\frac{C_1(A)}{C_0(A)} - 1\big)\bar{f}^V$ $\forall A \in [0, \bar{A}]$, then the interim probabilities of investigation are as follows.*

(a) *p-Segment 1: If* $p \in (0, p^0]$*, then the interim probabilities of investigation are* $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ *and* $q_1(A) = 0$ $\forall A \in [0, \bar{A}]$.

(b) *p-Segment 2: If* $p \in (p^0, 1)$*, then the interim probabilities of investigation are* $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ *and* $q_1(A) = 0$ $\forall A \in [0, A^0(p)]$*, and* $q_0(A) = 1$ *and* $q_1(A) = 1 - \frac{\bar{f}^V}{p(\bar{f}^V + A)}$ $\forall A \in (A^0(p), \bar{A}]$.

Proof. The proof follows the same logic than the proof of Proposition 2 of the benchmark model. ■

The next sections are focused on Environment 1, p-Segment 2.1 ($p \in (\tilde{p}, \bar{p}]$).[58]

Adjustment Procedures

This section discusses technical aspects of the adjustment procedures that can be implemented on the interim probabilities of investigation to satisfy the liable injurer's incentive-compatibility constraint.

Following the methodology presented in the benchmark model, we first characterize $\Omega_i(A)$ $(i = 1, 2)$.

$$\Omega_1(A) = \frac{\bar{f}^I}{(1-p)\left[C_0(A) - \frac{C_1(A)(1+\alpha_1\Lambda)}{\left(1+\frac{A}{\bar{f}^V}\right)}\right]}.$$

$$\Omega_2(A) = \frac{\bar{f}^I}{(1-p)C_0(A)}.$$

Lemma 7. *(1) Suppose* $C_0(A) \geq C_1(A)(1+\Lambda)$ $\forall A \in (0, \bar{A}]$*. If and only if* $C_0(0) > C_1(0)(1+\Lambda)$*,* $\Omega_1(A) > 0$ *exists* $\forall A \in [0, \bar{A}]$ *and* $\forall \alpha_1 \in [0, 1]$*. (2) If and only if* $C_0(0) > 0$*,* $\Omega_2(A) > 0$ *exists* $\forall A \in [0, \bar{A}]$.

Proof. The proof follows the same logic than the proof of Lemma 3 of the benchmark model. ■

[58] Formal analysis of p-Segment 1 and p-Segment 2.2 are available from the authors upon request.

Proposition 9. *(1) Suppose* $C_0(A) \geq C_1(A)(1+\Lambda)$ $\forall A \in (0,\bar{A}]$ *and* $C_0(0) > C_1(0)(1+\Lambda)$*. If* $\frac{\partial \frac{C_1(A)}{C_0(A)}}{\partial A} \leq 0$ $\forall A \in [0,\bar{A}]$*, then the implementation of Procedure 1 should start at the lowest value of A. (2) Suppose* $C_0(0) > 0$*. The implementation of Procedure 2 should start at the lowest value of A*

Proof. The proof follows the same logic than the proof of Proposition 3 of the benchmark model. ■

Proposition 10. *Suppose* $C_0(A) \geq C_1(A)(1+\Lambda)$ $\forall A \in (0,\bar{A}]$*,* $C_0(0) > C_1(0)(1+\Lambda)$*,* $C_0(0) > 0$ *and* $\frac{\partial \frac{C_1(A)}{C_0(A)}}{\partial A} \leq 0$ $\forall A \in [0,\bar{A}]$*. If and only if* $C_0(A) - C_0(0) < \frac{C_1(A)\bar{f}^V}{A+\bar{f}^V}$ $\forall A \in [0,\bar{A}]$ *and* $\forall \alpha_1 \in [0,1]$*, Procedure 1 is more efficient than Procedure 2 across victim's types.*

Proof. The proof follows the same logic than the proof of Proposition 4 of the benchmark model. ■

Step 2: Analysis of Liable Injurer's Incentive-Compatibility Constraint and Verification of the Non-Liable Injurer Incentive-Compatibility Constraint

The liable injurer's incentive-compatibility constraint, evaluated at the interim probabilities of investigation, is not satisfied:

$$\int_0^{\bar{A}} Ag(A)dA + (1-\alpha_1)\int_0^{\bar{A}} \left(\frac{A}{\bar{f}^V + A}\right) C_1(A)g(A)dA > \bar{f}^I \int_0^{\bar{A}} 0g(A)dA.$$

Therefore, adjustments on the interim probabilities of investigation are required. By Claim 7, the non-liable injurer's incentive-compatibility is satisfied.

The characterization of $\tilde{p}$, $\bar{p}$, and the threshold $A^{2.1}(p,\alpha_1)$ follows the procedure applied in the benchmark model.

Consider first $\tilde{p}$. Following the benchmark model, $\tilde{p}$ is defined as the level of p such that, after exhausting Procedure 1 $\forall A \in [0,\bar{A}]$, the liable injurer's incentive-compatibility constraint evaluated at the adjusted interim probabilities of investigation is satisfied as an equality. Given that $\forall A \in [0,\bar{A}]$,

$q_0(A)$ is increased from zero to $(\frac{p}{1-p})\frac{A}{\bar{f}^V}$ and $q_1(A)$ is decreased from $\frac{A}{\bar{f}^V+A}$ to zero, the relevant terms of the liable injurer's incentive-compatibility constraint that define $\tilde{p}$ are the same as in the benchmark model:

$$\int_0^{\bar{A}} Ag(A)dA + (1-\alpha_1)\int_0^{\bar{A}} 0C_1(A)g(A)dA \leq \bar{f}^I \int_0^{\bar{A}} \left(\frac{\tilde{p}}{1-\tilde{p}}\right)\frac{A}{\bar{f}^V}g(A)dA.$$

Hence, $\tilde{p} = \frac{\bar{f}^V}{\bar{f}^V+\bar{f}^I}$. Proposition 11 shows that $\tilde{p} < p^0$.

Consider now $\bar{p}$. Following the benchmark model, $\bar{p}$ is defined as the p-value such that, after exhausting the implementation of Procedure 1 for $A \in [0, A^0(\bar{p})]$, the liable injurer's incentive-compatibility constraint evaluated at the adjusted interim probabilities of investigation is satisfied as an equality, where $A^0(\bar{p}) = \left(\frac{1-\bar{p}}{\bar{p}}\right)\bar{f}^V$:

$$\int_0^{\bar{A}} Ag(A)dA + (1-\alpha_1)\times$$

$$\times\left[\int_0^{A^0(\bar{p})} 0(A)C_1(A)g(A)dA + \int_{A^0(\bar{p})}^{\bar{A}} \left(\frac{A}{\bar{f}^V+A}\right)C_1(A)g(A)dA\right] =$$

$$= \bar{f}^I\left[\int_0^{A^0(\bar{p})} (\frac{\bar{p}}{1-\bar{p}})\frac{A}{\bar{f}^V}g(A)dA + \int_{A^0(\bar{p})}^{\bar{A}} 0g(A)dA\right].$$

Given that the liable injurer's incentive-compatibility constraint now includes a term in α_1, $\bar{p}(\alpha_1)$. This p- hreshold is not the same as $\bar{p}$ in the benchmark model.

Proposition 11. *Suppose $C_0(A) \geq C_1(A)(1+\Lambda)$ $\forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)(1+\Lambda)$ and $p \in (0,1)$. There are three p-segments: p-Segment 1 where $p \in (0, \tilde{p}]$, p-Segment 2.1 where $p \in (\tilde{p}, \bar{p}]$, and p-Segment 2.2 where $p \in (\bar{p}, 1)$.*

Proof. The proof follows the same logic than the proof of Proposition 5 of the benchmark model. ■

Finally, consider threshold $A^{2.1}(p)$. Following the benchmark model, in p-Segment 2.1, adjustment Procedure 1 should be exhausted only for $A \in [0, A^{2.1}(p)]$, where $A^{2.1}(p)$ corresponds to the A-threshold such that the liable injurer's incentive-compatibility constraint evaluated at the adjusted interim probabilities of investigation holds as an equality:

$$\int_0^{\bar{A}} Ag(A)dA + (1-\alpha_1)\times$$

$$\times\left[\int_0^{A^{2.1}(p)} 0C_1(A)g(A)dA + \int_{A^{2.1}(p)}^{\bar{A}} \left(\frac{A}{\bar{f}^V + A}\right) C_1(A)g(A)dA\right] =$$

$$= \bar{f}^I \left[\int_0^{A^{2.1}(p)} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA + \int_{A^{2.1}(p)}^{\bar{A}} 0g(A)dA\right].$$

Given that the liable injurer's incentive-compatibility constraint now includes a term in α_1, $A^{2.1}(p, \alpha_1)$. This A-threshold is not the same as $A^{2.1}(p)$ in the benchmark model.

Lemma 8. *Suppose* $C_0(A) \geq C_1(A)(1+\Lambda)$ $\forall A \in (0, \bar{A}]$*,* $C_0(0) > C_1(0)(1+\Lambda)$ *and* $p \in (\tilde{p}, \bar{p}]$*. There exists a unique* $0 \leq A^{2.1}(p, \alpha_1) < \bar{A}$*.*

Proof. The proof follows the same logic than the proof of Lemma 5 of the benchmark model. ■

The adjusted interim probabilities of investigation are summarized in the main text of the paper, Corollary 1.

Next, we verify whether the non-liable incentive-compatibility constraint is still satisfied. At the adjusted interim probabilities of investigation, the liable injurer's incentive-compatibility constraint is satisfied as an equality. Hence, the non-liable injurer incentive-compatibility constraint is still satisfied, by Claim 7.

Step 3: Analysis of Victim's Individual-Rationality Constraint and Analysis of Victim's Incentive-Compatibility Constraint (Part 2)

Lemma 9. *Suppose* $C_0(A) \geq C_1(A)(1+\Lambda)$ $\forall A \in (0, \bar{A}]$*,* $C_0(0 > C_1(0)(1+\Lambda)$ *and* $p \in (\tilde{p}, \bar{p}]$*. The victim's individual-rationality constraint is satisfied under the interim probabilities of investigation presented in Step 2.*

Proof. For $A \in [0, A^{2.1}(p,\alpha_1)]$, $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$. Then, the victim's individual rationality constraint is: $\alpha_0 C_0(A) \leq \bar{f}^V$. Remember that $\bar{f}^V = W^V$ and $W^V > C_0(\bar{A})$ by assumption. Therefore, this constraint is satisfied for $A \in [0, A^{2.1}(p,\alpha_1)]$ and no further adjustment is required. For $A \in (A^{2.1}(p,\alpha_1), \bar{A}]$, $q_0(A) = 0$ and $q_1(A) = \frac{A}{\bar{f}^V + A} < 1$. Then, the victim's individual rationality constraint is: $\alpha_1 C_1(A) \leq A + \bar{f}^V$. Remember that $\bar{f}^V = W^V$, and by assumption, $W^V + A \geq C_1(A)$ for $A \in [0, \bar{A}]$. Therefore, this constraint is satisfied for $A \in (A^{2.1}(p,\alpha_1), \bar{A}]$ and no further adjustment is required. ■

Lemma 10 shows that the victim's incentive-compatibility constraint that can be expressed in terms of two constraints, the victim's incentive-compatibility constraint of the benchmark model and the victim's individual-rationality constraint. It also shows that the victim's incentive-compatibility constraint holds when both constraints hold.

Lemma 10. *Suppose* $C_0(A) \geq C_1(A)(1+\Lambda)$ $\forall A \in (0, \bar{A}]$*,* $C_0(0) > C_1(0)(1+\Lambda)$ *and* $p \in (\tilde{p}, \bar{p}]$*. The victim's incentive compatibility constraint is satisfied under the interim probabilities of investigation presented in Step 2.*

Proof. By Lemma 9, the victim's individual rationality constraint is satisfied. By Step 1 and given that additional adjustments are not implemented in Step 3, the victim's incentive-compatibility constraint of the benchmark model is still satisfied. Hence, the victim's incentive-compatibility constraint is satisfied. ■.

Step 4: Verification and Further Adjustments

Given that no further adjustment was required in Step 3, the interim probabilities of investigation found in Step 2 still satisfy the victim's incentive-compatibility constraint and the liable injurer's incentive-compatibility constraint. Hence, the interim probabilities of investigation presented in Step 2 still hold: For $A \in [0, A^{2.1}(p,\alpha_1)]$, $q_0(A) = \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V}$ and $q_1(A) = 0$; for $A \in (A^{2.1}(p,\alpha_1), \bar{A}]$, $q_0(A) = 0$ and $q_1(A) = \frac{A}{\bar{f}^V + A} < 1$. $A^{2.1}(p,\alpha_1)$ is determined implicitly by the liable injurer's incentive compatibility constraint written as equality: $\int_0^{\bar{A}} Ag(A)dA + \int_{A^{2.1}(p,\alpha_1)}^{\bar{A}} (1-\alpha_1)\frac{A}{\bar{f}^V + A} C_1(A)g(A)dA = \bar{f}^I \int_0^{A^{2.1}(p,\alpha_1)} \left(\frac{p}{1-p}\right)\frac{A}{\bar{f}^V} g(A)dA$. The interim probabilities of investigation in this model correspond to the optimal probabilities of investigation of the benchmark model. In contrast to the benchmark model, now the threshold $A^{2.1}(p,\alpha_1)$ also depends on α_1.

Step 5: Optimal Cost Allocation and Optimal Production of Evidence

We characterize the optimal α_i $(i = 0, 1)$ and then, evaluate $A^{2.1}(p, \alpha_1)$ at the optimal α_1 to characterize the optimal production of evidence.

The social planner problem is $\min_{\alpha_0 \in [0,1], \alpha_1 \in [0,1]} \{\mathbb{E}[C(A)] + \Lambda \mathbb{E}[\xi(A)]\}$. To simplify notation, we denote $\mathbb{E}[C(A)] + \Lambda \mathbb{E}[\xi(A)]$ as S, and use this notation in the proofs of Claim 9 and Proposition 9.

The next claim shows that $\mathbb{E}[C(A)]$ negatively depends on α_1 and $\mathbb{E}[\xi(A)]$ positively depends on α_1.

Claim 11. *Suppose* $C_0(A) \geq C_1(A)(1+\Lambda)$ $\forall A \in (0, \bar{A}]$, $C_0(0) > C_1(0)(1+\Lambda)$ *and* $p \in (\tilde{p}, \bar{p}]$. *(1)* $\frac{\partial \mathbb{E}[C(A)]}{\partial \alpha_1} < 0$. *(2)* $\frac{\partial \Lambda \mathbb{E}[\xi(A)]}{\partial \alpha_1} > 0$.

Proof. Although $A^{2.1}(p, \alpha_1)$, to simplify notation, we use $A^{2.1}$ in the proof.

1.
$$\mathbb{E}[C(A)] = (1-p) \int_0^{A^{2.1}} \frac{p}{1-p} \frac{A}{\bar{f}^V} C_0(A) g(A) dA +$$

$$+ p \int_{A^{2.1}}^{\bar{A}} \frac{A}{A + \bar{f}^V} C_1(A) g(A) =$$

$$= p \Bigg[\int_0^{A^{2.1}} \frac{A}{\bar{f}^V} C_0(A) g(A) dA + \int_{A^{2.1}}^{\bar{A}} \frac{A}{A + \bar{f}^V} C_1(A) g(A) \Bigg].$$

By the Chain Rule,
$$\frac{\partial \mathbb{E}[C(A)]}{\partial \alpha_1} =$$

$$= p \left[\frac{A^{2.1}}{\bar{f}^V} C_0(A^{2.1}) g(A^{2.1}) - \Big(\frac{A^{2.1}}{A^{2.1} + \bar{f}^V} \Big) C_1(A^{2.1}) g(A^{2.1}) \right] \frac{\partial A^{2.1}}{\partial \alpha_1},$$

where

$$\frac{\partial A^{2.1}}{\partial \alpha_1} = - \frac{\int_{A^{2.1}}^{\bar{A}} \frac{A}{A + \bar{f}^V} C_1(A) g(A) dA}{\Bigg[\frac{A^{2.1}}{\bar{f}^V} \frac{p}{1-p} \bar{f}^I + (1 - \alpha_1) \frac{A^{2.1}}{A^{2.1} + \bar{f}^V} C_1(A^{2.1}) \Bigg] g(A^{2.1})}.$$

$\frac{\partial \mathbb{E}[C(A)]}{\partial \alpha_1}$ can be rewritten as:

$$\frac{\partial \mathbb{E}[C(A)]}{\partial \alpha_1} = -p \left[\frac{C_0(A^{2.1})}{\bar{f}^V} - \frac{C_1(A^{2.1})}{A^{2.1} + \bar{f}^V} \right] \times$$

$$\times \frac{\int_{A^{2.1}}^{\bar{A}} \frac{A}{A+\bar{f}^V} C_1(A) g(A) dA}{\left[\frac{1}{\bar{f}^V} \frac{p}{1-p} \bar{f}^I + (1-\alpha_1) \frac{1}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})\right]}.$$

The expression

$$\left[\frac{C_0(A^{2.1})}{\bar{f}^V} - \frac{C_1(A^{2.1})}{A^{2.1}+\bar{f}^V}\right] > 0$$

by $C_0(A^{2.1}) > C_1(A^{2.1})$ and $\bar{f}^V < A^{2.1} + \bar{f}^V$. Hence, $\frac{\partial \mathbb{E}[C(A)]}{\partial \alpha_1} < 0$.

2.

$$\mathbb{E}[\xi(A)] = p \int_0^{\bar{A}} \alpha_1 q_1(A) C_1(A) g(A) dA = p \int_{A^{2.1}}^{\bar{A}} \alpha_1 \frac{A}{A+\bar{f}^V} C_1(A) g(A) dA.$$

$$\frac{\partial \Lambda \mathbb{E}[\xi(A)]}{\partial \alpha_1} =$$

$$= \Lambda p \left[\int_{A^{2.1}}^{\bar{A}} \frac{A}{A+\bar{f}^V} C_1(A) g(A) dA - \alpha_1 \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1}) g(A^{2.1}) \frac{\partial A^{2.1}}{\partial \alpha_1}\right],$$

where

$$\frac{\partial A^{2.1}}{\partial \alpha_1} = -\frac{\int_{A^{2.1}}^{\bar{A}} \frac{A}{A+\bar{f}^V} C_1(A) g(A) dA}{\left[\frac{A^{2.1}}{\bar{f}^V} \frac{p}{1-p} \bar{f}^I + (1-\alpha_1) \frac{A^{2.1}}{A^{2.1}+\bar{f}^V} C_1(A^{2.1})\right] g(A^{2.1})} < 0.$$

Hence, $\frac{\partial \Lambda \mathbb{E}[\xi(A)]}{\partial \alpha_1} > 0$.

■

Proposition 12, included in the main text of the paper, characterizes the optimal cost allocation (α_0, α_1). The proof is included in Appendix A. The optimal production of evidence is characterized by simply evaluating $A^{2.1}(p, \alpha_1)$ and $\bar{p}(\alpha_1)$ at the optimal α_1.

D. Model with Endogenous Cost Allocation – Uniform Distribution Model and Numerical Example

This Appendix presents the model with a uniform distribution of damages (Section D.1) and the numerical example (Section D.2) for the model with endogenous cost allocation. We focus on Environment 1 and p-Segment 2.1 where $p \in (\tilde{p}, \bar{p}]$.

D.1 Model with a Uniform Distribution of Damages

Assume that A is uniformly distributed over $A \in [0, \bar{A}]$, where $g(A) = \frac{1}{\bar{A}}$ $\forall A \in [0, \bar{A}]$, $G(A) = \frac{A}{\bar{A}}$, and $\int_0^{\bar{A}} Ag(A)dA = \frac{\bar{A}}{2}$.

Thresholds $\bar{\mathbf{p}}$ and $\mathbf{A^{2.1}}$.– In contrast to the benchmark model, closed-form solutions for $\bar{p}(\alpha_1)$ and $A^{2.1}(p, \alpha_1)$ cannot be obtained.

Consider $\bar{p}$. When $p = \bar{p}$, the liable injurer's incentive-compatibility constraint is:

$$\int_0^{\bar{A}} A\frac{1}{\bar{A}}dA+(1-\alpha_1)\int_{\frac{(1-\bar{p})\bar{f}^V}{\bar{p}}}^{\bar{A}} \frac{A}{A+\bar{f}^V}\frac{C_1(A)}{\bar{A}}dA = \bar{f}^I\int_0^{\frac{(1-\bar{p})\bar{f}^V}{\bar{p}}} \frac{\bar{p}A}{(1-\bar{p})\bar{f}^V}\frac{1}{\bar{A}}dA,$$

where $A^0(\bar{p}) = \frac{\bar{p}A}{(1-\bar{p})\bar{f}^V}$. The last equation implicitly defines $\bar{p}(\alpha_1)$ and cannot be solved analytically.

Next, consider $A^{2.1}(p)$. Suppose $p \in (\tilde{p}, \bar{p}]$. The liable injurer's incentive-compatibility constraint is:

$$\int_0^{\bar{A}} \frac{A}{\bar{A}}dA + (1-\alpha_1)\int_{A^{2.1}}^{\bar{A}} \frac{A}{A+\bar{f}^V}\frac{C_1(A)}{\bar{A}}dA = \bar{f}^I\int_0^{A^{2.1}} \frac{pA}{(1-p)\bar{f}^V}\frac{1}{\bar{A}}dA.$$

The last equation implicitly defines $A^{2.1}(p, \alpha_1)$ and cannot be solved analytically.

Social Welfare Loss Function.– Given the optimal mechanism and the assumptions regarding the uniform distribution of damages, the social welfare loss function for p-Segment 2.1 is as follows.

$$SWL^{2.1} = H(A) + \mathbb{E}[C(A)] + \Lambda \mathbb{E}[\xi(A)],$$

where $H(A) = \mathbb{E}[A] = \int_0^{\bar{A}} Ag(A)dA$,

$$\mathbb{E}[C(A)] = \int_0^{A^{2.1}} \frac{pA}{(1-p)\bar{f}^V} \frac{C_0(A)}{\bar{A}} dA + \int_{A^{2.1}}^{\bar{A}} \frac{A}{A+\bar{f}^V} \frac{C_1(A)}{\bar{A}} dA$$

and

$$E[\xi(A)] = p \int_{A^{2.1}}^{\bar{A}} \alpha_1 \frac{A}{A+\bar{f}^V} \frac{C_1(A)}{\bar{A}} dA.$$

D.2 Numerical Example

The model with a uniform distribution of damages is used to construct this numerical example. Although $A^{2.1}(p, \alpha_1)$, we use $A^{2.1}$ to simplify notation.

Assume $C_0(A) = C_0(A) + c_0 A$ and $C_1(A) = C_1 + c_1 A$. We use the same set of exogenous parameters used for Environment 1 in the numerical example for the benchmark model

$$\{C_0, C_1, c_0, c_1, \bar{f}^V, \bar{f}^I, \bar{A}, p\} = \{1528, 690, 0.3, 0.01, 1800, 3600, 1200, 0.45\}.$$

In addition, we use three Λ-values: $\Lambda \in \{0.30, 0.40, 0.50\}$. For the three Λ-values, $\bar{A} = 1200$. Hence, $H = \frac{\bar{A}}{2} = 600$.

Main Model Conditions.– The model conditions are satisfied under the sets of exogenous parameters.

- The condition for Environment 1 is $C_0(A) \geq (1+\Lambda)C_1(A)\ \forall A \in [0, \bar{A}]$.
  1. $\Lambda = 0.30$: $1528 + 0.3A \geq (1 + 0.30)(690 + 0.01A)$. After simplification, $0.29A > -631$ holds $\forall A \in [0, 1200]$.
  2. $\Lambda = 0.40$: $1528 + 0.3A \geq (1 + 0.40)(690 + 0.01A)$. After simplification, $0.29A > -562$ holds $\forall A \in [0, 1200]$.
  3. $\Lambda = 0.50$: $1528 + 0.3A \geq (1 + 0.50)(690 + 0.01A)$. After simplification, $0.29A > -493$ holds $\forall A \in [0, 1200]$.
- The condition for Lemma 9 and first condition for Propositions 9 and 10 is $C_0(0) > C_1(0)(1 + \Lambda)$.

1. $\Lambda = 0.30$: $1528 > 690(1+0.30)$. After simplification, $1528 > 897$.
2. $\Lambda = 0.30$: $1528 > 690(1+0.40)$. After simplification, $1528 > 966$.
3. $\Lambda = 0.30$: $1528 > 690(1+0.50)$. After simplification, $1528 > 1035$.

- The second condition for Proposition 9 is $\frac{\partial \frac{C_1(A)}{C_0(A)}}{\partial A} \leq 0 \ \forall A \in [0, \bar{A}]$. Given that this condition does not include Λ, it is the same across Λ-values.

  $$\frac{\partial \frac{C_1 + c_1 A}{C_0 + c_0 A}}{\partial A} = \frac{c_1 C_0 - c_0 C_1}{(C_0 + c_0 A)^2} = \frac{0.01(1528) - 0.3(690)}{(1528 + 0.3A)^2} = \frac{-191.72}{(1528 + 0.3A)^2} < 0$$

  $\forall A \in [0, 1200]$.

- The third condition for Proposition 9 is $\frac{\partial C_0(A)}{\partial A} > 0 \ \forall A \in [0, \bar{A}]$. Given that this condition does not include Λ, it is the same across Λ-values: $c_0 = 0.3 > 0 \ \forall A \in [0, 1200]$.

- The second condition for Proposition 10 is $C_0(A) - C_0(0) < \frac{C_1(A)\bar{f}^V}{A + \bar{f}^V}$. Given that this condition does not include Λ, it is the same across Λ-values. The left-hand side of the inequality is $C_0(A) - C_0(0) = c_0 A = 0.3A$. The right-hand side of the inequality is $\frac{(690+0.01A)(1800)}{A+1800}$. Evaluate the left-hand side at the highest A-value, the numerator of the right-hand side at the lowest A-value and the denominator of the right-hand side at the highest A-value. After simplification, $360 < 414 = \frac{(690)(1800)}{3000} \ \forall A \in [0, 1200]$.

Thresholds $\bar{\mathbf{p}}$ and $\mathbf{A^{2.1}}$.– $\bar{p}(\alpha_1)$ and $A^{2.1}(p, \alpha_1)$ cannot be analytically computed.

Consider $\bar{p}$. When $p = \bar{p}$, the liable injurer's incentive-compatibility constraint is:

$$\int_0^{\bar{A}} A \frac{1}{\bar{A}} dA + (1 - \alpha_1) \int_{\frac{(1-\bar{p})\bar{f}^V}{\bar{p}}}^{\bar{A}} \frac{A}{A + \bar{f}^V} (C_1 + c_1 A) \frac{1}{\bar{A}} dA =$$

$$= \bar{f}^I \int_0^{\frac{(1-\bar{p})\bar{f}^V}{\bar{p}}} \frac{\bar{p} A}{(1 - \bar{p})\bar{f}^V} \frac{1}{\bar{A}} dA.$$

After computing the integrals,

$$\frac{\bar{A}}{2} + \frac{(1-\alpha_1)}{\bar{A}}\Big[\frac{c_1}{2}(\bar{A}^2 - (A^0(\bar{p}))^2) + (C_1 - c_1\bar{f}^V)(\bar{A} - A^0(\bar{p})) -$$

$$-(C_1 - c_1\bar{f}^V)\bar{f}^V(\log(\bar{A}+\bar{f}^V) - \log(A^0(\bar{p})+\bar{f}^V))\Big] = \bar{f}^I\frac{A^0(\bar{p})}{2\bar{A}}, \qquad (1D)$$

where $A^0(\bar{p}) = \frac{(1-\bar{p})\bar{f}^V}{\bar{p}}$. Equation (1D) implicitly defines $\bar{p}$. Using the set of exogenous parameters, it can be solved numerically.

Next, consider $A^{2.1}(p)$. Suppose $p \in (\tilde{p}, \bar{p}]$. The liable injurer's incentive-compatibility constraint is:

$$\int_0^{\bar{A}} \frac{A}{\bar{A}}dA + (1-\alpha_1)\int_{A^{2.1}}^{\bar{A}} \frac{A}{A+\bar{f}^V}(C_1 + c_1A)\frac{1}{\bar{A}}dA = \bar{f}^I\int_0^{A^{2.1}} \frac{pA}{(1-p)\bar{f}^V}\frac{1}{\bar{A}}dA.$$

After computing the integrals:

$$\frac{\bar{A}}{2} + \frac{1-\alpha_1}{\bar{A}}\Big[\frac{c_1}{2}(\bar{A}^2 - (A^{2.1})^2) + (C_1 - c_1\bar{f}^V)(\bar{A} - A^{2.1}) - (C_1 - c_1\bar{f}^V)\bar{f}^V(\log(\bar{A}+\bar{f}^V) -$$

$$-\log(A^{2.1}+\bar{f}^V))\Big] = \frac{\bar{f}^I p (A^{2.1})^2}{2(1-p)\bar{f}^V\bar{A}}. \qquad (2D)$$

Equation (2D) implicitly defines $A^{2.1}(p, \alpha_1)$. Using the set of exogenous parameters, it can be solved numerically.

Optimal Cost-Allocation Rules.– In this numerical example,

$$\mu = \frac{C_1(A)\bar{f}^V(C_0(A) + \frac{p}{1-p}\bar{f}^I)}{(A+\bar{f}^V)\bar{f}^V(C_0(A) + \frac{p}{1-p}\bar{f}^I)} +$$

$$\frac{(A+\bar{f}^V)^2 C_0'(A)\frac{p}{1-p}\bar{f}^I + (A+\bar{f}^V)\bar{f}^V C_0'(A)C_1(A)}{(A+\bar{f}^V)\bar{f}^V(C_0(A) + \frac{p}{1-p}\bar{f}^I)}.$$

Therefore, $\forall A \in [0, \bar{A}]$,

$$\mu > \frac{C_1(A)\bar{f}^V(C_0(A) + \frac{p}{1-p}\bar{f}^I)}{(A+\bar{f}^V)\bar{f}^V(C_0(A) + \frac{p}{1-p}\bar{f}^I)} = \frac{C_1(A)}{A+\bar{f}^V}.$$

After simplification,

$$\frac{C_1(A)}{A+\bar{f}^V} < \mu.$$

Evaluate the numerator of the right-hand side of the inequality at the lowest A-value and the denominator of the right-hand side of the inequality at the highest A-value: $\frac{C_1(0)}{\bar{A}+\bar{f}^V} = \frac{690}{1200+1800} = 0.23$.

$$\frac{\partial C_1(A)}{\partial A} = 0.01 < 0.253 = \frac{C_1(0)}{\bar{A}+\bar{f}^V} < \frac{C_1(A)}{A+\bar{f}^V} < \mu.$$

Therefore, $\frac{\partial C_1(A)}{\partial A} = 0.01 < 0.23 < \mu \; \forall A \in [0, 1200]$. Hence, $\frac{\partial C_1(A)}{\partial A} < \mu \; \forall A \in [0, 1200]$.

In this numerical example,

$$\Lambda^0(\alpha_1) = \frac{(A^{2.1}(\alpha_1)+\bar{f}^V)(C_0 + cA^{2.1}(\alpha_1)) - \bar{f}^V(C_1 + c_1 A^{2.1}(\alpha_1))}{(A^{2.1}(\alpha_1)+\bar{f}^V)\frac{p}{1-p}\bar{f}^I + \bar{f}^V(C_1 + c_1 A^{2.1}(\alpha_1))}.$$

Given that $\frac{\partial C_1(A)}{\partial A} < \mu \; \forall A \in [0, 1200]$, $\frac{\partial \Lambda^0(\alpha_1)}{\partial \alpha_1} < 0$. Hence, $\underline{\Lambda}^0 = \Lambda^0(\alpha_1 = 1)$ and $\bar{\Lambda}^0 = \Lambda^0(\alpha_1 = 0)$:

$$\underline{\Lambda}^0 = \frac{(A^{2.1}(1)+\bar{f}^V)(C_0 + cA^{2.1}(1)) - \bar{f}^V(C_1 + c_1 A^{2.1}(1))}{(A^{2.1}(1)+\bar{f}^V)\frac{p}{1-p}\bar{f}^I + \bar{f}^V(C_1 + c_1 A^{2.1}(1))} = 0.396.$$

$$\bar{\Lambda}^0 = \frac{(A^{2.1}(0)+\bar{f}^V)(C_0 + cA^{2.1}(0)) - \bar{f}^V(C_1 + c_1 A^{2.1}(0))}{(A^{2.1}(0)+\bar{f}^V)\frac{p}{1-p}\bar{f}^I + \bar{f}^V(C_1 + c_1 A^{2.1}(0))} = 0.402.$$

The Λ-values that apply for Cases 1, 2, and 3(b)iii of Proposition 11 are $\Lambda \in \{0.30, 0.40, 0.50\}$, respectively. The conditions for the three cases are satisfied.

- The condition for Proposition 11, Case 1 is $\Lambda < \underline{\Lambda}^0$. It applies to Set 1 where $\Lambda = 0.30$ (Column 2 of Table 4). $\Lambda = 0.30 < 0.396 = \underline{\Lambda}^0$.
- The first condition for Proposition 11, Case 3(b)iii is $\frac{\partial C_1(A)}{\partial A} < \mu \; \forall A \in [0, \bar{A}]$. It applies to Set 1 where $\Lambda = 0.40$ (Column 3 of Table 5). As showed before, across Λ-values, $\frac{\partial C_1(A)}{\partial A} = 0.01 < 0.253 < \mu \; \forall A \in [0, 1200]$.

- The second condition for Proposition 11, Case 3(b)iii is $\Lambda \in (\underline{\Lambda}^0, \bar{\Lambda}^0)$. It applies to Set 2 where $\Lambda = 0.40$ (Column 3 of Table 5). $\Lambda = 0.40 \in (0.396, 0.402) = (\underline{\Lambda}^0, \bar{\Lambda}^0)$.
- The condition for Proposition 11, Case 2 is $\Lambda > \bar{\Lambda}^0$. It applies to Set 3 where $\Lambda = 0.50$ (Column 4 of Table 4). $\Lambda = 0.50 > 0.402 = \bar{\Lambda}^0$.

Social Welfare Loss Function.– Given the optimal mechanisms and the functional forms for $C_0(A)$ and $C_1(A)$, $SWL^{2.1}$ is as follows.

$$SWL^{2.1} = H + \mathbb{E}[C(A)] + \theta\mathbb{E}[\eta(A)] + \Lambda\mathbb{E}[\xi(A)],$$

where $H = 600$, $\theta\mathbb{E}[\eta(A)] = \theta(0) = 0$ because all victims get access to justice under the optimal mechanism,

$$\mathbb{E}[C(A)] = \int_0^{A^{2.1}} \frac{pA}{(1-p)\bar{f}^V}(C_0+c_0A)\frac{1}{\bar{A}}dA + \int_{A^{2.1}}^{\bar{A}} \frac{A}{A+\bar{f}^V}(C_1+c_1A)\frac{1}{\bar{A}}dA =$$

$$= \frac{p}{(1-p)\bar{f}^V\bar{A}}\left[\frac{c_0(A^{2.1})^3}{3} + \frac{C_0(A^{2.1})^2}{2}\right] +$$

$$+\frac{1}{\bar{A}}\Big[\frac{c_1}{2}(\bar{A}^2 - (A^{2.1})^2) + (C_1 - c_1\bar{f}^V)(\bar{A} - A^{2.1}) -$$

$$-(C_1 - c_1\bar{f}^V)\bar{f}^V(\log(\bar{A} + \bar{f}^V) - \log(A^{2.1} + \bar{f}^V))\Big]$$

and

$$E[\xi(A)] = \frac{\alpha_1}{\bar{A}}\Big[\frac{c_1}{2}(\bar{A}^2 - (A^{2.1})^2) + (C_1 - c_1\bar{f}^V)(\bar{A} - A^{2.1}) -$$

$$-(C_1 - c_1\bar{f}^V)\bar{f}^V(\log(\bar{A} + \bar{f}^V) - \log(A^{2.1} + \bar{f}^V))\Big].$$

Using the set of exogenous parameters, the $A^{2.1}(p, \alpha_1)$-values and the optimal α_1 for each Λ-values, the optimal $SWL^{2.1}$ for each Λ-value is computed.

Ландео К. М., Никитин М., Измалков С.

Оптимальный дизайн гражданско-правовой системы [Электронный ресурс]: препринт WP9/2024/02 / К. М. Ландео, М. Никитин, С. Измалков; Нац. исслед. ун-т «Высшая школа экономики». – Электрон. текст. дан. (700 Кб). – М.: Изд. дом Высшей школы экономики, 2024. – 128 с. (На англ. яз.)

Мы изучаем оптимальный дизайн гражданско-правовой системы в условиях двусторонней асимметрии информации. Используя методологию дизайна механизмов, мы идентифицируем черты, которые должны присутствовать в оптимальной гражданско-правовой системе, чтобы обеспечить доступ к правосудию и максимальную компенсацию потерпевших с минимальными затратами на получение доказательств. Мы демонстрируем, что полное выявление частной информации требует получения доказательств только в части гражданско-правовых споров. «Американское правило» появляется эндогенно в качестве оптимального правила распределения судебных издержек только при определенных условиях. Мы демонстрируем возможность судебной реформы, которая реализовывает оптимальный механизм в реальных условиях.

Ландео Клаудия М.[*] – Университет Альберты, факультет экономики; Канада, AB T6G 2H4, Эдмонтон, здание Генри Маршалла Тори 7-25; landeo@ualberta.ca

Никитин Максим – Международный институт экономики и финансов НИУ ВШЭ; Россия, Москва, Покровский бульвар, 11; mnikitin@hse.ru

Измалков Сергей – Российская экономическая школа; Россия, Москва, Сколково, ул. Нобеля, 3; sizmalkov@nes.ru

*Мы благодарим Эмилиано Катонини, Татьяну Майскую, Пейо Зуазо-Гарина и Джека Окса за содержательные обсуждения и комментарии. Мы благодарны за предложения от участников Фестиваля 2024 в Центре теории игр Стоуни-Брук и семинара по микроэкономической теории НИУ ВШЭ.

Препринт WP9/2024/02

Серия WP9

Исследования по экономике и финансам

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Оптимальный дизайн гражданско-правовой системы

(*на английском языке*)

Публикуется в авторской редакции

Изд. № 2947